Queues with Redundancy: Latency-Cost Analysis

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1. INTRODUCTION

A major advantage of cloud computing and storage is the large-scale sharing of resources, which provides scalability and flexibility. A side-effect of resource-sharing is the variability in the latency experienced by the user due to various factors for e.g., virtualization, server outages, and network congestion. The problem is aggravated when a job consists of several parallel tasks, because the task run on the slowest machine becomes the latency bottleneck.

A promising method to reduce latency is to assign a task to multiple machines and wait for one to finish. Similarly, in cloud storage systems requests to download the content can be assigned to multiple replicas, such that it is only sufficient to download one replica. Although studied actively in systems in the past few years, there is little work on rigorous analysis of how redundancy affects latency. The effect of redundancy in queueing systems was first analyzed only recently in [2–4, 6], assuming exponential service time. Other service distributions, in particular the effect of the tail in systems with redundancy is considered in [7, 8].

This work analyzes the trade-off between latency and the cost of computing resources in queues with redundancy, without assuming exponential service time. We study a generalized fork-join queueing model where finishing any k out of n tasks is sufficient to complete a job. The redundant tasks can be canceled when any k tasks finish, or earlier, when any k tasks start service. For the $k = 1$ case, we get an elegant latency and cost analysis by identifying equivalences between systems with the normal and early cancellation policies and the $M/G/1$ and $M/G/n$ queues respectively. For general k we derive bounds on the latency and cost.

2. PROBLEM SETUP

Consider a distributed system with n statistically identical servers. Jobs arrive according to a rate λ Poisson process. The scheduler forks each incoming job into n tasks, and assigns them respectively to first-come first-serve queues at the n servers. The n tasks are designed such that completion of any k tasks is sufficient to complete the job. The case $k = 1$ corresponds to running replicas of a job on multiple machines. General k arise in approximate computing, or in content download from coded distributed storage.

The time taken to serve a task, is modeled by the random variable X, with distribution F_X, and is assumed to be i.i.d. across requests and servers. Dependence across servers due to the job size can be modeled by adding a constant proportional to average job size to service time X.

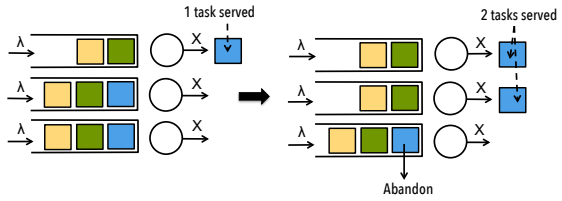

Fig. 1: The $(3, 2)$ fork-join system. When any 2 tasks of a job finish, the third task abandons its queue.

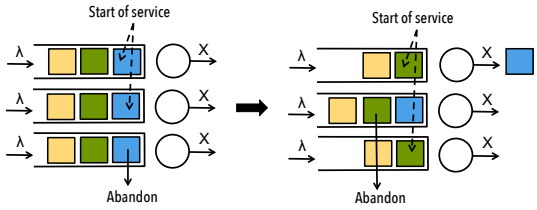

Fig. 2: The $(3, 2)$ fork-early cancel system. When any 2 tasks of a job start service, the third task abandons its queue. The job is complete when the 2 tasks finish.

When any k out of the n tasks of a job are served, the scheduler immediately cancels the remaining $n-k$ redundant tasks, as illustrated in Fig. 1. We refer to this system as the (n, k) fork-join system, defined formally as follows.

DEFINITION 1 ((n, k) FORK-JOIN SYSTEM). *An incoming job is forked into n tasks that join queues at each of the n servers. When any k tasks finish service, all remaining tasks are canceled and abandon their queues immediately.* [1]

Instead of waiting for k tasks to finish, we could cancel the redundant tasks as soon as k tasks start service. This variant, called the (n, k) fork-early cancel system is formally defined as follows.

DEFINITION 2 ((n, k) FORK-EARLY CANCEL SYSTEM). *An incoming job is forked into n tasks that join queues at the n servers. When any k tasks start service, we cancel the redundant tasks immediately. If more than k tasks start service simultaneously, we retain any k chosen uniformly at random. The job is complete when these k tasks finish.*

Fig. 1 and Fig. 2 illustrate the (n, k) fork-join and fork-early cancel systems respectively for $n = 3$ and $k = 2$. Early

[1] The (n, k) fork-join system is a generalization of the well known fork-join queue, which corresponds to the $k = n$ case.

Table 1: Summary of Results on Latency-Cost Analysis. We get exact analysis for $k = 1$, and bounds for general k

	Replicated System ($k = 1$)		General k	
	Normal Join	**Early Cancel**	**Normal Join**	**Early Cancel**
Latency $\mathbb{E}[T]$	Thm. 1, using $\equiv$ to $M/G/1$ queue	Thm. 2, using $\equiv$ to $M/G/n$ queue	Bounds in Thm. 3	Upper Bound (generalizing [5])
Cost $\mathbb{E}[C]$	$n\mathbb{E}[X_{1:n}]$	$\mathbb{E}[X]$	Bounds in Thm. 4 (Tight for $k = 1, n$)	$k\mathbb{E}[X]$

cancellation of redundant tasks can save computing cost, but could result in higher latency because of loss of diversity. In this work we develop insights into when early cancellation is better than normal join.

Using more redundant tasks (larger n) reduces latency, but generally results in additional cost of computing resources. We now define the latency and computing cost metrics, and analyse their trade-off afterwards.

DEFINITION 3 (LATENCY). *The latency $\mathbb{E}[T]$ is defined as the expected time from when a job arrives until when k of its tasks are complete.*

DEFINITION 4 (COMPUTING COST). *The computing cost $\mathbb{E}[C]$ is the expected total time spent serving the tasks of a job, not including the waiting time in queue.*

We now express the service capacity of the system in terms of $\mathbb{E}[C]$, the average total server time utilized per job.

CLAIM 1 (SERVICE CAPACITY IN TERMS OF $\mathbb{E}[C]$). *For an (n, k) system with normal join or early cancellation of redundancy, the maximum λ for which $\mathbb{E}[T] < \infty$ is*

$$\lambda_{max} = \frac{n}{\mathbb{E}[C]}. \tag{1}$$

Thus $\mathbb{E}[C]$ can be used to compare systems in the high λ regime. We will illustrate this new technique in Fig. 4, comparing the normal join and early cancellation policies.

Table 1 summarizes the key results of the latency-cost analysis presented in Sections 3 and 4 below. We use the notation $X_{i:n}$ to denote the i^{th} smallest of i.i.d. random variables $X_1, \ldots X_n$, with distribution F_X.

3. REPLICATED SYSTEM ($k = 1$)

Observing that the $(n, 1)$ fork-join system is equivalent to an $M/G/1$ queue, and the $(n, 1)$ fork-early cancel system is equivalent to an $M/G/n$ queue will help us derive the latency and the cost of these systems.

THEOREM 1. *The latency and computing cost of an $(n, 1)$ fork-join system is given by*

$$\mathbb{E}[T] = \mathbb{E}[X_{1:n}] + \frac{\lambda\mathbb{E}\left[X_{1:n}^2\right]}{2(1 - \lambda\mathbb{E}[X_{1:n}])}, \tag{2}$$

$$\mathbb{E}[C] = n\mathbb{E}[X_{1:n}]. \tag{3}$$

To prove Thm. 1 we identify that in the $(n, 1)$ fork-join system, all tasks of a job start service simultaneously. Thus, it is equivalent to an $M/G/1$ queue with service time $X_{1:n}$, whose latency is given by the Pollaczek-Khinchine formula (2). Fig. 3 shows the latency-cost trade-off when the service time $X = \Delta + \text{Exp}(\mu)$, a shifted exponential with $\mu = 0.5$,

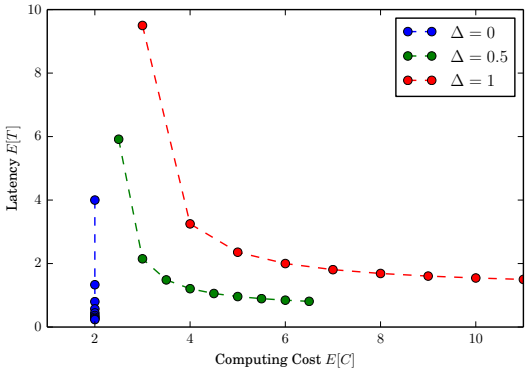

Fig. 3: The service time $X \sim \Delta + \text{Exp}(\mu)$, with $\mu = 0.5$, $\lambda = 0.25$ and different values of Δ. Latency decreases and cost increases with n increasing along each curve. But for $\Delta = 0$ latency reduces at no additional cost.

and $\lambda = 0.25$. As n increases along each curve, $\mathbb{E}[T]$ decreases and $\mathbb{E}[C] = n\mathbb{E}[X_{1:n}]$ increases. Only when X is a pure exponential (generally not true in practice), we can reduce latency without any additional cost. For heavy tailed X (eg. Pareto), $\mathbb{E}[C]$ may decrease with n, and as a result, give a simultaneous reduction in both latency and cost.

THEOREM 2. *The latency and cost of the $(n, 1)$ fork- early cancel system are given by*

$$\mathbb{E}[T] = \mathbb{E}\left[T^{M/G/n}\right] \approx \mathbb{E}[X] + \frac{\mathbb{E}\left[X^2\right]}{2\mathbb{E}[X]^2}\mathbb{E}\left[W^{M/M/n}\right], \tag{4}$$

$$\mathbb{E}[C] = \mathbb{E}[X], \tag{5}$$

where $\mathbb{E}\left[W^{M/M/n}\right]$ is the expected waiting time in an $M/M/n$ queueing system with service time $X \sim F_X$.

To prove Thm. 2 we identify that in the $(n, 1)$ fork-early cancel system, one task of each job joins the shortest queue available, and the other tasks are canceled before they begin service. Thus, it is equivalent to an $M/G/n$ system whose latency is given by the well-known approximation (4). Since the cost is $\mathbb{E}[C] = \mathbb{E}[X]$ which is independent of n, there is no latency-cost trade-off similar to Fig. 3.

In Fig. 4, we plot the latency of normal and early cancellation systems vs. λ for shifted exponential service time $X \sim 1 + \text{Exp}(0.5)$. Early cancellation gives lower latency in the high λ regime. This can be inferred from Claim 1, since $\mathbb{E}[C]$ with early cancellation is $\mathbb{E}[X]$, which is smaller than $n\mathbb{E}[X_{1:n}]$ for shifted exponential X. Latency behavior similar to Fig. 4 can be observed when we plot $\mathbb{E}[T]$ vs. Δ, the constant part of the service time. Early cancellation gives lower latency for higher Δ (more light tailed F_X).

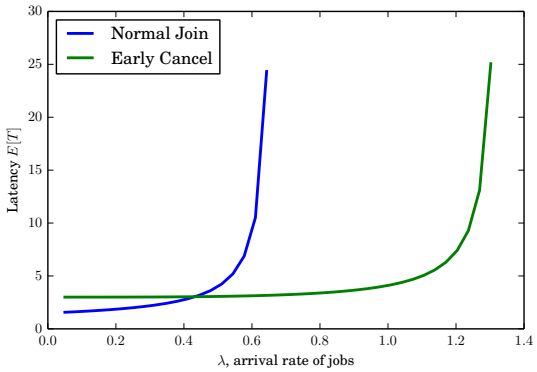

Fig. 4: Comparison of $\mathbb{E}[T]$ of the $(4,1)$ system with normal join and early cancellation, vs. λ for $X \sim 1+\text{Exp}(0.5)$. Early cancellation is gives higher service capacity (by Claim 1).

4. GENERAL CASE: $1 \leq k \leq n$

In the traditional fork-join queue ($k = n$ case in Def. 1 with exponential service time), an exact expression for latency can be found only for $n = 2$ [1]. The analysis is hard and only bounds are known for general k. We determine the first latency and cost bounds with general k and general service distribution F_X in Thm. 3 and Thm. 4 below.

THEOREM 3. *The latency $\mathbb{E}[T]$ of the (n,k) fork-join system is bounded as*

$$\mathbb{E}[T] \leq \mathbb{E}[X_{k:n}] + \frac{\lambda \mathbb{E}\left[X_{k:n}^2\right]}{2(1-\lambda \mathbb{E}[X_{k:n}])}, \tag{6}$$

$$\mathbb{E}[T] \geq \mathbb{E}[X_{k:n}] + \frac{\lambda \mathbb{E}\left[X_{1:n}^2\right]}{2(1-\lambda \mathbb{E}[X_{1:n}])}. \tag{7}$$

To get (6), we use the split-merge system, in which no two jobs are served simultaneously. In (7), we use the waiting time of the $(n,1)$ fork-join system to lower bound that of the (n,k) system. Fig. 5 shows the latency bounds and simulation values vs. k for $n = 10$, $\lambda = 0.5$, and X following the Pareto distribution with $x_m = 0.5$ and $\alpha = 2.5$. For $k = n$, we can get a tighter bound than (6) by generalizing the approach used in [5]. The same approach can be used to upper bound $\mathbb{E}[T]$ of the (n,k) fork-early cancel system.

THEOREM 4. *The computing cost $\mathbb{E}[C]$ of the (n,k) fork-join system is bounded as*

$$\mathbb{E}[C] \leq (k-1)\mathbb{E}[X] + (n-k+1)\mathbb{E}[X_{1:n-k+1}], \tag{8}$$

$$\mathbb{E}[C] \geq \sum_{i=1}^{k} \mathbb{E}[X_{i:n}] + (n-k)\mathbb{E}[X_{1:n-k+1}]. \tag{9}$$

The key idea for proving Thm. 4 is our observation that for each job, some $n-k+1$ of its tasks start service simultaneously, which allowed us to analyze them separately. The bounds are tight for $k = 1$ and $k = n$ as seen in Fig. 6.

For the (n,k) fork-early cancel system, since exactly k tasks start and finish service, it follows that $\mathbb{E}[C] = k\mathbb{E}[X]$.

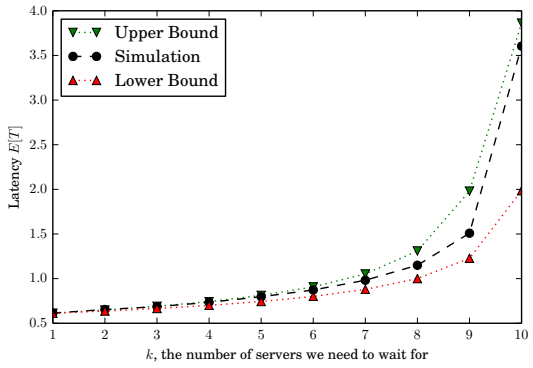

Fig. 5: Bounds $\mathbb{E}[T]$ vs. k, with service time $X = \text{Pareto}(0.5, 2.5)$ with $n = 10$, and $\lambda = 0.5$. The $k = n$ upper bound is determined by generalizing the approach in [5].

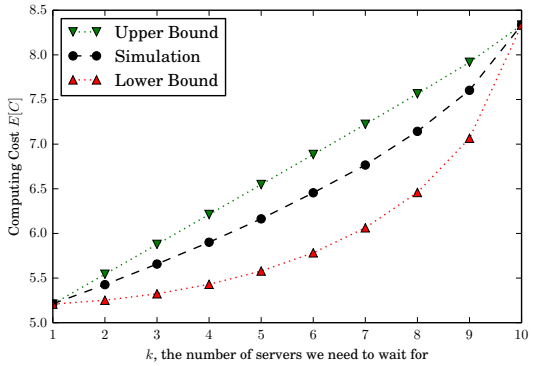

Fig. 6: Bounds on $\mathbb{E}[C]$ vs. k, for the same parameters as Fig. 5. The bounds are tight for $k = 1$ and $k = n$.

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