Quasi-Static Assignment of Voltages and Optional Cycles in Imprecise-Computation Systems With Energy Considerations

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***Abstract*—For some realtime systems, it is possible to tradeoff precision for timeliness. For such systems, typically considered under the imprecise computation model, a function assigns reward to the application depending on the amount of computation allotted to it. Also, these systems often have stringent energy constraints since many such applications run on battery powered devices. We address in this paper, the problem of maximizing rewards for imprecise computation systems that have energy constraints, more specifically, the problem of determining the voltage at which each task runs as well as the number of optional cycles such that the total reward is maximal while time and energy constraints are satisfied. We propose a quasi-static approach that is able to exploit, with low online overhead, the dynamic slack that arises from variations in the actual number of task execution cycles. In our quasi-static approach, the problem is solved in two steps: first, at design-time, a set of voltage/optional-cycles assignments are computed and stored (offline phase); second, the selection among the precomputed assignments is left for runtime, based on actual completion times and consumed energy (online phase). The advantages of the approach are demonstrated through numerous experiments with both synthetic examples and a real life application.**

***Index Terms*—Energy management, imprecise computation, quasi-static, realtime.**

I. Introduction

THERE exists several application areas, such as image processing and multimedia, in which approximate, but timely, results are acceptable. For example, fuzzy images in time are often preferable to perfect images too late. In these cases it is, thus, possible to tradeoff precision for timeliness. Such systems have been studied in the frame of imprecise computation (IC) techniques [23], [14]. These techniques assume that tasks are composed of mandatory and optional parts: both parts must be finished by the deadline but the optional part can be left incomplete at the expense of the quality of results. There is a function associated with each task that assigns a reward as a function of the amount of computation allotted to the optional part: the more the optional part executes, the more reward it produces.

Also, power and energy consumption have become very important design considerations for embedded systems, in particular for battery powered devices with stringent energy constraints. The availability of vast computational capabilities at low cost has promoted the use of embedded systems in a wide variety of application areas where power and energy consumption play an important role.

The tradeoff between energy consumption and performance has extensively been studied under the framework of dynamic voltage scaling (DVS): by lowering the supply voltage quadratic savings in dynamic energy consumption can be achieved, while the performance is approximately degraded in linear fashion. One of the earliest papers in this area is by Yao *et al.* [29], where the case of a single processor with continuous voltage scaling is addressed. The discrete voltage selection for minimizing energy consumption in monoprocessor systems was formulated as an integer linear programming (ILP) problem by Okuma *et al.* [19]. DVS techniques have been applied to distributed systems [9], [12], [15], and even considering overheads caused by voltage transitions [31] and leakage energy [1]. DVS has also been considered under fixed [25] and dynamic priority assignments [13].

While DVS techniques have mostly been studied in the context of hard realtime systems, IC approaches have, until now, disregarded the power/energy aspects. Rusu *et al.* [20] recently proposed the first approach in which energy, reward, and deadlines are considered under a unified framework. The goal of the approach is to maximize the reward without exceeding deadlines or the available energy. This approach, however, statically solves at compile-time the optimization problem and, therefore, considers only worst cases, which leads to overly pessimistic results. A similar approach (with similar drawbacks) for maximizing the total reward subject to energy constraints, but considering that tasks have fixed priority, was presented in [30]. Such approaches can only exploit the static slack, which is due to the fact that at nominal voltage the processor runs faster than needed.

Most of the techniques proposed in the frame of DVS, for instance, are static approaches in the sense that they can only exploit the static slack [19], [20], [29]. Nonetheless, there has been a recent interest in dynamic approaches [3], [9], [24], that is, techniques aimed at exploiting the dynamic slack, which is caused by tasks executing less number of clock cycles than their worst case. Solutions by static approaches are computed offline, but have to make pessimistic assumptions, typically in the form

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of worst case execution times. Dynamic approaches recompute solutions at runtime in order to exploit the slack that arises from variations in the execution time, but these online computations are typically nontrivial and, consequently, their overhead is high.

In this paper, we focus on realtime systems for which it is possible to tradeoff precision for timeliness as well as energy consumption for performance. We study such systems under the IC model. The goal is to find the voltages at which each task runs and the number of optional cycles, such that the objective function is optimized and the constraints are satisfied. That is, we aim at finding a voltage/optional (V/O)-cycles assignment (actually a set of assignments, as explained later), such that the total reward is maximal while guaranteeing the deadlines and the energy budget. An important contribution of our work is that we exploit the dynamic time and energy slack caused by variations in the actual number of execution cycles. Furthermore, we take into consideration the time and energy overhead incurred during voltage transitions.

In this paper, static V/O assignment refers to finding one assignment of voltages and optional cycles that makes the reward maximal at design-time while guaranteeing the time and energy constraints (this is the problem addressed by [20]). Dynamic V/O assignment refers to finding at runtime, every time a task completes, a new assignment of voltages and optional cycles for those tasks not yet started, but considering the actual execution times by tasks already completed. On the one hand, static V/O assignment causes no online overhead but is rather pessimistic because actual execution times are typically far off from worst case values. On the other hand, dynamic V/O assignment exploits information known only after tasks complete and accordingly computes new assignments; however, the energy and time overhead for online computations is high, even if polynomial-time algorithms can be used. We propose a quasi-static approach that is able to exploit, with low online overhead, the dynamic slack: first, at design-time, a set of V/O assignments are computed and stored (offline phase); second, the selection among the precomputed assignments is left for runtime, based on actual completion times and consumed energy (online phase).

Quasi-static scheduling has been previously studied, mostly in the context of formal synthesis and without considering an explicit notion of time, but only the partial order of events [5], [21], [26]. Recently, in the context of realtime systems, Shih *et al.* have proposed a template-based approach that combines offline and online scheduling for phase array radar systems [22], where templates for schedules are computed offline considering performance constraints, and tasks are scheduled online such that they fit in the templates. The online overhead, though, can be significant when the system workload is high. Quasi-static scheduling for real-time systems with hard and soft tasks was recently discussed [7], but without any energy consideration.

The problem of quasi-static voltage scaling for energy minimization in hard realtime systems was recently addressed [2]. This approach prepares and stores at design-time a number of voltage settings, which are used at runtime for adapting the processor voltage based on actual execution times. Another, somehow similar, approach in which a set of voltage settings is precalculated was discussed in [28]. It considers that the application is given as a task graph composed of subgraphs, some of which might not execute for a certain activation of the system. The selection of a particular voltage setting is, thus, done online based on which subgraphs will be executed at that activation. For each subgraph worst case values are assumed and, consequently, no dynamic slack is exploited. Such approaches, however, target only hard realtime systems and, as opposed to our work, do not consider the reward produced by the soft component of the system (in our case the optional part of tasks).

To the best of our knowledge, the quasi-static approach presented in this paper is the first of its type, that is, it is the first one in which reward, energy, and deadlines are considered together in a quasi-static framework. The chief merit of our approach is its ability to exploit the dynamic slack, caused by tasks completing earlier than in the worst case, at a very low online overhead. This is possible because a set of solutions are prepared and stored at design-time, leaving only for runtime the selection of one of them.

The rest of this paper is structured as follows. Section II introduces notations and definitions used in this paper. In Section III, we motivate the advantages of our approach through an example. The precise formulation of the problem addressed in this paper is presented in Section IV. We propose, in Section V, a method for computing at design-time the set of V/O assignments. Our approach is extensively evaluated in Section VI through numerous synthetic benchmarks and a realistic application. Finally, conclusions are drawn in Section VII.

II. Preliminaries

A. Task and Architectural Models

We consider that the functionality of the system is captured by a directed acyclic graph $G = (\mathbf{T}, \mathbf{E})$ where the nodes $\mathbf{T} = \{T_1, T_2, \ldots, T_n\}$ correspond to the computational tasks and the edges $\mathbf{E}$ indicate the data dependencies between tasks. For the sake of convenience in the notation, we assume that tasks are named according to a particular execution order (as explained later in this subsection) that respects the data dependencies [8]. That is, task T_{i+1} executes immediately after T_i, $1 \leq i < n$.

Each task T_i is composed of a mandatory part and an optional part, characterized in terms of the number of CPU cycles M_i and O_i, respectively. The actual number of mandatory cycles M_i of a task T_i at a certain activation of the system is unknown beforehand but lies in the interval bounded by the best case number of cycles M_i^{bc} and the worst case number of cycles M_i^{wc}, that is, $M_i^{\mathrm{bc}} \leq M_i \leq M_i^{\mathrm{wc}}$. The optional part of a task executes immediately after its corresponding mandatory part completes. For each task T_i, there is a deadline d_i by which both mandatory and optional parts of T_i must be completed.

For each task T_i, there is a reward function $R_i(O_i)$ that takes as an argument the number of optional cycles O_i assigned to T_i; we assume that $R_i(0) = 0$. We consider nondecreasing concave[1] reward functions as they capture the particularities of most real

[1]A function $f(x)$ is concave iff $f''(x) \leq 0$, that is, the second derivative is negative.

life applications [20]. Also, as detailed in Section IV, the concavity of reward functions is exploited for obtaining solutions to particular optimization problems in polynomial time. We assume also that there is a value $O_i^{\max}$, for each T_i, after which no extra reward is achieved, that is, $R_i(O_i) = R_i^{\max}$ if $O_i \geq O_i^{\max}$. The total reward is the sum of individual reward contributions and is denoted $R = \sum_{T_i \in \mathbf{T}} R_i(O_i)$.

We consider that tasks are nonpreemptable and have equal release time ($r_i = 0, 1 \leq i \leq n$). All tasks are mapped onto a single processor and executed in a fixed order, determined offline, that respects the data dependencies and according to an earliest deadline first (EDF) policy. For nonpreemptable tasks with equal release time and running on a single processor, EDF gives the optimal execution order [6].[2] T_i denotes the i-th task in this sequence.

Observe that the nonpreemption assumption is exploited in order to characterize the space from which possible V/O assignments can be selected (as explained in Section V-A) and, thus, obtain a good-quality solution. Note also that our approach deals with voltage levels and optional cycles but not with executing orders. Scheduling and voltage scaling together requires a solution significantly different to the one discussed here and, thus, beyond the scope of this paper.

The target processor supports voltage scaling and we assume that the voltage levels can be varied in a continuous way in the interval $[V^{\min}, V^{\max}]$. If only a discrete set of voltages are supported by the processor, our approach can easily be adapted by using well-known techniques for determining the discrete voltage levels that replace the calculated continuous one [19].

In our quasi-static approach we compute a number of V/O-cycles assignments. The set of precomputed V/O assignments is stored in a dedicated memory in the form of lookup tables, one table LUT_i for each task T_i. The maximum number of V/O assignments that can be stored in memory is a parameter fixed by the designer and is denoted $N^{\max}$.

B. Energy and Delay Models

The power consumption in CMOS circuits is the sum of dynamic, static (leakage), and short-circuit power. The short-circuit component is negligible. The dynamic power is at the moment the dominating component. However, the leakage power is becoming an important factor in the overall power dissipation. For the sake of simplicity and clarity in the presentation of our ideas, we consider only the dynamic energy consumption. Nonetheless, the leakage energy and adaptive body biasing (ABB) techniques [1] can easily be incorporated into the formulation without changing our general approach. The amount of dynamic energy consumed by task T_i is given by the following expression [16]:

$$E_i = C_i V_i^2 (M_i + O_i) \tag{1}$$

[2]By optimal in this context, we mean the task execution order that, among all feasible orders, admits the V/O assignment which yields the highest total reward. We have demonstrated in [6] that an EDF execution order is the one that least constraints the space of V/O solutions and, henceforth, optimal in the above sense.

where C_i is the effective switched capacitance, V_i is the supply voltage, and $M_i + O_i$ is the total number of cycles executed by the task. The energy overhead caused by switching from V_i to V_j is as follows [16]:

$$\mathcal{E}_{i,j}^{\Delta V} = C_r (V_i - V_j)^2 \tag{2}$$

where C_r is the capacitance of the power rail. We also consider, for the quasi-static solution, the energy overhead $\mathcal{E}_i^{\mathrm{sel}}$ originated from the need to look up and select one of the precomputed V/O assignments. The way we store the precomputed assignments makes the lookup and selection process take $\mathcal{O}(1)$ time. Therefore, $\mathcal{E}_i^{\mathrm{sel}}$ is a constant value. Also, this value is the same for all tasks ($\mathcal{E}_i^{\mathrm{sel}} = \mathcal{E}^{\mathrm{sel}}$, for $1 \leq i \leq n$). For consistency reasons, we keep the index i in the notation of the selection overhead $\mathcal{E}_i^{\mathrm{sel}}$. The energy overhead caused by online operations is denoted $\mathcal{E}_i^{\mathrm{dyn}}$. In a quasi-static solution the online overhead is just the selection overhead ($\mathcal{E}_i^{\mathrm{dyn}} = \mathcal{E}_i^{\mathrm{sel}}$) [8].

The total energy consumed up to the completion of task T_i (including the energy consumed by the tasks themselves as well as the overheads) is denoted $\mathrm{EC}_i = \mathrm{EC}_{i-1} + \mathcal{E}_{i-1,i}^{\Delta V} + \mathcal{E}_i^{\mathrm{dyn}} + E_i$. We consider a given energy budget, denoted $E^{\max}$, that imposes a constraint on the total amount of energy that can be consumed by the system.

The execution time of a task T_i executing $M_i + O_i$ cycles at supply voltage V_i is [16]

$$\tau_i = k \frac{V_i}{(V_i - V_{\mathrm{th}})^{\alpha}} (M_i + O_i) \tag{3}$$

where k is a constant dependent on the process technology, α is the saturation velocity index (also technology dependent, typically $1.4 \leq \alpha \leq 2$), and V_{th} is the threshold voltage. The time overhead, when switching from V_i to V_j, is given by the following expression [1]:

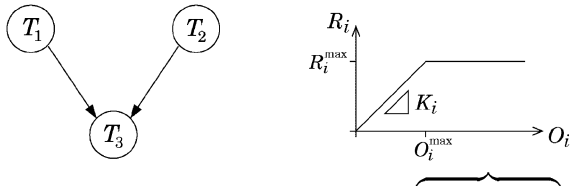
$$\delta_{i,j}^{\Delta V} = p|V_i - V_j| \tag{4}$$

where p is a constant. The time overhead for looking up and selecting one V/O assignment in the quasi-static approach is denoted δ_i^{sel} and, as explained above, is constant and is the same value for all tasks.

The starting and completion times of a task T_i are denoted s_i and t_i, respectively, with $s_i + \delta_i + \tau_i = t_i$, where δ_i captures the total time overheads. $\delta_i = \delta_{i-1,i}^{\Delta V} + \delta_i^{\mathrm{dyn}}$, where δ_i^{dyn} is the online overhead. Note that in a quasi-static solution this online overhead is just the lookup and selection time, that is, $\delta_i^{\mathrm{dyn}} = \delta_i^{\mathrm{sel}}$.

III. Motivational Example

Let us consider the motivational example shown in Fig. 1. The nondecreasing reward functions are of the form $R_i(O_i) = K_i O_i$, $O_i \leq O_i^{\max}$. The energy constraint is $E^{\max} = 1$ mJ and the tasks run, according to a schedule fixed offline in conformity to an EDF policy, on a processor that permits continuous voltage scaling in the range 0.6–1.8 V. For clarity reasons, in this example, we assume that transition overheads are zero.



Task	M_i^{bc}	M_i^{wc}	C_i [nF]	d_i [μs]	K_i	$R_i^{\max}$
T_1	20000	100000	0.7	250	0.00014	7
T_2	70000	160000	1.2	600	0.0002	16
T_3	100000	180000	0.9	1000	0.0001	6

Fig. 1. Motivational example.

TABLE I
OPTIMAL STATIC V/O ASSIGNMENT

Task	V_i [V]	O_i
T_1	1.654	35
T_2	1.450	19925
T_3	1.480	11

The optimal static V/O assignment for this example is given by Table I. It produces a total reward $R^{\text{st}} = 3.99$. The assignment gives, for each task T_i, the voltage V_i at which T_i must run and the number of optional cycles O_i that it must execute in order to obtain the maximum total reward, while guaranteeing that deadlines are met and the energy constraint is satisfied.

The V/O assignment given by Table I is optimal in the static sense. It is the best possible that can be obtained offline without knowing the actual number of cycles executed by each task. However, the actual number of cycles, which are not known in advance, are typically far off from the worst case values used to compute such a static assignment. The implication of such a situation is illustrated by the following case. The first task starts executing at $V_1 = 1.654$ V, as required by the static assignment. Assume that T_1 executes $M_1 = 60\,000$ (instead of $M_1^{\text{wc}} = 100\,000$) mandatory cycles and then its assigned $O_1 = 35$ optional cycles. At this point, knowing that T_1 has completed at $t_1 = \tau_1 = 111.73$ μs and that the consumed energy is $\text{EC}_1 = E_1 = 114.97$ μJ, a new V/O assignment can accordingly be computed for the remaining tasks, aiming at obtaining the maximum total reward for the new conditions. We consider, for the moment, the ideal case in which such an online computation takes zero time and energy. Observe that, for computing the new assignments, the worst case for tasks not yet completed has to be assumed as their actual number of executed cycles is not known in advance. The new assignment gives $V_2 = 1.446$ V and $O_2 = 51\,396$. Then T_2 runs at $V_2 = 1.446$ V and let us assume that it executes $M_2 = 100\,000$ (instead of $M_2^{\text{wc}} = 160\,000$) mandatory cycles and then its newly assigned $O_2 = 51\,396$ optional cycles. At this point, the completion time is $t_2 = \tau_1 + \tau_2 = 461.35$ μs and the energy so far consumed is $\text{EC}_2 = E_1 + E_2 = 494.83$ μJ. Again, a new assignment can be computed taking into account the information about completion time and consumed energy. This new assignment gives $V_3 = 1.472$ V and $O_3 = 60\,000$.

For such a situation, in which $M_1 = 60\,000$, $M_2 = 100\,000$, $M_3 = 150\,000$, the V/O assignment computed dynamically (considering $\delta^{\text{dyn}} = 0$ and $\mathcal{E}^{\text{dyn}} = 0$) is summarized in Table II(a). This assignment delivers a total reward $R^{\text{dyn}^{\text{ideal}}} = 16.28$. In reality, however, the online overhead caused by computing new assignments is not negligible. When considering time and energy overheads, using for example $\delta^{\text{dyn}} = 65$ μs and $\mathcal{E}^{\text{dyn}} = 55$ μJ, the V/O assignment computed dynamically is evidently different, as given by Table II(b). This assignment delivers a total reward $R^{\text{dyn}^{\text{real}}} = 6.26$. The values of δ^{dyn} and $\mathcal{E}^{\text{dyn}}$ are, in practice, several orders of magnitude higher than the ones used in this hypothetical example. For instance, for a system with 50 tasks, computing one such V/O assignment using a commercial solver takes a few seconds. Even online heuristics, which produce approximate results, have long execution times [20]. This means that a dynamic V/O scheduler might produce solutions that are actually inferior to the static one (in terms of total reward delivered) or, even worse, a dynamic V/O scheduler might not be able to fulfill the given time and energy constraints, due to the overheads.

TABLE II
DYNAMIC V/O ASSIGNMENTS (FOR $M_1 = 60\,000$, $M_2 = 100\,000$, $M_3 = 150\,000$)

(a)

Task	V_i [V]	O_i
T_1	1.654	35
T_2	1.446	51396
T_3	1.472	60000

(b)

Task	V_i [V]	O_i
T_1	1.654	35
T_2	1.429	1303
T_3	1.533	60000

TABLE III
PRECOMPUTED SET OF V/O ASSIGNMENTS

Task	Condition	V_i [V]	O_i
T_1	—	1.654	35
T_2	if $t_1 \leq 75$ μs $\wedge$ $EC_1 \leq 77$ μJ	1.444	66924
	else if $t_1 \leq 130$ μs $\wedge$ $EC_1 \leq 135$ μJ	1.446	43446
	else	1.450	19925
T_3	if $t_2 \leq 400$ μs $\wedge$ $EC_2 \leq 430$ μJ	1.380	60000
	else if $t_2 \leq 500$ μs $\wedge$ $EC_2 \leq 550$ μJ	1.486	46473
	else	1.480	11

In our quasi-static solution, we compute at design-time a number of V/O assignments, which are selected at runtime by the so-called quasi-static V/O scheduler (at very low overhead) based on the information about completion time and consumed energy after each task completes.

We can define, for instance, a quasi-static set of assignments for the example discussed in this section, as given by Table III. Upon completion of each task, V_i and O_i are selected from the precomputed set of V/O assignments, according to the given condition. The assignments were computed considering the selection overheads $\delta^{\text{sel}} = 0.3$ μs and $\mathcal{E}^{\text{sel}} = 0.3$ μJ.

For the situation $M_1 = 60\,000$, $M_2 = 100\,000$, $M_3 = 150\,000$ and the set given by Table III, the quasi-static V/O scheduler would do as follows. Task T_1 is run at $V_1 = 1.654$ V and is allotted $O_1 = 35$ optional cycles. Since, when completing T_1, $t_1 = \tau_1 = 111.73 \leq 130$ μs and $\text{EC}_1 = E_1 = 114.97 \leq 135$ μJ, $V_2 = 1.446/O_2 = 43\,446$ is selected by the quasi-static V/O scheduler. Task T_2 runs under this assignment so that, when it finishes, $t_2 = \tau_1 + \delta_2^{\text{sel}} + \tau_2 = 442.99$ μs and $\text{EC}_2 = E_1 + \mathcal{E}_2^{\text{sel}} + E_2 = 474.89$ μJ. Then $V_3 = 1.486/O_3 = 46\,473$ is selected and task T_3 is executed accordingly. Table IV summarizes the selected assignment. The total reward delivered

TABLE IV
QUASI-STATIC V/O ASSIGNMENT (FOR $M_1 = 60\,000$, $M_2 = 100\,000$, $M_3 = 150\,000$) SELECTED FROM THE PRECOMPUTED SET OF TABLE III

Task	V_i [V]	O_i
T_1	1.654	35
T_2	1.446	43446
T_3	1.486	46473

by this V/O assignment is $R^{\mathrm{qs}} = 13.34$ (compare to $R^{\mathrm{dyn^{ideal}}} = 16.28$, $R^{\mathrm{dyn^{real}}} = 6.26$, and $R^{\mathrm{st}} = 3.99$). It can be noted that the quasi-static solution qsoutperforms the dynamic one $\mathrm{dyn^{real}}$ because of the large overheads of the latter.

IV. PROBLEM FORMULATION

We tackle the problem of maximizing the total reward subject to a limited energy budget, in the framework of DVS. In the following, we present the precise formulation of certain related problems and of the particular problem addressed in this paper. Recall that the task execution order is predetermined, with T_i being the ith task in this sequence (see Section II-A).

STATIC V/O ASSIGNMENT: Find, for each task T_i, $1 \leq i \leq n$, the voltage V_i and the number of optional cycles O_i that

$$\text{maximize} \quad \sum_{i=1}^{n} R_i(O_i) \tag{5}$$

$$\text{subject to} \quad V^{\min} \leq V_i \leq V^{\max} \tag{6}$$

$$s_{i+1} = t_i = s_i + \underbrace{p|V_{i-1} - V_i|}_{\delta^{\Delta V}_{i-1,i}} + \underbrace{k\frac{V_i}{(V_i - V_{\mathrm{th}})^{\alpha}}(M_i^{\mathrm{wc}} + O_i)}_{\tau_i} \leq d_i \tag{7}$$

$$\sum_{i=1}^{n} \Big(\underbrace{C_r(V_{i-1} - V_i)^2}_{\mathcal{E}^{\Delta V}_{i-1,i}} + \underbrace{C_i V_i^2 (M_i^{\mathrm{wc}} + O_i)}_{E_i}\Big) \leq E^{\max}. \tag{8}$$

The previous formulation can be explained as follows. The total reward, as given by (5), is to be maximized. For each task, the voltage V_i must be in the range $[V^{\min}, V^{\max}]$ [(6)]. The completion time t_i is the sum of the start time s_i, the voltage-switching time $\delta^{\Delta V}_{i-1,i}$, and the execution time τ_i, and tasks must complete before their deadlines [(7)]. The total energy is the sum of the voltage-switching energies $\mathcal{E}^{\Delta V}_{i-1,i}$ and the energy E_i consumed by each task, and cannot exceed the available energy budget $E^{\max}$ [(8)]. Note that a static assignment must consider the worst case number of mandatory cycles M_i^{wc} for every task [(7) and (8)].

For tractability reasons, when solving the above problem, we consider O_i as a continuous variable and then we round the result down. By this, without generating the optimal solution, we obtain a solution that is very near to the optimal one because one clock cycle is a very fine-grained unit (tasks execute typically hundreds of thousands of clock cycles). We can rewrite the above problem as "minimize $\sum R'_i(O_i)$," where $R'_i(O_i) = -R_i(O_i)$. It can, thus, be noted that: $R'_i(O_i)$ is convex since $R_i(O_i)$ is a concave function; the constraint functions are also convex.[3] Therefore, we have a convex nonlinear programming (NLP) formulation [27] and, hence, the problem can be solved using polynomial-time methods [18].

Dynamic V/O Scheduler: The following is the problem that a dynamic V/O scheduler must solve every time a task T_c completes. It is considered that tasks $T_1, \ldots, T_c$ have already completed (the total energy consumed up to the completion of T_c is EC_c and the completion time of T_c is t_c).

Find V_i and O_i, for $c+1 \leq i \leq n$, that

$$\text{maximize} \quad \sum_{i=c+1}^{n} R_i(O_i) \tag{9}$$

$$\text{subject to} \quad V^{\min} \leq V_i \leq V^{\max} \tag{10}$$

$$s_{i+1} = t_i = s_i + \delta_i^{\mathrm{dyn}} + \delta^{\Delta V}_{i-1,i} + \tau_i \leq d_i \tag{11}$$

$$\sum_{i=c+1}^{n} \left(\mathcal{E}_i^{\mathrm{dyn}} + \mathcal{E}^{\Delta V}_{i-1,i} + E_i\right) \leq E^{\max} - \mathrm{EC}_c \tag{12}$$

where δ_i^{dyn} and $\mathcal{E}_i^{\mathrm{dyn}}$ are, respectively, the time and energy overhead of computing dynamically V_i and O_i for task T_i.

Observe that the problem solved by the dynamic V/O scheduler corresponds to an instance of the static V/O assignment problem (for $c+1 \leq i \leq n$ and taking into account t_c and EC_c), but considering δ_i^{dyn} and $\mathcal{E}_i^{\mathrm{dyn}}$.

The total reward R^{ideal} delivered by a dynamic V/O scheduler, in the ideal case $\delta_i^{\mathrm{dyn}} = 0$ and $\mathcal{E}_i^{\mathrm{dyn}} = 0$, represents an upper bound on the reward that can practically be achieved without knowing in advance how many mandatory cycles tasks will execute and without accepting risks regarding deadline or energy violations.

Although the dynamic V/O assignment problem can be solved in polynomial-time, the time and energy for solving it are in practice very large and, therefore, unacceptable at runtime for practical applications. In our approach, we prepare offline a number of V/O assignments, one of which is to be selected (at very low online cost) by the *quasi-static V/O scheduler*.

Every time a task T_c completes, the quasi-static V/O scheduler checks the completion time t_c and the total energy EC_c, and looks up an assignment in the table for T_c. From the lookup table LUT_c it obtains the point (t'_c, EC'_c), which is the closest to (t_c, EC_c), such that $t_c \leq t'_c$ and $\mathrm{EC}_c \leq \mathrm{EC}'_c$, and selects V'/O' corresponding to (t'_c, EC'_c), which are the voltage and number of optional cycles for the next task T_{c+1}. Our aim is to obtain a reward, as delivered by the quasi-static V/O scheduler, that is maximal. This problem we discuss in the rest of the paper as follows.

[3]Observe that the function *abs* cannot be used directly in mathematical programming because it is not differentiable in 0. However, there exist established techniques for obtaining equivalent formulations that can be used in NLP problems [1].

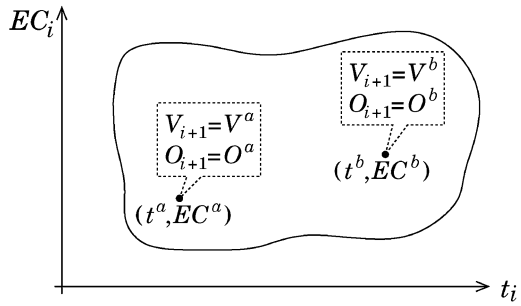


Fig. 2. Time-energy space.

Set of V/O Assignments: Find a set of N assignments such that: $N \leq N^{\max}$; the V/O assignment selected by the quasi-static V/O scheduler guarantees the deadlines $(s_i + \delta_i^{\text{sel}} + \delta_{i-1,i}^{\Delta V} + \tau_i = t_i \leq d_i)$ and the energy constraint $(\sum_{i=1}^{n} \mathcal{E}_i^{\text{sel}} + \mathcal{E}_{i-1,i}^{\Delta V} + E_i \leq E^{\max})$, and yields a total reward R^{qs} that is maximal.

As will be discussed in Section V, for a task T_i, potentially there exist infinitely many possible values for t_i and EC_i. Therefore, in order to approach the theoretical limit R^{ideal}, it would be needed to compute an infinite number of V/O assignments, one for each (t_i, EC_i). The problem is, thus, how to select at most $N^{\max}$ points in this infinite space such that the respective V/O assignments produce a reward as close as possible to R^{ideal}.

V. Computing the Set of V/O Assignments

For each task T_i, there exists a time-energy space of possible values of completion time t_i and energy EC_i consumed up to the completion of T_i (see Fig. 2). Every point in this space defines a V/O assignment for the next task T_{i+1}, that is, if T_i completed at t^a and the energy consumed was EC^a, the assignment for the next task would be $V_{i+1} = V^a / O_{i+1} = O^a$. The values V^a and O^a are those that an ideal dynamic V/O scheduler would give for the case $t_i = t^a, \text{EC}_i = \text{EC}^a$ (recall that we aim at matching the reward R^{ideal}). Observe that different points (t_i, EC_i) define different V/O assignments. Note also that for a given value t_i there might be different valid values of EC_i, and this is due to the fact that different previous V/O assignments can lead to the same t_i but still different EC_i.

We need first to determine the boundaries of the time-energy space for each task T_i, in order to select N_i points in this space and accordingly compute the set of N_i assignments. N_i is the number of assignments to be stored in the lookup table LUT_i, after distributing the maximum number $N^{\max}$ of assignments among tasks. A straightforward way to determine these boundaries is to compute the earliest and latest completion times as well as the minimum and maximum consumed energy for task T_i, based on the values $V^{\min}$, $V^{\max}$, M_j^{bc}, M_j^{wc}, and $O_j^{\max}$, $1 \leq j \leq i$. The earliest completion time $t_i^{\min}$ occurs when each of the previous tasks T_j (inclusive T_i) execute their minimum number of cycles M_j^{bc} and zero optional cycles at maximum voltage $V^{\max}$, while $t_i^{\max}$ occurs when each task T_j executes $M_j^{\text{wc}} + O_j^{\max}$ cycles at $V^{\min}$. Similarly, $\text{EC}_i^{\min}$ happens when each task T_j executes M_j^{bc} cycles at $V^{\min}$, while $\text{EC}_i^{\max}$ happens when each task T_j executes $M_j^{\text{wc}} + O_j^{\max}$ cycles at $V^{\max}$. The intervals $[t_i^{\min}, t_i^{\max}]$ and $[\text{EC}_i^{\min}, \text{EC}_i^{\max}]$ bound the time-energy space as shown in Fig. 3. However, there are points in this space that cannot happen. For instance, $(t_i^{\min}, \text{EC}_i^{\min})$ is not feasible because $t_i^{\min}$ requires all tasks running at $V^{\max}$ whereas $\text{EC}_i^{\min}$ requires all tasks running at $V^{\min}$.

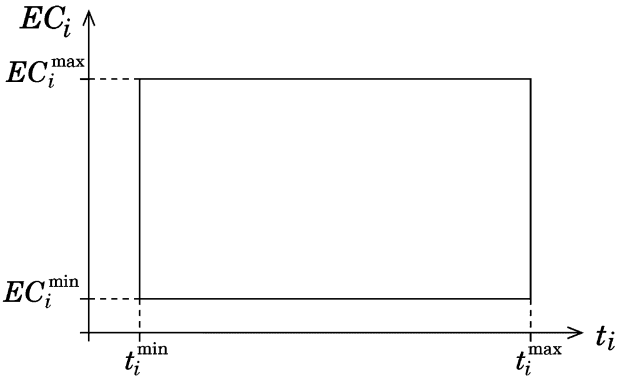


Fig. 3. Pessimistic boundaries of the time-energy space.

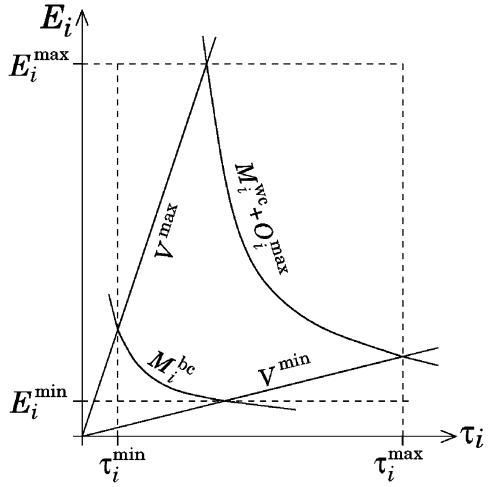


Fig. 4. τ_i-E_i space for task T_i.

A. Characterization of the Time-Energy Space

In this section, we make a characterization of the time-energy space in order to determine how the points in this space (for which the V/O assignments are to be computed offline) should be selected so that good quality results are achieved. We elaborate on the different steps along this characterization.

We now take a closer look at the relation between the energy E_i consumed by a task and its execution time τ_i as given, respectively, by (1) and (3). In this section, we consider that the execution time is inversely proportional to the supply voltage $(V_{\text{th}} = 0, \alpha = 2)$, an assumption commonly made in the literature [19]. Observe, however, that we make such an assumption only in order to make the illustration of our point simpler, yet the drawn conclusions are valid, in general, and do not rely on this assumption.

After some simple algebraic manipulations on (1) and (3), we get the following expression:

$$E_i = \frac{C_i V_i^3}{k} \tau_i \tag{13}$$

which, in the τ_i-E_i space, gives a family of straight lines, each for a particular V_i. Thus, $E_i = C_i (V^{\min})^3 \tau_i / k$ and $E_i =$

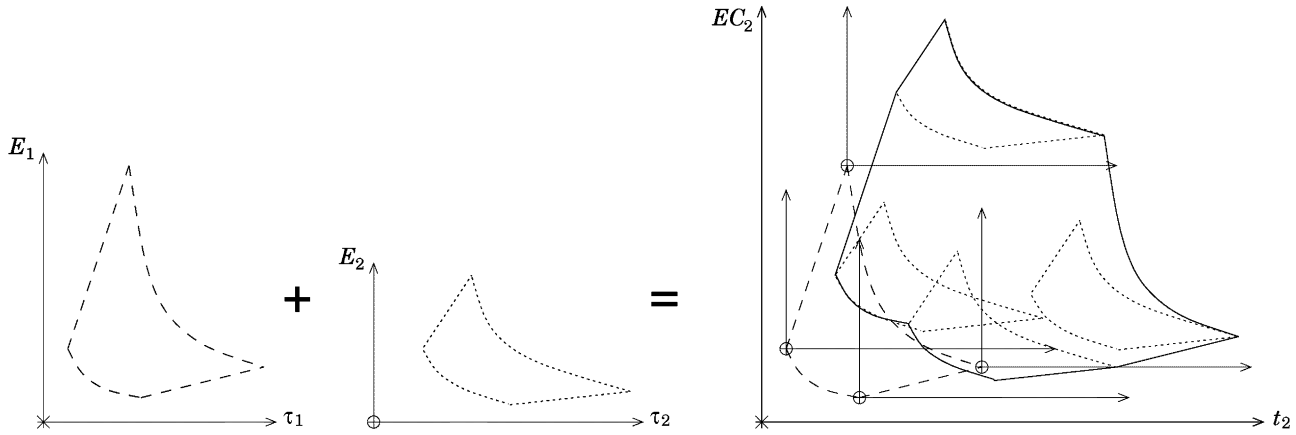


Fig. 5. Illustration of the "sum" of spaces τ_1-E_1 and τ_2-E_2.

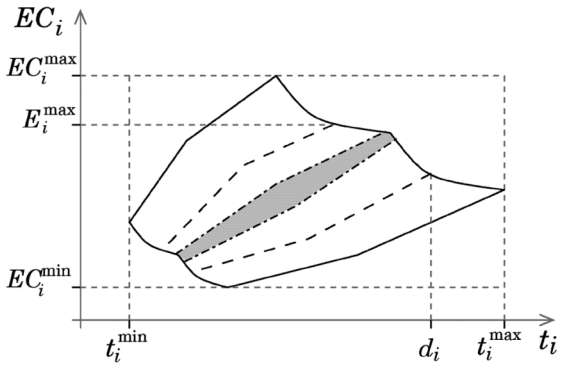


Fig. 6. Realistic boundaries of the time-energy space.

$C_i(V^{\max})^3\tau_i/k$ define two boundaries in the τ_i-E_i space. We can also write the following equation:

$$E_i = C_i k^2 (M_i + O_i)^3 \frac{1}{\tau_i^2} \tag{14}$$

which gives a family of curves, each for a particular $M_i + O_i$. Thus, $E_i = C_i k^2 (M_i^{\mathrm{bc}})^3/\tau_i^2$ and $E_i = C_i k^2 (M_i^{\mathrm{wc}} + O_i^{\max})^3/\tau_i^2$ define other two boundaries, as shown in Fig. 4. Note that Fig. 4 represents the energy consumed by one task (energy E_i if T_i executes for τ_i time), as opposed to Fig. 3 that represents the energy by all tasks up to T_i (total energy EC_i consumed up to the moment t_i when task T_i finishes).

In order to obtain a realistic, less pessimistic, view of the diagram in Fig. 3, we must "sum" the spaces τ_j-E_j previously introduced. The result of this summation, as illustrated in Fig. 5, gives the time-energy space t_i-EC_i we are interested in. In Fig. 5, the t_2-EC_2 space is obtained by sliding the τ_2-E_2 space with its coordinate origin along the boundaries of τ_1-E_1. The "south-east" (SE) and "north-west" (NW) boundaries of the t_i-EC_i space are piecewise linear because the SE and NW boundaries of the individual spaces τ_j-E_j, $1 \leq j \leq i$, are straight lines (see Fig. 4). Similarly, the NE and SW boundaries of the t_i-EC_i space are piecewise parabolic because the NE and SW of the individual spaces τ_j-E_j are parabolic. The shape of the t_i-EC_i space, obtained as a result of such a summation, is depicted by the solid lines in Fig. 6.

In order to further refine the t_i-EC_i space, one has to consider, in addition, deadlines d_i as well as energy constraints $E_i^{\max}$. Note that, for each task, the deadline d_i is explicitly given as part of the system model, whereas $E_i^{\max}$ is an implicit constraint induced by the overall energy constraint $E^{\max}$. The energy constraint $E_i^{\max}$ imposed upon the completion of task T_i comes from the fact that future tasks will consume at least a certain amount of energy $F_{i+1\to n}$ so that $E_i^{\max} = E^{\max} - F_{i+1\to n}$. The deadline d_i and the induced energy constraint $E_i^{\max}$ further restrict the time-energy space, as depicted by the dashed lines in Fig. 6.

The time-energy space can be narrowed down even further if we take into consideration that we are only interested in points as calculated by the ideal dynamic V/O scheduler. This is explained in the following. Let us consider two different activations of the system. In the first one, after finishing task T_{i-1} at t'_{i-1} with a total consumed energy EC'_{i-1}, task T_i runs under a certain assignment V'_i/O'_i. In the second activation, after T_{i-1} completes at t''_{i-1} with total energy EC''_{i-1}, T_i runs under the assignment V''_i/O''_i. Since the points $(t'_{i-1}, \mathrm{EC}'_{i-1})$ and $(t''_{i-1}, \mathrm{EC}''_{i-1})$ are, in general, different, the assignments V'_i/O'_i and V''_i/O''_i are also different. The assignment V'_i/O'_i defines in the t_i-EC_i space a segment of straight line L'_i that has slope $C_i(V'_i)^3/k$, with one end point corresponding to the execution of $M_i^{\mathrm{bc}} + O'_i$ cycles at V'_i and the other end corresponding to the execution of $M_i^{\mathrm{wc}} + O'_i$ cycles at V'_i [see (13)]. V''_i/O''_i defines analogously a different segment of straight line L''_i. The lines L'_i and L''_i defined, respectively, by the assignments V'_i/O'_i and V''_i/O''_i are depicted in Fig. 7. It must be observed that solutions to the dynamic V/O assignment problem, though, tend toward letting tasks consume as much as possible of the available energy and finish as late as possible without risking energy or deadline violations: there is no gain by consuming less energy or finishing earlier than needed, as the goal is to maximize the reward. Both solutions V'_i/O'_i and V''_i/O''_i (that is, the straight lines L'_i and L''_i in Fig. 7) will, thus, converge in the t_i-EC_i space when $M'_i = M''_i = M_i^{\mathrm{wc}}$ (which is the value of mandatory cycles that has to be assumed when computing the V/O values to be assigned to T_i after T_{i-1} has terminated). Therefore, if T_i under the assignment V'_i/O'_i completes at the same time as under the second assignment V''_i/O''_i ($t'_i = t''_i$), the respective energy values EC'_i and EC''_i

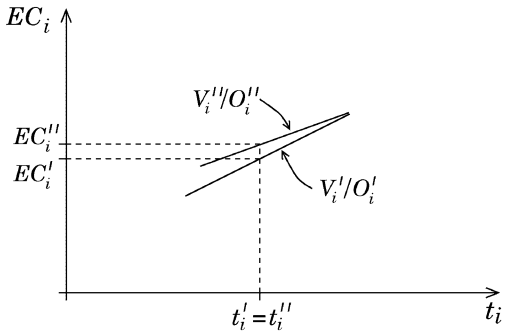


Fig. 7. V_i'/O_i' and V_i''/O_i'' converge.

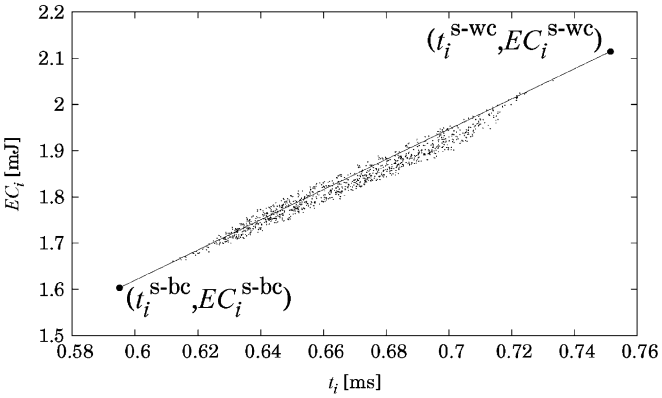


Fig. 8. Actual points in the time-energy space.

will actually be very close as shown in Fig. 7. This means that in practice the t_i-EC_i space is a narrow area, as depicted by the dash-dot lines and the gray area enclosed by them in Fig. 6.

We have conducted a number of experiments in order to determine how narrow the area of points in the time-energy space actually is. For each task T_i, we consider a segment of straight line, called in the sequel the diagonal D_i, defined by the points $(t_i^{s-\mathrm{bc}}, \mathrm{EC}_i^{s-\mathrm{bc}})$ and $(t_i^{s-\mathrm{wc}}, \mathrm{EC}_i^{s-\mathrm{wc}})$. The point $(t_i^{s-\mathrm{bc}}, \mathrm{EC}_i^{s-\mathrm{bc}})$ corresponds to the solution given by the ideal dynamic V/O scheduler in the particular case when every task T_j, $1 \leq j \leq i$, executes its best case number of mandatory cycles M_j^{bc}. Analogously, $(t_i^{s-\mathrm{wc}}, \mathrm{EC}_i^{s-\mathrm{wc}})$ corresponds to the solution in the particular case when every task T_j executes its worst case number of mandatory cycles M_j^{wc}. We have generated 50 synthetic examples, consisting of between 10 and 100 tasks, and simulated for each of them the ideal dynamic V/O scheduler for 1000 cases, each case S being a combination of executed mandatory cycles $M_1^S, M_2^S, \ldots, M_n^S$. Note that we have used various distributions for the number of mandatory cycles. For each task T_i of the different benchmarks and for each set S of mandatory cycles, we obtained the actual point (t_i^S, EC_i^S) in the t_i-EC_i space, as given by the ideal dynamic V/O scheduler. Each point (t_i^S, EC_i^S) was compared with the point $(t_i^S, \mathrm{EC}_i^{D_i})$ (a point with the same abscissa t_i^S, but on the diagonal D_i so that its ordinate is $\mathrm{EC}_i^{D_i}$) and the relative deviation $e = |\mathrm{EC}_i^S - \mathrm{EC}_i^{D_i}|/\mathrm{EC}_i^S$ was computed. We found through our experiments average deviations of around 1% and a maximum deviation of 4.5%. These results show that the t_i-EC_i space is indeed a narrow area. For example, Fig. 8 shows the actual points (t_i^S, EC_i^S), corresponding to the simulation of the 1000 sets S of executed mandatory cycles, in the time-energy space of a particular task T_i as well as the diagonal D_i. It can also be mentioned that Fig. 8 corresponds to an experiment where the number of mandatory cycles M_i were considered to be uniformly distributed in its interval $[M_i^{\mathrm{bc}}, M_i^{\mathrm{wc}}]$.

Condition	V_{i+1}	O_{i+1}
if $t_i \leq t^a \wedge EC_i \leq EC^a$	V^a	O^a
else if $t_i \leq t^b \wedge EC_i \leq EC^b$	V^b	O^b
else if $t_i \leq t^c \wedge EC_i \leq EC^c$	V^c	O^c
else	V^d	O^d

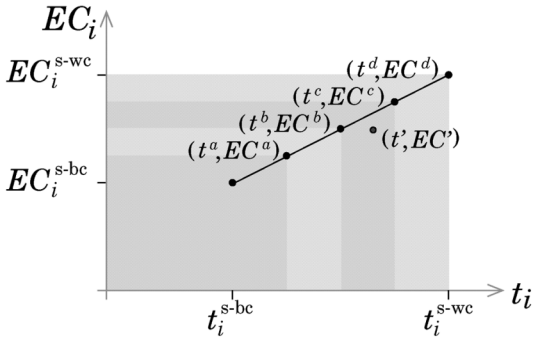


Fig. 9. Regions.

B. Selection of Points and Computation of Assignments

From the discussion in Section V-A, we can draw the conclusion that the points in the t_i-EC_i space are concentrated in a relatively narrow area along the diagonal D_i. This observation is crucial for selecting the points for which we compute, at design-time, the V/O assignments.

We take N_i points (t_i^j, EC_i^j) along the diagonal D_i in the t_i-EC_i space of task T_i, and then we compute and store the respective assignments V_{i+1}^j/O_{i+1}^j that maximize the total reward when T_i completes at t_i^j and the total energy is EC_i^j. It should be noted that for the computation of the assignment V_{i+1}^j/O_{i+1}^j, the time and energy overheads $\delta_{i+1}^{\mathrm{sel}}$ and $\mathcal{E}_{i+1}^{\mathrm{sel}}$ (needed for selecting assignments at run time) are taken into account.

Each one of the points, together with its corresponding V/O assignment, covers a region as indicated in Fig. 9. The quasi-static V/O scheduler selects one of the stored assignments based on the actual completion time and consumed energy. Referring to Fig. 9, for example, if task T_i completes at t' and the consumed energy is EC', the quasi-static V/O scheduler will select the precomputed V/O assignment corresponding to (t^c, EC^c).

The pseudocode of the procedure we use for computing, offline, the set of V/O assignments is given by Algorithm 1. First, the maximum number $N^{\max}$ of assignments that are to be stored is distributed among tasks (line 1). A straightforward approach is to distribute them uniformly among the different tasks, so that each lookup table contains the same number of assignments. However, as shown by the experimental evaluation of Section VI, it is more efficient to distribute the assignments according to the size of the time-energy space of tasks (we use the length of the diagonal D as a measure of this size), in such a way that the lookup tables of tasks with larger spaces get more points.

After distributing $N^{\max}$ among tasks, the solutions $\mathrm{V/O}^{s-\mathrm{bc}}$ and $\mathrm{V/O}^{s-\mathrm{wc}}$ are computed (lines 2-3). $\mathrm{V/O}^{s-\mathrm{bc}}$ ($\mathrm{V/O}^{s-\mathrm{wc}}$) is a structure that contains the pairs $V_i^{s-\mathrm{bc}}/O_i^{s-\mathrm{bc}}$ ($V_i^{s-\mathrm{wc}}/O_i^{s-\mathrm{wc}}$), for each task T_i, $1 \leq i \leq n$. $V_i^{s-\mathrm{bc}}/O_i^{s-\mathrm{bc}}$

and $V_i^{s-\mathrm{wc}}/O_i^{s-\mathrm{wc}}$ are the values that the dynamic V/O scheduler would compute when every task executes its best and worst case number of cycles, respectively. In order to obtain the values $V_i^{s-\mathrm{bc}}/O_i^{s-\mathrm{bc}}$ and $V_i^{s-\mathrm{wc}}/O_i^{s-\mathrm{wc}}$, $1 \leq i \leq n$ (and, thus, the structures $\mathrm{V/O}^{s-\mathrm{bc}}$ and $\mathrm{V/O}^{s-\mathrm{wc}}$), we solve the problem, as formulated by (9)–(12), $n-1$ times (corresponding to each task completion) first, considering the best case of mandatory cycles for all tasks, and second, considering the worst case of mandatory cycles for all tasks.

Since the assignment V_1/O_1 is invariably the same (line 4), this is the only one stored for the first task (line 5).

For every task T_i, $1 \leq i \leq n-1$, we compute: 1) $t_i^{s-\mathrm{bc}}$ ($t_i^{s-\mathrm{wc}}$) as the sum of execution times $\tau_k^{s-\mathrm{bc}}$ ($\tau_k^{s-\mathrm{wc}}$)—given by (3) with $V_k^{s-\mathrm{bc}}$, M_k^{bc}, and $O_k^{s-\mathrm{bc}}$ ($V_k^{s-\mathrm{wc}}$, M_k^{wc}, and $O_k^{s-\mathrm{wc}}$)—and time overheads δ_k (line 7); 2) $\mathrm{EC}_i^{s-\mathrm{bc}}$ ($\mathrm{EC}_i^{s-\mathrm{wc}}$) as the sum of energies $E_k^{s-\mathrm{bc}}$ ($E_k^{s-\mathrm{wc}}$)—given by (1) with $V_k^{s-\mathrm{bc}}$, M_k^{bc}, and $O_k^{s-\mathrm{bc}}$ ($V_k^{s-\mathrm{wc}}$, M_k^{wc}, and $O_k^{s-\mathrm{wc}}$)—and energy overheads $\mathcal{E}_k$ (line 8).

Having the endpoints $(t_i^{s-\mathrm{bc}}, \mathrm{EC}_i^{s-\mathrm{bc}})$ and $(t_i^{s-\mathrm{wc}}, \mathrm{EC}_i^{s-\mathrm{wc}})$ that define the diagonal D_i (as shown in Fig. 9), for every task T_i, we take N_i equally-spaced points (t_i^j, EC_i^j) along D_i and, for each such point, we compute the respective assignment V_{i+1}^j/O_{i+1}^j (we solve the dynamic V/O assignment problem as formulated by (9)–(12), assuming that the total energy consumed up to the completion of T_i is EC_i^j and the completion time of T_i is t_i^j) and store it accordingly in the jth position in the particular lookup table LUT_i (lines 10–12).

Algorithm 1: OffLinePhase

input: The maximum number $N^{\max}$ of assignments
output: The set of V/O assignments
1: distribute $N^{\max}$ among tasks (T_i gets N_i points)
2: $\mathrm{V/O}^{s-\mathrm{bc}}$:= sol. by dyn. scheduler when $M_k = M_k^{\mathrm{bc}}$, $1 \leq k \leq n$
3: $\mathrm{V/O}^{s-\mathrm{wc}}$:= sol. by dyn. scheduler when $M_k = M_k^{\mathrm{wc}}$, $1 \leq k \leq n$
4: $V_1 := V_1^{s-\mathrm{bc}} = V_1^{s-\mathrm{wc}}$; $O_1 := O_1^{s-\mathrm{bc}} = O_1^{s-\mathrm{wc}}$
5: store V_1/O_1 in $\mathrm{LUT}_1[1]$
6: **for** $i \leftarrow 1, 2, \ldots, n-1$ **do**
7: $t_i^{s-\mathrm{bc}} := \sum_{k=1}^{i}(\tau_k^{s-\mathrm{bc}} + \delta_k)$; $t_i^{s-\mathrm{wc}} := \sum_{k=1}^{i}(\tau_k^{s-\mathrm{wc}} + \delta_k)$
8: $\mathrm{EC}_i^{s-\mathrm{bc}} := \sum_{k=1}^{i}(E_k^{s-\mathrm{bc}} + \mathcal{E}_k)$; $\mathrm{EC}_i^{s-\mathrm{wc}} := \sum_{k=1}^{i}(E_k^{s-\mathrm{wc}} + \mathcal{E}_k)$
9: **for** $j \leftarrow 1, 2, \ldots, N_i$
10: $t_i^j := [(N_i - j)t_i^{s-\mathrm{bc}} + j\, t_i^{s-\mathrm{wc}}]/N_i$
11: $\mathrm{EC}_i^j := [(N_i - j)\mathrm{EC}_i^{s-\mathrm{bc}} + j\mathrm{EC}_i^{s-\mathrm{wc}}]/N_i$
12: compute V_{i+1}^j/O_{i+1}^j for (t_i^j, EC_i^j) and store it in $\mathrm{LUT}_i[j]$
13: **end for**
14: **end for**

The set of V/O assignments, prepared offline, is used online by the quasi-static V/O scheduler as outlined by Algorithm 2. Note that the energy EC_i consumed up to the completion of task T_i can be calculated based on the energy EC_{i-1} consumed up to the previous task and the actual execution time τ_i as given by line 1. Upon completing task T_i, the lookup table LUT_i is consulted. If the point (t, EC) lies above the diagonal D_i (line 2) the index j of the table entry is simply calculated as in line 3, else as in line 5. Computing directly the index j, instead of searching through the table LUT_i, is possible because the points (t_i^j, EC_i^j) stored in LUT_i are equally-spaced. Finally, the V/O assignment stored in $\mathrm{LUT}_i[j]$ is retrieved (line 7). Observe that Algorithm 2 has a time complexity $\mathcal{O}(1)$ and, therefore, the online operation performed by the quasi-static V/O scheduler takes constant time and energy. Also, this lookup and selection process is several orders of magnitude cheaper than the online computation by the dynamic V/O scheduler.

Algorithm 2: OnLinePhase

input: Actual completion time $t = t_i$ of T_i, and lookup table LUT_i (contains N_i assignments and the diagonal D_i—defined as $\mathrm{EC}_i = A_i t_i + B_i$)
output: The assignment V_{i+1}/O_{i+1} for the next task T_{i+1}
1: $EC = \mathrm{EC}_i = \mathrm{EC}_{i-1} + \mathcal{E}_{i-1,i}^{\Delta V} + \mathcal{E}_i^{\mathrm{sel}} + C_i V_i \frac{(V_i - V_{th})^{\alpha}}{k}\tau_i$
2: **if** $EC > A_i t + B_i$ **then**
3: $j := \lceil N_i(\mathrm{EC} - \mathrm{EC}_i^{s-\mathrm{bc}})/(\mathrm{EC}_i^{s-\mathrm{wc}} - \mathrm{EC}_i^{s-\mathrm{bc}})\rceil$
4: **else**
5: $j := \lceil N_i(t - t_i^{s-\mathrm{bc}})/(t_i^{s-\mathrm{wc}} - t_i^{s-\mathrm{bc}})\rceil$
6: **end if**
7: return V/O assignment stored in $\mathrm{LUT}_i[j]$

VI. Experimental Evaluation

In order to evaluate the presented approach, we performed a large number of experiments using numerous synthetic benchmarks. We generated task graphs containing between 10 and 100 tasks. Each point in the plots of the experimental results (Figs. 10–12) corresponds to 50 automatically-generated task graphs, resulting overall in more than 4000 performed evaluations. The technology-dependent parameters were adopted from [16] and correspond to a processor in a 0.18-μm CMOS fabrication process. The reward functions we used along the experiments are of the form $R_i(O_i) = \alpha_i O_i + \beta_i\sqrt{O_i} + \gamma_i\sqrt{[3]O_i}$, with coefficients α_i, β_i, and γ_i randomly chosen. The experiments in this section were performed using uniform probability distributions for the number of mandatory cycles.

The first set of experiments was performed with the goal of investigating the reward gain achieved by our quasi-static approach compared to the optimal static solution (the approach proposed in [20]). In these experiments we consider that the time and energy overheads needed for selecting the assignments by the quasi-static V/O scheduler are $\delta^{\mathrm{sel}} = 450$ ns and $\mathcal{E}^{\mathrm{sel}} = 400$ nJ. These are realistic values as selecting a precomputed assignment takes only tens of cycles and the access time and energy consumption (per access) of, for example, a low-power Static RAM are around 70 ns and 20 nJ, respectively, [17].[4] Fig. 10(a) shows the reward (normalized with respect to the

[4]Besides, in order to get a realistic estimation of the number of clock cycles needed for executing Algorithm 2, we used the MPARM simulation environment [4]: we found out that it requires about 200 clock cycles in a processor with floating-point unit, which in the case of the Crusoe 5600 processor at 600 MHz means less than 350 ns.

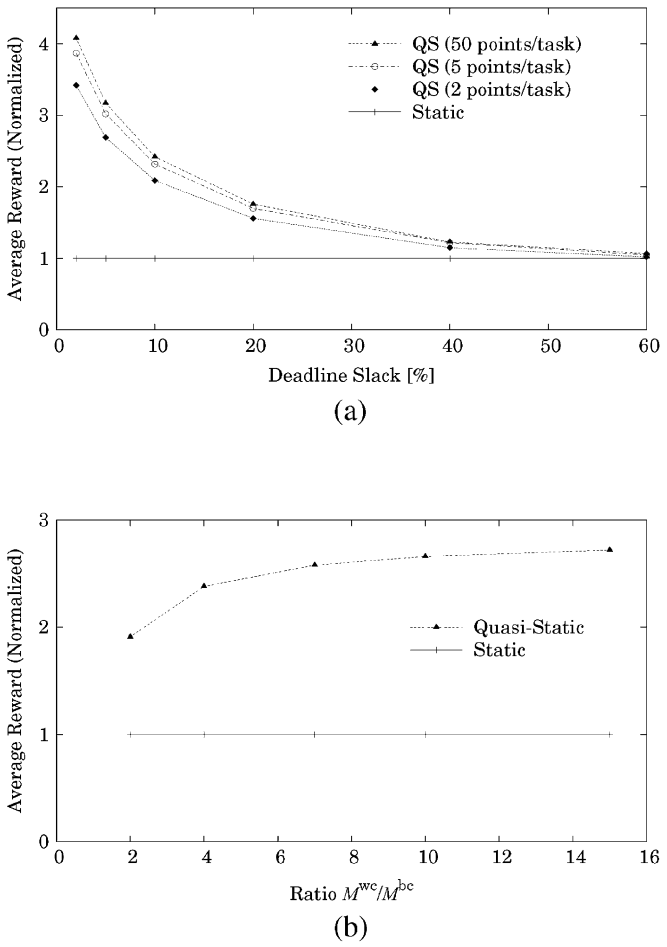


Fig. 10. Comparison of the quasi-static and static solutions. (a) Influence of the deadline slack. (b) Influence of the ratio $M^{\mathrm{wc}}/M^{\mathrm{bc}}$.

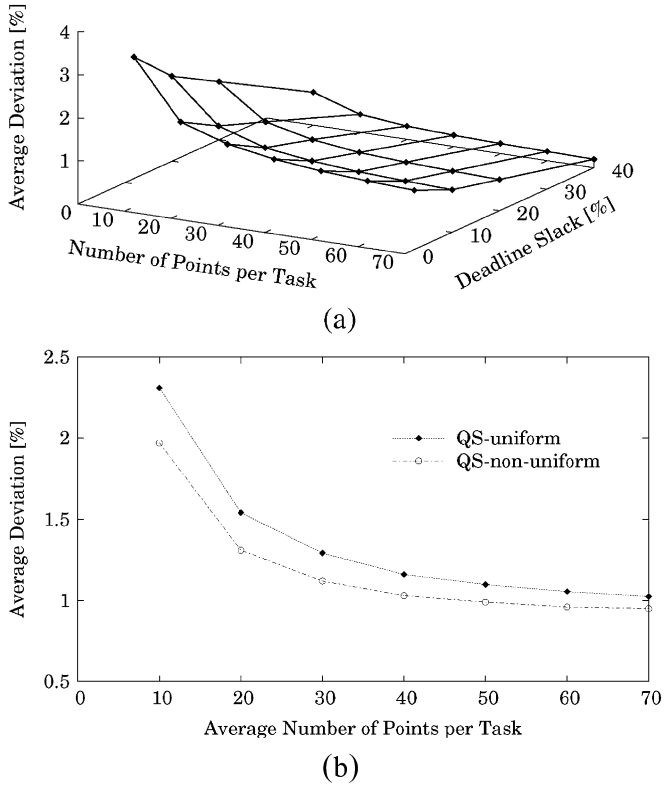


Fig. 11. Comparison of the quasi-static and ideal dynamic solutions. (a) Influence of the deadline slack and number of points. (b) Influence of the distribution of points among lookup tables.

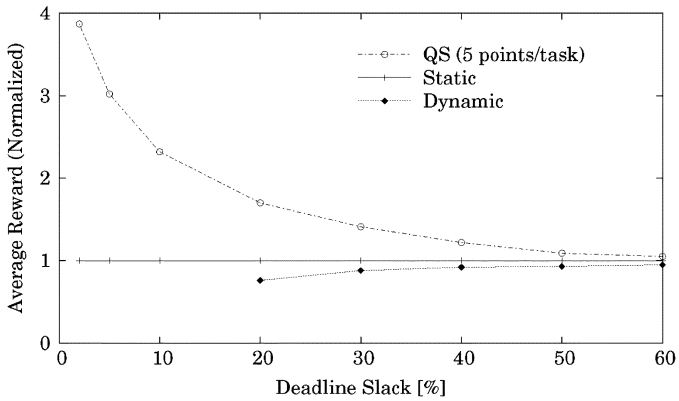


Fig. 12. Comparison considering realistic overheads.

reward given by the static solution) as a function of the deadline slack (the relative difference between the deadline and the completion time when worst case number of mandatory cycles are executed at the maximum voltage that guarantees the energy constraint). Three cases for the quasi-static approach (2, 5, and 50 points per task) are considered in this figure. Very significant gains in terms of total reward, up to four times, can be obtained with the quasi-static solution, even with few points per task. The highest reward gains are achieved when the system has very tight deadlines (small deadline slack). This is so because, when large amounts of slack are available, the static solution can accommodate most of the optional cycles (recall there is a value $O_i^{\max}$ after which no extra reward is achieved) and, therefore, the difference in reward between the static and quasi-static solutions is not big in these cases.

The influence of the ratio between the worst case number of cycles M^{wc} and the best case number of cycles M^{bc} has also been studied and the results are presented in Fig. 10(b). In this case, we have considered systems with a deadline slack of 10% and 20 points per task in the quasi-static solution. The larger the ratio $M^{\mathrm{wc}}/M^{\mathrm{bc}}$ is, the more the actual number of execution cycles deviate from the worst case value M^{wc} (which is the value that has to be considered by a static solution). Thus, the dynamic slack becomes larger and, therefore, there are more chances to exploit such a slack and, consequently, improve the reward.

The second set of experiments was aimed at evaluating the quality of our quasi-static approach with respect to the theoretical limit that could be achieved without knowing in advance the exact number of execution cycles (the reward delivered by the ideal dynamic V/O scheduler, as discussed in Section IV, in which the dynamic V/O assignment problem formulated by (9)–(12) is solved assuming zero overheads δ_i^{dyn} and $\mathcal{E}_i^{\mathrm{dyn}}$). For the sake of comparison fairness, since the reward produced by an ideal dynamic V/O scheduler with no overheads is taken as reference point, in this set of experiments, we have also considered zero time and energy overheads δ^{sel} and $\mathcal{E}^{\mathrm{sel}}$ for the proposed quasi-static approach (as opposed to the previous experiments).

Fig. 11(a) shows the deviation $\mathrm{dev} = (R^{\mathrm{ideal}} - R^{\mathrm{qs}})/R^{\mathrm{ideal}}$ as a function of the number of precomputed assignments (points per task) and for various degrees of deadline tightness. The ordinate in Fig. 11(a), as well as in Fig. 11(b), shows the average deviation of the reward given by a quasi-static approach in relation to the reward produced by a dynamic V/O scheduler in the ideal case of zero time and energy overheads. More points per task produce higher reward, closer to the theoretical limit (smaller deviation). Nonetheless, with relatively few points per task, we can still get very close to the theoretical limit, for instance, in systems with deadline slack of 20% and for 30 points per task the average deviation is around 1.3%. As mentioned previously, when the deadline slack is large even a static solution (which corresponds to a quasi-static solution with just one point per task) can accommodate most of the optional cycles. Hence, the deviation gets smaller as the deadline slack increases, as shown in Fig. 11(a).

In the previous experiments, it has been considered that, for a given system, the lookup tables have the same size, that is, contain the same number of assignments. When the number $N^{\max}$ of assignments is distributed among tasks according to the size of their time-energy spaces (more assignments in the lookup tables of tasks that have larger spaces), better results are obtained, as shown in Fig. 11(b). This figure plots the case of equal-size lookup tables (QS-uniform) and the case of assignments distributed nonuniformly among tables (QS-nonuniform), as described above, for systems with a deadline slack of 20%. The abscissa is the average number of points per task.

In a third set of experiments, we took into account the online overheads of the dynamic V/O scheduler (as well as the quasi-static one) and compared the static, quasi-static, and dynamic approaches in the same graph. Fig. 12 shows the total reward normalized with respect to the one produced by the static solution. It shows that, in a realistic setting, the dynamic approach performs poorly, even worse than the static one. More importantly, for systems with tight deadlines (small deadline slack), the dynamic approach cannot guarantee the time and energy constraints because of its large overheads (this is why no data is plotted for benchmarks with deadline slack less than 20%). In this set of experiments, the overhead values that have been considered for the dynamic case correspond actually to overheads by heuristics [20] and not the overheads incurred by exact methods, although in these experiments the reward values produced by the optimal solutions were considered. This means that, even in the optimistic case of an online algorithm that delivers exact solutions in a time frame similar to one of the existing heuristic methods (which naturally produce values of less quality than the exact ones), the quasi-static approach outperforms the dynamic one.

We have also measured the execution time of Algorithm 1, used for computing at design-time the set of V/O assignments. Fig. 13 shows the average execution time as a function of the number of tasks in the system, for different values of $N^{\max}$ (total number of assignments). It can be observed that the execution time is linear in the number of tasks and in the total number of assignments. The time needed for computing the set of assignments, though considerable, is affordable since Algorithm 1 is run offline.

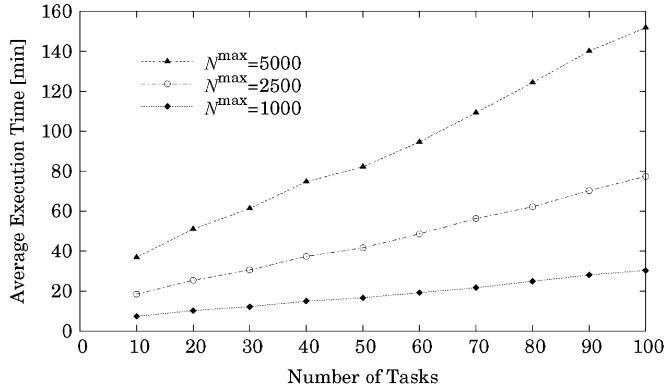


Fig. 13. Execution time of Algorithm 1.

In addition to the synthetic benchmarks previously discussed, we have also evaluated our approach by means of a real-life application, namely the navigation controller of an autonomous rover for exploring a remote place [11]. The rover is equipped, among others, with two cameras and a topographic map of the terrain. Based on the images captured by the cameras and the map, the rover must travel toward its destination avoiding nearby obstacles. This application includes a number of tasks described briefly as follows. A frame acquisition task captures images from the cameras. A position estimation task correlates the data from the captured images with the one from the topographic map in order to estimate the rover's current position. Using the estimated position and the topographic map, a global path planning task computes the path to the desired destination. Since there might be impassable obstacles along the global path, there is an object detection task for finding obstacles in the path of the rover and a local path planning task for adjusting accordingly the course in order to avoid those obstacles. A collision avoidance task checks the produced path to prevent the rover from damaging itself. Finally, a steering control task commands the motors, the direction, and speed of the rover.

For this application, the total reward is measured in terms of how fast the rover reaches its destination [11]. Rewards produced by the different tasks (all but the steering control task which has no optional part) contribute to the overall reward. For example, higher-resolution images by the frame acquisition task translates into a clearer characterization of the surroundings of the rover. This, in turn, implies a more accurate estimation of the location and, consequently, makes the rover get faster to its destination (that is, higher total reward). Similarly, running the global path planning task longer results in a better path which, again, implies reaching the desired destination faster. The other tasks make, in a similar manner, their individual contribution to the global reward, in such a way that the amount of computation allotted to each of them has a direct impact on how fast the destination is reached.

The navigation controller is activated periodically every 360 ms and tasks have a deadline equal to the period.[5] The energy budget per activation of the controller is 360 mJ (average power consumption 1 W) during the night and 540 mJ (average power 1.5 W) during the daytime [10].

[5]At its maximum speed of 10 km/h the rover travels in 360 ms a distance of 1 m, which is the maximum allowed without recomputing the path.

We use two memories, one for the assignments used during daytime and the other for the set used during the night (these two sets are different because the energy budget differs), and assume that $N^{\max} = 512$ assignments can be stored in each memory. We computed, for both cases, $E^{\max} = 360$ mJ and $E^{\max} = 540$ mJ, the sets of assignments using Algorithm 1. When compared to the respective static solutions, our quasi-static solution delivers rewards that are in average 3.8 times larger for the night case and 1.6 times larger for the day case. This means that a rover using the precomputed assignments can reach its destination faster than in the case of a static solution and, thus, explore a larger region under the same energy budget.

The significant difference between the night and day modes can be explained by the fact that, for more stringent energy constraints, fewer optional cycles can be accommodated by a static solution and, therefore, its reward is smaller. Thus, the relative difference between a quasi-static solution and the corresponding static one is significantly larger for systems with more stringent energy constraints.

VII. Conclusion

We have addressed the problem of maximizing rewards for realtime systems with energy constraints, in the frame of the imprecise computation model. We have proposed a quasi-static approach, whose chief merit is the ability to exploit the dynamic slack at very low online overhead. This is possible because, in our quasi-static approach, a set of solutions are prepared and stored at design-time, leaving for runtime only the selection of one of them.

The number of assignments that can be stored is limited by the resources of the target system. Therefore, a careful selection of assignments is crucial because it has a large impact on the quality of the solution.

We considered that the voltage can continuously be varied. If only discrete voltages are supported, the approach can easily be adapted by using well-known techniques for obtaining the discrete voltage levels that replace the calculated ideal one [19].

We evaluated our approach through numerous synthetic benchmarks and a real-life application. We found that significant gains, up to four times, in terms of reward can be obtained by the quasi-static approach. We showed also that, due to its large online overheads, a dynamic approach performs poorly.

The dynamic slack can efficiently be exploited only if adversely high overheads are avoided, as done by our approach. The methods proposed in this paper succeed in exploiting the dynamic slack, yet having small online overhead, because the complex time- and energy-consuming parts of the computations are performed offline, at design-time, leaving for runtime only simple lookup and selection operations.

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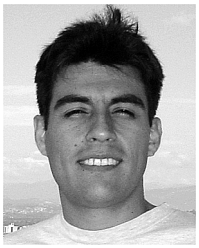

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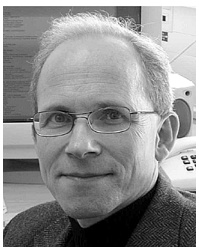

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