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# Quintessence Compact Stars Satisfying Karmarkar Condition

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# Quintessence Compact Stars Satisfying Karmarkar Condition

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## Abstract

In this research article, authors have presented the modelling of quintessence compact stars which satisfies the Karmarkar conditions. For this purpose, we have formulated the set of Einstein field equations with the static metric, anisotropic perfect fluid and quintessence field. The equation of state $p_r = \alpha\rho$ and Karmarkar condition  have been used to solve the set of field equations. The unknown constant in the metric functions (appearing due to the Karmarkar conditions) have been found by matching the interior metric with the Schwarzschild  exterior  metric.  The observed value of mass and radius of some well  known class of a star has been used.  The fluid variables density,  radial and transverse pressures  and anisotropic parameter  have plotted graphically.  The first and second derivatives of density and  radial pressure have  been evaluated to discuss  the regularity of the  model.  The speed of  sound for the radial  and transverse directions determine the stability of the proposed model. More-over the  redshift for the proposed model of the star has been discussed.

**Keywords**: General  relativity, Compact star, Embendding  class, anisotropy.

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# 1    Introduction

Hundred years back, the first exact solution of the Einstein field equations was gained by Karl Schwarzchild [1]. After this a lot of exact solutions have been represented by different mathematicians with small changes. Scwarzchild's constant density sphere is established to include physically observed phenomenon like the pressure anisotropy, the electromagnetic field, dissipation, rotation and deviation from spherical symmetry. Lake and Degaty [2] carried out comprehensive and systematic study of solutions of Einstein field equations. They studied one hundred and twenty seven exact solutions of Einstein field equations and only nine solutions meet the physical possibility condition. It shows the difficulty to get exact solution of Einstein field equations describing realizable astrophysical objects. This encourages the researchers to find out solutions which are more physically applicable but more importantly, they should also be good approximation to observational data.

Canuto [3] and Ruderman [4] showed that when the nuclear density is much lower than the matter density, matter will be anisotropic in nature. Comprehensive studies on impact of anisotropy on self-gravitational configurations were carried out by Bowers and Liang [5] and Herrera and Santos [ 6]. The anisotropy may take place due to existence of type 3- A superfluids [7], phase transformation [8] with in the care or due to electromagnetic field [9]-[11].

Models of pseudo-spheroidal, relativistic stars on spherical and paraboloidal spacetime had been studied by Tikekar and Thomas [12], Vaidya and Tikekar [13] and Tikekar and Jatonia [14] respectively. Petal and Kopper [15], Gupta and Kumar [16], Sharma et. al. [17] and Komathiraj and Maharaj [18] have studied charged stars on spheroidal spacetime. Tikekar and Thomas [19], Chattopadhay and Paul [20] and Thomas et al. [21] have studied the compact objects on pseudo-spherical spacetime. The paraboidal spacetime is a specific case of the Finch and Skea [22] spacetime. Sharma and Ratanpal [23] studied the relativistic star model accepting the quadratic equation of state on paraboidal spacetime. These studies propose that geometrically significant spacetime may be used to explain the physically realistic stars.

The embedding problem is one of the interesting problem on geometrically significant spacetimes that was first delivered by Schlai [24]. The 1st isometric embedding theorem was provided by Nash [25]. The condition for embedding 4-dimensional spacetime metric in 5-dimensional Euclidean spaces was derived by Karmarker [26]. Karmarker categorized these spacetimes like class- 1 spacetime. For a spherically symmetric spacetime, Karmarker condition in the form of curvature components takes the form

$$R_{1414}R_{2323} = R_{1212}R_{3434} + R_{1224}R_{1334}. \qquad (1)$$

Recently, attention amongst a lot of researchers working on modeling compact objects, Exact solutions of Einstein field equation and stability analyses of self- gravitating objects. The study of charged compact stars fulfilling compact stars Karmarker's condition was begun by Maurya et. al. [27]. This led to fruitful of models of compact objects fulfilling Karmarker' s condition [28, 38]. In the current work, we have studied the spherically symmetric spacetime metric of embedding class-1 and gained the singularity free solution of Einstein field equations for anisotropic fluid distribution. We have to proved that the model fulfill all the physical

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plausibility conditions and also stable. Zubair et al.[39]-[42]. The work is arranged as follows: Section-2 presents the Einstein's field equations, Karmarker condition for spherically symmetric spacetimes and TOV equation necessary for this analysis. We show the anisotropic solution of embedding class-1 for compact stars in section-3. We suppose the matching condition of the interior spacetime to the vacuum Schwarzschild exterior solution in section- 4. The physical properties of our model are expressed in section-5. We finish with a discussion of result in section 6.

# 2　Anisotropic Source and Field Equations

We start with the static spherically symmetric spacetime metric that is given by

$$ds^2 = e^{\nu(r)} dt^2 - e^{\lambda(r)} dr^2 - r^2(d\theta^2 + \sin^2\theta \, d\phi^2), \tag{2}$$

With the metric (2) together with the energy- momentum (4) Einstein's field equations take the form

$$G_{\mu\nu} = \kappa \left( T_{\mu\nu} + \tau_{\mu\nu} \right) \tag{3}$$

where $G_{\mu\nu}$ is the Einstein tensor, $\kappa = \frac{8\pi G}{c^4}$ is the coupling constant and $T_{\mu\nu}$, $\tau_{\mu\nu}$ are respectively the energy- momentum tensor of the ordinary matter which is anisotropic perfect fluid and quintessence like field characterized by a parameter $\omega_q$ with $\omega_q - 1 < \omega_q < -1/3$. Now Kiselev [43] has proved that the components of this tensor need to fulfill the conditions of additivity and linearity . In consideration of the different signature used in line elements, the components can be expressed as follows

$$\tau_t^t = \tau_r^r = -\rho_q \tag{4}$$

$$\tau_\theta^\theta = \tau_\phi^\phi = \frac{3\omega_q + 1}{2}\rho_q \tag{5}$$

and the corresponding energy-momentum tensor can be stated as

$$T_\nu^\mu = (\rho + p_r) u^\mu u_\nu - p_t g_\nu^\mu + (p_r - p_t)\eta^\mu \eta_\nu, \tag{6}$$

with $u^i u_j = 1 = -\eta^i \eta_j$ and $u^i \eta_j = 0$. Here spacelike vector which is orthogonal to $u^i$, $\rho$, $p_t$ and $p_r$ denote matter density, tangential pressure and the radial pressure respectively.

$$8\pi (\rho + \rho_q) = e^{-\lambda} \left[ \frac{\lambda'}{r} - \frac{1}{r^2} \right] + \frac{1}{r^2} \tag{7}$$

$$8\pi (p_r - \rho_q) = e^{-\lambda} \left[ \frac{v'}{r} + \frac{1}{r^2} \right] - \frac{1}{r^2}, \tag{8}$$

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$$8\pi \left( p_t \; + \frac{3\omega_q + \; 1}{2}\rho_q \right) = \; e^{-\lambda} \left[ \frac{v''}{2} \; - \frac{\lambda' v'}{4} \; + \frac{v'^2}{4} + \; \frac{v' - \lambda'}{2r} \right], \tag{9}$$

Also, anisotropic parameter is

$$\Delta \;\; = \;\; p_t - \;\; p_r = \frac{e^{-\lambda}}{16\pi} \left[ \frac{1}{2}\acute{v}^2 \; + v'' - \; \frac{1}{2}\lambda' v' - \; \frac{1}{r}(\lambda' - \nu') \; - \frac{2\alpha}{\alpha + 1}(\frac{\lambda'}{r} + \; \frac{\nu'}{r}) \right]$$
$$- \; \frac{1}{2}(3\omega_q + 1)\rho \tag{10}$$

## 2.1 Karmarkar condition:

In general, the spherically symmetric spacetime metric (2) is of class two. If the metric (2) satisfies the Karmarkar condition (1) It will then show a spacetime of embedding class one. The components of the Riemann curvature tensor $R_{hijk}$ for metric (2) are given by $R_{2323} \;\; = \;\; \frac{sin^2\theta(e^\lambda - 1)\, r^2}{e^\lambda}$, $R_{1212} \;\; = \frac{\lambda' r}{2}, \;\; R_{2424} = \;\; \frac{e^{(\nu - \lambda)}}{2}$,

$$R_{1224} = 0, \; R_{1414} \;\; = \frac{e^\nu}{4}\, [2\,\nu'' + \; \nu'^2 - \; \lambda'\,\nu'], \;\; R_{3434} = \;\; sin^2\theta \, R_{2424}.$$

By inserting the components of $R_{hijk}$ into the Karmarkar condition (1), we obtain the following differential equation,

$$\frac{\nu''}{\nu'} \; + \frac{\nu'}{2} \;\; = \frac{\lambda'\lambda}{2\,(e^\lambda - 1)}. \tag{11}$$

This gives the gravitational potential $\nu$, as follows

$$\nu \; = 2\,ln \left[ A_1 + \; B_1 \int \; \sqrt{(e^{\lambda(r)} - \; 1)}dr \right], \tag{12}$$

where $A_1$ and $B_1$ are non-zero arbitrary constants of integration.

## 2.2 Anisotropic solution of embedding class one for compact star:

It is interesting to note that the solution of Einstein field equations for anisotropic matter distribution depends on one of the metric functions $\nu$ or $\lambda$ because the Karmarkar condition gives a direct relation between the metric functions. For this purpose , we use the ansatz as already used in Maurya et al.[38] for $e^\lambda$,

$$e^\lambda = 1 + \frac{(a - b)r^2}{1 + br^2} \tag{13}$$

where, $a \neq 0, b \neq 0$. If $a = \; 0 = b$ then the spacetime can be written as

$$ds^2 \; = -dr^2 - \; r^2(d\theta^2 + \; sin^2\,\theta\,d\phi^2) + \; e^{\nu(r)}\,dt^2, \tag{14}$$

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for which Karmarkar's condition (1) is satisfied but spacetime metric (14) is not of class 1 as shown by Pandey and Sharma (1998), hence we take $a$ and $b$ as positive constants. The metric potential $e^\lambda$ chosen here does not give rise to paraboloidal, spheroidal or pseudo-spheroidal spacetimes. Substituting Eq.(13) into Eq.(12), we get

$$\nu = 2\ln\left[Ab + B\sqrt{(a-b)}\sqrt{1+br^2}\right] - 2\ln b \tag{15}$$

where $A$ and $B$ are constants .

For a physically viable model the metric functions $e^\lambda$ and $e^\nu$ must be finite at the center while both should be monotonically increasing functions of $r$ as shown in **Fig.(1)**. We observe from Eqs.(13) and (15), $(e^\lambda)_{r=0} = 1$ and $(e^\nu)_{r=0} = \frac{1}{b^2}[Ab + B\sqrt{(a-b)}]^2$, which are finite and free from singularity.

To solve the above system of equations let's assume that the radial pressure $p_r$ is proportional to the matter density $\rho$

$$p_r = \alpha\rho, \qquad\qquad 0 < \alpha < 1,$$

where $\alpha$ is the equation of state parameter. This equation corresponds to a polytropic equation of state of the second class. .

From the Eqs.(7)-(10) together with Eqs.(13) and (15), we get $\rho$, $p_r$, $p_t$, $\rho_q$ and $\Delta$,

$$4\pi\rho = \frac{\sqrt{a-b}}{(1+\alpha)}\left[\frac{Ab\sqrt{a-b} + aB(1+br^2)^{\frac{3}{2}}}{(1+ar^2)^2(Ab+B\sqrt{a-b}\sqrt{1+br^2})}\right], \tag{16}$$

$$4\pi p_r = \frac{\alpha\sqrt{a-b}}{(1+\alpha)}\left[\frac{Ab\sqrt{a-b} + aB(1+br^2)^{\frac{3}{2}}}{(1+ar^2)^2(Ab+B\sqrt{a-b}\sqrt{1+br^2})}\right], \tag{17}$$

$$8\pi\rho_q = \sqrt{a-b}\left[-2bB\sqrt{1+br^2}(1+ar^2) + \left(Ab\sqrt{a-b} + B(a-b)\sqrt{1+br^2}\right)\right.$$
$$\left.\times\left((3\alpha+1) + ar^2(\alpha+1)\right)\right] \times \frac{1}{(\alpha+1)(1+ar^2)^2(Ab+B\sqrt{a-b}\sqrt{1+br^2})}$$
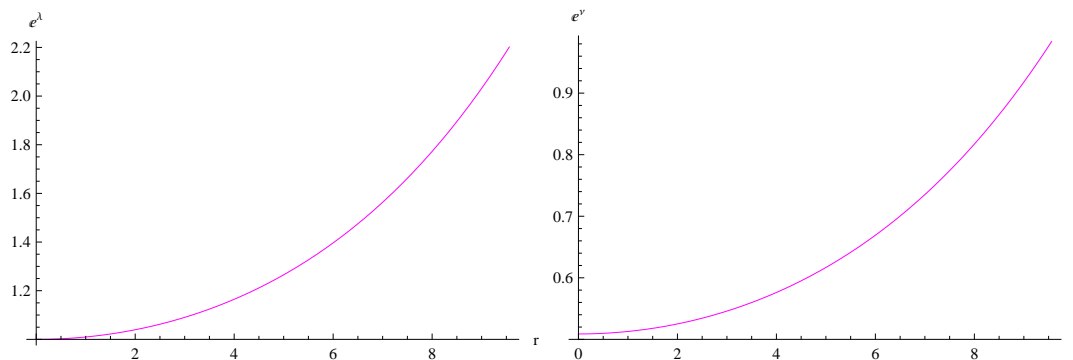$$\tag{18}$$

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Figure 1: First and second graphs represent the behavior of the metric function for star Vela X-1

$$
\begin{aligned}
8\pi p_t &= \frac{\sqrt{a-b}}{(1+ar^2)(Ab+b\sqrt{a-b}\sqrt{1+br^2})}\Big[\frac{b^2B^2r^2\sqrt{a-b}}{(Ab+b\sqrt{a-b}\sqrt{1+br^2})}\Big) \\
&+ \frac{bB}{1+br^2}\Big(\frac{Ab+B\sqrt{a-b}\sqrt{1+br^2}(1-br^2)}{Ab+b\sqrt{a-b}\sqrt{1+br^2}}\Big) - \frac{bBr^2(a-b)}{(1+ar^2)\sqrt{1+br^2}} \\
&+ bB\sqrt{1+br^2} - \frac{\sqrt{a-b}(Ab+B\sqrt{a-b}\sqrt{1+br^2})}{1+ar^2} - \frac{3\omega_q+1}{2(\alpha+1)(1+ar^2)} \\
&\times (Ab\sqrt{a-b}+B(a-b)\sqrt{1+br^2})(3\alpha+1+ar^2(\alpha+1)) - 2bB\sqrt{1+br^2}(1+ar^2)\Big]
\end{aligned}
$$

$$(19)$$

$$
\begin{aligned}
\Delta &= \frac{\sqrt{a-b}}{(1+ar^2)(Ab+b\sqrt{a-b}\sqrt{1+br^2})}\Big[\frac{b^2B^2r^2\sqrt{a-b}}{(Ab+b\sqrt{a-b}\sqrt{1+br^2})}\Big) \\
&+ \frac{bB}{1+br^2}\Big(\frac{Ab+B\sqrt{a-b}\sqrt{1+br^2}(1-br^2)}{Ab+b\sqrt{a-b}\sqrt{1+br^2}}\Big) - \frac{bBr^2(a-b)}{(1+ar^2)\sqrt{1+br^2}} \\
&+ bB\sqrt{1+br^2} - \frac{\sqrt{a-b}(Ab+B\sqrt{a-b}\sqrt{1+br^2})}{1+ar^2} - \frac{3\omega_q+1}{2(\alpha+1)(1+ar^2)} \\
&\times (Ab\sqrt{a-b}+B(a-b)\sqrt{1+br^2})(3\alpha+1+ar^2(\alpha+1)) \\
&- 2bB\sqrt{1+br^2}(1+ar^2) - \frac{2\alpha}{(\alpha+1)(1+ar^2)}\times Ab\sqrt{a-b}+aB(1+br^2)^{\frac{3}{2}}\Big]
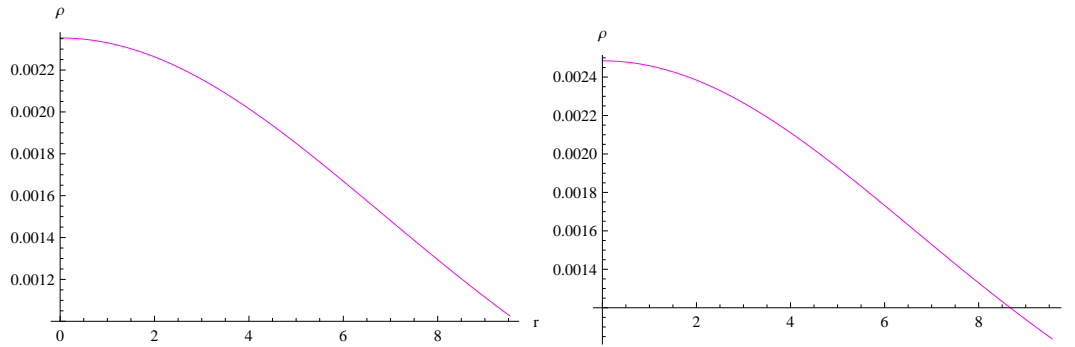\end{aligned}
$$

$$(20)$$

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Figure 2: First and second graphs represent the density variation of Strange star candidate Vela X-1 and 4U1608-52 respectively.
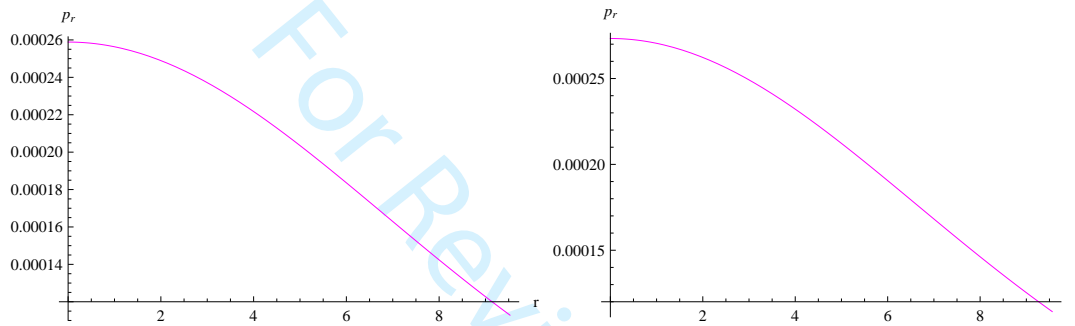


Figure 3: First and second graphs represent the radial pressure of Strange star candidate Vela X-1 and 4U1608-52 respectively.

$$\frac{d\rho}{dr} = \frac{\sqrt{a-b}}{4\pi(\alpha+1)(1+ar^2)^3(Ab+\sqrt{a-b}\sqrt{1+br^2})^2}$$
$$\times \Big[3abBr(1+br^2+ar^2+abr^4)(Ab+\sqrt{a-b}\sqrt{1+br^2}) - (Ab\sqrt{a-b}+aB(1+br^2)^{\frac{3}{2}})$$
$$(bBr+abBr^3+4ar\sqrt{1+br^2}(Ab+\sqrt{a-b}\sqrt{1+br^2}))\Big] \tag{21}$$

$$\frac{dp_r}{dr} = \frac{\sqrt{a-b}}{4\pi(\alpha+1)(1+ar^2)^3(Ab+\sqrt{a-b}\sqrt{1+br^2})^2}\Big)$$
$$\times[3abBr(1+br^2+ar^2+abr^4)(Ab+\sqrt{a-b}\sqrt{1+br^2})$$
$$- (Ab\sqrt{a-b}+aB(1+br^2)^{\frac{3}{2}})(bBr+abBr^3+4ar\sqrt{1+br^2}(Ab+\sqrt{a-b}\sqrt{1+br^2})] \tag{22}$$
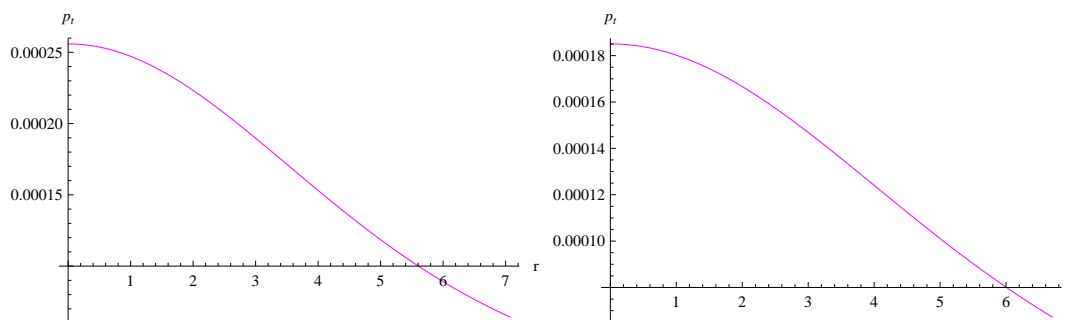
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Figure 4: First and second graphs represent the tangential pressure of Strange star candidate Vela X-1 and 4U1608-52 respectively.
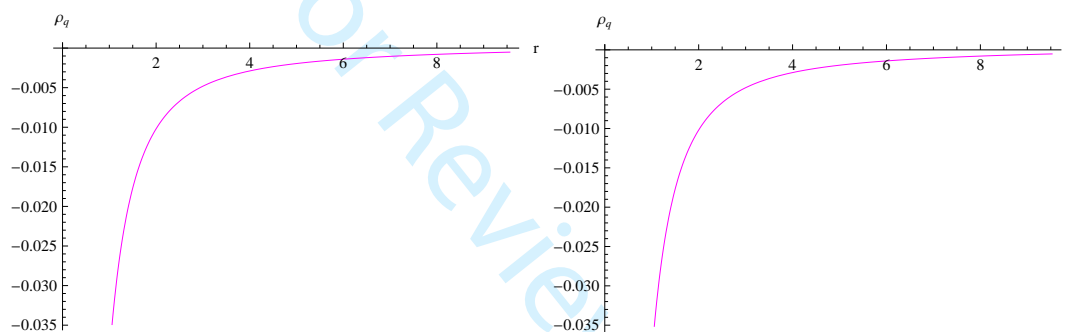


Figure 5: First and second graphs represent energy density of the quintessence field $\rho_q$ of strange star candidates Vela X-1 and 4U1608-52 respectively.
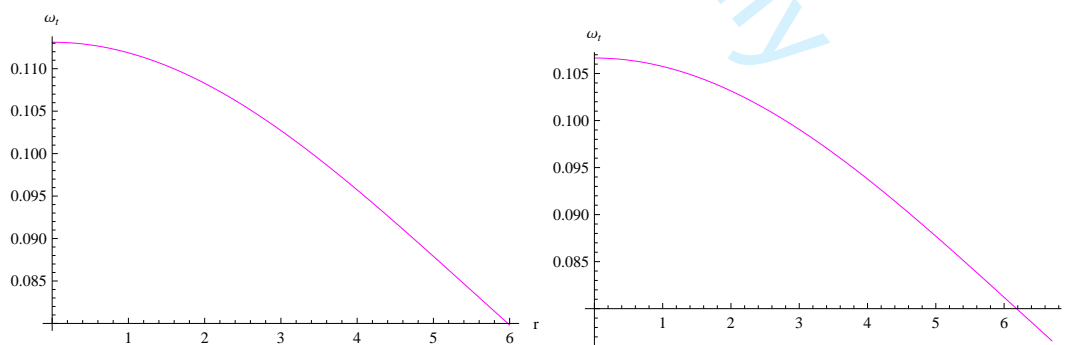


Figure 6: First and second graphs represent the EOS parameter $\omega_t$ star candidate Vela X-1 and 4U1608-52 respectively.
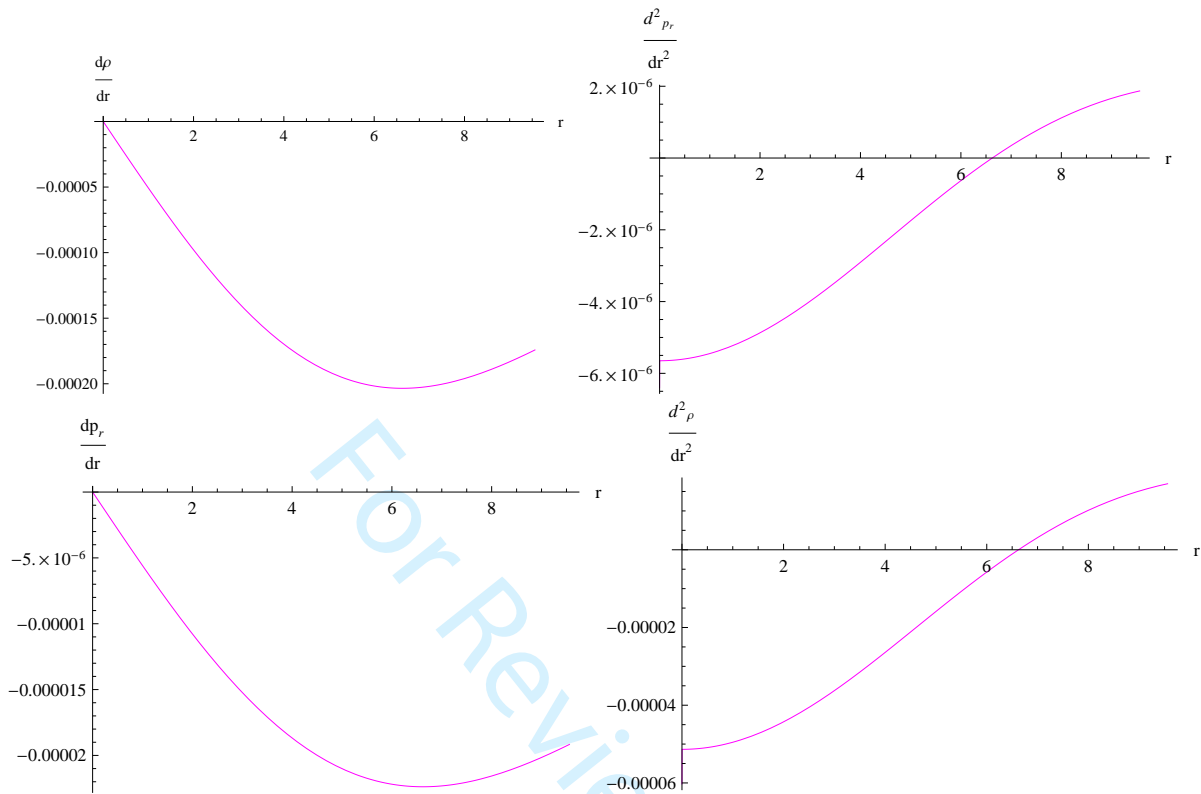
8

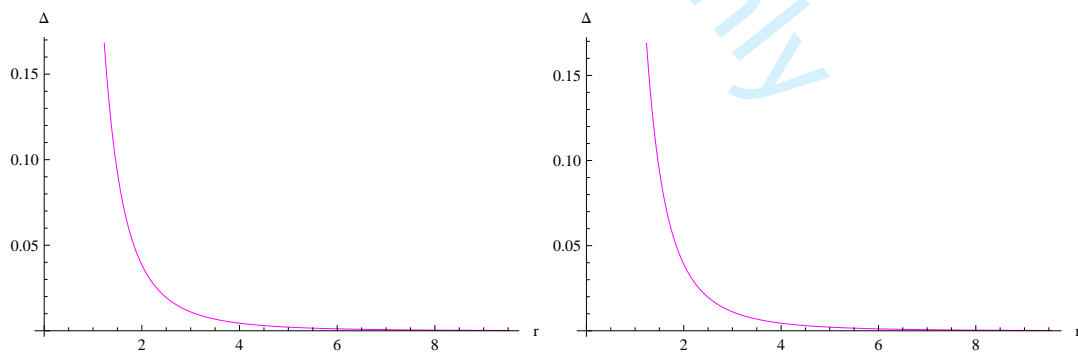Figure 7: All these graphs has been plotted only for the data of Vela X-1.



Figure 8: First and second graphs represent the variation of anisotropy $\Delta$ of strange s-star candidates Vela X-1 and 4U1608-52 respectively.
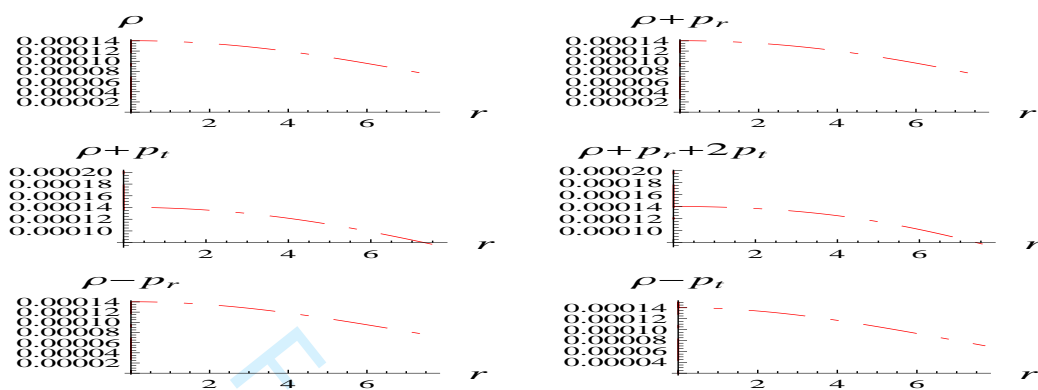
9

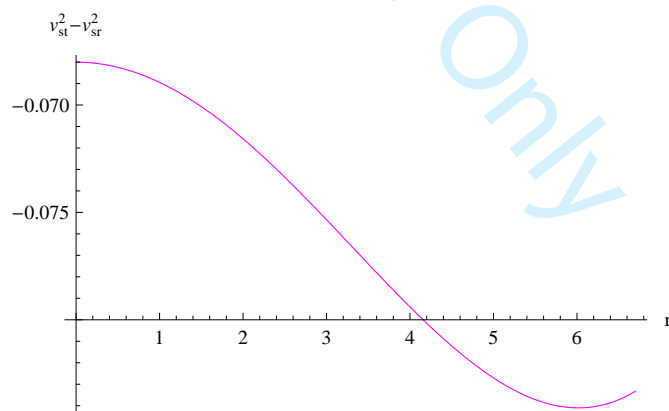Figure 9: Evolution of energy constraints at the stellar interior of strange star Vela X-1.



Figure 10: All these graphs has been plotted only for the data of Vela X-1.

$$\frac{d^2\rho}{dr^2} = \frac{\sqrt{a-b}}{4\pi}[(1+ar^2)^3 \tag{23}$$

$$\times(Ab+B\sqrt{a-b}\sqrt{1+br^2})(\frac{3bBr\sqrt{a-b}}{\sqrt{1+br^2}}(abBr+ab^2Br^3+a^2bBr^3+a^2b^2Br^5))$$

$$+(3abB+9ab^2Br^2+9a^2bBr^2+15a^2b^2Br^4)(Ab+B\sqrt{a-b}\sqrt{1+br^2}$$

$$-(3abBr\sqrt{1+br^2})(bBr+abBr^3+4abAr\sqrt{1+br^2}+4arB\sqrt{a-b}+4ar^3bB\sqrt{a-b}))$$

$$-((3abBr+3ab^2Br^3+3a^2bBr^3+3a^2b^2Br^5)$$

$$-(Ab\sqrt{a-b}+aB(1+br^2)^{\frac{3}{2}})(bBr+abBr^3+4ar\sqrt{1+br^2}(Ab+B\sqrt{a-b}\sqrt{1+br^2})))$$

$$\times(6ar(1+ar^2)^2(Ab+B\sqrt{a-b}\sqrt{1+br^2})^2+(\frac{4bBr\sqrt{a-b}}{1+br^2})(Ab+B\sqrt{a-b}\sqrt{1+br^2}))]$$

$$\tag{24}$$

$$\frac{d^2p_r}{dr^2} = \frac{\alpha\sqrt{a-b}}{4\pi}[(1+ar^2)^3(Ab+B\sqrt{a-b}\sqrt{1+br^2}) \tag{25}$$

$$\times(\frac{3bBr\sqrt{a-b}}{\sqrt{1+br^2}}(abBr+ab^2Br^3+a^2bBr^3+a^2b^2Br^5))$$

$$+(3abB+9ab^2Br^2+9a^2bBr^2+15a^2b^2Br^4)(Ab+B\sqrt{a-b}\sqrt{1+br^2}$$

$$-(3abBr\sqrt{1+br^2})(bBr+abBr^3++4abAr\sqrt{1+br^2}+4arB\sqrt{a-b}+4ar^3bB\sqrt{a-b}))$$

$$-((3abBr+3ab^2Br^3+3a^2bBr^3+3a^2b^2Br^5)$$

$$-(Ab\sqrt{a-b}+aB(1+br^2)^{\frac{3}{2}})(bBr+abBr^3+4ar\sqrt{1+br^2}(Ab+B\sqrt{a-b}\sqrt{1+br^2})))$$

$$(6ar(1+ar^2)^2(Ab+B\sqrt{a-b}\sqrt{1+br^2})^2+(\frac{4bBr\sqrt{a-b}}{1+br^2})(Ab+B\sqrt{a-b}\sqrt{1+br^2}))]$$

$$\tag{26}$$

The pressure anisotropy $\Delta$ is zero at at the boundary. However it can be made zero everywhere inside the star only when $a=b$ ( which implies $B=0$). In this scene the metric roll out to be flat and all the physical parameters like the radial pressure, tangential pressure and density disappear. The physical behavior of all parameters is presented in figs.(2)-( 5). The equation of state parameter is $0<\omega_t<1$, which implies that the matter is ordinary matter as shown in figure (6). Figure (7) gives the regularity of density and pressure and Fig.( 8) indicates that the anisotropy parameter is positive at each interior point of the matter configuration, ie., $p_t>p_r$. This indicates that the force due to local anisotropy is repulsive and may lead to more massive, stable configurations.

# 3   Boundary conditions for the solution:

The obtained interior solution must match continuously with the Schwarzschild exterior solution

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$$ds^2 = \left(1 - \frac{2M}{r}\right) dt^2 - \left(1 - \frac{2M}{r}\right)^{-1} dr^2 - r^2(d\theta^2 + \sin^2\theta\, d\phi^2), \tag{27}$$

at the boundary of stellar configuration $r = R$, where $M$ is total mass of anisotropic stellar configuration hold within a sphere of radius $R$. By matching the first fundamental form (continuity of $e^\nu$ and $e^\lambda$) and second fundamental forms (continuity of $\frac{\partial g_{tt}}{\partial r}$ i.e. $(p_r)_R = 0$ )of the interior solution with exterior Schwarzschild solution at the boundary of the star.

$$1 - \frac{2M}{R} = e^{\nu_R} = [\frac{Ab + B\sqrt{a-b}\sqrt{1+bR^2}}{b}]^2 \tag{28}$$

$$1 - \frac{2M}{R} = e^{-\lambda_R} = \frac{1 + BR^2}{1 + AR^2} \tag{29}$$

$$(p_r)_R = 0. \tag{30}$$

By solving the above boundary conditions we get the constants as:

$$A = \frac{a}{b}(\frac{1+bR^2}{1+aR^2})^{\frac{3}{2}} \tag{31}$$

$$B = \frac{\sqrt{a-b}}{(1+aR^2)^{\frac{3}{2}}} \tag{32}$$

$$M = \frac{R}{2}[1 - \frac{1+bR^2}{1+aR^2}] \tag{33}$$

## 3.1 Stability criterion via cracking

For a physically acceptable model of anisotropic fluids here one must have the radial and transverse velocity of sound should be less than 1. Where the radial velocity $(v_{sr}^2)$ and transverse velocity $(v_{st}^2)$ of sound can be attained as

$$v_{sr}^2 = \frac{dp_r}{d\rho} = \alpha \tag{34}$$

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$$v_{st}^2 \;=\; \frac{dp_t}{d\rho} \;=\; \frac{\sqrt{a-b}}{8\pi\sqrt{a+br^2}(1+ar^2)^2(Ab+B\sqrt{a-b}\sqrt{1+br^2})}\Big)$$

$$\times\Big[(1+ar^2)(Ab+b\sqrt{a-b}\sqrt{1+br^2})$$

$$\times\,\frac{(Ab+b\sqrt{a-b}\sqrt{1+br^2})(2b^2B^2r\sqrt{a-b}\sqrt{1+br^2})-b^3B^2r^3(a-b)}{(Ab+B\sqrt{a-b}\sqrt{1+br^2})^2}$$

$$-\frac{br(Ab+2B\sqrt{a-b}\sqrt{1+br^2})}{(1+br^2)(Ab+B\sqrt{a-b}\sqrt{1+br^2})}$$

$$+\sqrt{1+br^2}(Ab+B\sqrt{a-b}\sqrt{1+br^2})(b^2B^2r\sqrt{a-b}(1-br^2)-2b^2B^2r\sqrt{a-b}(1+br^2)$$

$$-b^2B^2r\sqrt{a-b}\sqrt{1+br^2}(1-br^2)(Ab+2B\sqrt{a-b}\sqrt{1+br^2}))$$

$$-\frac{bBr(a-b)(2-r-2ar^3-br^3-3abr^5}{(1+br^2)(1+ar^2)^3}+b^2Br-bBr(a-b)(1+ar^2)$$

$$-\frac{2ar\sqrt{a-b}\sqrt{1+br^2}(Ab+B\sqrt{a-b}\sqrt{1+br^2})}{(1+ar^2)^2}-(\frac{3\omega_q+1}{2})(\frac{\sqrt{1+br^2}}{(\alpha+1)(1+ar^2)^2})$$

$$\times((1+ar^2)(-2b^2r-4abBr(1+br^2)-2qb^2Br^3+bBr(3\alpha+1)(a-b)$$

$$+2aAbr\sqrt{a-b}\sqrt{1+br^2}(\alpha+1)+B(a-b)(1+br^2))+2ar\sqrt{1+br^2}(2bB\sqrt{1+br^2})$$

$$-(Ab\sqrt{a-b}+B\sqrt{1+br^2}(a-b))(3\alpha+1+ar^2(\alpha+1))))$$

$$-(bBr\sqrt{a-b}(1+ar^2)+2ar\sqrt{1+br^2}(Ab+B\sqrt{a-b}\sqrt{1+br^2}))\Big)$$

$$+\frac{bB}{1+br^2}(\frac{Ab+B\sqrt{a-b}\sqrt{1+br^2}(1-br^2)}{Ab+b\sqrt{a-b}\sqrt{1+br^2}})-\frac{bBr^2(a-b)}{(1+ar^2)\sqrt{1+br^2}}$$

$$+bB\sqrt{1+br^2}-\frac{\sqrt{a-b}(Ab+B\sqrt{a-b}\sqrt{1+br^2})}{1+ar^2}$$

$$-\frac{3\omega_q+1}{2(\alpha+1)(1+ar^2)}(Ab\sqrt{a-b}+B(a-b)\sqrt{1+br^2})(3\alpha+1+ar^2(\alpha+1))$$

$$-2bB\sqrt{1+br^2}(1+ar^2)\Big]. \tag{36}$$

Herrera [44] introduced a proposal to check the stability of anisotropic gravitating source. Currently, this technique is termed as cracking concept which states that if radial speed of sound is greater than the transverse speed of sound then such a region is a potentially stable region, otherwise unstable region. From fig.(10) it is clear that our model satisfies the condition $v_{st}^2-v_{sr}^2<0$. So we conclude that our model is potentially stable.

## 3.2   Energy conditions:

The stellar configuration must satisfy the null energy condition (NEC), weak energy condition (WEC) and strong energy condition (SEC). These conditions respectively are

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Table 1: Numerical values of the constants for different compact stars

| Compact star | $a(km^{-2})$ | $b(km^{-2})$ | $A$ | $B(km^{-1})$ | $Z_s$ |
|---|---|---|---|---|---|
| Vela X-1 | 0.00033 | 0.19 | -44.3761 | 44.2876 | 2.213532 |
| 4U1608-52 | 0.000325 | 0.1845 | -45.3428 | 45.2910 | 3.34025 |

$$NEC : \rho(r) \ge 0, \tag{37}$$

$$WEC : \rho(r) - p_r(r) \ge 0 \quad and \quad \rho(r) - p_t(r) \ge 0, \tag{38}$$

$$SEC : \quad \rho - p_r(r) - 2p_t(r) \ge 0. \tag{39}$$

Figure (9) clearly shows that all the energy conditions are satisfied for each of our stellar models. We should point out that the cracking scenario derived by Abreu et al.[45] in terms of the relative sound speeds within the fluid configuration requires that the strong and dominant energy conditions must be satisfied. The surface redshift $(Z_s)$ in this case is given by as

$$1 + Z_s = \left(\frac{1 + aR^2}{a + bR^2}\right)^{-\frac{1}{2}}. \tag{40}$$

For the $VelaX - 1$, and $4U1608 - 52$ the surface redshift is given in table 1.

# 4   Conclusions

In this research article, we have explored the possibility of formation of compact star in the presence of quintessence field. The compact stars are assumed to satisfy the Karmarkar conditions. We have evaluated the matter density, radial and transverse pressures, quintessence energy density and anisotropic parameter of the model. Using the observational data of Vela X- 1 ( radius=7.07 km), 4U1608- 52 (radius=10 km), we have plotted the energy density, pressure and quintessence density at center $r = 0$ to the boundary of the corresponding star. All this results have been shown in figure 1-5. The first and second derivatives of density and pressures shown in figures 7, indicate that these quantities have maximum values at the center and minimum values at boundary. The graphical behavior of quintessence density $\rho_q$ does not change in this case when star satisfy the Karmarkar condition, but there occur a deviation in numerical values 4. The constraint on the EoS parameter is given by $0 < \omega_r < 1$ (as shown in figure 6) which is in agreement with normal matter distribution. We have investigated that for our model $\Delta > 0$ ( as shown in figure 8) and a repulsive force due to anisotropy results to the formation of more massive stars. The proposed model satisfy the energy conditions, as an example we have shown in figure 9 that these conditions are satisfied. We have shown that $v_{st}^2 < v_{sr}^2$ (see figure 10), hence our model is potentially stable. The range of surface redshift $Z_s$ for the compact star candidate Vela X-1 is given in table 1.

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