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Quasi Oppositional Population Based Global Particle Swarm Optimizer with Inertial Weights (QGPSO-w) for Solving Economic Load Dispatch Problem

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ABSTRACT In recent years, power companies have shown increasing interest in making strategic decisions to maintain profitable energy systems. Economic Load Dispatch (ELD) is a complex decision-making process where the output power of the entire power generating units must be set in a way that results in the overall economic operation of the power system. Moreover, it is a constrained multi-objective optimization problem. Now a days, there is a tendency to use metaheuristic methods to deal with the complexity of the ELD problem. Particle swarm optimization (PSO) is a subclass of metaheuristic methods inspired by fish schooling and bird flocking behaviors. However, the optimization performance of the PSO is highly dependent on fitness landscapes and can lead to local optima stagnation and premature convergence. Therefore, in the proposed study, two new variants of the PSO called global particle swarm optimizer with inertia weights (GPSO-w) and quasi-oppositional population based global particle swarm optimizer with inertia weights (QGPSO-w) are proposed to address the complexity of the ELD problem. The ELD problem is formulated as an optimization problem and validation of the proposed methods is performed on IEEE standards (3, 6, 13, 15, 40 & 140) unit Korean grid ELD test systems under numerous constraints and the obtained results are compared with the several recent techniques presented in the literature. The results obtained with convex systems showed excellent cost-effectiveness, while for non-convex systems sequential quadratic programming (SQP) optimizer was added to discover global minima even more efficiently. The proposed techniques were successful in solving the ELD problem and yielded better results compared to the reported results in the selected cases. It is further inferred that the proposed techniques with less algorithmic parameters reflected improved exploration and convergence characteristics.

INDEX TERMS Economic Load Dispatch, Quasi-population, PSO, Swarm intelligence, Optimization

I. INTRODUCTION

The world is currently going through uncertain times. The corona pandemic has halted economic progress worldwide. The energy sector, which was a backbone of economic progress, also had serious consequences [1]. The year 2020 saw a huge energy drop of 5%, which a few years ago could be considered unrealistic or even desecrated [2]. Although the consumption of household electricity is increasing due to the recession in industry and the situation in economic sector has proved that the overall demand has decreased [3]. The impact of this decline was felt so severely by some entities in the power sector that alone 19 companies in America went bankrupt [4]. The current situation seems bleak, but all is not lost, as the post-corona scenario could give an impetus

to the global energy sector and revive it to its former glory. The disruption in development will significantly motivate investment firms to strengthen their strategy and use people's panic-buying psychology due to Covid-19 to make a profit, thus stimulating the demand for energy in the process. Such positive trends for energy demand can be seen in the automotive sector which has increased in demand after corona due to people's preferences to avoid public transport [5], [6]. To meet the growing demand for energy in future and create a sustainable energy infrastructure for our future generations, renewable energy sources must be vigorously integrated into the energy infrastructure. The penetration of renewable energy sources has seen a rising trend, even during the Covid-19 situation, and it is expected to retain its trajectory. The current

positive trends of renewable integration are largely attributed to the incentives and policies of governments around the world. In 2022, the future for renewable energy looks a bit uncertain, as most of these incentives and policies are likely to end. The post-Covid-19 energy situation and uncertainty over global energy policy, coupled with a very volatile energy demand, are forcing the need for a backup plan. Global energy generation is still largely attributed to fossil fuels that have been in service for decades and have a proven track record of resilience, reliability, and efficient grid integration. To ensure energy supply, these thermal generation sources cannot be overlooked and must be utilized in an efficient and cost-effective manner. Economic load dispatch (ELD) is a well-documented optimization problem related to the economy of the power system, which involves the control of thermal resources so that maximum power can be extracted at the lowest possible fuel cost [7]–[9].

Economic load dispatch of thermal resources deals with non-linear, non-convex fuel cost curves under constraints such as energy balance, valve point effect, generators limits and prohibited operating zones. All these limitations make the ELD problem extremely challenging for optimization engineers, making it ideal for research. Simply put, ELD is the transmission of power in an economical way for a given load demand, so that no restriction is violated. ELD has been the subject of research for decades. The ongoing literature in ELD proves its necessity and importance in the overall energy mix. The ELD problem was initially addressed by traditional solution techniques such as lambda iteration method, quadratic programming, linear programming, gradient method, Newton's method and similar mathematical methods to find gradient or iterative search techniques. These approaches can solve the ELD problem, but with increasing dimensions and system size, these methods face severe performance degradation. It has also been noted in several studies that conventional approaches are lacking in adequate global exploration and are more prone to local exploitation. This tendency of trapping in local optima severely hampered their overall effectiveness. When conventional approaches did not yield the desired results, evolutionary and swarm-based approaches took the lead to solve the complex real world optimization problems [10]–[15]. Among these categories, swarm intelligence-based approaches have been found to be very effective in achieving the optimal solution of the ELD problem. These swarm-based techniques are a subclass of metaheuristic methods and can be further divided into two groups, such as global swarm optimizers (GSOs) and local swarm optimizers (LSOs). Among them the GSOs are more popular; the GSO techniques mimic the behavior of fish, insects, animals, or birds [16]–[19].

These techniques start with some random initial population and achieve the best position in the transition by learning from the personal experience of each solution as well as the experience of best solution in a coordinated manner. GSO techniques are widely recognized as the black-box problem solvers; many of them have few control parameters and are

easy to implement in a computer code. Although most of these methods show promising results on unimodal optimization problems, many of them are challenged by multimodal and other complex fitness landscapes of the real-world optimization problems. To overcome the aforesaid shortcomings several new variants of swarm-based techniques were proposed like modified particle swarm (MPSO) [20], stochastic modified particle swarm (SM-PSO) [21], [22], hybrid mutated particle swarm (HMPSO) [23], improved particle swarm (IPSO) [24] and particle swarm with random drift (PSORD) [25], an expanded particle swarm (XPSO) [26], triple archives particle swarm (TAPSO) [27] and U-based particle swarm (U-PSO) [28].

One such global PSO variant namely global inertial PSO (GPSO-w) has been proposed in [29]. GPSO-w is an enhanced and advanced PSO algorithm with better exploration, exploitation, and convergence capabilities that can handle many fitness landscapes. In this study, to explore broader regions of the search space and to find the best solutions for the selected ELD cases, GPSO-w augmented with a quasi-oppositional population called the QPGPSO-w is used on IEEE standards (3, 6, 13, 15, 40 & 140) unit ELD test systems and the results obtained are compared with the results of several other meta-heuristic techniques reported in recent literature.

The core contributions of the study are as follows:

- 1) The study seeks to identify two new meta-heuristic methods to solve the ELD problem under various constraints.
- 2) A new quasi-oppositional global particle swarm optimizer with inertial weights (QPGPSO-w) is being developed by incorporating the quasi-oppositional population in the global particle swarm optimizer with inertial weights (GPSO-w).
- 3) The validation of the proposed methods for the solution of IEEE standards (3, 6, 13, 15, 40 & 140) unit Korean grid ELD test systems under different constraints have been carried out.

The paper is structured into the following sections. Section II describes problem formulation and briefs on solution techniques. Section III explains the overview of the Quasi-Oppositional GPSO-w. Section IV provides details of simulation and discussion of results. And section V draws some useful conclusions.

II. PROBLEM FORMULATION

ELD is a well-known optimization problem in power system with the aim of assigning optimal load to thermal units to reduce the total fuel cost, subject to the operational limitations of power system. Mathematically, ELD contains non-convexity and non-linearity, also ELD is a challenging mathematical problem due to hard binding constraints such as power balance and soft constraints such as generator limits, prohibited operating zones (POZs) and valve point effect. The computation of transmission losses at each dispatch level assigned to thermal units can also be part of the ELD

problem. Numerically, the principal goal is to minimize the operating cost of generation units that can be modeled as formulated in Eq. 1 and Eq. 2.

minimize

$$\sum_{i=1}^{N_x} F_{P_i} = \sum_{i=1}^{N_x} aP_i^2 + bP_i + c \quad (1)$$

$$\sum_{i=1}^{N_x} F_{P_i} = \sum_{i=1}^{N_x} aP_i^2 + bP_i + c + (e * abs(\sin(f * (P_{il} - P_i)))) \quad (2)$$

The quadratic approach of cost curves for thermal unit fuel without valve point effect is described by Eq. 1. Detailed costs including the impact of valve point are described by Eq. 2. In the above equations, a, b, c, e and f represent cost coefficients; N_x indicates the total number of generating units available for scheduling, P_i represents the i th power output of the generating unit and P_{il} shows the minimum power generating limit of the i th generating unit [9].

These objective functions are subjected to following equality and inequality constraints. Equality constraints include the power generation balance as described by Eq. 3.

$$P_{generated} = P_{required} + P_{loss} \quad (3)$$

In Eq. 3 $P_{generated}$ represents the total scheduled power, $P_{required}$ indicates the total power demand of the system in megawatts (MW) and P_{loss} shows loss of the transmission network of the system in MW.

Inequality constraints include generation limits and prohibited operating zones (POZs) described by Eq. 4 and Eq. 5.

$$P_{il} < P_i < P_{ih} \quad (4)$$

$$\left\{ \begin{array}{l} P_{il} < P_i < P_{i1} \\ P_{i2} < P_i < P_{i(n-1)} \\ P_{in} < P_i < P_{ih} \end{array} \right. \quad (5)$$

Where P_{il} and P_{ih} are the minimum and maximum power generation limits of the i th generator, P_i represents power scheduled on the i th generation unit and P_{i1} to P_{in} represent the workable operating zones of the i th generation unit.

The transmission losses can be calculated from loss coefficient matrix B using following Eq. 6.

$$P_{loss} = \sum_{i=1}^{N_x} \sum_{k=1}^{N_x} (P_i B_{ik} P_k) + \sum_{i=1}^{N_x} (B_{i0} P_i) + B_{00} \quad (6)$$

Where B_{ik} , B_{i0} and B_{00} are transmission loss coefficients.

The overall fitness function including equality constraints and objective can be defined as:

$$fitness = penalty * abs\left(\sum_{i=1}^{N_x} P_i - P_{required} - P_{Loss}\right) + objective \quad (7)$$

Penalty in the above mentioned equation is a constant value that is usually excess than 100.

A. OVERVIEW OF GPSO-W

The authors in [29] introduced GPSO-w by augmenting the working of PSO. In GPSO-w N distinct solutions are initialized having N distinct velocity vectors. The initial values are randomly selected within the range of search space. The solutions then progressively iterate to optimal solution by updating their location and velocity vectors based on following equations.

$$V_i(j) = w(j) V_i(j-1) + c_1 r_1(j) [G_{pi}(j-1) - X_i(j-1)] + c_2 r_2(j) [G_B(j-1) - X_i(j-1)] \quad (8)$$

$$X_i(j) = X_i(j-1) + V_i(j) \quad (9)$$

Where c_1 & c_2 are constants, $r_1(j)$ & $r_2(j)$ are random numbers in the range $[0,1]$, G_{pi} represents the optimal position vectors of each solution and G_B is the best optimal position achieved with solutions at iteration j . Also, $V_i(j)$ and $X_i(j)$ represent the velocity and position of i th solution at the j th iteration. From the above comparison, it is very clear that position vectors are updated in the direction of most optimal solution, based on their own position as well as the position of the most optimal solution. The contribution of each in the overall reproduction of solutions can be controlled by means of c_1 & c_2 . The solution strategy of GPSO-w algorithm is described by the pseudocode as follows:

Algorithm 1 GPSO-w

- 1: Initialize N , d dimensional velocity vectors $V_i(j)$ and solutions $X_i(j)$ within search space range.
 - 2: Initialize iteration $j=0$
 - 3: **for** 1st iteration **do**
 - 4: Initialize N , $G_{pi}(0) = X_i(0)$
 - 5: **end for**
 - 6: **for** $i=1:N$ **do**
 - 7: Evaluate initial fitness as per Eq. 7
 - 8: Find $fit(G_B(0)) = \min(fit(G_{pi}(0)))$
 - 9: Find $G_B(0)$ corresponding to $index\ fit(G_B(0))$
 - 10: **end for**
 - 11: **for** $j=1: maxiter$ **do**
 - 12: Determine $w(j) = 0.9 - 0.5/maxiter$
 - 13: Update $V_i(j)$ and $X_i(j)$ using Eq. 8 and Eq. 9.
 - 14: Update

$$G_{pi}(j) = \begin{cases} X_i(j) & \text{if } fit(X_i(j)) < fit(G_{pi}(j-1)) \\ G_{pi}(j-1) & \text{else} \end{cases}$$
 - 15: Update $fit(G_B(j)) = \min(fit(G_{pi}(j)))$
 - 16: Update $G_B(j)$ corresponding to $fit(G_B(j))$
 - 17: **end for**
 - 18: Print best solution $G_B(maxiter)$ and best fitness $fit(G_B(maxiter))$
-

III. OVERVIEW OF QUASI OPPOSITIONAL GPSO-W (QPGPSO-W)

Quasi oppositional population strategy was presented by M. Basu et al in [30]. In this strategy both the current

population and its quasi opposite population are taken into account simultaneously for obtaining an optimum candidate solution. This optimization technique depends on quasi opposition initialization and quasi opposition generation jumping, by which optimum initial candidate solutions may be achieved by making use of opposite points, even without the availability of previous information about the solutions. A similar methodology can be employed consistently to every arrangement in the current population [31]. Since then quasi population has been extensively integrated to improve the exploitation capabilities of many algorithms. The quasi opposite population progresses as follows:

For each solution in the population an opposite number is defined as

$$X_0 = \min + \max - X_i \quad (10)$$

$$X_{qoi} = \text{rand}\left[\frac{\min + \max}{2}, (X_0)\right] \quad (11)$$

Where X_i is the i th d -dimensional solution at a particular iteration. X_0 is the opposite number of this i th solution and X_{qoi} is the respective opposite population. Quasi population is integrated with GPSO-w, as illustrated by the pseudo code provided below.

Algorithm 2 QUASI POPULATION BASED GPSO-w (QPGPSO-w)

```

1: Initialize N, d dimensional velocity vectors  $V_i(j)$  and
   solutions  $X_i(j)$  within search space range.
2: Initialize iteration  $j=0$ 
3: for 1st iteration do
4:   initialize N,  $G_{pi}(0) = X_i(0)$ 
5: end for
6: for  $i=1:N$  do
7:   Evaluate initial fitness as per Eq. 7
8:    $fit(G_B(0)) = \min(fit(G_{pi}(0)))$ 
9:    $G_B(0)$  corresponding to  $index\ fit(G_B(0))$ 
10: end for
11: for  $j=1: \text{maxiter}$  do
12:   Determine  $w(j) = 0.9 - 0.5/\text{maxiter}$ 
13:   Update  $V_i(j)$  and  $X_i(j)$  using Eq. 8 and Eq. 9.
14:   Find opposite number  $X_0$  and corresponding quasi
       opposite population  $X_{qoi}$  using Eqs. 10 and 11.
15:   Update
        $X_i(j) = \begin{cases} X_{qoi} & \text{if } fit(X_{qoi}) < fit(X_i(j)) \\ X_i(j) & \text{else} \end{cases}$ 
16:   Update
        $G_{pi}(j) = \begin{cases} X_i(j) & \text{if } fit(X_i(j)) < fit(G_{pi}(j-1)) \\ G_{pi}(j-1) & \text{else} \end{cases}$ 
17:   Update  $fit(G_B(j)) = \min(fit(G_{pi}(j)))$ 
18:   Update  $G_B(j)$  corresponding to  $fit(G_B(j))$ 
19: end for
20: Print best solution  $G_B(\text{maxiter})$  and best fitness
      $fit(G_B(\text{maxiter}))$ 

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The effect of proposed QPGPSO-w can also be seen graphically in the distribution of the solution obtained by applying it to solve the following equation:

$$f(x, y) = 9 + (x - 2)^2 + (y + 3)^2 \quad (12)$$

Eq. 12 is solved for 100 iterations by both GPSO-w and QPGPSO-w. It can be seen from Figure. 1 that QPGPSO-w outperforms GPSO-w by achieving optimum results in 32 iterations as compared to 60 iterations taken by the latter technique.

Eq. 12 is solved for 100 iterations by both GPSO-w and QPGPSO-w. From Figure. 1 it can be seen that QPGPSO-w performs better than GPSO-w by achieving optimal results in 32 iterations compared to 60 iterations taken by the latter technique. 2 shows the solution transition in QPSO-w during iteration 2 and iteration 4 respectively. We can see that the solution space is thoroughly utilized by a quasi population leading to the improvement of convergence and local optima stagnation.

IV. SIMULATION RESULTS

Both GPSO-w and proposed QPGPSO-w were used to implement standard ELD-IEEE test systems. The included test systems consist of:

- 1) 3 thermal unit convex system at a load demand of 150 MW proposed by A.J. Wood in [32].
- 2) IEEE standard 6 thermal unit test convex system having POZs constraint at a load demand of 1263 MW including transmission losses. The data is taken from [7].
- 3) IEEE standard 15 thermal unit convex system having POZs constraint at a load demand of 2630 MW including transmission losses. The data is taken from [33].
- 4) IEEE standard 13 thermal unit non-convex system having valve point constraint at load demand of 1800 MW. The data is taken from [34].
- 5) IEEE standard 40 thermal unit non-convex system having valve point constraint at a load demand of 10500 MW. The data is taken from [35].
- 6) Korean 140 Unit convex test system at a load demand of 49342 MW taken from [24].

MATLAB 2016 software was used to perform simulations. The framework used had 8 GB ram and Intel core i5 processor. The results achieved were compared with other similar results in the literature and are resented below.

A. SMALL SCALE CONVEX TEST SYSTEMS:

The convex systems of (3, 6 & 15) thermal units were simulated for 20 runs and the iterations were kept at 500 per run for each system. The most optimal results obtained by GPSO-w and QPGPSO-w are shown in Tables. A1, A2 and A3. Table. 1 shows a comparison of the results obtained with other techniques available in the literature. It is clear from Table. 1 that for 3 unit test system QPGPSO-w has achieved better results in terms of cost ranging from 0.0602

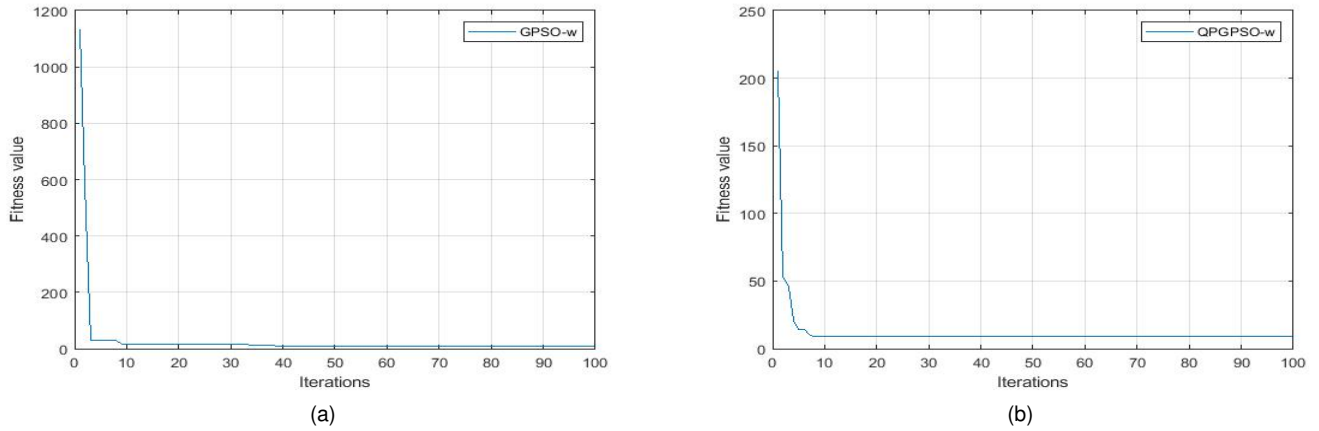


FIGURE 1: (a) Convergence curve GPSO-w. (b) Convergence curve QPGPSO-w.

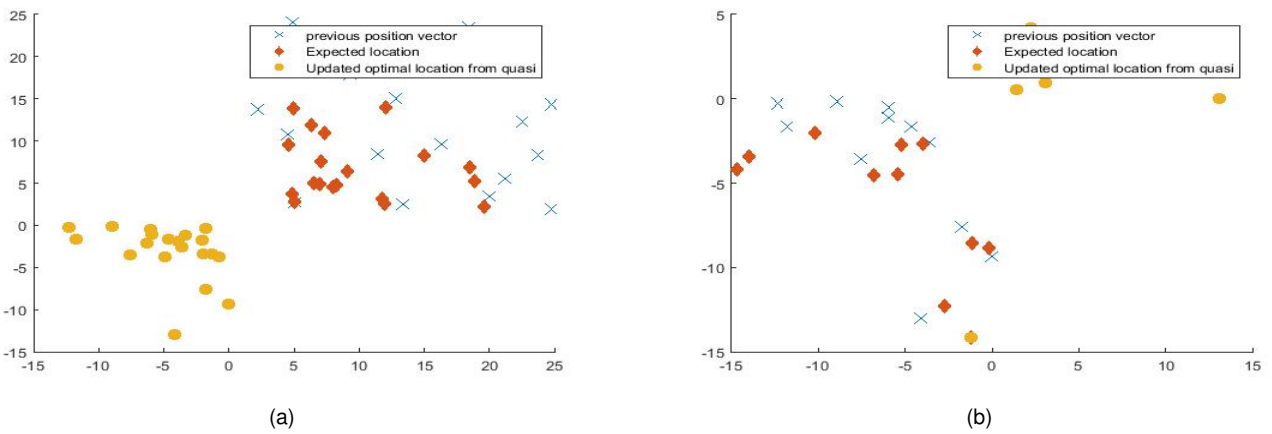


FIGURE 2: (a) Solution scattering iteration 2. (b) Solution scattering iteration 4.

to 0.0472 0.0472 \$/hr compared to the lambda iteration method (LI), teaching learning based optimization (TLBO), disruption based symbiotic organism search (DSOS), and the GPSO-w methods. For 6 unit test system, QPGPSO-w showed an improvement in the range of 9.42 to 0.094\$/hr compared to PSO, MPSO, SOH-PSO, CPSO, PSORD, PSO-WPF, D-MPSO and GPSO-w methods. Whereas the cost improvement was between 14.31 to 1.56 \$/hr, as opposed to other modern techniques such as TS, CBA, MABC, DSOS, Jaya, VSA, PBO, QPBO, SSO and SSA. Finally, QPGPSO-w for 15 unit convex test systems showed an improvement in the range of 309.81 to 0.43 \$/hr and 564.81 to 143.7 \$/hr compared to PSO, MPSO, CPSO, PSORD, PSO-WPF, D-MPSO and other modern techniques such as GA, DSOS, EMA, IFEP and FCEP respectively. Convergence characteristics of GPSO-w and QPGPSO-w are shown in Figure. 3.

From Figure. 3 it can be noted that QPGPSO-w initially has a faster convergence rate and can later achieve more optimal solutions in the iterations. This search trend confirms the benefit of using a quasi population strategy.

B. NON CONVEX TEST SYSTEMS:

Similarly, (13 & 40) unit non-convex thermal systems were also simulated for 20 runs and the iterations were kept at 500 per run for each system. The most optimal results obtained by GPSO-w and QPGPSO-w are shown in Tables. A4 and A5. From Tables. A4 and A5, it can be seen that both GPSO-w and QPGPSO-w were able to achieve an optimal solution of given test systems at a reasonable solution cost. The results obtained with QPGPSO-w demonstrate its efficacy compared to GPSO-w. Although QPGPSO-w was able to outperform GPSO-w, the results obtained are only a marginal improvement. To ensure that QPGPSO-w reaches the global optimum, the results were further optimized using the MATLAB SQP optimizer. The best results obtained after QPGPSO-w-SQP are shown in Table. A6. Table. 2 shows comparison of results with other techniques available in the literature. It is clear from Table. 2 that QPGPSO-w-SQP achieved better results in terms of cost from 145.44 to 0.2894 \$/hr for a non-convex system of 13 units compared to TLBO, PSO, IFEP, EP-SQP, PSO-SQP, QMPSO, GPSO-w

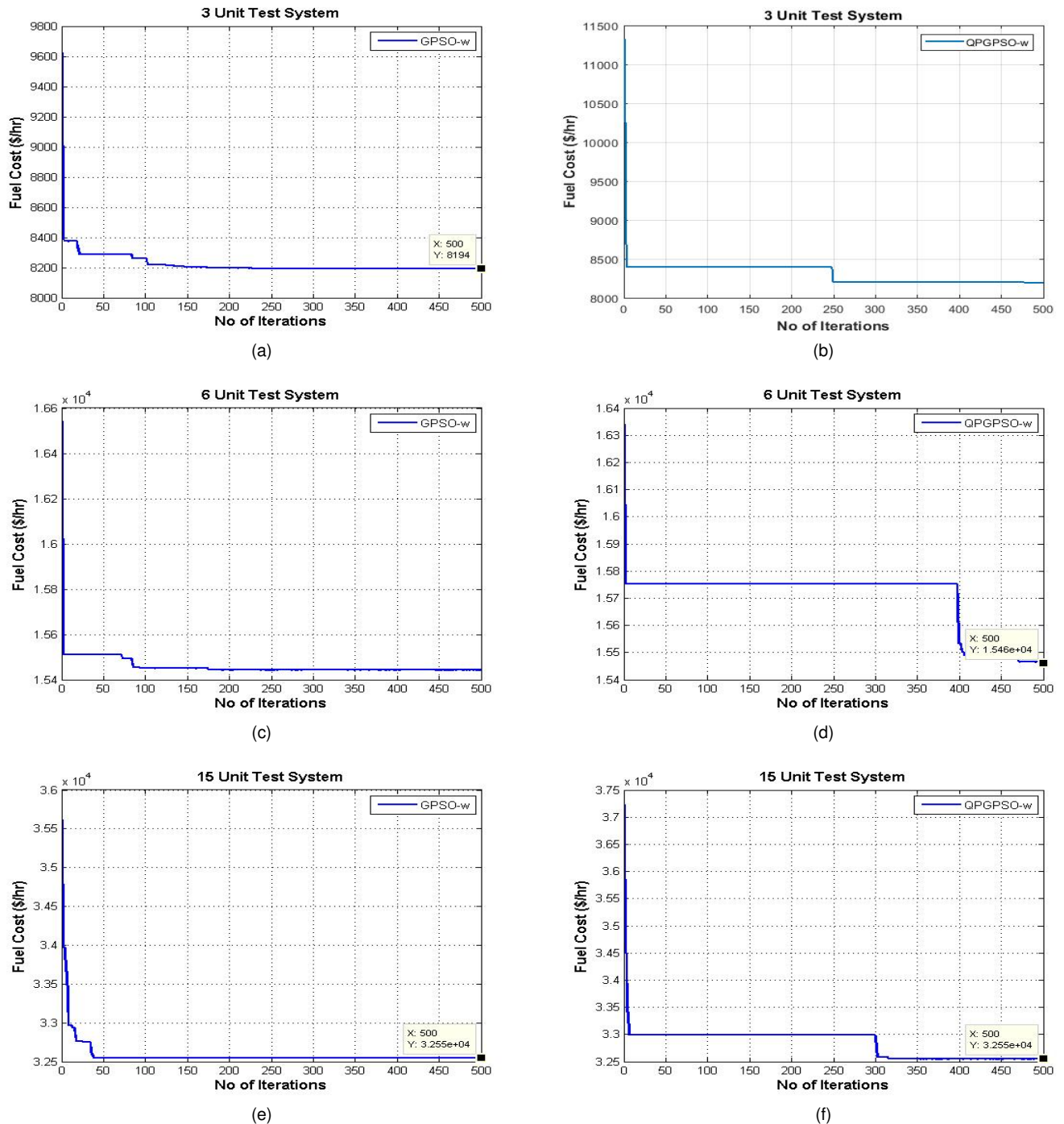


FIGURE 3: Convergence characteristics of small scale convex test system.

and QPGSO-w. While QPGSO-w-SQP for 40 unit non-convex test system showed better results in terms of cost from 7420.36 to 13.91 \$/hr compared to PSORD, PSO, ACO, NPSO, NPSO-LRS, SOH-PSO, GA-PS-SQP, GPGSO-w and QPGSO-w.

Convergence properties of the best solution of GPGSO-w and QPGSO-w are presented in Figure. A5. Figure. 4 shows convergence properties of QPGSO-w-SQP. From

Table. 2 we can also observe a significant improvement in execution times, with the proposed techniques showing an improvement of up to 45 seconds in the total execution time per run.

C. LARGE SCALE CONVEX TEST SYSTEMS:

The 140 unit Korean convex ELD test system consists of a combination of coal, oil fuel, LNG, and nuclear units. Twenty

TABLE 1: Comparison table for small scale convex test systems.

Technique	Best Cost (\$/hr)	Average Cost (\$/hr)	Worst Cost (\$/hr)	Average Time (Sec)
3 Unit Test System				
LI [32]	8194.36	-	-	-
TLBO [36]	8194.3561	-	-	-
DSOS [37]	8194.3561	-	-	-
GPSO-w	8194.347	8194.347	8194.347	0.933
QPGPSO-w	8194.2998	8195.299	8200.941	1.2168
6 Unit Test System				
TS [38]	15454.89	15488.98	15498.05	-
CBA [39]	15450.238	15454.76	15518.6588	-
PSO [33]	15450	-	-	-
MABC [40]	15449.899	15449.899	15449.899	-
DSOS [37]	15,449.694	-	-	-
Jaya [41]	15448.74	-	-	-
MPSO [20]	15447	-	-	-
VSA [42]	15447	-	-	-
SOH-PSO [43]	15446.02	-	-	-
CPSO [44]	15,446	15,449	15,490	-
PBO [7]	15444.43	15456.09	15483.06	6.2178
PSORD [25]	15442.7813	15453.7265	15484.86	-
PSO-WPF [45]	15442.6601	15442.6613	15442.6658	-
SSO [46]	15442.4	15442.6	-	-
SSA [47]	15442.4	15442.6	15443.2764	-
QPBO [9]	15442.14	-	-	-
D-MPSO [48]	15440.674	15441.478	15456.821	-
GPSO-w	15442.63	15477.334	15534.198	0.934
QPGPSO-w	15440.58	15488.449	15634.021	2.463
15 Unit Test System				
GA [33]	33113	-	-	-
PSO [33]	32858	-	-	-
CPSO [44]	32,834	33,021	33,318	-
MPSO [20]	32708	-	-	-
DSOS [37]	32,706.76	-	-	-
EMA [35]	32704.45	32704.45	32704.451	-
IFEP [49]	32,701.92	32,703.35	32,705.23	-
PSO-WPF [45]	32700.668	32700.669	32700.79	-
FCEP [49]	32,691.89	32,691.94	32,692.14	-
PSORD [25]	32652.34	32744.59	23959.7951	-
D-MPSO [48]	32560.280	32562.696	32580.765	-
GPSO-w	32548.62	32699.19289	32828.69	1.029
QPGPSO-w	32548.19	32589.541	32644.31	1.094

runs of 500 iterations each were performed for both GPSO-w and QPGPSO-w. The best results obtained with GPSO-w and QPGPSO-w are shown in Table. A6. The results obtained show that the QPGPSO-w compared to GPSO-w was not only able to obtain an optimal solution but also achieved a cost comparable to the global solution available in the literature. QPGPSO-w achieved the best cost of 1665379 \$/hr at an average cost of 1735099 \$/hr in a run time of 40 seconds per run. The optimum cost achieved by QPGPSO-w is only 0.59% higher than the current global minimum.

V. CONCLUSION AND FUTURE WORK

In this study, the global particle swarm optimizer with inertial weights algorithm (GPSO-w) and quasi-opposition-based global particle swarm optimizer with inertial weights

(QPGPSO-w), which are new variants of swarm-inspired (SI) metaheuristic algorithms, were used to solve the IEEE standards (3, 6, 13, 15, 40 & 140) unit Korean grid ELD test systems under various constraints. The optimization potential of the proposed metaheuristic techniques has been validated by comparing their searching performance with several recent approaches reported in the literature. From the simulation results it could be seen that the energy generation costs of the systems have decreased significantly by augmenting the GPSO-w with quasi-oppositional population QPGPSO-w in the selected cases. In addition, the convergence characteristics and optimization potential of the QPGPSO-w algorithm are significantly better than the GPSO-w algorithm. Results for small-scale IEEE standards (3, 6 & 15) thermal unit

TABLE2: Comparison table for non-convex test systems.

Technique	Best Cost (\$/hr)	Average Cost (\$/hr)	Worst Cost (\$/hr)	Average Time (Sec)
13 Unit Test System				
TLBO [36]	18115	*	*	*
PSO [50]	18,030.72	18,205.78	*	77.37
IFEP [49]	17994.07	18127.06	18267.42	*
EP-SQP [50]	17,991.03	18,106.93	*	121.93
PSO-SQP [50]	17,969.93	18,029.99	*	33.97
QMPSO [51]	17969.85	18081.05	18154.15	*
GPSO-w	17978.62	18437.12	18723.5	0.56
QGPSO-w	17971.81	18033.85	18082.74	0.618
QGPSO-w-SQP	17969.56061	18165.52462	18439.48317	2.6
40 Unit Test System				
PSORD [25]	128864.4525	129482.097	131129.0861	*
PSO [52]	121751.339	122020.754	122607.9145	19
NPSO [53]	121704.7391	122221.3697	122995.0976	*
NPSO-LRS [53]	121664.43	122209.31	122981.59	*
ACO [54]	121532.41	121606.45	121679.64	*
SOH-PSO [43]	121501.14	*	*	*
GA-PS-SQP [55]	121458	122039	*	46.98
GPSO-w	127404.2737	130283.5476	140212.1534	1.765
QGPSO-w	123780.6993	128601.7061	140212.1534	2.02
QGPSO-w-SQP	121444.0924	124679.6987	128196.6197	3.131768925

convex system show that QGPSO-w achieved better results in terms of cost up to 0.000735%, 0.093% and 1.735% for IEEE standards (3, 6 & 15) units respectively. The proposed approaches have also reduced the likelihood of a premature convergence on the ELD problem due to improved searching properties. Furthermore, for non-convex systems, sequential quadratic programming (SQP) optimizer searched the global minimum even more effectively. For non-convex system, the operation of QGPSO-w was enhanced by passing the results through the SQP optimizer, and the resulting QGPSO-w - SQP achieved better results in terms of cost up to 0.81% and 6.11%, for (13 & 40) thermal unit non-convex test systems respectively. For 140-units, the Korean ELD test system, the QGPSO-w, outperformed the GPSO-w and found better results in relatively less iterations. In the future, this improved modification of the GPSO-w may be tested on other practical ELD issues with more operational constraints, for example prohibited operating zones, different fuel options and transmission losses, as well as with renewable energy sources. In addition, emerging metaheuristic methods can also be validated in the selected ELD cases.

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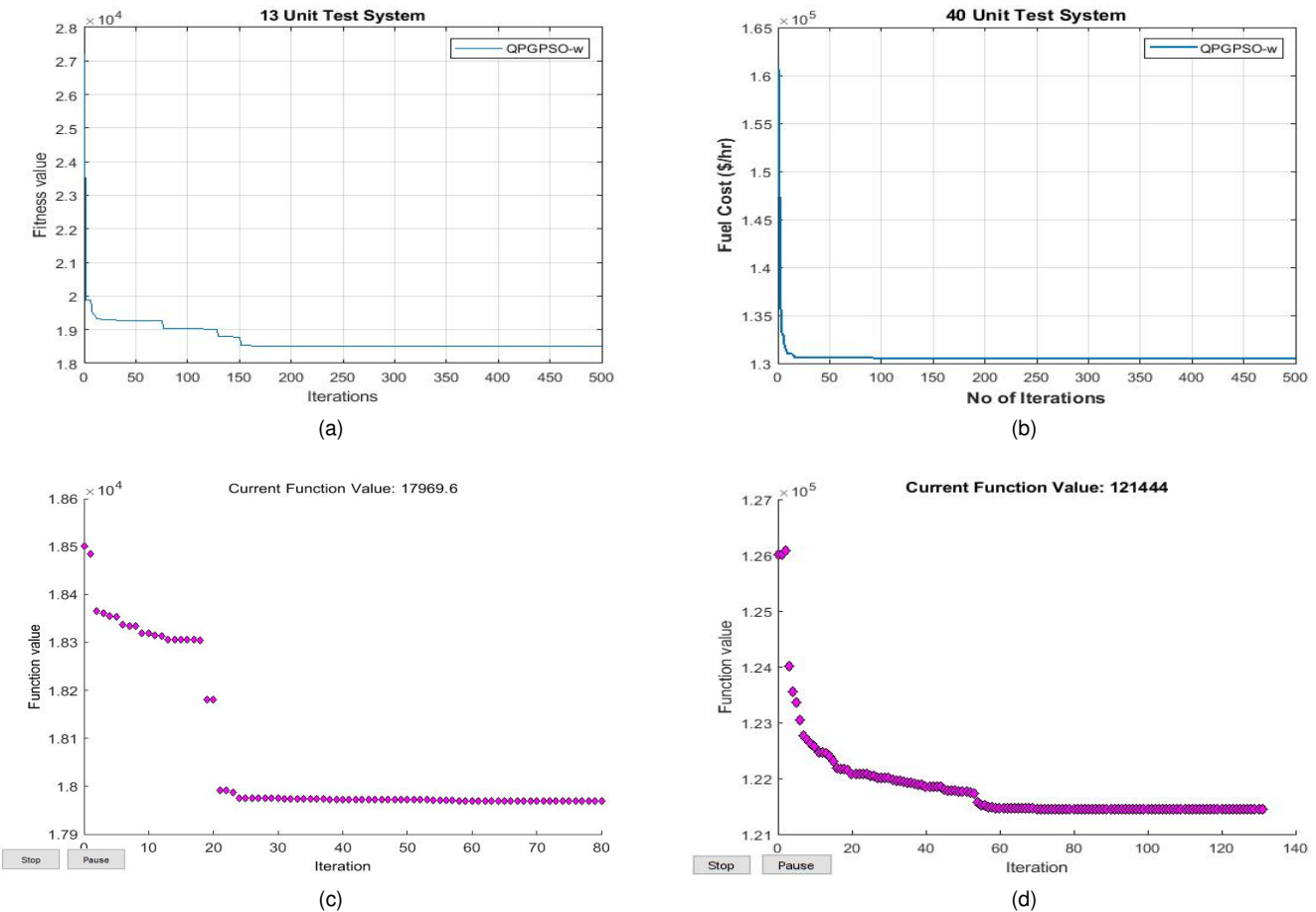


FIGURE 4: Convergence characteristics of non convex test system. a) and b) Convergence curve best solution for QPGPSO-w before SQP. c) and d) Convergence curve for SQP.

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APPENDIX. RESULT TABLES APPENDIX. CONVERGENCE CHARACTERISTICS

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TABLEA1: Best solution for 3 thermal unit convex test system.

Units	$P_{i,min}$	$P_{i,max}$	GPSO-w		QPGPSO-w	
			Generation (MW)	Fuel Cost (\$/hr)	Generation (MW)	Fuel Cost (\$/hr)
1	100	600	393.1694	3916.359	391.7621	3903.488118
2	50	200	334.6034	3153.838	335.2335	3159.602647
3	100	400	122.2263	1124.151	122.9975	1131.209065
Total Generation (MW)/ Total Fuel Cost (\$/hr)			849.999 MW / 8194.347 \$/hr		849.993 MW / 8194.2998 \$/hr	

TABLEA2: Best solution for 6 thermal unit convex test system.

Units	$P_{i,min}$	$P_{i,max}$	GPSO-w		QPGPSO-w	
			Generation (MW)	Fuel Cost (\$/hr)	Generation (MW)	Fuel Cost (\$/hr)
1	100	500	445.4241	4746.787	459.4323	4933.573
2	50	200	170.9543	2187.184	190.7573	2453.263
3	80	300	261.2511	3054.904	273.752	3221.353
4	50	150	150	2052.5	142.0687	1944.407
5	50	200	161.5584	2125.172	150	1975
6	50	120	85.89575	1276.085	58.13596	912.9799
Total Transmission Line Losses (MW)			12.08364 MW		11.14633 MW	
Total Generation (MW)/ Total Fuel Cost (\$/hr)			1275.084 MW / 15442.63 \$/hr		1274.146 MW / 15440.58 \$/hr	

TABLEA3: Best solution for 15 thermal unit convex test system.

Units	$P_{i,min}$	$P_{i,max}$	GPSO-w		QPGPSO-w	
			Generation (MW)	Fuel Cost (\$/hr)	Generation (MW)	Fuel Cost (\$/hr)
1	150	455	455	5328.4	455	5328.4
2	150	455	455	5252.886	455	5252.886
3	20	130	130	1537.029	130	1537.029
4	20	130	130	1537.029	130	1537.029
5	150	470	236.9708	2937.008	235.9043	2925.813
6	135	460	460	5339.692	460	5339.692
7	135	465	465	5183.706	465	5183.706
8	60	300	60	900.2168	60	900.2168
9	25	162	25	453.5044	25	453.5044
10	25	160	25	443.2519	25	443.2519
11	20	80	80	1024.95	80	1024.95
12	20	80	80	1057.283	80	1057.283
13	25	85	25	552.7319	25	552.7319
14	15	55	15	490.934	15	490.934
15	15	55	15	510.0006	15	510.0006
Total Transmission Line Losses (MW)			26.97077 MW		26.9708 MW	
Total Generation (MW)/ Total Fuel Cost (\$/hr)			2656.971 MW / 32548.62 \$/hr		2656.971 MW / 32548.19 \$/hr	

TABLE4: Best solution for 13 thermal unit non-convex test system.

Units	$P_{i,min}$	$P_{i,max}$	GPSO-w			QPGPSO-w		
			Generation (MW)	Fuel (\$/hr)	Cost	Generation (MW)	Fuel (\$/hr)	Cost
1	0	680	628.3185	5749.92		628.3185	5749.92	
2	0	360	224.3707	2154.836		223.3356	2154.884	
3	0	360	148.7126	1529.544		298.6696	2779.514	
4	60	180	60	716.064		109.8547	1129.488	
5	60	180	109.8665	1129.476		60	716.064	
6	60	180	109.6557	1129.687		60	716.064	
7	60	180	60	716.064		60	716.064	
8	60	180	159.7306	1559.003		109.8666	1129.476	
9	60	180	109.5848	1129.757		60	716.064	
10	40	120	40	474.544		40	474.544	
11	40	120	40	474.544		40	474.544	
12	55	120	55	607.591		55	607.591	
13	55	120	55	607.591		55	607.591	
Total Generation (MW)/ Total Fuel Cost (\$/hr)			1800.24 MW / 17978.62 \$/hr			1800.045 MW / 17971.81 \$/hr		

TABLE5: Best solution for 40 thermal unit non-convex test system.

Units	$P_{i,min}$	$P_{i,max}$	GPSO-w			QPGPSO-w		
			Generation (MW)	Fuel (\$/hr)	Cost	Generation (MW)	Fuel (\$/hr)	Cost
1	36	114	38.19017491	380.082317		114	978.156289	
2	36	114	114	978.1562885		114	978.156289	
3	60	120	60	806.748		120	1544.65338	
4	80	190	190	2353.689135		190	2353.68914	
5	47	97	97	853.1776164		97	853.177616	
6	68	140	140	1596.46432		68	822.53608	
7	110	300	110	1214.207		300	3216.42404	
8	135	300	135	1425.297		300	3052.30951	
9	135	300	135	1451.18925		300	3071.98951	
10	130	300	130	2502.065		130	2502.065	
11	94	375	375	6335.335317		160.4312594	2906.17241	
12	94	375	168.7998251	2977.455099		94	1908.16684	
13	125	500	125	2541.68125		125	2541.68125	
14	125	500	500	8219.407332		500	8219.40733	
15	125	500	125	2982.675		125	2982.675	
16	125	500	500	8232.307332		125	2982.675	
17	220	500	500	5525.293739		489.2793703	5296.71075	
18	220	550	550	6224.075875		550	6224.07588	
19	242	550	550	6271.211113		511.2793703	5540.92922	
20	242	550	550	6271.191113		550	6271.19111	
21	254	550	550	5575.329273		550	5575.32927	
22	254	550	550	5575.329273		550	5575.32927	
23	254	550	550	5558.049273		550	5558.04927	
24	254	550	550	5558.049273		550	5558.04927	
25	254	550	550	5785.664273		550	5785.66427	
26	254	550	550	5785.664273		550	5785.66427	
27	10	150	10	1140.524		10	1140.524	
28	10	150	10	1140.524		10	1140.524	
29	10	150	10	1140.524		10	1140.524	
30	47	97	97	853.1776164		97	853.177616	
31	60	190	190	1643.991252		190	1643.99125	
32	60	190	190	1643.991252		190	1643.99125	
33	60	190	190	1643.991252		190	1643.99125	
34	90	220	220	2228.37304		220	2228.37304	
35	90	220	90	893.19		220	2164.48304	
36	90	220	220	2164.48304		220	2164.48304	
37	25	110	110	1220.166122		110	1220.16612	
38	25	110	110	1220.166122		110	1220.16612	
39	25	110	110	1220.166122		110	1220.16612	
40	242	550	550	6271.211113		550	6271.21111	
Total Generation (MW)/ Total Fuel Cost (\$/hr)			10499.99MW / 127404.3\$/hr			10499.99 MW / 123780.699253442 \$/hr		

TABLEA6: Best solution by QPGPSO-w-SQP for non-convex test system.

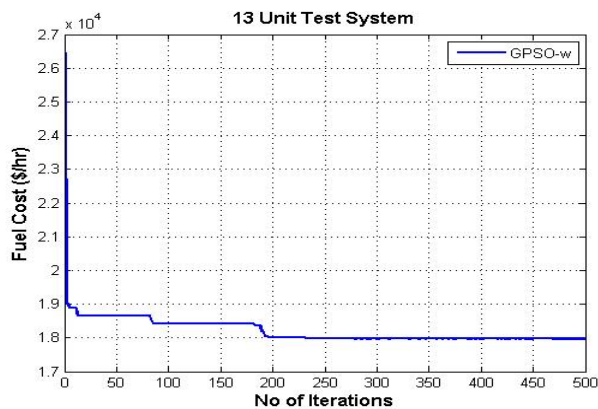
Units	$P_{i,min}$	$P_{i,max}$	QPGPSO-w-SQP			QPGPSO-w-SQP		
			Generation (MW)	Fuel (\$/hr)	Cost	Generation (MW)	Fuel (\$/hr)	Cost
1	36	114	628.3183974	5749.919941		110.8030054	925.149352	
2	36	114	299.1679552	2782.644557		110.7994898	925.096419	
3	60	120	222.7840634	2149.514536		97.40070414	1190.5639	
4	80	190	109.863185	1129.479391		80	1083.718	
5	47	97	109.866399	1129.476183		87.79944357	706.500945	
6	68	140	60	716.064		140	1596.46432	
7	110	300	60	716.064		287.5616471	3076.53137	
8	135	300	60	716.064		284.6011536	2779.86395	
9	135	300	60	716.064		284.5990143	2798.22935	
10	130	300	40	474.544		130	2502.065	
11	94	375	40	474.544		94	1893.3054	
12	94	375	55	607.591		94	1908.16684	
13	125	500	55	607.591		125	2541.68125	
14	125	500	*	*		394.2796052	6414.86636	
15	125	500	*	*		394.2792982	6436.58598	
16	125	500	*	*		394.2792932	6436.58596	
17	220	500	*	*		489.2796938	5296.71771	
18	220	550	*	*		550	6224.07588	
19	242	550	*	*		511.2793717	5540.92925	
20	242	550	*	*		511.2793081	5540.90918	
21	254	550	*	*		523.2799349	5071.30113	
22	254	550	*	*		523.2802114	5071.30673	
23	254	550	*	*		523.2796202	5057.22813	
24	254	550	*	*		523.2797869	5057.23148	
25	254	550	*	*		523.2797348	5275.096	
26	254	550	*	*		523.2794273	5275.0897	
27	10	150	*	*		10	1140.524	
28	10	150	*	*		10	1140.524	
29	10	150	*	*		10	1140.524	
30	47	97	*	*		87.8000905	706.503159	
31	60	190	*	*		190	1643.99125	
32	60	190	*	*		190	1643.99125	
33	60	190	*	*		190	1643.99125	
34	90	220	*	*		220	2228.37304	
35	90	220	*	*		220	2164.48304	
36	90	220	*	*		220	2164.48304	
37	25	110	*	*		110	1220.16612	
38	25	110	*	*		110	1220.16612	
39	25	110	*	*		110	1220.16612	
40	242	550	*	*		511.280166	5540.94647	
Total Generation (MW)/ Total Fuel Cost (\$/hr)			1800 MW / 17969.56 \$/hr			10500 MW / 121444.1 \$/hr		

TABLE 7: Best solution for large scale 140 thermal unit convex test system.

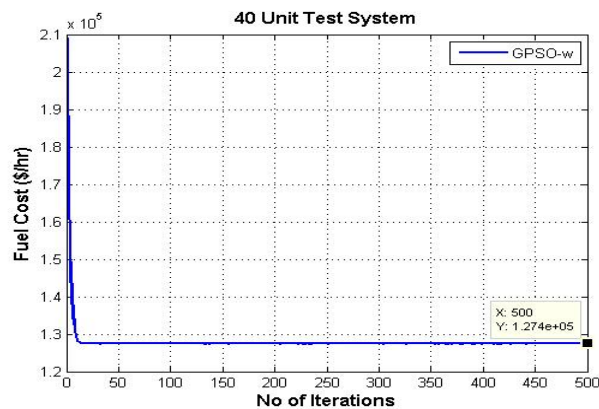
Units	$P_{i,min}$	$P_{i,max}$	GPSO-w		QGPSO-w	
			Generation (MW)	Fuel Cost (\$/hr)	Generation (MW)	Fuel Cost (\$/hr)
1	71	119	103.4771919	6875.172488	119	7784.169968
2	120	189	123.4998791	6516.633916	189	9377.84288
3	125	190	130.8988088	7002.162518	125	6723.178625
4	125	190	163.7478315	8560.65467	190	9812.1369
5	90	190	93.32543515	7408.508111	190	13815.5438
6	90	190	90.541433	7003.481506	190	13510.1482
7	280	490	438.4310344	8340.888379	490	9285.9529
8	280	490	392.3538992	7514.852085	490	9250.1959
9	260	496	382.0407877	7112.47845	496	9126.311008
10	260	496	260.7443225	5113.461637	496	9126.311008
11	260	496	260	5101.6548	260	5101.6548
12	260	496	327.7198411	6239.307102	496	9171.395832
13	260	506	320.791024	6248.981843	506	9543.166436
14	260	509	358.4338356	6896.839331	509	9598.727981
15	260	506	322.8587608	6284.281994	506	9543.166436
16	260	505	456.4802845	8636.181373	505	9524.661525
17	260	506	290.693167	5774.491757	260	5253.1128
18	260	506	275.2388001	5511.407817	506	9558.614148
19	260	505	442.2989117	8432.158608	505	9569.9231
20	260	505	329.2925276	6454.711552	505	9569.9231
21	260	505	375.1213859	7245.307976	505	9569.9231
22	260	505	313.0651966	6178.482688	505	9569.9231
23	260	505	262.9028162	5333.658726	505	9558.9041
24	260	505	260	5288.7294	505	9569.9231
25	280	537	321.6265673	6373.370878	537	10174.95241
26	280	537	394.1924458	7590.681938	537	10176.15032
27	280	549	316.1236137	6179.498126	549	10385.72719
28	280	549	312.4920739	6114.757884	549	10385.72719
29	260	501	306.864563	5831.454507	260	5049.9478
30	260	501	276.4515124	5310.568999	501	9171.044547
31	260	506	442.5258213	8420.927353	506	9588.387512
32	260	506	420	8013.4788	506	9588.387512
33	260	506	384.3563799	7376.098505	260	5222.8292
34	260	506	370.0202193	7122.276754	506	9588.387512
35	260	500	269.5683993	5277.854468	500	9057.46
36	260	500	437.5401245	7975.210533	500	9031.992
37	120	241	127.5333368	2566.471475	241	4566.79495
38	120	241	204.3698442	3912.73271	241	4566.79495
39	423	774	745.9422431	14161.71093	774	14667.43132
40	423	769	586.2421115	11251.02778	769	14532.92774
41	3	19	15.03766959	2008.842724	3	904.50124
42	3	28	7.678390435	1135.736148	3	524.168655
43	160	250	170.2728508	13777.69062	160	13113.0128
44	160	250	213.8256416	16727.84529	250	19186.772
45	160	250	161.5155357	13402.98901	250	19144.4675
46	160	250	246.0318128	18657.59154	250	18920.3925
47	160	250	196.4613102	15369.46049	160	12982.8064
48	160	250	173.4054988	14033.38407	250	18990.004
49	160	250	160.1193027	13262.88079	160	13255.258
50	160	250	160.566897	13472.38816	250	19151.925
51	165	504	311.6912848	24214.7986	165	13710.9729
52	165	504	215.8253838	17350.10882	504	37987.9277
53	165	504	393.2682907	30056.93736	165	13710.9729
54	165	504	199.5743445	16186.4974	504	37987.9277
55	180	471	229.9283594	22178.53986	180	17876.8972
56	180	561	272.629374	26696.13058	180	18568.7094
57	103	341	182.6191903	18559.28934	103	12787.34055
58	198	617	428.9470381	38484.91617	198	19288.43091
59	100	312	140.1635881	14639.37089	312	25603.86862
60	153	471	214.8747914	21802.04812	153	18018.69237
61	163	500	191.1266192	18644.8066	163	16407.9663
62	95	302	200.4630555	18028.74799	95	9510.0751
63	160	511	370.2327945	29081.07346	160	14331.2814
64	160	511	486.5433677	37243.01651	160	14331.2814
65	196	490	253.3552963	20931.1965	196	17587.38203
66	196	490	236.2572999	19450.63608	196	16628.80643
67	196	490	403.8675709	30295.09716	196	17587.38203
68	196	490	196	17587.38203	196	17587.38203

Units	$P_{i,min}$	$P_{i,max}$	GPSO-w		QGPSO-w	
			Generation (MW)	Fuel Cost (\$/hr)	Generation (MW)	Fuel Cost (\$/hr)
69	130	462	404.7493151	36851.67183	130	13512.834
70	130	432	270.0671681	25955.83579	236.999	23829.97667
71	137	455	311.5661223	26758.08313	137	13688.22065
72	137	455	354.835374	30190.96539	455	37577.69688
73	195	541	233.7978582	17608.79711	195	14852.73888
74	175	536	353.6776056	26262.00808	175	13579.0675
75	175	540	392.8891721	29045.70969	175	13579.0675
76	175	538	296.688519	22276.83373	175	13646.86688
77	175	540	478.6737421	35070.05536	175	13534.048
78	330	574	429.4611749	32670.13301	330	25591.7723
79	160	531	332.8808913	13438.12411	531	21026.89468
80	160	531	415.0802058	16592.43842	531	21025.11258
81	200	542	202.0428371	16111.86728	542	39487.75314
82	56	132	57.31559794	6924.080032	56	6801.9908
83	115	245	131.5800952	14468.6914	115	13016.5838
84	115	245	115.1718941	12861.34752	115	12845.5183
85	115	245	115	12845.5183	245	25742.8807
86	207	307	211.4533349	23687.41843	307	33377.99012
87	207	307	220.2403935	24718.57504	207	23389.95569
88	175	345	186.0767282	16731.02596	175	15925.65763
89	175	345	248.2617782	21320.62213	175	15901.90313
90	175	345	197.4234624	17427.20452	175	15828.3225
91	175	345	198.9533872	17665.36825	175	15902.68275
92	360	580	489.2670116	1632.616308	580	1896.6886
93	415	645	457.7903879	1784.394637	645	2387.9403
94	795	984	828.7085196	2737.777249	795	2632.21155
95	795	978	810.8518544	2633.938214	978	3148.646648
96	578	682	587.9260801	1407.768057	682	1592.792172
97	615	720	631.6417739	1459.173802	720	1630.2022
98	612	718	616.6103777	1530.166763	718	1740.64566
99	612	720	632.2509601	1585.051091	720	1770.1832
100	758	964	816.5322747	2724.921897	964	3177.595128
101	755	958	870.6041811	2894.874096	958	3165.122508
102	750	1007	763.9431892	2339.177774	1007	3122.983876
103	750	1006	750	2422.785	1006	3293.713384
104	713	1013	898.7434511	2869.58226	1013	3202.53761
105	718	1020	872.5486673	2634.271412	1020	3037.279
106	791	954	882.7452169	2754.226866	954	2957.514028
107	786	952	804.1100235	2609.612659	952	3041.260656
108	795	1006	795	2758.011525	1006	3421.408396
109	795	1013	891.6401628	3061.816667	1013	3445.617371
110	795	1021	801.3170467	2661.983807	1021	3303.760787
111	795	1015	806.1730342	2975.210389	1015	3653.36385
112	94	203	148.5791414	14932.89382	94	9839.99278
113	94	203	111.8698193	11498.04864	94	9839.99278
114	94	203	151.814677	15237.49411	94	9839.99278
115	244	379	266.4058513	24196.44011	244	22413.54058
116	244	379	260.3064802	23708.08497	379	33615.93751
117	244	379	247.9578613	22726.26939	379	33615.93751
118	95	190	106.960232	10660.84118	190	17575.0697
119	95	189	101.431908	10604.27396	189	17807.96518
120	116	194	116	11391.08332	194	18254.26042
121	175	321	184.352256	17956.54582	321	29637.34621
122	2	19	7.902060551	1073.549795	19	2228.418923
123	4	59	9.092100642	2262.284014	59	7856.510164
124	15	83	48.06171589	5989.552086	83	9496.265052
125	9	53	33.77007148	4291.176169	53	6159.457432
126	12	37	18.48472916	2652.719505	12	1994.469128
127	10	34	10.53106722	1723.971267	34	4224.868136
128	112	373	206.4319326	20272.58691	373	35467.98142
129	4	20	17.76878581	2419.291695	20	2659.631
130	5	38	30.2087826	3604.35828	38	4529.996792
131	5	18	8.065652087	1133.199146	5	734.353875
132	50	98	86.09581967	8153.863782	50	5029.432
133	5	10	6.413512314	971.5740127	5	796.217125
134	42	74	61.39772249	6729.594952	42	4933.834632
135	42	74	44.097629	5042.536438	74	7852.685804
136	41	105	95	16721.90678	75	13493.04938

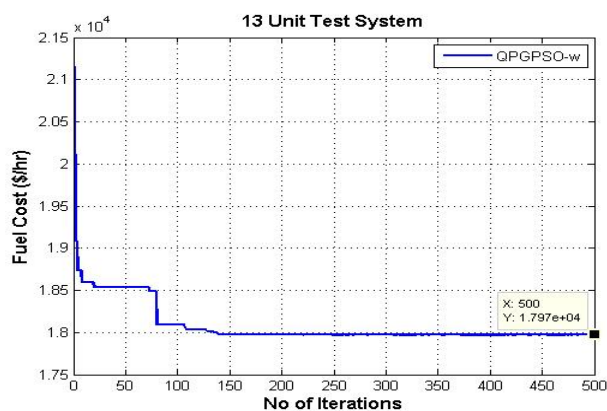
Units	$P_{i,min}$	$P_{i,max}$	GPSO-w		QGPSO-w	
			Generation (MW)	Fuel Cost (\$/hr)	Generation (MW)	Fuel Cost (\$/hr)
137	17	51	41.50417866	11202.72475	17	7229.718805
138	7	19	15.24037892	1407.333254	7	664.102563
139	7	19	16.89877087	1560.764119	7	664.102563
140	26	40	26	2428.328528	40	3680.9218
Total Generation (MW)/ Total Fuel Cost (\$/hr)			42767.023 MW / 1625345.038 \$/hr		49342 MW / 1665379 \$/hr	



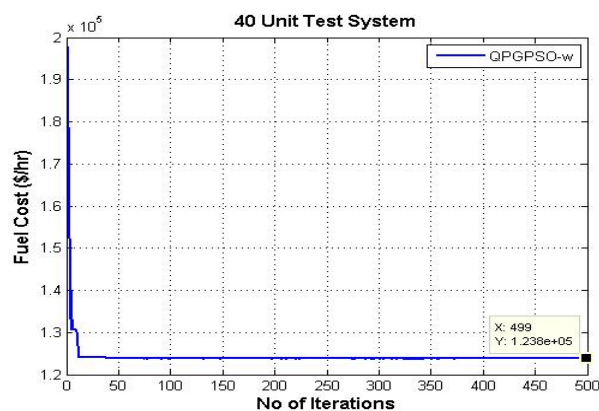
(a)



(b)



(c)



(d)

FIGURE 5: Convergence characteristics of non-convex test system. a) and b) Convergence characteristics GPSO-w. c) and d) Convergence characteristics of QGPSO-w.