Canadian Science Publishing | Canadian Journal of Physics

Quintessence Compact Stars with the Vaidya Tikekar type g_rr for anisotropic fluid

Journal:	*Canadian Journal of Physics*
Manuscript ID	cjp-2019-0596.R1
Manuscript Type:	Article
Date Submitted by the Author:	09-Dec-2019
Complete List of Authors:	Abbas, G.; The Islamia University of Bahawalpur Pakistan, Mathematics Shahzad, M.R.; The Islamia University of Bahawalpur Pakistan Faculty of Science, Mathematics
Keyword:	General Relativity, Quintessence field, Stability, Compact star, Killing vectors
Is the invited manuscript for consideration in a Special Issue? :	Not applicable (regular submission)

Quintessence Compact Stars with the Vaidya Tikekar type g_{rr} for anisotropic fluid

G. Abbas[1] *and M. R. Shahzad [1†]
[1] Department of Mathematics The Islamia University of Bahawalpur, Bahawalpur, Pakistan.

Abstract

The present study provides a new solution to the Einstein field equations for anisotropic matter configuration in static and spherically symmetric space-time. By taking benefit from the conformal Killing vector (CKV) technique and quintessence field specified by a parameter ω_q as $-1 < \omega_q < -\frac{1}{3}$, we generate an exact solution to the field equations. For this investigation we have used a specific form of metric potential taken from the Vaidya Tikekar (J Astrophys Astron 3:325, 1982) geometry. To canvass the physical plausibility of presented solution, we explored some analytical expressions such as energy conditions, TOV equation, stability analysis and equation of state parameters. We present graphical analysis of necessary analytical expressions which revealed that our presented solution satisfy the necessary physical conditions.

Keywords: Compact stars, Conformal motion, Quintessence field.
PACS: 04.70.Bw; 04.70.Dy.

1 Introduction

During the last century, General theory of relativity becomes a substantial tool for explaining and understanding the gravitational systems. Particularly, in relativistic astrophysics, it is a significant issue to achieve a regular solution for the interior of compact astrophysical objects. Intuitively, high density objects of different types such as quark stars, white dwarfs and neutron stars are considered as compact objects, which is the last stage of stellar evolutionary phase. Therefore, in modern astrophysics, to study such compact objects and properties of dense matter is one of the important issues. In fact, different authors illustrated the difficulties in connection to construct the theoretical models of astrophysical compact objects for inclusive description of their interior configuration. Thus with in the dense cores, the

*abbasg91@yahoo.com
†rezwan.chaudhery@gmail.com

understanding of particle physics and description of compact stars necessitated for the search of more realistic solutions to the field equations. The inclusion of charge, an equation of state, bulk viscosity, pressure anisotropy, multilayered fluids and consideration of different symmetries lend to the espial of many exact solutions describing the interior of the relativistic compact stars in the static limit [1, 2, 3, 4, 5]. It is noteworthy to notice here that by modifying the stipulation of perfect fluid and permitting the pressure anisotropy and charge with in the interior matter configuration of stellar distribution, provides measurable and observable properties related to the interior of compact stars. Bowers and Liang [6] initiated the search for the anisotropic compact objects and then numerous researchers have divulged the effects of pressure anisotropy on the interior of the relativistic compact objects [7, 8, 9, 10, 11, 12, 13]. The inclusion of charge leads to the modification of the Buchdahl limit [14] and other interior properties of compact object whereas pressure anisotropy results in the large surface red-shift [15, 16]. Pant and his collaborators [17]-[22] also studied the effect of charge and anisotropy on the interior configuration of celestial objects. In theoretical particle physics, the linear equation of state $p = \alpha\rho$ has been generalized from observations. A wide range of exact solutions to the field equations have been propounded by taking benefit from so called MIT bag model, in which the equation of state read as $p = \alpha(\rho - 4B)$, where B is bag constant [23, 24, 25]. These solutions predicted the radii and masses of compact stars which are consistent with the observational constraints. The recent evidences about the accelerating expansion of our universe, stimulated the researchers to incorporate the dark energy, which is a suitable candidate to interpret this accelerated expansion. Thus the subject of dark energy has gained a considerable attention of researchers during the last few decades. To incorporate the dark energy, researchers extended the scope of α in $p = \alpha\rho$ (called dark energy equation of state) to include $\alpha < 0$, where α refers to as dark energy parameter. For accelerated expansion α should satisfy $\alpha < -\frac{1}{3}$ whereas for quintessence field it should be constrained as $-1 < \alpha < -\frac{1}{3}$. The value of $\alpha < -1$ refers to as phantom regime which has a particular property of infinitely increasing energy density. For $\alpha = -1$, dark energy equation of state takes the form $p = -\rho$ describing the equation of state of the shell of a Gravastar [26, 27]. Bhar [28] formulated the mathematical model of compact stars by using the equation of state for the quintessence field. Her obtained model is consistent with the observed values. In the same year (2015), Bhar [29] also studied a new model for compact objects composed of the mixture of matters by taking benefit from the MIT bag model equation of state. She also compared their results with the observational values and observed the consistency which support the more realistic behavior of her presented model. Recently, we (Abbas and Shahzad) [30] have also constructed a new model of compact stars with the quintessence field in the Rastall theory of gravity. We have presented a comparative analysis of obtained results with the observational constraints and General Relativity and revealed that our model is more consistent with the observational values. Moreover, some interesting studies incorporating the dark energy equation of state (in particular quintessence field) can be found in [31]-[35]. Kumar et al. [36] studied a charge compact stars model by using the Vaidya-Tikekar [37] geometry for one of the metric potentials for isotropic fluid. Recently, Thirukkanesh et al. [38] generalized the Vaidya-Tikekar model for superdense stars by considering the specific choices of the metric potential and pressure anisotropy. *Bhar [39] has studied the compact star model using the conformal Killing vector technique in the presence of quintessence field but actually ignored the contribution of quintessence field. Indeed, in the present study, we have studied the effects of the quintessence field in the modeling of compact objects.*

In the present study, motivated by the above discussion and ever growing interest of the dark matter, we develop a new mathematical model for compact celestial objects in the quintessence field by considering the conformal motion in the Vaidya-Tikekar geometry. Several physical investigations have been done to observe the physical plausibility of the presented solution. We organize our study as: next section deals with the formulation of CKVs and the explicit expression for the gravitational potential corresponding to these vectors. In sec. 3, we formulate the field equations in the quintessence field using the CKVs generated in the previous section. In sec. 4, we will presented some physical behavior of obtained solution to analyze the necessary physical conditions and the last section contains the concluding remarks of our findings.

2 The Conformal Killing Vector (CKV)

To pursuit the natural link between geometry and matter using the field equations, an efficient approach is to use inheritance symmetry. The symmetry arises from the $CKVs$ is usually considered as inheritance symmetry. Although, one of the important features of the CKV technique is that, the partial differential equations, which are highly nonlinear can be converted to the ordinary differential equations and hence can be dealt easily. The conformal Killing equation has the form:

$$L_\zeta g_{ij} = \zeta_{i;j} + \zeta_{j;i} = \Upsilon g_{ij}, \tag{1}$$

In the above equation L represents the Lie derivative of g_{ij}, depicting the interior gravitational field of the celestial object with respect to the vector field ζ and Υ denoting the conformal factor. It is worthwhile to discuss here that, even with the static metric, ζ and Υ need not to be static [40, 41, 42]. However in the present context, we used the static ζ and Υ [43, 44]. Moreover, the above equation provides killing vector for $\Upsilon = 0$, and for $\Upsilon = constant$, Eq. (1) yields homothetic vector, whereas for $\Upsilon = \Upsilon(x, t)$ in Eq. (1) produces conformal vectors. It can also be noticed here that for $\Upsilon = 0$, the considered spacetime becomes asymptotically flat, inferring that the Weyl tensor will also vanishes. Thus the $CKVs$ offer a profound understanding about spacetime geometry.

Now, to generate explicit gravitational potential with the aid of eq. (1), we consider a static and spherically symmetric space-time as

$$ds^2 = -e^{\mu(r)}dt^2 + e^{\lambda(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2), \tag{2}$$

where $\mu(r)$ and $\lambda(r)$ represents the gravitational potential functions. Manipulation of eq. (1) along with eq. (2) generates the following relations

$$\zeta^1 \mu' = \Upsilon, \tag{3}$$

$$\zeta^1 = \frac{\Upsilon r}{2}, \tag{4}$$

$$\zeta^4 = B_1, \tag{5}$$

$$\zeta^1 \lambda' + 2\zeta^1{}_{,1} = \Upsilon, \tag{6}$$

B_1 is a constant.
Where 1 and 4 represent r and t coordinates respectively. The above four equations provide the following relations

$$e^{\mu} = B_2^2 r^2. \tag{7}$$

$$e^{\lambda} = (\frac{B_3}{\Upsilon})^2, \tag{8}$$

$$\zeta^i = B_1 \delta_4^i + (\frac{\Upsilon r}{2})\delta_1^i. \tag{9}$$

Here B_2 and B_3 are integrating constants.

3 Field Equations and their solution

In this section, we formulate the Einstein field equations in the presence of quintessence like field defined by the parameter ω_q satisfying $-1 < \omega_q < -\frac{1}{3}$. Thus the field equations take the following form:

$$G_{\iota\kappa} = K(T_{\iota\kappa} + \tau_{\iota\kappa}). \tag{10}$$

here $\tau_{\iota\kappa}$ is the energy momentum tensor of quintessence field, $K = \frac{8\pi G}{c^4}$ is a coupling constant and $G_{\iota\kappa}$ is the Einstein tensor. It has been argued by the Kiselev [45] that the components of the tensor $\tau_{\iota\kappa}$ need to fulfill the condition of linearity and additivity. Thus Corresponding to the different signatured line element, the components of $\tau_{\iota\kappa}$ can be written as

$$\tau_t^t = \tau_r^r = \rho_q, \tag{11}$$

$$\tau_\theta^\theta = \tau_\phi^\phi = \frac{3\omega_q + 1}{2}\rho_q, \tag{12}$$

here ω_q is the quintessence parameter with $-1 < \omega_q < -\frac{1}{3}$.
The energy momentum tensor for the normal matter can be obtained as:

$$T_{\iota\kappa} = (\rho + p_r)u_\iota u_\kappa + p_t g_{\iota\kappa} + (p_r + p_t)\zeta_\iota \zeta_\kappa \tag{13}$$

Here u_ι vector is the fluid 4-velocity and η^ι is the space like vector which is perpendicular to u_ι which satisfying the constraints $u^\iota u_\kappa = -\zeta^\iota \zeta_\kappa = 1$ and $u^\iota \eta_\kappa = 0$. p_t, p_r and ρ are respectively transverse pressure , radial pressure and energy density. Thus by using the relativistic units $G = c = 1$ and from eq. (10) the Einstein field equation can be written as:

$$e^{-\lambda}\left(\frac{\lambda' r - 1}{r^2}\right) + \frac{1}{r^2} = 8\pi(\rho + \rho_q), \tag{14}$$

$$e^{-\lambda}\left(\frac{\mu' r + 1 - e^{\lambda}}{r^2}\right) = 8\pi(p_r - \rho_q), \tag{15}$$

$$\frac{e^{-\lambda}}{2}\left(\frac{\mu'}{2}(\mu' - \lambda) + \frac{1}{r}(\mu' - \lambda) + \mu''\right) = 8\pi[\frac{\rho_q}{2}(3\omega_q + 1) + p_t], \tag{16}$$

The manipulation of eqs. (7)-(9) with eqs. (14)-(16) provide the following set of equations.

$$-\frac{2\Upsilon\Upsilon'}{rB_3^2} + \frac{1}{r^2} - \frac{\Upsilon^2}{r^2 B_3^2} = 8\pi(\rho + \rho_q), \tag{17}$$

$$\frac{3\Upsilon^2}{r^2 B_3^2} - \frac{1}{r^2} = 8\pi(p_r - \rho_q), \tag{18}$$

$$\frac{2r\Upsilon\Upsilon' + \Upsilon^2}{B_3^2 r^2} = 8\pi\left(\frac{\rho_q}{2}(3\omega_q + 1) + p_t\right). \tag{19}$$

The above system of equations contains three independent equations having five unknowns viz. Υ, ρ_q, p_t, p_r and ρ which we have to solve simultaneously to attain the required result. As it is difficult task to achieve an exact solution to the Einstein field equations due to their high nonlinearity. Therefore, we reduce the number of unknown functions by considering the well studied form of metric potential [37]

$$e^{\lambda} = \frac{1 - k\frac{r^2}{R^2}}{1 - \frac{r^2}{R^2}}, \tag{20}$$

where k and R be the parameters characterizing the geometry of the star. This ansatz of Vaidya-Tikekar is well motivated physically and has been used by different authors to study the relativistic compact objects, Tikekar [46] and Maharaj and Leach [47] have studied the same ansatz for uncharged superdense star whereas Kumar et al. [48] have used this ansatz for charged superdense star. Thus by using this ansatz model has been facilitated in an interesting geometric mode by deviating from sphericity of 3-space geometry. It may be notice here that the metric potential confine the geometry of 3-dimensional hypersurface ($t = const.$) to be spheroidal and this hypersurface turn into spherical for $k = 0$ and provides a Schwarzschild interior solution, whereas the hypersurface ($t = const.$) becomes flat for $k = 1$. Thus, here $k < 1$ and the metric potential is non-singular at the center and is well-behaved for $r < R$.

Using eqs.(8) and (20), we obtain the value of Υ as

$$\Upsilon^2 = \frac{B_3^2(r^2 - R^2)}{kr^2 - R^2}. \tag{21}$$

Using eq. (21) in (17)-(19) we get the following system of equations

$$\frac{(k-1)(kr^2 - 3R^2)}{(R^2 - kR^2)^2} = 8\pi(\rho + \rho_q), \tag{22}$$

$$\frac{2R^2 + r^2(k-3)}{r^2(R^2 - kr^2)} = 8\pi(p_r - \rho_q), \tag{23}$$

$$\frac{1}{r^2}\left(\frac{r^2 - R^2}{kr^2 - R^2}\right) + \frac{2R^2(k-1)}{(R^2 - kr^2)^2} = 8\pi\left(p_t + (\frac{3\omega_q + 1}{2})\rho_q\right). \tag{24}$$

Again note that from (22) -(24), we have three independent equations containing four unknowns namely, matter density, radial pressure, transverse pressure and quintessence density, i.e., ρ, p_r, p_t and ρ_q respectively .
Now to solve the above system of equations, we consider a usual linear equation of state, i.e., radial pressure (p_r) is proportional to the matter density (ρ):

$$p_r = \beta\rho, \tag{25}$$

here β is the equation of the state parameter satisfying the constraint $0 < \beta < 1$.
Using the above equation of state along with the eqs.(22)-(24), one can get the explicit relations for the matter variables as follows:

$$\rho = \frac{1}{8\pi(\beta+1)}\left(\frac{r^2(k-3) - 2R^2}{r^2\left(kr^2 - R^2\right)} + \frac{(k-1)\left(kr^2 - 3R^2\right)}{\left(kr^2 - R^2\right)^2}\right) \tag{26}$$

$$p_r = \frac{\beta}{8\pi(\beta+1)}\left(\frac{r^2(k-3) - 2R^2}{r^2\left(kr^2 - R^2\right)} + \frac{(k-1)\left(kr^2 - 3R^2\right)}{\left(kr^2 - R^2\right)^2}\right) \tag{27}$$

$$\rho_q = \frac{(k-1)\left(kr^2 - 3R^2\right)}{8\pi\left(kr^2 - R^2\right)^2} - \frac{1}{8\pi(\beta+1)}\left(\frac{r^2(k-3) - 2R^2}{r^2\left(kr^2 - R^2\right)} + \frac{(k-1)\left(kr^2 - 3R^2\right)}{\left(kr^2 - R^2\right)^2}\right) \tag{28}$$

$$\begin{aligned} p_t &= \frac{R^2(k-1)}{4\pi\left(kr^2 - R^2\right)^2} + \frac{r^2 - R^2}{8\pi r^2\left(kr^2 - R^2\right)} - \frac{(k-1)\left(3\omega_q + 1\right)\left(kr^2 - 3R^2\right)}{16\pi\left(kr^2 - R^2\right)^2} \\ &+ \frac{(k-1)\left(3\omega_q + 1\right)\left(kr^2 - 3R^2\right)}{16\pi(\beta+1)\left(kr^2 - R^2\right)^2} + \frac{(3\omega_q + 1)(2R^2 + r^2(k-3))}{16\pi r^2(\beta+1)(kr^2 - R^2)}. \end{aligned} \tag{29}$$

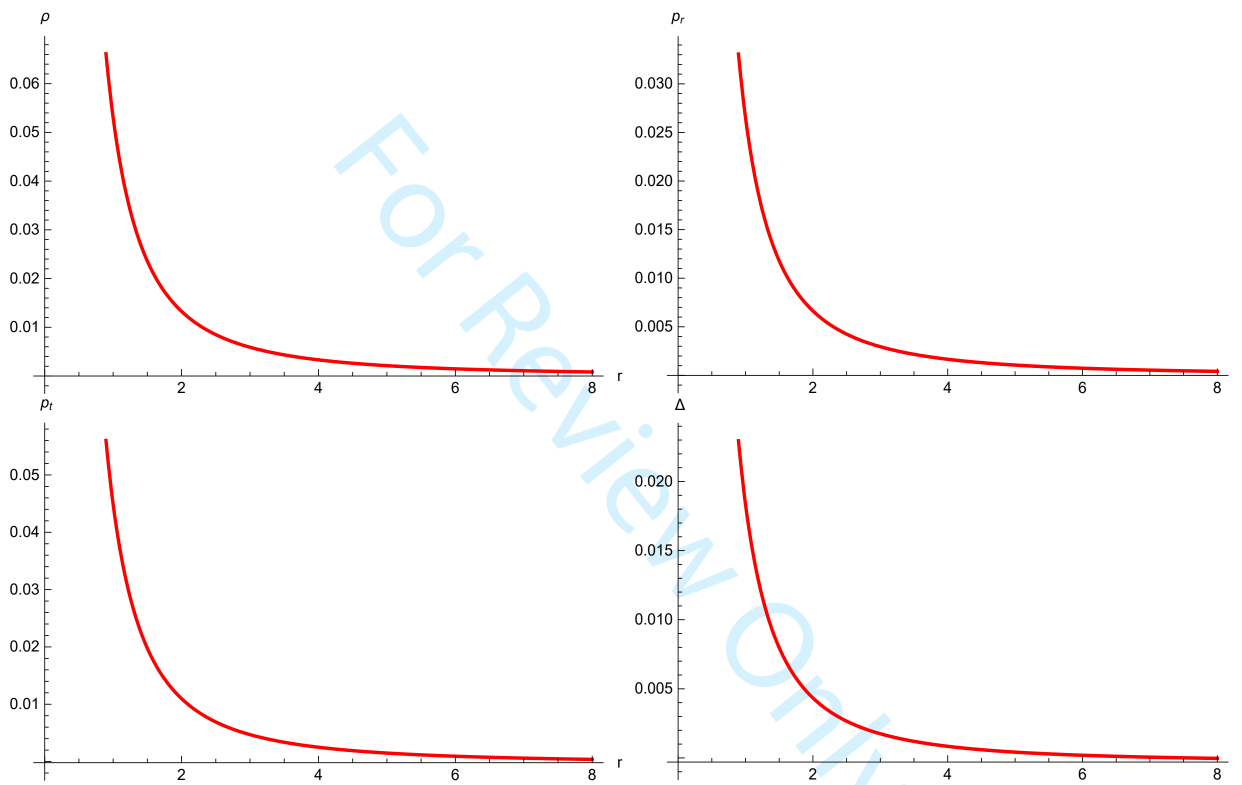

Figure 1: Variation of density, Pressure, and anisotropy. In the present study we take the values of parameters as: $R = 19$, $k = -0.049$, $\beta = 0.5$, such that the obtained solution remain physically acceptable.

4 Some physical features

In this section, we will describe some physical aspects of the presented solution to check the physical plausibility. In this connection we will compute some analytical expressions to examine the physical features analytically as well as graphically.

4.1 Evolution of Energy density and pressure

For a physically acceptable solution, energy density and pressure (p_r and p_t in the present case) should be positive throughout the configuration of the stellar object and monotonically decrease towards the surface. To observe the graphical behavior of density and pressure, We have plotted these function in Fig. 1 which revealed that pressure and density remains positive throughout the configuration and gradually diminish towards the surface. Moreover, we have shown the graphical nature of quintessence energy density in Fig. 3 which is increasing from center to surface with negative value.

4.2 Energy Equation

For any physically plausible configuration of the compact object some conditions depending on the connections between the pressure (anisotropic in the present case) and energy density must be satisfied within the entire region of the sphere. These relationship between density and pressure normally termed as energy conditions including null energy conditions (NEC), weak energy conditions (WEC) and strong energy conditions (SEC). These energy conditions actually asserted that the total energy should be positive, as the negative energy would not support the stable configuration. Indeed, these energy conditions are satisfied if the following inequalities hold simultaneously:

$$\rho \geq 0. \tag{30}$$

$$\rho + p_r \geq 0. \tag{31}$$

$$\rho + p_t \geq 0. \tag{32}$$

$$\rho + p_r + 2p_t \geq 0. \tag{33}$$

To check the behavior of the above energy condition for the present system we have plotted the above inequalities in Fig. 2. Manifestly it can be seen (from Fig. 2) that all the relations between energy density and pressure defined above are positive throughout the configuration and hence the energy conditions are satisfied for our system which is a necessary condition for physically reasonable solution.

4.3 TOV Equation

In this subsection, we will check the hydrostatic equilibrium of forces acting on the system, i.e., gravitational force $\mathcal{F}_g$, hydrostatic force $\mathcal{F}_h$ and anisotropic force $\mathcal{F}_a$. In the case of

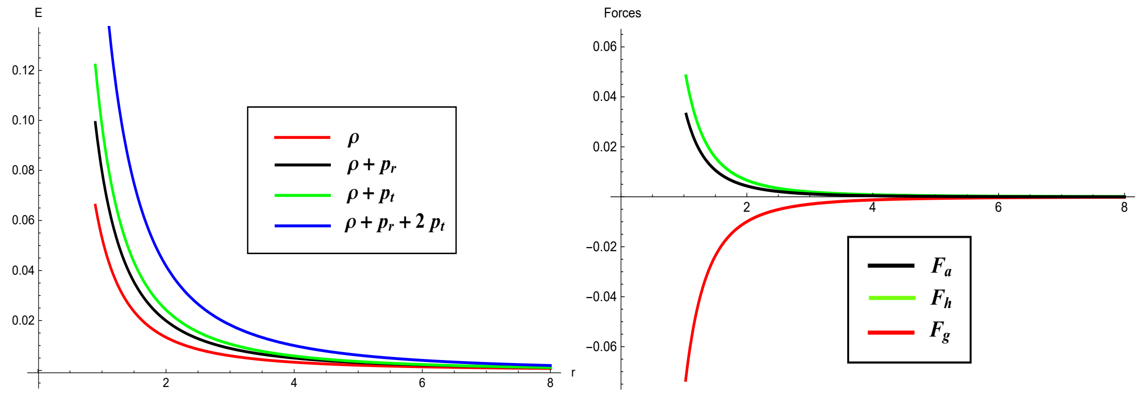

Figure 2: Variation of energy conditions (Left plot) and behavior of different forces (involved in the TOV equation) (Right plot).

uncharged and isotropic matter content this equilibrium condition has been studied by using the so called Tolman-Oppenheimer-Volkov (TOV) equation in [49, 50]. According to this study, the system will be in the equilibrium if all the forces acting on the system will cancel the effect of each other and the net force remains zero. In the present case, for an anisotropic matter content the TOV equation can be modified by using the conservation of the energy momentum tensor as:

$$-\frac{M_G(\rho+p_r)}{r^2}e^{\frac{\lambda-\mu}{2}}-\frac{dp_r}{dr}+\frac{2}{r}(p_t-p_r)=0, \tag{34}$$

here $M_G(r)$ is the gravitational mass inside the fluid sphere of radius r can be obtained as:

$$M_G^r=\frac{1}{2}r^2e^{\frac{\mu-\lambda}{2}}\mu'. \tag{35}$$

The above expression can be generate from the Tolman-Whittaker mass formula as:

$$M_G(r)=\int_0^r 4\pi r^2(T_0^0-T_1^1-T_2^2-T_3^3)e^{\frac{(\lambda+\mu)}{2}}dr \tag{36}$$

plugging the eq. (35) in (34), we get the modified form of TOV equation.

$$\mathcal{F}_g+\mathcal{F}_h+\mathcal{F}_a=0, \tag{37}$$

where the explicit form of the above forces can be defined as

$$\mathcal{F}_g=-\frac{\mu'}{2}(p_r+\rho). \tag{38}$$

$$\mathcal{F}_h=-\frac{dp_r}{dr}. \tag{39}$$

$$\mathcal{F}_a = -\frac{2}{r}(p_r - p_t). \tag{40}$$

The evolution of the above forces has been sown in Fig. 2 (right graph). We observed that the gravitational force $\mathcal{F}_g$ counterbalanced the combined effect of anisotropic and hydrostatic forces, i.e., $\mathcal{F}_a$ and $\mathcal{F}_h$ respectively. Thus under the combined effect of three forces our system attains hydrostatic equilibrium (see Fig. 2).

4.4 Stability Analysis

4.4.1 Stability via Sound speed

To check the stability of the presented solution, we firstly used the causality condition and secondly the cracking concept due to Herrera [51] and Abreu et al. [52]. Causality concept demands that the squared sound speeds (transverse and radial in the present context) should be positive and remain less than the speed of light (In the present context, we have used the relativistic units in which the speed of light is unity i.e., $c = 1$) throughout the stellar body. It means that, ν_{st}^2 and ν_{sr}^2 should be consistent with the constraints, $0 < \nu_{st}^2 = \frac{dp_t}{d\rho} \leq 1$ and $0 < \nu_{sr}^2 = \frac{dp_r}{d\rho} \leq 1$ respectively. To check the range of the squared sound speeds for causality, we have plotted their explicit relations in Fig. 3 (upper row) which eminently show that squared sound speeds remain in the causality limit for the present solution which is an important criteria to check the stability. Moreover, according to Abreu et al. [52] investigations regarding the stability, the sound speed ν_{st}^2 and ν_{sr}^2 should satisfy the constraint $|\nu_{st}^2 - \nu_{sr}^2| < 1$, for a physically reasonable model. For the present study, Fig. 3 (lower graph) shows that the mentioned inequality hold good within the entire configuration indicating that there is no cracking and our system remains stable through the radius. Thus the present solution is consistent with two important restrictions for stability.

For the present anisotropic configuration, squared radial velocity (ν_{sr}^2) and squared transverse velocity (ν_{st}^2) of sound can be formulated as:

$$\nu_{sr}^2 = \frac{dp_r}{d\rho} = \beta = 0.5 < 1. \tag{41}$$

$$\nu_{st}^2 = \frac{f_1(r)}{4\left(k^2r^6 + (1-4k)kr^4R^2 + 3kr^2R^4 - R^6\right)},$$

where

$$\begin{aligned} f_1(r) &= 3\omega_q\left(k^2r^6(k(-\beta)+k+\beta-3) + kr^4R^2(k(5\beta+3)-5\beta+3) - 6kr^2R^4 + 2R^6\right) \\ &\quad -2R^4\left(3kr^2+\beta+1\right) + kr^2R^2\left(r^2(13(k-1)\beta+11k-5)+6(\beta+1)\right) \\ &\quad +kr^4(kr^2(k(-\beta)+k+\beta-3)-4(\beta+1)) + 2R^6 \end{aligned} \tag{42}$$

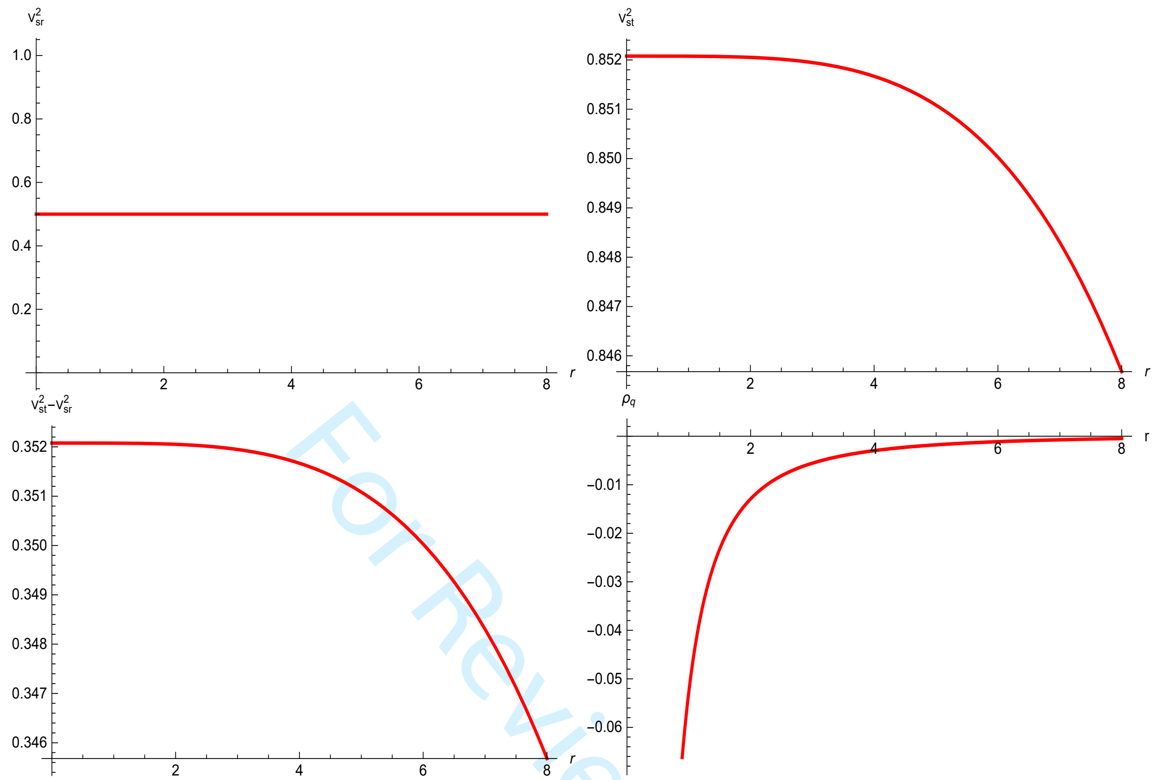

Figure 3: Sound velocities and quintessence density.

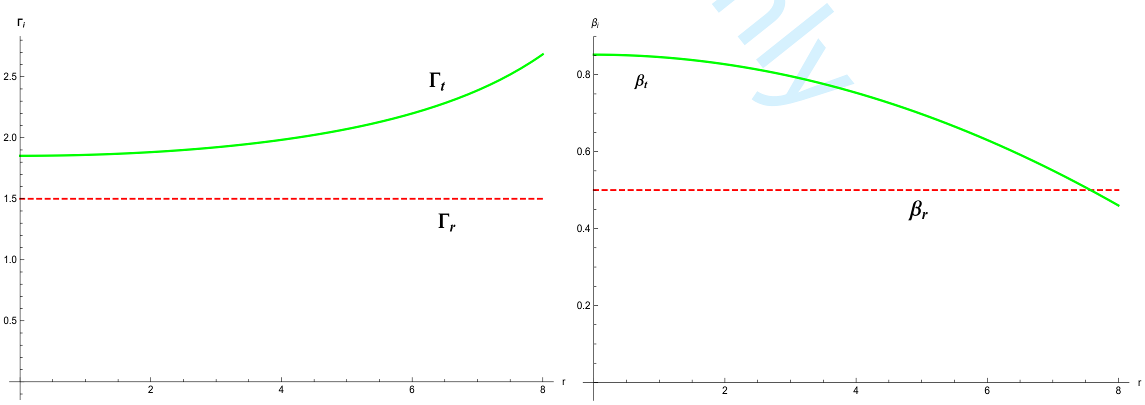

Figure 4: Evolution of adiabatic indices (Left Plot) and equation of state parameters (Right Plot)

4.4.2 Stability via Adiabatic Index

An other condition to check the dynamical stability of the system corresponding to the radial adiabatic perturbation is suggested by Chandrasekhar in [53]. According to that criteria the adiabatic index Γ should be greater than $\frac{4}{3}$. In the presence of anisotropy, Γ can be split into Γ_r and Γ_t, radial adiabatic index and tangential adiabatic index respectively. For a dynamically stable anisotropic and relativistic system Chan et al. [54] and Heinzmann [55] suggested that these adiabatic indices should be $> \frac{4}{3}$. Indeed, the adiabatic index denotes the ratio of two specific heats defined as:

$$\Gamma_r = \nu_{sr}^2 \left(\frac{\rho + p_r}{p_r} \right) \tag{43}$$

$$\Gamma_t = \nu_{st}^2 \left(\frac{\rho + p_t}{p_t} \right) \tag{44}$$

To check the validity of this condition for stability, we graphically evaluate the explicit expressions of Γ_r and Γ_t in Fig. 4 (Left Plot). It can be observed that both expressions meet the condition of dynamical stability, i.e., $\Gamma_r > \frac{4}{3}$ and $\Gamma_t > \frac{4}{3}$. Thus our solution fulfill another important condition for stability.

4.5 Equation of State (EoS)

The barotropic equation of state in its simplest form can be expressed as $p_i = \beta_i \rho$, where β_i is EoS parameter corresponds to tangential and radial directions. Thus in the present study for anisotropic matter configuration, both tangential and radial EoS parameters can be defined as:

$$\beta_t = \frac{p_t}{\rho}, \qquad \beta_r = \frac{p_r}{\rho} \tag{45}$$

For any physically acceptable solution, the values of these parameters remain in the range $(0, 1)$. We have shown the evolution of these parameter in Fig. 4 (Right Plot) and observed that the values of β_r and β_t are consistent with the interval $(0, 1)$ and also the matter content in our consideration is non-exotic in nature [56].

4.6 Measure of Anisotropy

One of the important features for the physical solution is the nature of anisotropy which can be measure as $\Delta = p_t - p_r$ and obtained as follows:

$$\Delta = \frac{f_2(r)}{16\pi(\beta+1)r^2 \left(R^2 - kr^2\right)^2}, \tag{46}$$

$$\begin{aligned} f_2(r) \quad &= 3\omega_q \left(kr^4(k(-\beta)+k+\beta-3)+r^2R^2(3(k-1)\beta+k+3)-2R^4\right) \\ &+kr^4(-(k+3)\beta+k-3)+r^2\left(R^2(15k\beta+5k-7\beta-1)-2(\beta+1)\right) \\ &+2R^2\left(-(2\beta+1)R^2+\beta+1\right) \end{aligned}$$

For positive Δ, i.e., $p_r < p_t$, the force produced due to anisotropy will be repulsive and directed outward, whereas for negative Δ, i.e., $p_r > p_t$, the anisotropic force will be attractive in nature and inward directed. We present the profile of anisotropy for our system in **Fig.1** (Lower right plot), which illustrates the positive behavior of Δ and produces repulsive anisotropic force, which support the configuration of more massive object [57].

5 Conclusion

Motivated from the recent evidences about the accelerated expansion of our cosmos, many researchers have devoted their attention to develop the dark energy models during the last few decades, which is a suitable candidate to explain this phenomenon. In this sequence, in the present study, we develop a new exact solution of Einstein field equations in the Vaidya-Tikekar geometry in the presence of quintessence field characterized by the parameter ω_q constrained as $-1 < \omega_q < -\frac{1}{3}$. For this investigation, we have taken benefit from the conformal Killing vector technique and have used a specific form of the metric potential. We have explored some matter variables to inspect their behavior analytically as well as graphically. Some salient features of the present analysis are given below:

- **Density and pressure:** From Fig.1, we observe that as radius approaches to 0, pressure and density blow up which infer that the core of the star is highly compact and the presented solution is valid outside the core of the star. We could not find the surface density as there is no cut on the r-axis (radius of the star) in the plot of the radial pressure whereas density and pressure (both transverse and radial) remains positive within the entire region of the star.

- **Energy conditions:** In Fig.2 (Left plot), we have shown that the relationship between pressure (radial and transverse in the present case) and density defined in eqs. (30)-(33) are satisfied for our present system which is also an important condition for physically plausible solution.

- **TOV equation:** Our system also attained the hydrostatic equilibrium which we have studied by using Tolman-Oppenheimer-Volkoff equation. It can be seen that there are three forces involved in the system namely, gravitational ($\mathcal{F}_g$), hydrostatic ($\mathcal{F}_h$) and anisotropic ($\mathcal{F}_a$). We observed that the combined effect of repulsive forces including hydrostatic $\mathcal{F}_h$ and anisotropic $\mathcal{F}_a$ has been counterbalanced by a single attractive force $\mathcal{F}_g$. Thus the net effect of applied forces on the system vanishes and hence the presented solution maintains the equilibrium (see Fig.2 (Right plot)).

- **Stability:** For the present solution, we have analyzed the stability via. causality condition, cracking concept and adiabatic index. From Fig.3 (upper row) it can be seen that the quantities ν_{sr}^2 and ν_{st}^2 remain in the limit $[0, 1]$ which confirm the causality condition and the inequality $|\nu_{st}^2 - \nu_{sr}^2| \leq 1$ hold good which confirm that there is no cracking (lower left plot). Moreover, we have examined from Fig.4 (Left plot) that the adiabatic indices Γ_r and Γ_t remain greater than $\frac{4}{3}$ which also indicate the stability of the presented solution.

- **Anisotropy:** We have analyzed the nature of the anisotropy factor Δ in Fig.1 (2nd row right plot). One can see that $\Delta > 0$ throughout the configuration, which show that the anisotropic force produced due to the anisotropy is repulsive and support the formation of more ultra-compact object.

- **Equation of State:** We inspected the demeanor of equation of state parameters corresponding to the transverse and radial directions β_t and β_r respectively (Fig.4 (Right plot)). We studied that β_t and β_r remain within the limit (0,1), indicating that the matter content is non-exotic in nature, which is another indication for good behavior of presented solution.

References

[1] R.L. Bowers, E.P.T. Liang, Astrophys. J. 188, 657(1974)

[2] J.D. Bekenstein, Phys. Rev. D. 4, 2185(1971)

[3] S.D. Maharaj, R. Maartens, Gen. Relativ. Gravit. 21, 899(1989)

[4] R. Tikekar, K. Jotania, Int. J. Mod. Phys. D. 14, 1037(2005)

[5] M. Esculpi et al., Gen. Relativ. Gravit. 39, 633(2007)

[6] R.L. Bower, E.P.T. Liang, Astrophys. J. 188, 657(1974)

[7] S.K. Maurya, A. Banerjee, P. Channuie, Chin. Phys. C 42, 055101(2018)

[8] S.K. Maurya et al., Ann. Phys. 385, 532(2017)

[9] S. Thirukkanesh, F.C. Ragel, Chin. Phys. C 41, 015102(2017)

[10] G.Abbas, M.Tahir, Adv. High Ener. Phys. 2018, 1 (7420546)(2018)

[11] G. Abbas, H. Nazar, Eur. Phys. J. C 78, 510(2018)

[12] G. Abbas, H. Nazar, Eur. Phys. J. C 78, 957(2018)

[13] F. Rahaman et al., Eur. Phys. J. C 72, 2071(2012)

[14] H. Andreasson, J. Phys.: Conference Series, 189, 012001(2009)

[15] P. Bhar, Astrophys. Space Sci. 356, 309 (2015)

[16] K.N. Singh, N. Pant, Astrophys. Space Sci. 358, 44(2015)

[17] N. Pant, N. Pradhan and M. H. Murad, Astrophys. Space Sci. 355, 137(2015)

[18] N. Pant, N. Pradhan, R. K. Bansal, Astrophys. Space Sci. 361, 41(2016)

[19] S. Gedela, N. Pant, J. Upreti, R. P. Pant, Eur. Phys. J. C 79, 566(2019)

[20] R. P. Pant, S. Gedela, R. K. Bisht, N. Pant, Eur. Phys. J. C 79, 602(2019)

[21] S. Gedela, R. K. Bisht, and N. Pant, Eur. Phys. J. A 54, 207(2018)

[22] S. Gedela, R. K. Bisht, N. Pant, Mod. Phys. Lett. A, 34, (20), 1950157(2019)

[23] P. Takisa Mafa, S. Ray, S. D. Maharaj, Astrophys. Space Sci. 350,733(2014)

[24] S.A.Ngubelanga, S.D.Maharaj, S.Ray,Astrophys. Space Sci. 357, 74(2015)

[25] M. Govender, S. Thirukkanesh, Astrophys. Space Sci. 358, 16 (2015)

[26] P., Mazur, Mottola, E.: arXiv:gr-qc/0109035 (2001). Report number:LA-UR-01-5067

[27] P., Mazur, Mottola, E.: Proc. Natl. Acad. Sci. USA 101, 9545(2004)

[28] P. Bhar, Astrophys Space Sci 356, 365(2015)

[29] P. Bhar, Astrophys Space Sci 357, 46(2015)

[30] G. Abbas and M.R. Shahzad, Eur. Phys. J. A 54, 211(2018)

[31] P. Saha, U. Debnath, Eur. Phys. J. C 79, 919(2019)

[32] M. Kalam, F, Rahaman, S. Molla, S. Monowar Hossein, Astrophys Space Sci 349, 865(2014)

[33] G. Abbas, M. Zubair, G. Mustafa, Astrophys. Space Sci. 358, 26(2015)

[34] P. Saha, U. Debnath, Adv. High Energy Phys. 2018, 3901790(2018)

[35] Francisco S. N. Lobo, Class. Quantum Grav. 23, 1525(2006)

[36] J. Kumar, A.K. Prasad, S.K. Maurya, A. Banerjee, Eur. Phys. J. C 78, 540(2018)

[37] P.C. Vaidya, R. Tikekar, J. Astrophys. Astron. 3, 325 (1982).

[38] S. Thirukkanesh, R. Sharma, and S. D. Maharaj, Eur. Phys. J. Plus 134, 378(2019)

[39] P. Bhar, Eur. Phys. J. C 75, 123(2015)

[40] Bohmer, C.G., Mussa, A., Tamanini, N.: Class. Quantum Gravity 28, 245020 (2011)

[41] Mafa Takisa, P., Maharaj, S.D., Manjonjo, A.M., Moopanar, S.: Eur. Phys. J. Plus 77, 713 (2017)

[42] Manjonjo, A.M., Maharaj, S.D., Moopanar, S.: Class. Quantum Gravity 35, 045015 (2018)

[43] A. Das, F.Rahaman, B.K.Guha, S.Ray, Eur. Phys. J. C 76, 654(2016)

[44] G. Abbas M.R. Shahzad, Astrophys. Space Sci.363, 251(2018)

[45] V.V. Kiselev, Class. Quantum Gravit. 20, 1187 (2003)

[46] R. Tikekar, J. Math. Phys. 31, 24542458 (1990)

[47] S.D. Maharaj, P.G.L. Leach, J. Math. Phys. 37, 430437 (1996)

[48] J. Kumar, A.K.Prasad, S.K. Maurya, A. Banerjee, Eur. Phys. J. C 78, 540(2018)

[49] R.C. Tolman, Static Solutions of Einsteins Field Equations for Spheres of Fluid, Phys. Rev. 55, 364(1939).

[50] J.R. Oppenheimer, G.M. Volkoff, On Massive Neutron Cores, Phys. Rev. 55, 374(1939).

[51] L. Herrera, Phys. Lett. A 165, 206 (1992).

[52] H. Abreu, H. Hernandez, L.A. Nunez, Class. Quantum Grav. 24, 4631(2007).

[53] S. Chandrasekhar, Astrophys. J. 140, 417 (1964)

[54] R. Chan, L. Herrera, N. O. Santos, Mon. Not. R. Astron. Soc. 265, 533 (1993)

[55] H. Heintzmann, W. Hillebrandt, Astron. Astrophys. 24, 51 (1975)

[56] D. Shee, F. Rahaman, B.K. Gupta, S. Ray, Astrophys.Space Sci. 361, 167 (2016)

[57] M.K. Gokhro, A.L. Mehra, Gen. Relativ. Gravit. **26**, 75(1994)