

# Quaternion Image Watermarking using the Spatio-Chromatic Fourier Coefficients Analysis

Tsz Kin Tsui  
Ryerson University  
350 Victoria Street  
Toronto, Canada  
ttsui@ee.ryerson.ca

Xiao-Ping Zhang  
Ryerson University  
350 Victoria Street  
Toronto, Canada  
xzhang@ee.ryerson.ca

Dimitri Androutsos  
Ryerson University  
350 Victoria Street  
Toronto, Canada  
dimitri@ee.ryerson.ca

## ABSTRACT

In this paper, a new color watermarking algorithm that uses the quaternion Fourier transform (QFT) to mark the  $La^*b^*$  components of color images is presented. First, we propose an interpretation of the QFT coefficients using the Spatio-Chromatic Fourier analysis, so the effects of any changes to the coefficients can be predicted. Next, watermark casting is performed by modifying the positive and negative coefficients together. The idea is twofold: Robustness is achieved by embedding a color watermark in the coefficient with positive frequency, which spreads it to all components in the spatial domain. On the other hand, invisibility is satisfied by modifying the coefficient with negative frequency, such that the combined effects of the two are insensitive to human eyes.

**Categories and Subject Descriptors:** I.4.9 [Image Processing and Computer Vision]: Applications

**General Terms:** Algorithms, Security

**Keywords:** Color image watermarking, Quaternion Fourier Transform, Spatio-Chromatic Image Processing

## 1. INTRODUCTION

Digital Watermarking is a method to embed secret information in multimedia data, such as videos and images. There are two main classes of watermarking algorithms: spatial domain-based and transform domain-based techniques. Recent researches have shown that transform domain-based techniques are more robust against common digital signal processing attacks such as histogram equalization, sharpening, blurring etc. Examples of watermarking techniques in transform domain can be found in [1]. However, most of the existing algorithms only mark the channels of color images separately, which does not consider the correlation of the image components. To overcome this problem, researchers have proposed models to process the color channels intrinsically. For example, Reed *et al.* proposed a system that takes advantage of the low sensitivity of the human visual system to high frequency changes along the yellow-blue axis, to place most of the watermark in the yellow component of the image [3]. Bas *et al.* proposed a digital color image watermarking scheme using the hypercomplex numbers representation and the Quaternion Fourier Transform (QFT) [4]. Tsui [9] also proposed a color watermarking scheme us-

ing the Spatio-Chromatic Fourier Transform (SCDFT). It first encodes only the color information of an image (for example,  $a^*$  and  $b^*$  channels in the  $La^*b^*$  color space) to complex numbers  $a + jb$ , and transform it via one complex frequency transform. The advantage is that it embeds a watermark into the chromatic components, and maximizes the strength by incorporating some characteristics of human visual system (HVS). However, it ignores the luminance component. Attacks such as color to greyscale conversion would destroy the watermark. This paper addresses this issue by introducing a new color watermarking algorithm using the QFT.

This paper is organized as follows: the theory of the hypercomplex representation and the QFT is first presented, followed by an interpretation of the QFT spectral coefficients. Finally, a new color image watermarking scheme using the QFT is presented and evaluated.

## 2. QUATERNION FOURIER TRANSFORM

Sangwine and Ell [2] defined a new Fourier Transform, called the Quaternion Fourier Transform (QFT) to transform a quaternion image  $F(m, n)$  to the frequency domain. It is defined as:

$$F(u, v) = \frac{1}{MN} \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} e^{-i2\pi(\frac{mu}{M})} F(m, n) e^{-j2\pi(\frac{nv}{N})}, \quad (1)$$

and the inverse QFT is formulated as:

$$F(m, n) = \frac{1}{MN} \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} e^{i2\pi(\frac{mu}{M})} F(u, v) e^{j2\pi(\frac{nv}{N})}, \quad (2)$$

where  $M$  and  $N$  are the size of the image.

## 3. SPECTRUM INTERPRETATION

The first question in mind is: How does embedding watermarks in the QFT coefficients affect the image  $F(m, n)$  in the spatial domain? Before answering this question, we need a general framework to interpret the quaternion spectral points of color images. Unfortunately, this interpretation has not yet been developed. However in 1997, Dr. McCabe *et al* [5] proposed an interpretation for the Spatio-Chromatic Fourier Transform (SCDFT) coefficients using phasors. We will extend his idea, and apply it to do watermarking in the QFT domain.

### 3.1 Spatio-Chromatic Image Processing

Spatio-chromatic image processing was proposed by Dr. McCabe. The motivation was to develop a framework to process the chromatic contents of color images in the frequency domain. It first encodes only the color information of an image (for example,  $a^*$  and  $b^*$  channels in the  $La^*b^*$  color space) to complex numbers  $a + jb$ , and analyze it via one complex frequency transform - Spatio-chromatic Discrete Fourier Transform (SCDFT). His idea of *phasors* gives an interpretation that each pair of frequency values results in rainbow gratings of different frequency and orientation in the image, which are combined to form the color image [5]. McCabe also pointed out that it is possible to generate spatial frequencies with modulations operating in either the red-green or yellow-blue color opponency paths in the color space, if the frequency coefficients satisfy some conditions.

### 3.2 Interpretation of the QFT coefficients

Having introduced the concept of phasors, let's expand (2). Let  $\theta_1 = 2\pi(\frac{mu}{M})$ ,  $\theta_2 = 2\pi(\frac{nv}{N})$ ,  $F(u, v) = a_1 + b_1i + c_1j + d_1k$  and  $F(-u, -v) = a_2 + b_2i + c_2j + d_2k$ ,

$$e^{i\theta_1} F(u, v) e^{j\theta_2} = a_1 \cos \theta_1 \cos \theta_2 - c_1 \cos \theta_1 \sin \theta_2 - b_1 \sin \theta_1 \cos \theta_2 + d_1 \sin \theta_1 \sin \theta_2 + i(b_1 \cos \theta_1 \cos \theta_2 - d_1 \cos \theta_1 \sin \theta_2 + a_1 \sin \theta_1 \cos \theta_2 - c_1 \sin \theta_1 \sin \theta_2) + j(a_1 \cos \theta_1 \sin \theta_2 + c_1 \cos \theta_1 \cos \theta_2 - b_1 \sin \theta_1 \sin \theta_2 - d_1 \sin \theta_1 \cos \theta_2) + k(b_1 \cos \theta_1 \sin \theta_2 + d_1 \cos \theta_1 \cos \theta_2 + a_1 \sin \theta_1 \sin \theta_2 + c_1 \sin \theta_1 \cos \theta_2), \quad (3)$$

$$e^{-i\theta_1} F(-u, -v) e^{-j\theta_2} = a_2 \cos \theta_1 \cos \theta_2 + c_2 \cos \theta_1 \sin \theta_2 + b_2 \sin \theta_1 \cos \theta_2 + d_2 \sin \theta_1 \sin \theta_2 + i(b_2 \cos \theta_1 \cos \theta_2 + d_2 \cos \theta_1 \sin \theta_2 - a_2 \sin \theta_1 \cos \theta_2 - c_2 \sin \theta_1 \sin \theta_2) + j(-a_2 \cos \theta_1 \sin \theta_2 + c_2 \cos \theta_1 \cos \theta_2 - b_2 \sin \theta_1 \sin \theta_2 + d_2 \sin \theta_1 \cos \theta_2) + k(-b_2 \cos \theta_1 \sin \theta_2 + d_2 \cos \theta_1 \cos \theta_2 + a_2 \sin \theta_1 \sin \theta_2 - c_2 \sin \theta_1 \cos \theta_2), \quad (4)$$

We will demonstrate that considerable simplifications can be made to (3) and (4), and an interpretation of the QFT coefficients can be drawn if we assign the parameters  $a_1, a_2, b_1, b_2, \dots$  in certain ways. Let  $F(\pm u, \pm v)$  denotes  $F(u, v) = F(-u, -v)$ ,

**Case 1:**  $F(\pm u, \pm v) = dk$ , then the summation of (3), and (4) gives

$$2d \sin \theta_1 \sin \theta_2 + k2d \cos \theta_1 \cos \theta_2, \quad (5)$$

(5) shows that if  $F(\pm u, \pm v) = dk$ , then the combined effect of the positive and negative coefficients is the same as a cosine function with magnitude of  $2d$  varying along the  $k$  component (we ignore the real component of the image).

**Case 2:**  $F(\pm u, \pm v) = bi$ , then the summation of (3), and (4) gives

$$i2b \cos \theta_1 \cos \theta_2 - j2b \sin \theta_1 \sin \theta_2, \quad (6)$$

(6) shows that if  $F(\pm u, \pm v) = bi$ , then the combined effects of the positive and negative coefficients is the same as a circle with diameter  $2b$ , starting at angle 0, rotating in the negative direction on a complex plane.

**Case 3:**  $F(\pm u, \pm v) = cj$ , then the summation of (3), and (4) gives

$$-i2c \sin \theta_1 \sin \theta_2 + j2c \cos \theta_1 \cos \theta_2 \quad (7)$$

(7) shows that if  $F(\pm u, \pm v) = cj$ , then the combined effects of the positive and negative coefficients is the same as a circle with diameter  $2c$ , starting at angle  $\pi/2$ , rotating in the negative direction on a complex plane.

**Case 4:**  $F(\pm u, \pm v) = bi + cj + dk$  can be interpreted as the combinations of case 1, 2 and 3.

The next section will demonstrate how to apply the interpretation to do watermarking.

## 4. APPLYING THE INTERPRETATION TO COLOR IMAGE WATERMARKING

### 4.1 Color Spaces

The CIE  $La^*b^*$  color model proposed in 1976 is the chromaticity model chosen for this paper [7]. This color space provides two advantages:

First of all, it is a uniform color space used to measure the color difference  $\Delta E$  at each pixel, which is defined as:

$$\Delta E = \sqrt{\Delta L^2 + (\Delta a^*)^2 + (\Delta b^*)^2}. \quad (8)$$

Experiments have shown that  $\Delta E < 1$  is not detectable by humans, and  $\Delta E < 3$  is not apparent. In C.H. Chou's paper [6],  $\Delta E = 3$  is called *Uniform Just-Noticeable Color Difference (UJNCD)*, and he derived a JND model, which he called *Non-Uniform Just-Noticeable Color Difference (NUJNCD)* after considering some masking effects. This paper uses his model to bound the distortion generated by embedding the watermarks. Secondly, if we encode a color image as  $F(m, n) = a^*(m, n)i + b^*(m, n)j + L^*(m, n)k$ , using the interpretation from section 3,

**Case 1:**  $F(\pm u, \pm v) = dk$  can be interpreted as adding a cosine function in the luminance component of the image.

**Case 2 and 3:**  $F(\pm u, \pm v) = bi$  or  $F(\pm u, \pm v) = cj$  enables us to use the interpretations provided by McCabe, as in the SCDFT domain, to add a function, in the form of a rainbow grating in the chromatic components of the image.

**Case 4:**  $F(\pm u, \pm v) = bi + cj + dk$  can be interpreted as a cosine grating embedded in the luminance component and two rainbow gratings rotating with the same direction and different initial angles in the chrominance components.

### 4.2 Human Visual System

A good model of the HVS should be used to ensure watermark invisibility. The importance of the HVS cannot be overemphasized and much research exists on its characteristics and behavior [8]. This paper uses the following characteristic of the HVS to satisfy the invisibility requirement: 1) The eye is about a fifth less sensitive to middle-high frequencies in blue-yellow than luminance [10].

### 4.3 New Adaptive Color Watermarking Algorithm

In the context of digital image watermarking, our goal is to design a watermark  $W(u, v)$ , that is as strong as possible, such that the visible distortion to the image is perceptually minimal. Mathematically speaking, the watermarked image  $WF(m, n)$  can be formulated as:

$$\begin{aligned} WF(m, n) &= IQFT(F(u, v) + \alpha W(u, v)) \\ &= IQFT(F(u, v)) + IQFT(\alpha W(u, v)) \\ &= F(m, n) + W(m, n). \end{aligned} \quad (9)$$

Let  $W(\pm u, \pm v) = b_3i + c_3j + d_3k$ . To apply the characteristics of the HVS, some constraints are imposed to the values  $b_3$ ,  $c_3$ , and  $d_3$ . From (6) and (7), if  $b_3 = c_3$ , the summation of them becomes

$$\begin{aligned} &i2b_3(\cos \theta_1 \cos \theta_2 - \sin \theta_1 \sin \theta_2) + \\ &j2b_3(\cos \theta_1 \cos \theta_2 - \sin \theta_1 \sin \theta_2) = \\ &2b_3(\cos \theta_1 \cos \theta_2 - \sin \theta_1 \sin \theta_2)(i + j), \end{aligned} \quad (10)$$

as shown in Figure 1.

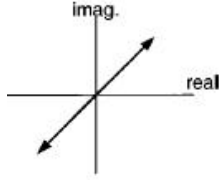


Figure 1:  $W(\pm u, \pm v) = b_3i + b_3j$ .

The conclusion is the watermark would vary along the 45 degree axis. To satisfy the requirement of varying along the yellow-blue axis, we transform the  $a^*b^*$  by 45 degrees before taking the QFT of the image  $F(m, n)$ .

Since the sensitivity of human eyes to yellow-blue component is approximately  $\frac{1}{5}$  to the luminance component. Thus, we set  $d_3 = \frac{1}{5}b_3$ . As a result,  $W(\pm u, \pm v) = b_3i + b_3j + \frac{1}{5}b_3k$ . To ensure invisibility of the watermark, we define a distortion metric  $D(n, m, u, v)$  that measures the difference in the spatial domain generated by adding the coefficients  $W(u, v)$  and  $W(-u, -v)$  in the frequency domain, defined as:

$$\begin{aligned} D(n, m, u, v) &= \frac{\alpha}{N^2}(i2b_3(\cos \theta_1 \cos \theta_2 - \sin \theta_1 \sin \theta_2) \\ &+ j2b_3 \cos \theta_1 \cos \theta_2 - \sin \theta_1 \sin \theta_2 + k\frac{2}{5}b_3 \cos \theta_1 \cos \theta_2) \\ &\leq UNJNCD \end{aligned} \quad (11)$$

if we embed the watermarks  $W(\pm u, \pm v) = b_3i + b_3j + \frac{1}{5}b_3k$  into the image. Watermark invisibility is achieved if  $\|D(n, m, u, v)\|^2 \leq (NUJNCD)^2 \forall n, m$ , that is,

$$\begin{aligned} \|D(n, m, u, v)\|^2 &= \frac{\alpha^2}{N^4}(8(b_3(\cos \theta_1 \cos \theta_2 - \sin \theta_1 \sin \theta_2))^2 \\ &+ \frac{4}{25}(b_3 \cos \theta_1 \cos \theta_2)^2) \leq (NUJNCD)^2. \end{aligned} \quad (12)$$

Depending on the perceptual requirement of the applications, we have two options to determine  $\alpha$ . One is to make the **average** distortions of a block less than  $NUJNCD$  (loose condition), and the other one is to make the distortion of **every** pixel less than  $NUJNCD$  (tight condition):

**Approach 1:** Calculate  $\alpha$  such that the average distortions of a block are less than  $NUJNCD$

$$\begin{aligned} \|D(u, v)\|^2 &= \\ \frac{\alpha^2}{N^4} \sum_{n=0}^{N-1} \sum_{m=0}^{N-1} &8(b_3(\cos \theta_1 \cos \theta_2 - \sin \theta_1 \sin \theta_2))^2 \\ &+ \frac{4}{25}(b_3 \cos \theta_1 \cos \theta_2)^2 = N^2(NUJNCD)^2. \end{aligned} \quad (13)$$

Let  $x = \cos \theta_1 \cos \theta_2 - \sin \theta_1 \sin \theta_2$  and  $y = \cos \theta_1 \cos \theta_2$ ,

$$\alpha = \sqrt{\frac{N^6(NUJNCD)^2}{\sum_{n=0}^{N-1} \sum_{m=0}^{N-1} 8(b_3x)^2 + \frac{4}{25}(b_3y)^2}}. \quad (14)$$

**Approach 2:** Recursively decrease  $\alpha$  until the distortions of all pixels are smaller than  $NUJNCD$ . We assign an initial  $\alpha$ , and calculate  $\|D(n, m, u, v)\|^2$  for every pixel of the block. If some pixel distortion exceeds  $(NUJNCD)^2$ , we decrease  $\alpha$  until  $\|D(n, m, u, v)\|^2 < (NUJNCD)^2 \forall n, m$ .

### 4.4 Embedding and Decoding Procedures

Given an image of size  $N \times N$ , we divide the image into blocks of size  $8 \times 8$ , followed by taking the QFT of each block. We embed a pair of watermarks  $W(u', v')$  and  $W(-u', -v')$  into the coefficients  $F(u', v')$  and  $F(-u', -v')$  in the following way:

$$\begin{aligned} WF(u', v') &= F(u', v') \pm \alpha W(u', v') \\ WF(-u', -v') &= F(-u', -v') \pm \alpha W(-u', -v'), \end{aligned} \quad (15)$$

where  $\alpha$  is the watermark strength to be maximized. To apply the characteristic of the HVS, we set  $W(u', v') = (0, b_3, b_3, \frac{1}{5}b_3)$  and  $W(-u', -v') = (0, b_3, b_3, \frac{1}{5}b_3)$  to generate watermarks that vary along the luminance and yellow-blue component. Notice that the real component is ignored when converting the watermarked image back to the  $La^*b^*$  color space, therefore, we need to keep the real component  $Re(m, n)$ , as a key when extracting the watermark.

To decode the embedded watermark  $W(u, v)$ , we need the original image  $F(m, n)$ , the watermarked (possibly corrupted) image  $WF(m, n)$ , and  $Re(m, n)$ . First, we encode the image to quaternion format, and replace the real component of  $WF(m, n)$  with  $Re(m, n)$ , followed by taking the QFT of both images. We then calculate  $F(u', v') + \alpha W(u', v')$  and  $F(u', v') - \alpha W(u', v')$  from the original image, followed by a distance comparison to the coefficient extracted from  $WF(u', v')$ . If the extracted coefficient is closer to  $F(u', v') + \alpha W(u', v')$ , we assume bit 1 was embedded, otherwise, bit 0 was embedded.

## 5. EXPERIMENTAL RESULTS

We embedded 1 bit of information into each block imperceptibly using the above watermarking technique on  $AC(2, 2)$  and  $AC(6, 6)$ . The Lenna image with size  $128 \times 120$  was chosen for testing. We embedded 240 bits of information into the blocks. Figure 2 is the original image, and Figure 3 is the watermarked images using the QFT approach. In order to test the robustness of the algorithm, we applied some common attacks, and compared the results to the DCT watermarking algorithm that only marks the blue component. The PSNRs and the number of bits embedded using the QFT, and DCT algorithms are almost the same. The following table shows the Bit Error Rates (BERs) against histogram equalization, sharpening, resizing, gaussian noise, and JPEG compression.

Attack	QFT	DCT
Histogram equalization	0%	0%
Sharpening	0%	7.08%
Resizing by factor 2	1.25%	5.83%
10% Gaussian noise	1.67%	5.42%
JPEG compression	2.08%	7.08%



Figure 2: Original Image.



Figure 3: Watermarked Image.

The experimental results demonstrate that the QFT approach is more robust against common attacks. The underlying reason is that the proposed algorithm is a true multidimensional scheme that embeds the watermark into all components of the image.

## 6. CONCLUSION

In this paper, a new watermarking technique for digital color images has been presented. The QFT transform is chosen because the color content of a color image is processed holistically in the frequency domain. We proposed a framework of interpreting the QFT coefficients, and use it for adding a watermark into the image. It is more robust against most common attacks because the watermark is embedded in the positive frequency, which spreads it to all components of the image. Perceptual effects are minimized by modifying the negative frequency.

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