

Queue Automata of Constant Length

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Finite State Automata with Memory

Starting from DFA and NFA we can define

1. Pushdown Automata (PDA) by adding a LIFO storage
2. Queue Automata (QA) by adding a FIFO storage

Note

These automata models are simply Turing machines with restrictions of the access of the working tape.

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Restricting the storage sizes to a constant h we get

1. constant height PDA (h -PDA)
2. constant length QA (h -QA)

Measuring Constant Memory Automata

Measure for Constant Memory Automata

Our measure for constant memory automata is an ordered triple consisting of

1. the number of states of the finite control,
2. the size of memory alphabet,
3. the memory limit.

This definition was already used for results on constant height pushdown automata. [Geffert, CM, BP 2010]

Conversion h -PDA \rightarrow FA

The following trade-offs were shown
[Bednárová, Geffert, CM, BP 2012]

- \rightarrow h -NPDA \rightarrow NFA: exponential
- \rightarrow h -NPDA \rightarrow DFA: double exponential

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- \rightarrow h -NPDA \rightarrow DFA: double exponential
- \rightarrow h -DPDA \rightarrow NFA: exponential
- \rightarrow h -DPDA \rightarrow DFA: exponential

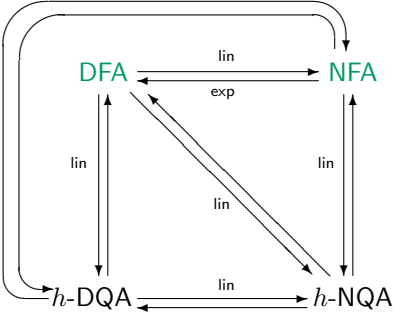
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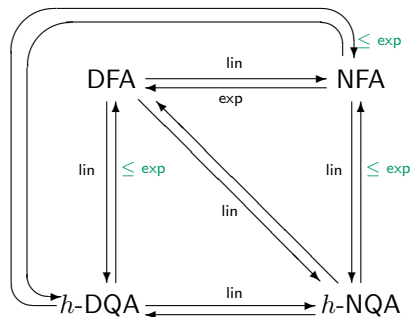
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What are the trade-offs for the conversions between **constant length queue automata** and finite automata?

Conversion $h\text{-QA} \rightarrow \text{FA}$



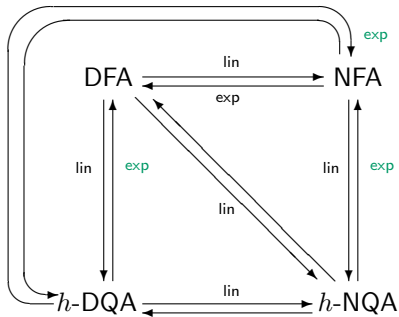
Conversion h -NQA \rightarrow NFA



Theorem

For each **constant length NQA** $A = \langle Q, \Sigma, \Gamma, \delta, q_0, \vdash, F, h \rangle$, there exists an equivalent **NFA** $A' = \langle Q', \Sigma, \delta', q'_0, F' \rangle$ with $|Q'| \leq |Q| \cdot |\Gamma^{\leq h}|$. Moreover, if A is a **DQA** then A' is a **DFA**.

Conversion h -NQA \rightarrow NFA



Optimality of the bound

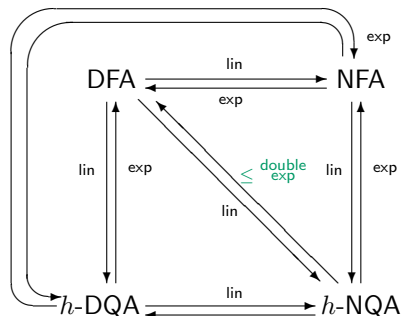
The language

$$D_{\Gamma, h} = \{w\#w : w \in \Gamma^{\leq h}\}$$

is accepted by

- a constant length DQA with 3 states, queue alphabet $\Gamma \cup \{\#, \vdash\}$, and constant length h ,
- an NFA with at least $|\Gamma^{\leq h}|$ states (fooling set).

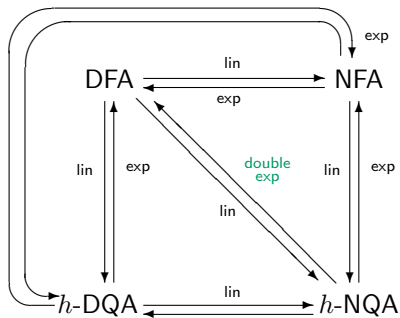
Conversion h -NQA \rightarrow DFA



Theorem

Any **constant length NQA** with state set Q , queue alphabet Γ , and queue length h can be converted into an equivalent **DFA** with $2^{|Q| \cdot |\Gamma|^{\leq h}}$ states.

Conversion h -NQA \rightarrow DFA

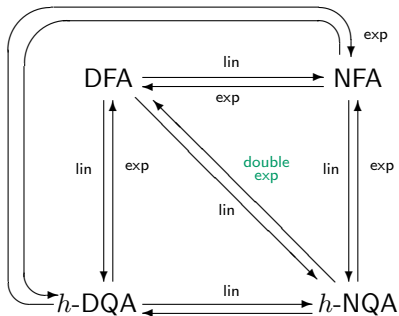


Optimality of the bound

The language

$$S_{\Gamma, h} = \{v_1 v_2 \dots v_r \# w_1 w_2 \dots w_t \mid v_i, w_j \in \Gamma^h, \exists i, j : v_i = w_j\}$$

Conversion h -NQA \rightarrow DFA

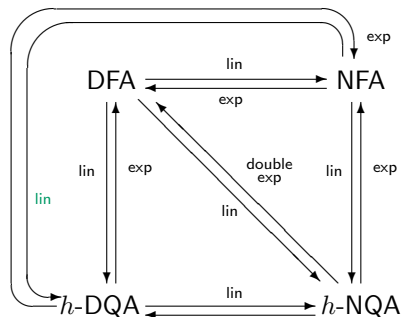


Optimality of the bound

is accepted by

- \rightarrow a constant length NQA with $O(h)$ states, queue alphabet $\Gamma \cup \{\vdash\}$, and constant length h ,
- \rightarrow a DFA with at least $2^{|\Gamma^h|}$ states.

Conversion NFA \rightarrow h -DQA



Theorem

For each NFA $A = \langle Q, \Sigma, \delta, q_1, F \rangle$ there exists an equivalent constant length DQA $A' = \langle Q', \Sigma, \Gamma, \delta', q_0, \vdash, F', h \rangle$ such that $|Q'| \in O(|Q| \cdot |\Sigma|)$, and $|\Gamma|, h \in O(|Q|)$.

Conversion NFA \rightarrow h -DQA

Key idea of the proof

- \rightarrow In its queue the constant length DQA stores the set of successor states the NFA A may be in on reading an input symbol.
- \rightarrow Let $\delta(s_1, a) = \{r_1, \dots, r_k\}$ be a transition of A and the queue content of A' be $s_1 s_2 \dots s_l \# t_1 \dots t_m \vdash$ the DQA converts its queue content in the following way by using λ -moves:

$$\begin{array}{l} s_1 s_2 \dots s_l \# t_1 \dots t_m \vdash \\ \xrightarrow{a} s_2 \dots s_l \# t_1 \dots t_m \vdash \end{array}$$

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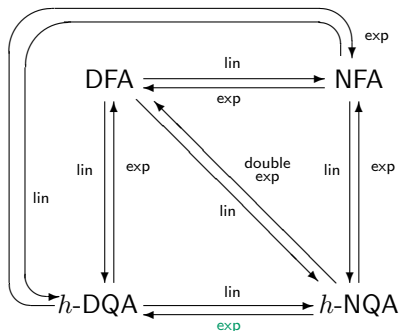
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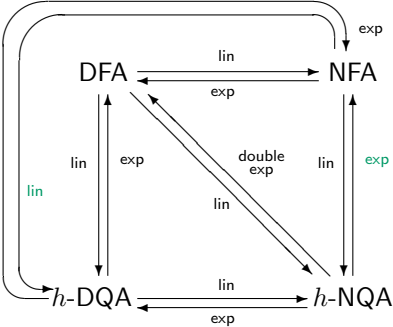
Determinization of h -NQA



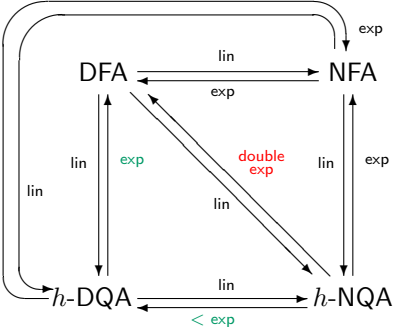
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Determinization of h -NQA

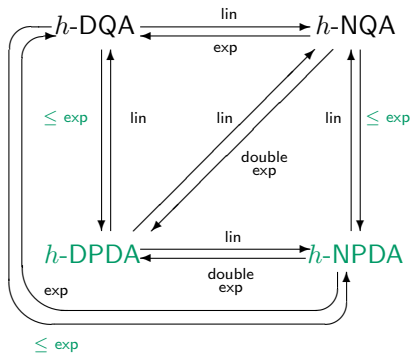


Determinization of h -NQA



Conversions between h -PDA and h -QA

Conversions between h -PDA and h -QA



Straight Line Programs for Regular Expressions

Straight line programs (SLP) are used for the representation of regular expressions. Given an alphabet Σ and a set of variables $X = \{x_1, \dots, x_l\}$, an SLP is defined to be a finite sequence of instructions of the form

→ $x_i := \emptyset$, $x_i := \lambda$, or $x_i := a$ for any $a \in \Sigma$

→ $x_i := x_j + x_k$, $x_i := x_j \cdot x_k$, or $x_i := x_j^*$ for $1 \leq j, k < i$

An SLP is always loopless. The variable x_l contains the regular expression.

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An SLP is always loopless. The variable x_l contains the regular expression.

Measure for an SLP P : $(\text{length}(P), \text{fan-out}(P))$

- $\text{length}(P)$: number of instructions in P
- $\text{fan-out}(P)$: maximum of occurrences of a reused variable

Conversion h -NPDA \rightarrow SLP

Theorem [Geffert, CM, BP 2010]

Let $A = \langle Q, \Sigma, \Gamma, \delta, q_0, \{q_f\}, \vdash, h \rangle$ be a **constant height NPDA**. Then there exists an **SLP** P_A such that $reg-exp(P)$ denotes $L(A)$, with $length(P_A) \leq O(h \cdot |Q|^4 \cdot |\Gamma| + |Q|^2 \cdot |\Sigma|)$ and $fan-out(P_A) \leq |Q|^2 + 1$.

Conversion h -NQA \rightarrow SLP

Theorem

For each constant length NQA $A = \langle Q, \Sigma, \Gamma, \delta, q_0, \vdash, F, h \rangle$, there is an equivalent SLP of length $O(|Q|^4 \cdot |\Gamma^{\leq h}|^4 \cdot |\Sigma|)$ and fan-out $O(|Q|^2 \cdot |\Gamma^{\leq h}|^2)$.

Conversion h -NQA \rightarrow SLP

The optimality of this bound is proved by the language

$$L_{\Gamma,h} = \bigcup_{u \in \Gamma^h} \{(\#u)^i \$: i \geq 1\}$$

Conversion h -NQA \rightarrow SLP

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$L_{\Gamma,h}$ is accepted by

- \rightarrow a constant length DQA with $O(h)$ states, queue alphabet $\Gamma \cup \{\#, \$\}$, and queue length $h + 1$.
- \rightarrow an SLP with at least $|\Gamma^h|$ variables.

Conversion h -NQA \rightarrow SLP

$$L_{\Gamma,h} = \bigcup_{u \in \Gamma^h} \{(\#u)^i\$: i \geq 1\}$$

- \rightarrow x is called a **star-variable** if it occurs in a **star-instruction** $x := y^*$. $L(x)$ denotes the language represented by the regular expression computed in x .

Conversion h -NQA \rightarrow SLP

$$L_{\Gamma,h} = \bigcup_{u \in \Gamma^h} \{(\#u)^i\$: i \geq 1\}$$

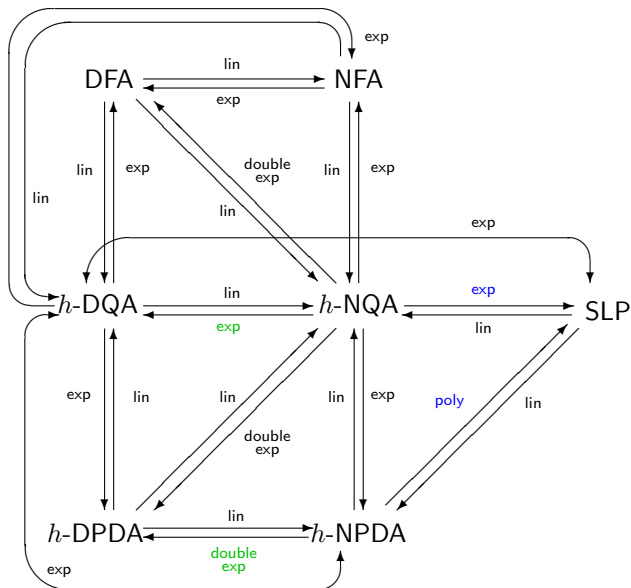
- \rightarrow Let the SLP P compute a regular expression describing $L_{\Gamma,h}$.
- \rightarrow The SLP P' is obtained from P by replacing every instruction $x := y^*$ by $x := \lambda$.
- \rightarrow P' describes a finite language. Let m denote the length of its longest word.
- \rightarrow For all words $z \in L_{\Gamma,h}$ with $|z| > m$, P must use a star-variable to produce it.

Conversion h -NQA \rightarrow SLP

$$L_{\Gamma,h} = \bigcup_{u \in \Gamma^h} \{(\#u)^i\$: i \geq 1\}$$

- \rightarrow Choosing $z_u = (\#u)^m\$ \in L_{\Gamma,h}$, $u \in \Gamma^h$, we know that there exists a star-variable x_u in P that produces one part of z_u .
- \rightarrow We can show that there exists a star-variable x_u such that $L(x_u)$ contains a word having the factor $\#u\#$.
- \rightarrow If P has less than Γ^h variables, there exist $u, v \in \Gamma^h$, $u \neq v$, such that $x_u = x_v$.
- \rightarrow Then P describes words of the form $\alpha\#u\#\beta\#v\#\gamma \notin L_{\Gamma,h}$.

Summary of the Results



Thank you for your attention