

N O T I C E

THIS DOCUMENT HAS BEEN REPRODUCED FROM
MICROFICHE. ALTHOUGH IT IS RECOGNIZED THAT
CERTAIN PORTIONS ARE ILLEGIBLE, IT IS BEING RELEASED
IN THE INTEREST OF MAKING AVAILABLE AS MUCH
INFORMATION AS POSSIBLE

NASA Contractor Report 156864

Radar Altimeter Mean Return Waveforms from Near-Normal-Incidence Ocean Surface Scattering

(NASA-CR-156864) RADAR ALTIMETER MEAN
RETURN WAVE FORMS FROM NEAR-NORMAL-INCIDENCE
OCEAN SURFACE SCATTERING Final Report
(Applied Science Associates, Inc., Apex, N.
C.) 27 p HC A03/MF A01

N80-22662

Unclas
46868
CSCI 2CN G3/35

George S. Hayne

April 1980

NASA

National Aeronautics and
Space Administration

Wallops Flight Center

Wallops Island, Virginia 23337
AC 804 824-3411

NASA Contractor Report 156864

**Radar Altimeter Mean Return Waveforms
from Near-Normal-Incidence Ocean Surface
Scattering**

George S. Hayne

Applied Science Associates, Inc.
105 East Chatham Street
Apex, North Carolina 27502

Prepared Under Contract No. NAS6-2810

NASA

National Aeronautics and
Space Administration

Wallops Flight Center

Wallops Island, Virginia 23337
AC 804 874 3111

I. INTRODUCTION

Two recent NASA earth-orbiting satellites, GEOS-3 and SEASAT-1, have carried pulsewidth-limited radar altimeters with provision for sampling a number of points in the individual radar return waveforms¹. For these and similar systems, the mean return waveform is [1] the convolution of: A) the average impulse response of the quasi-calm sea surface, B) the sea surface elevation distribution, and C) the radar system point-target response (transmitted pulse as affected by transmitter and receiver bandwidths;. The first term, A, includes effects of antenna beamwidth and off-nadir pointing angle; "quasi-calm" emphasizes that an incoherent surface scattering process is assumed but that the sea surface elevation distribution is separately written in B in the above convolution.

A number of papers have described the extraction of ocean significant waveheight (SWH) from altimeter waveform samples [2-7]. SWH is directly related to the second moment of the surface elevation distribution; there is also a preliminary altimeter measurement [5] of the surface skewness, related to the third moment of the surface elevation distribution. All of these papers use the same basic procedure: a specific probability distribution form (usually a simple Gaussian) is assumed for the ocean

This work was supported by the National Aeronautics and Space Administration, Wallops Flight Center, under Contract NAS6-2810.

¹ GEOS-3 had 16 waveform sample-and-hold (S&H) gates at 6.25 nanosecond spacing, while SEASAT-1 had 60 S&H gates at 3.125 ns spacing.

surface, amplitude and timing biases are removed from waveform sampler data if necessary, a suitable waveform sample averaging time is chosen, and the average sampled mean return waveform is "best-fitted" by a theoretical template through some process of varying the template parameters until some typical least-squares error criterion is satisfied². In several cases [3,7] differences between adjacent sampled waveform values are fitted to the derivative of the theoretical return waveform, but this is a minor modification of the basic template-fitting procedure. In the general template-fitting, either a simplified form is assumed for the different convolution terms so that the final expression will be simple and suited to easy computer implementation and fast computer running time, or a more general, more complete template is used at the expense of having to do several numerical convolutions for each sample point within the linefitting process.

This paper presents general expressions for the radar mean return waveform for the case in which the ocean surface elevation distribution and the radar system point-target response can both be represented by a modified Gaussian form including skewness and kurtosis terms, and the antenna pattern can be assumed to be Gaussian in angle; these expressions give the mean return waveform at the times of interest in the region of

² One exception to the template-fitting procedure is described in [8] in which Priester and Miller describe work intended to estimate the surface height probability density function without the necessity of assuming a specific functional form; this method is difficult to implement and interpret and will not be considered for purposes of this paper.

the risetime portion ("ramp portion") of the waveform for small off-nadir angles. These general expressions should be useful in either waveform parameter estimation from experimental data, or for waveform generation as part of design studies of system trade-offs for future radar altimeters. Comparable results can be obtained from numerical convolution but the expressions in this paper have the strong practical advantages of reducing computational time and of avoiding the detailed questions of sample density and round-off problems which should be part of any numerical convolution modeling study.

II. EVALUATION OF MEAN RETURN WAVEFORM

This work is based on Brown's 1977 paper [1] which reviewed the assumptions and limitations of the convolutional model for near-normal-incidence rough-surface backscattering of short-pulse radar waveforms. The general square-law-detected waveform $W(t)$ is given by the convolution

$$W(t) = P_{FS}(t) * q_s(t) * s_r(t) \quad (1)$$

where $P_{FS}(t)$ is the average flat-surface impulse response, $q_s(t)$ is related to the surface elevation probability density of scattering elements ("specular points"), and $s_r(t)$ is the radar system point target response which includes the transmitted pulse shape. Each of the three terms in (1) will be separately discussed; then through a small-argument series expansion for Brown's $P_{FS}(t)$, the $W(t)$ of (1) will be expressed as a power series for which the first four terms are evaluated in this paper.

A. Average Flat Surface Impulse Response Function

For cases of practical interest for satellite-carried radar altimeters, the flat surface impulse response is given by Brown's equation (9) as

$$P_{FS}(t) = A \exp(-\delta t) I_0(t^{1/2} \beta) U(t) \quad (2)$$

in which

$$\delta = (4/\gamma)(c/h) \cos |2\xi| \quad (3)$$

and

$$\beta = (4/\gamma)(c/h)^{1/2} \sin |2\xi|. \quad (4)$$

In (2), $U(t)$ is a unit step function, $I_0(t^{1/2} \beta)$ is a modified Bessel function, and ξ is the off-nadir pointing angle. In (3) and (4), c is the speed of light, h is the satellite altitude, and γ is an antenna beamwidth parameter defined as in Brown's equation (4) by a Gaussian approximation to the antenna gain of the form

$$G(\theta) = G_0 \exp[-(2/\gamma) \sin^2 \theta] \quad (5)$$

If θ_w is the usual antenna beamwidth, i.e., the angular full-width at half-power points, (3) and (4) can be written as

$$\delta = \frac{\ln 4}{\sin^2(\theta_w/2)} (c/h) \cos |2\xi| \quad (6)$$

and

$$\beta = \frac{\ln 4}{\sin^2(\theta_w/2)} (c/h)^2 \sin|2\xi|. \quad (7)$$

The amplitude term, A, in (2) actually contains a number of different constants;

$$A = \frac{G_0^2 \lambda^2 c \sigma^o(0)}{4(4\pi)^2 L_p h^3} \exp\left(-\frac{4}{\gamma} \sin^2 \xi\right) \quad (8)$$

where λ = radar wavelength,

$\sigma^o(0)$ = ocean surface backscattering cross section at normal incidence,

G_0 = radar antenna gain at nadir,

and L_p = two-way propagation loss.

In (8), the possible variation of σ^o with angle of incidence (or, equivalently, with increasing time in the return waveform) has been ignored; this is a reasonable assumption for sea surface scattering in the short-pulse (≤ 20 nanoseconds pulsewidth) radar altimeters considered here. Because radar return signals are normalized by a typical altimeter's AGC system, we will ignore all individual terms within the A in (8), and will use A as just a simple amplitude scaling term.

B. Radar-Observed Surface Elevation Density Function

Equation (1) includes a term $q_s(t)$ which is the surface specular

point density function written in the altimeter's time domain; the conversion factor from surface elevation measurements in meters to two-way ranging time in nanoseconds is $c/2 = -0.15$ m/ns. The negative sign indicates that an increase in ranging time corresponds to a decrease in surface elevation³. This paper makes the usual assumption that the surface specular point density function is identical to the true (geometric) surface elevation density function, but the possibility of a difference between the radar-observed and the true geometric elevation densities must remain an open issue for future work⁴.

³ This change of sign is important only for odd moments. Specifically, when a template-fitting procedure is used on satellite sampled waveform data to obtain a (time-domain) estimate of the skewness contribution from the surface, the desired surface elevation skewness will have the same magnitude but opposite sign from the time-domain-estimated skewness component.

⁴ Radar measurements by Yaplee et al. [15] can be interpreted [16] as showing that the mean of the scattering distribution is displaced downward from the true water level by 20% of the rms waveheight and recent theoretical work by Jackson [17], assuming infinitely long-crested waves, is in agreement with these measurements. However, preliminary results from the aircraft-borne Surface Contour Radar experiment [18] indicate that this bias may be in range of 4-8% of the rms waveheight with more measurements currently underway [19].

In previous work the ocean surface elevation density function $q_s(t)$ has been assumed to be skewed Gaussian form given in the time domain by

$$q_s(t) = \frac{1}{\sqrt{2\pi}\sigma_s} \left[1 + \frac{\lambda_s}{6} H_3(\tau/\sigma_s) \right] \exp[-\frac{1}{2}(t/\sigma_s)^2], \quad (9)$$

where σ_s is a risetime, λ_s is the skewness, and H_3 is a Hermite polynomial. Pierson and Mehr [9] discussed this form for radar altimeter analyses; this is a low order case of a general probability function by Longuet-Higgins [10] for a random variable that is weakly nonlinear. The first six Hermite polynomials for a general argument z are:

$$H_1(z) = z,$$

$$H_2(z) = z^2 - 1,$$

$$H_3(z) = z^3 - 3z,$$

(10)

$$H_4(z) = z^4 - 6z^2 + 3,$$

$$H_5(z) = z^5 - 10z^3 + 15z,$$

and $H_6(z) = z^6 - 15z^4 + 45z^2 - 15.$

The σ_s in (9) is related to the significant waveheight (SWH) by

$$SWH = 4(c/2)\sigma_s \quad (11)$$

in which the $c/2$ converts from ranging time to surface elevation, and

the factor of 4 is from the definition of SWH as four times the rms waveheight; since SWH is related to a second moment, there is no sign change from the time-domain-observed to the surface elevation quantity. Most of the radar altimeter SWH measurements to date have been based on the pure Gaussian resulting from setting $\lambda_s = 0$, but (9) is the general form used in attempts to recover a surface skewness [5].

Equation (9) is the result from taking the first two terms only in the general Gram-Charlier series [11]. Recent work by Huang and Long [12], based on laboratory measurements of the surface elevation density function for a wind-generated wave field, suggests that (9) is an inadequate form for the surface elevation density, and that it is necessary to use the four-term series given by

$$q_s(t) = \frac{1}{\sqrt{2\pi} \sigma_s} \left[1 + \frac{\lambda_s}{6} H_3(t/\sigma_s) + \frac{\kappa_s}{24} H_4(t/\sigma_s) + \frac{\lambda_s^2}{72} H_6(t/\sigma_s) \right] \exp[-\frac{1}{2}(t/\sigma_s)^2], \quad (12)$$

where κ_s is the kurtosis with other quantities as already defined. The general waveform result to be derived in the following portions of this paper will be based on (12), but λ_s^2 terms will be kept separate from the λ_s terms so that the final results can easily be converted to a surface elevation density of form (9) instead of (12) if desired.

C. Effect of Radar System Point Target Response

The radar system point target response $s_r(t)$ is primarily the transmitted radar pulse shape but also includes effects of the bandwidths of the transmitter and receiver. When the system point target response is nearly Gaussian, the same general form can be used as already discussed for the surface elevation density,

(13)

$$s_r(t) = \frac{1}{\sqrt{2\pi} \sigma_r} \left[1 + \frac{\lambda r}{6} H_3(t/\sigma_r) + \frac{\kappa_r}{24} H_4(t/\sigma_r) + \frac{\lambda r^2}{72} H_6(t/\sigma_r) \right] \exp[-\frac{1}{2}(t/\sigma_r)^2].$$

In this case, the convolution in (1) of $q_s(t) * s_r(t)$ can immediately be written as

$$B(t) = q_s(t) * s_r(t) = \frac{1}{\sqrt{2\pi} \sigma} \left[1 + \frac{\lambda}{6} H_3(t/\sigma) + \frac{\kappa}{24} H_4(t/\sigma) + \frac{\lambda^2}{72} H_6(t/\sigma) \right] \exp[-\frac{1}{2}(t/\sigma)^2], \quad (14)$$

in which σ , λ , and κ (with no subscripts) are the composite risetime, skewness, and kurtosis; these composite quantities can easily be shown to be related to the contributing surface elevation and point target quantities by

$$\sigma^2 = \sigma_s^2 + \sigma_r^2, \quad (15)$$

$$\lambda = \lambda_s (\sigma_s/\sigma)^3 + \lambda_r (\sigma_r/\sigma)^3, \quad (16)$$

$$\text{and} \quad \kappa = \kappa_s (\sigma_s/\sigma)^4 + \kappa_r (\sigma_r/\sigma)^4. \quad (17)$$

In a practical radar altimeter, the waveform sampling gates will be positioned by a range tracker having its own range-noise (jitter) characteristics, and the probability density function of the tracker jitter must be included as an additional term in the convolutional description of the mean return waveform. For this paper the jitter will be assumed negligible in comparison to the other terms⁵.

⁵ Usually when the jitter can not be treated as negligible, it can be described by a near-Gaussian form with its own σ_j , λ_j and κ_j and (15), (16), and (17) will each have one additional term of the same form as the two terms already written. For instance, (15) will become $\sigma^2 = \sigma_s^2 + \sigma_r^2 + \sigma_j^2$.

D. Expansion and Solution for W(t)

The waveform W(t) is given by the convolution of the P_{FS}(t) of (2) with the B(t) of (14),

$$\begin{aligned} W(t) &= P_{FS}(t) * B(t) \\ &= \int_{-\infty}^{\infty} P_{FS}(z) B(t-z) dz, \end{aligned} \quad (18)$$

but the presence of the Bessel function I₀ in P_{FS}(t) prevents the easy integration of the integral in (18). For the waveform regions of interest in the case of short-pulse radar altimeters in this paper, it is possible to expand the I₀ in a small-argument series expansion, given by expression 9.6.10 in [13],

$$I_0(z) = \sum_{n=0}^{\infty} (z^2/4)^n \left(\frac{1}{n!}\right)^2 \quad (19)$$

and this will lead to a term-by-term integration in (18) with the result

$$W(t) = \frac{A}{6} \exp[-d(\tau + d/2)] \sum_{n=0}^{\infty} \left(\frac{1}{n!}\right)^2 \left(\frac{\beta^2 \sigma}{4}\right)^n C_n(t) \quad (20)$$

where

$$C_n(t) = C_{n0} + \kappa C_{n1} + \lambda^2 C_{n2} \quad (21)$$

$$C_{n0} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\tau} (\tau-z)^n \left[6 + \lambda H_3(z+d) \right] e^{-z^2/2} dz \quad (22)$$

$$C_{n1} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\tau} (\tau-z)^n \left[H_4(z+d) \right] e^{-z^2/2} dz \quad (23)$$

$$\text{and } C_{n2} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\tau} (\tau-z)^n \left[H_6(z+d) \right] e^{-z^2/2} dz, \quad (24)$$

$$\text{with } \tau = \frac{t - t_0}{\sigma} - d, \quad (25)$$

$$\text{and } d = \Delta\sigma. \quad (26)$$

Notice that an arbitrary time origin shift, t_0 , has been written in (25); in template-fitting for parameter recovery from altimeter data, t_0 is one of the parameters to be varied for the final solution since in most altimeter designs the altitude tracker moves the waveform sampler set back and forth in (time) position relative to the true mean waveform.

Carrying out the integrations indicated in (22), (23), and (24) for any given n will produce results in terms of $G(\tau)$ and $P(\tau)$, where $G(\tau)$ is the Gaussian, $G(\tau) = (2\pi)^{-1/2} \exp(-\tau^2/2)$, and $P(\tau)$ is the Probability Integral⁶; two new functions of time, D_{nm} and E_{nm} , are then defined by

$$C_{nm} = D_{nm} P(\tau) + E_{nm} G(\tau), \quad \text{for } m=0,1, \text{ and } 2. \quad (27)$$

These D_{nm} and E_{nm} have been worked out for $n = 0,1,2$, and 3 , with the following results:

⁶ Often the error function $\text{erf}(\tau)$ is written instead of $P(\tau)$. These are related by $\text{erf}(\tau) = 2P(2^{1/2}\tau) - 1$. $P(\tau)$ can be implemented in a computer through the series expansions given by expression 26.2.17 in [13].

$$\begin{aligned}
D_{00} &= 6 + \lambda d^3, \\
E_{00} &= \lambda(1 - 3d^2 - 3d\tau - \tau^2), \\
D_{01} &= d^4/4, \\
E_{01} &= (d - d^3) + \left(\frac{3}{4} - \frac{3}{2}d^2\right)\tau - d\tau^2 - \frac{\tau^3}{4}, \\
D_{02} &= d^6/12, \\
E_{02} &= \left(-\frac{3}{2}d + \frac{5d^3}{3} - \frac{d^5}{2}\right) + \left(-\frac{5}{4} + \frac{15d^2}{4} - \frac{5d^4}{4}\right)\tau \\
&\quad + (3d - \frac{5}{3}d^3)\tau^2 + \left(\frac{5}{6} - \frac{5}{4}d^2\right)\tau^3 - \frac{d}{2}\tau^4 - \frac{\tau^5}{12},
\end{aligned} \tag{28}$$

$$\begin{aligned}
D_{10} &= -3\lambda d^2 + (6 + \lambda d^3)\tau, \\
E_{10} &= (6 + 3\lambda d + \lambda d^3) + \lambda\tau, \\
D_{11} &= -d^3 + \frac{d^4}{4}\tau, \\
E_{11} &= \left(-\frac{1}{4} - 3d^2 + \frac{d^4}{4}\right) + d\tau + \frac{\tau^2}{4}, \\
D_{12} &= -\frac{d^5}{2} + \frac{d^6\tau}{12}, \\
E_{12} &= \left(\frac{1}{4} - \frac{5}{4}d^2 + \frac{5}{4}d^4 + \frac{d^6}{12}\right) + \left(-\frac{3}{2}d + \frac{5}{3}d^3\right) \\
&\quad + \left(-\frac{1}{2} + \frac{5}{4}d^2\right)\tau^2 + \frac{d}{2}\tau^3 + \frac{\tau^4}{12},
\end{aligned} \tag{29}$$

$$\begin{aligned}
D_{20} &= (6 + 6\lambda d + \lambda d^3) - 6\lambda d^2\tau + (6 + \lambda d^3)\tau^2, \\
E_{20} &= (-2\lambda - 6\lambda d^2) + (6 + \lambda d^3)\tau, \\
D_{21} &= (3d^2 + \frac{d^4}{4}) - 2d^3\tau + \frac{d^4}{4}\tau^2, \\
E_{21} &= (-2d - 2d^3) + \left(-1 + \frac{d^4}{2}\right)\tau,
\end{aligned} \tag{30}$$

$$D_{22} = \left(\frac{5}{2}d^4 + \frac{d^6}{12} \right) - d^5\tau + \frac{d^6}{12}\tau^2, \quad (30 \text{ cont'd.})$$

$$E_{22} = \left(d - \frac{10}{3}d^3 - d^5 \right) + \left(\frac{1}{2} - \frac{5}{2}d^2 + \frac{d^6}{12} \right)\tau - d\tau^2 - \frac{\tau^3}{6},$$

$$D_{30} = (-6\lambda - 9\lambda d^2) + (18 + 18\lambda d + 3\lambda d^3)\tau - 9\lambda d^2\tau^2 + (6 + \lambda d^3)\tau^3,$$

$$E_{30} = (12 + 18\lambda d + 2\lambda d^3) - 9\lambda d^2\tau + (6 + \lambda d^3)\tau^2,$$

$$D_{31} = (-6d - 3d^3) + (9d^2 + \frac{3}{4}d^4)\tau - 3d^3\tau^2 + \frac{d^4}{4}\tau^3,$$

$$E_{31} = \left(\frac{3}{2} + 9d^2 + \frac{d^4}{2} \right) - 3d^3\tau + \frac{d^4}{4}\tau^2,$$

(31)

$$D_{32} = (-10d^3 - \frac{3}{2}d^5) + \left(-\frac{15}{2}d^4 + \frac{d^6}{4} \right)\tau - \frac{3d^5}{2}\tau^2 + \frac{d^6}{12}\tau^3,$$

$$E_{32} = \left(-\frac{1}{2} + \frac{15}{2}d^2 + 10d^4 + \frac{d^6}{6} \right) + \left(3d - \frac{3d^5}{2} \right)\tau + \left(\frac{1}{2} + \frac{d^6}{12} \right)\tau^2.$$

III. SUMMARY AND EXAMPLES

In brief review, the waveform is given by (20), with the number of terms required being a function of time, pointing angle, and antenna beamwidth⁷. For each term $C_n(t)$, (21) gives $C_n(t)$ in terms of c_{n0} , C_{n1} , and C_{n2} with these three quantities in turn given by (27) which uses (28), (29), and (30), or (31) for $n=0,1,2$, or 3, respectively. This procedure may appear cumbersome and it does assume the use of a computer, but these results are very much easier to use and produce answers in far less computer time than the numerical convolutions that would otherwise be needed⁸.

⁷ For instance, a single term is adequate for the GEOS-3 altimeter waveforms at any time within the span covered by the individual sampling gates for pointing angles within 1° of nadir, while SEASAT-1 with its smaller beamwidth and larger sampling gate span in time requires the first three terms in (20) for any position within the sampling gate span and for pointing angles within 1° of nadir.

⁸ One of the uses of the waveform expansion (20) is as the template to fit to radar altimeter data. Typical non-linear iterative least-squares linefitting routines require the derivatives of the waveform expression with respect to the individual parameters as evaluated at each input sample point. Such derivatives may be estimated by numerical differences, varying each parameter concerned. Alternatively, the derivatives may be obtained by differentiating (20). While the resulting expressions are probably not of sufficient general interest to justify their appearing in this paper, they have been worked out and checked and are being used in linefitting routines by the author; these derivative expressions will be mailed upon receipt of a request.

Figures 1-6 provide examples of mean return waveforms calculated from (20) for a radar altimeter having an antenna beamwidth $\theta_w = 1.6$ degrees at a height 8×10^5 meters above the ocean and having a pure Gaussian radar system point target response of 3.125 nanoseconds full-width at $\frac{1}{2}$ height so that $\sigma_r = 1.327$ ns, and $\lambda_r = \kappa_r = 0$ in (14). These numbers are nominal SEASAT-1 values [14], and these Figures predict SEASAT-1 measured mean return waveforms for the limit of infinite averaging time⁹.

⁹ The actual SEASAT-1 mean return waveforms will differ from this paper's Figures because of the distinctly non-Gaussian sidelobe structure in the transmitted pulse shape. MacArthur, on pg. 4-50 of [14], describes an attempt to treat this sidelobe structure by increasing the effective width of the Gaussian used to represent the transmitted pulse; however, a discussion of the full consequences of the actual SEASAT-1 transmitted pulse shape is beyond the scope of this paper.

REFERENCES

1. G. S. Brown, "The Average Impulse Response of a Rough Surface and its Applications," IEEE Trans. Antennas Propagat. Vol. AP-25, No. 1, pp. 67-74, January 1977; same paper also in IEEE J. Oceanic Eng., Vol. OE-2, No. 1, pp. 67-74, January 1977.
2. C. L. Rufenach and W. R. Alpers, "Measurement of Ocean Wave Heights Using the GEOS-3 Altimeter," J. Geophys. Res., Vol. 83, No. C10, pp. 5011-5018, October 1978.
3. N. Mognard and B. Lago, "The Computation of Wind Speed and Wave Heights from GEOS-3 Data," J. Geophys. Res., Vol. 84, No. B8, pp. 3979-3986, July 1979.
4. L. S. Fedor, T. W. Godbey, J. F. R. Gower, R. Guptill, G. S. Hayne, C. L. Rufenach, and E. J. Walsh, "Satellite Altimeter Measurements of Sea State - An Algorithm Comparison," J. Geophys. Res., Vol. 84, No. B8, pp. 3991-4001, July 1979.
5. E. J. Walsh, "Extraction of Ocean Waveheight and Dominant Wavelength from GEOS-3 Altimeter Data," J. Geophys. Res., Vol. 84, No. B8, pp. 4003-4010, July 1979.
6. C. L. Parsons, "GEOS-3 Waveheight Measurements: An Assessment During High Sea State Conditions in the North Atlantic," J. Geophys. Res., Vol. 84, No. B8, pp. 4011-4020, July 1979.

7. L. S. Fedor and D. E. Barrick, "Measurement of Ocean Waveheights with a Satellite Radar Altimeter," EOS Trans. AGU, Vol. 59, No. 9, pp. 843-846, Sept. 1978.
8. R. W. Priestler and L. S. Miller, "Estimation of Significant Wave Height and Wave Height Density Function Using Satellite Altimeter Data," J. Geophys. Res., Vol. 84, No. B8, pp. 4021-4026, July 1979.
9. W. J. Pierson, Jr. and E. Mehr, "Average Return Pulse Form and Bias for the S193 Radar Altimeter on Skylab as a Function of Wave Conditions," in The Use of Artificial Satellites for Geodesy, S. W. Henrickson, et al., eds, Geophysical Monograph Series, Vol. 15, American Geophysical Union, Washington, D.C., 1972, pp. 217-226.
10. M. S. Longuet-Higgins, "The Effect of Nonlinearities on Statistical Distributions in the Theory of Sea Waves," J. Fluid Mechanics, Vol. 17, part 3, pp. 459-480, 1963.
11. M. G. Kendall and A. Stuart, The Advanced Theory of Statistics, Volume 1: Distribution Theory, 2nd Edition, Charles Griffin & Co., Ltd; London, 1963, Sections 6.17-6.22.
12. N. E. Huang and S. R. Long, "A Study of the Surface Elevation Probability Distribution Function and Statistics of Wind Generated Waves," J. Fluid Mechanics (to be published).
13. M. Abramowitz and I. A. Stegun, eds., NBS Handbook of Mathematical Functions, Dover Publications, New York, NY 1972.

14. J. L. MacArthur, "SEASAT-A Radar Altimeter Design Description," SD0-5232, Applied Physics Laboratory, Laurel, MD, November 1978.
15. B. S. Yaplee, A. Shapiro, D. L. Hammond, B. D. Au, and E. A. Uliana, "Nanosecond Radar Observations of the Ocean Surface From a Stable Platform," IEEE Trans. Geosci. Electronics, Vol. GE-9, No. 3, pp. 170-174, July 1971.
16. A. Shapiro, E. A. Uliana, and B. S. Yaplee, "Radar Pulse Shape Versus Ocean Waveheight," in Sea Surface Topography from Space, J. R. Apel, ed., NOAA Tech. Rep. ERL-AOML 7, U. S. Dept. of Commerce, Washington, D.C., 1972.
17. F. C. Jackson, "The Reflection of Impulses from a Nonlinear Random Sea", J. Geophys. Res., Vol. 84, No. C8, pp. 4939-4943, August 1979.
18. E. J. Walsh and J. E. Kenney, "Determination of the Electromagnetic Bias at Nadir for 36 GHz Using the Surface Contour Radar," paper presented at URSI/National Radio Science Meeting, Boulder, CO, November 6-9, 1978.
19. E. J. Walsh (private communication).

Figure 1. Mean Return Waveforms in Idealized SEASAT-1 Radar Altimeter at 1.0° Pointing Angle, Showing Effects of Number of Terms in Waveform Expansion

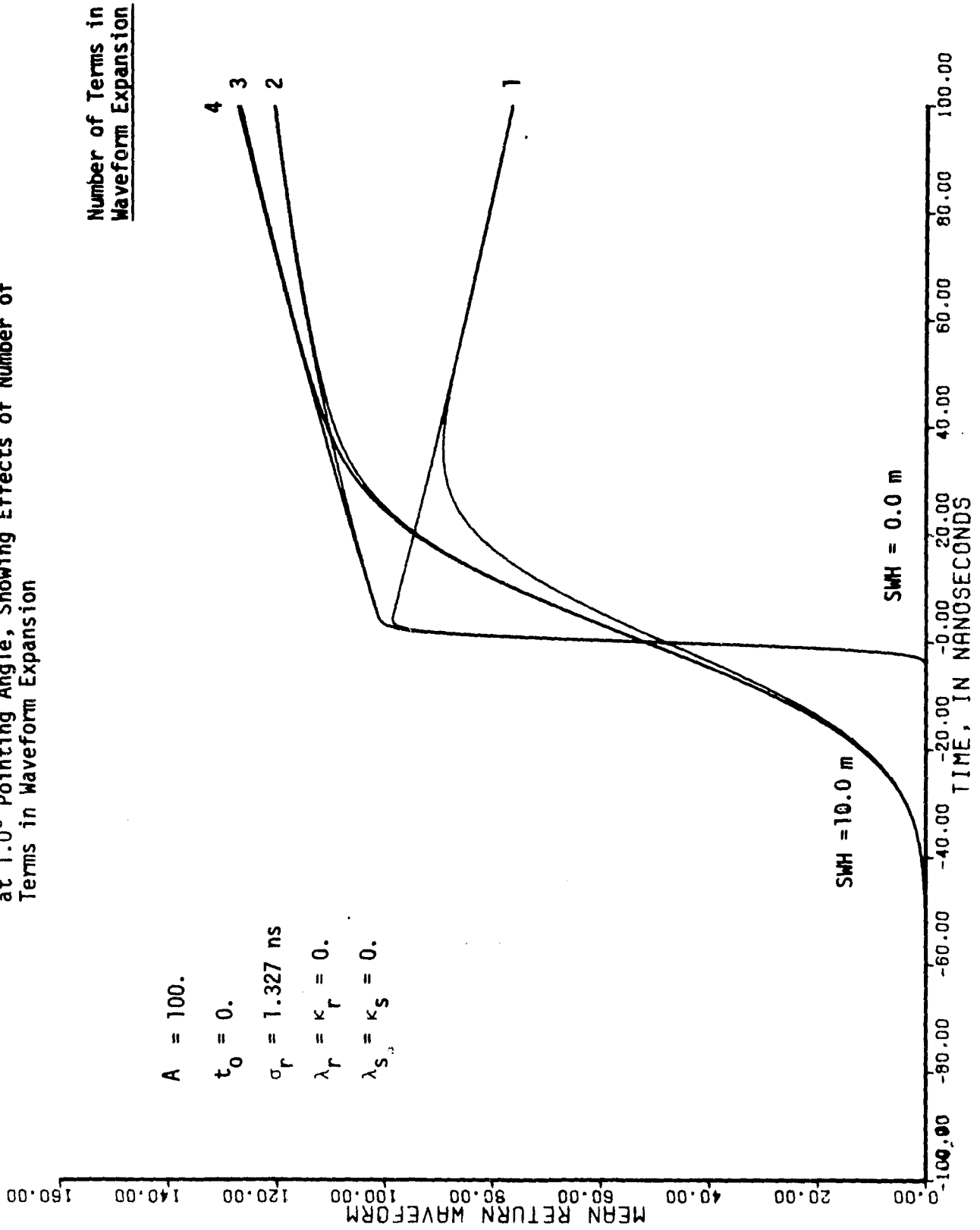


Figure 2. Mean Return Waveforms in Idealized SEASAT-1 Radar Altimeter at 0.4° Pointing Angle, Showing Effects of Number of Terms in Waveform Expansion.

$A = 100.$
 $t_0 = 0.$
 $\sigma_r = 1.327 \text{ ns}$
 $\lambda_r = \kappa_r = 0.$
 $\lambda_s = \kappa_s = 0.$

Number of Terms in
Waveform Expansion

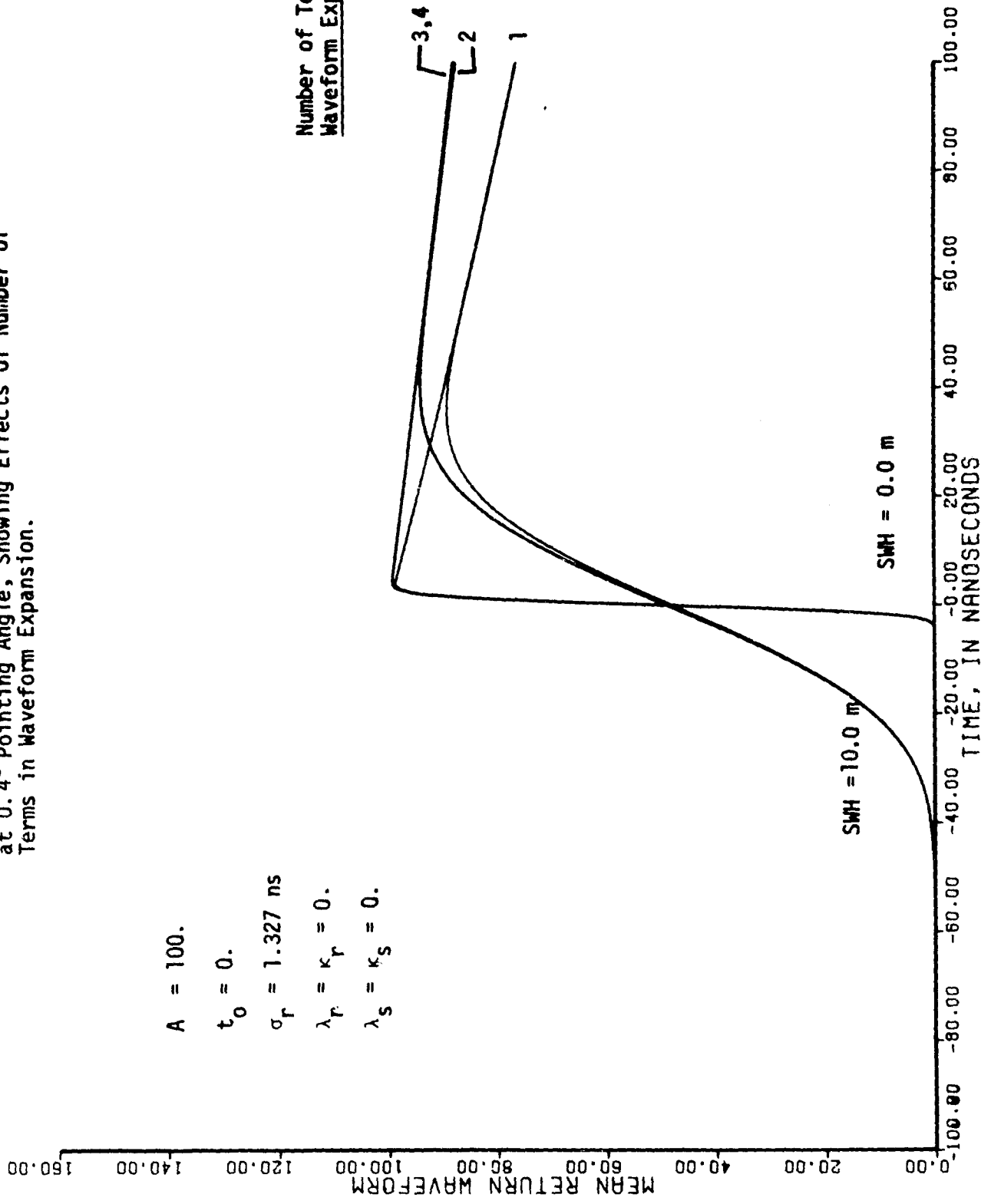
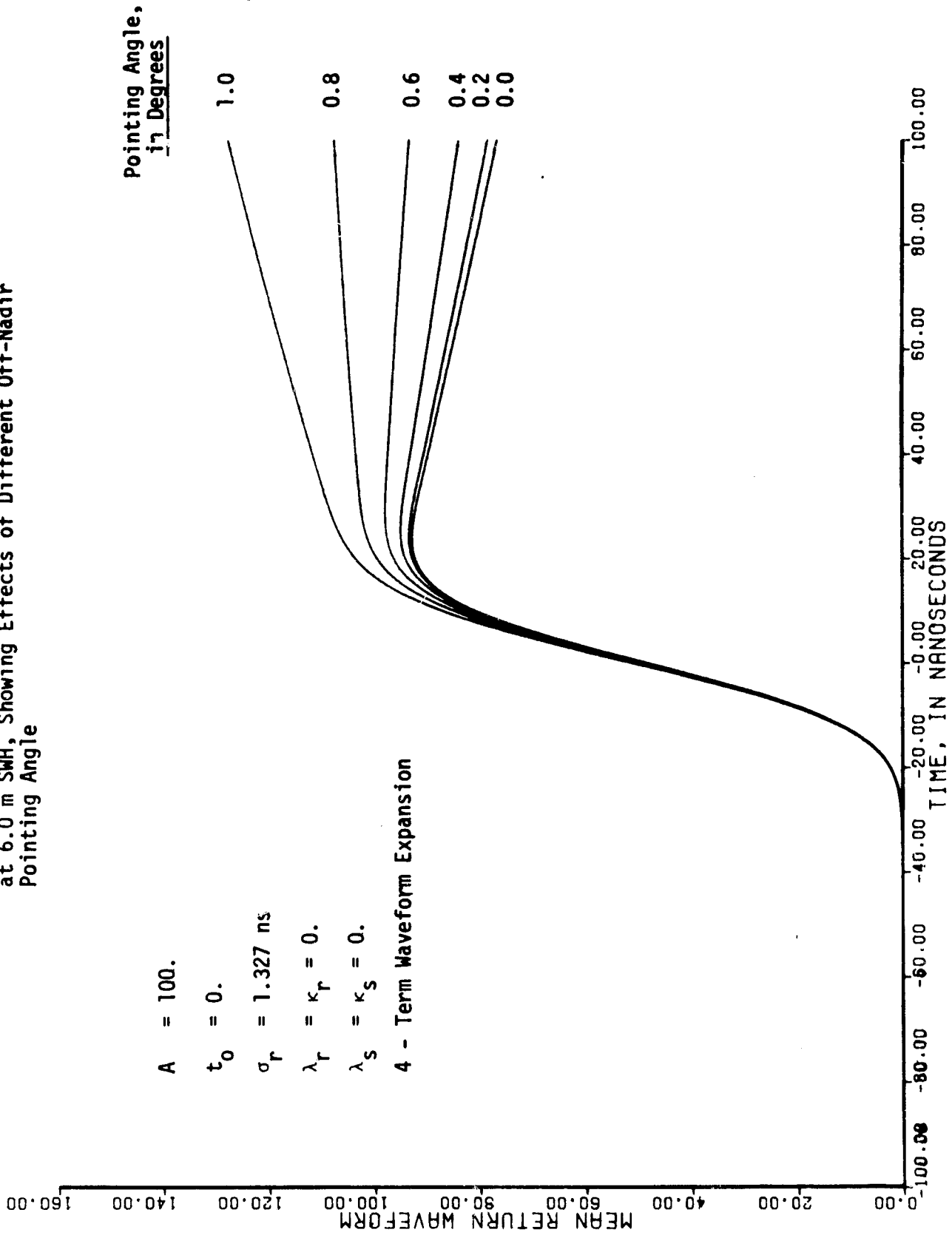


Figure 3. Mean Return Waveform in Idealized SEASAT-1 Radar Altimeter at 6.0 m SWH, Showing Effects of Different Off-Nadir Pointing Angle



$A = 100.$
 $t_0 = 0.$
 $\sigma_r = 1.327 \text{ ns}$
 $\lambda_r = \kappa_r = 0.$
 $\lambda_s = \kappa_s = 0.$

4 - Term Waveform Expansion

Figure 4. Mean Return Waveforms in Idealized SEASAT-1 Radar Altimeter at 0.0° Pointing Angle, Showing Effects of Different Ocean Significant Waveheight

$A = 100.$
 $t_0 = 0.$
 $\sigma_r = 1.327 \text{ ns}$
 $\lambda_r = \kappa_r = 0.$
 $\lambda_s = 0.$

4 - Term Waveform Expansion

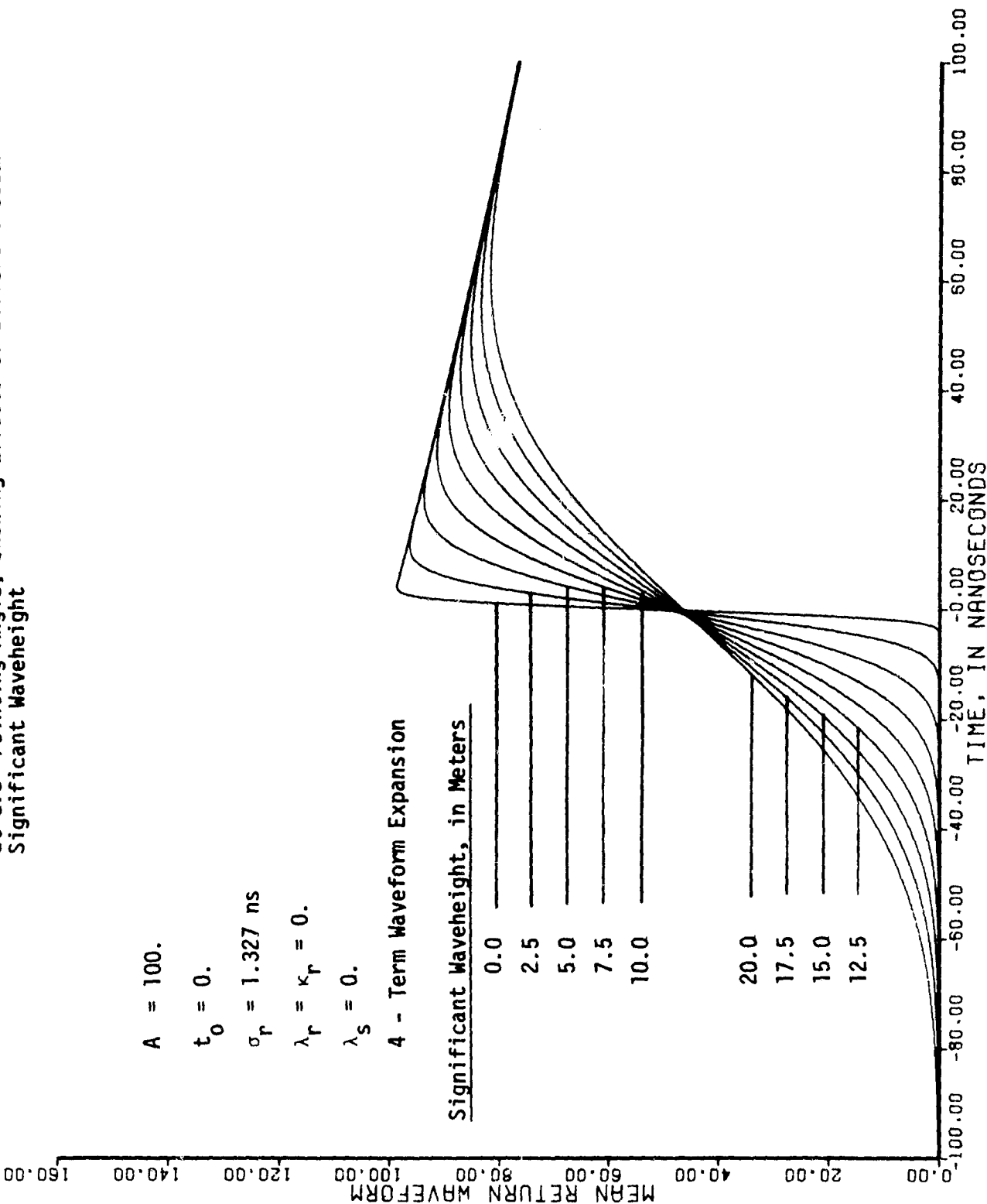


Figure 5. Mean Return Waveforms in Idealized SEASAT-1 Radar Altimeter at 6.0 m SWH, 0.0° Pointing Angle, Showing Effects of Skewness in Surface Elevation Probability Density Function

$A = 100.$

$t_0 = 0.$

$\sigma_r = 1,327 \text{ ns}$

$\lambda_r = \kappa_r = 0.$

$\kappa_s = 0.$

4 - Term Waveform Expansion

Curve a surface skewness $\lambda_s = 0.$

b $\lambda_s = -0.5$

c $\lambda_s = +0.5$

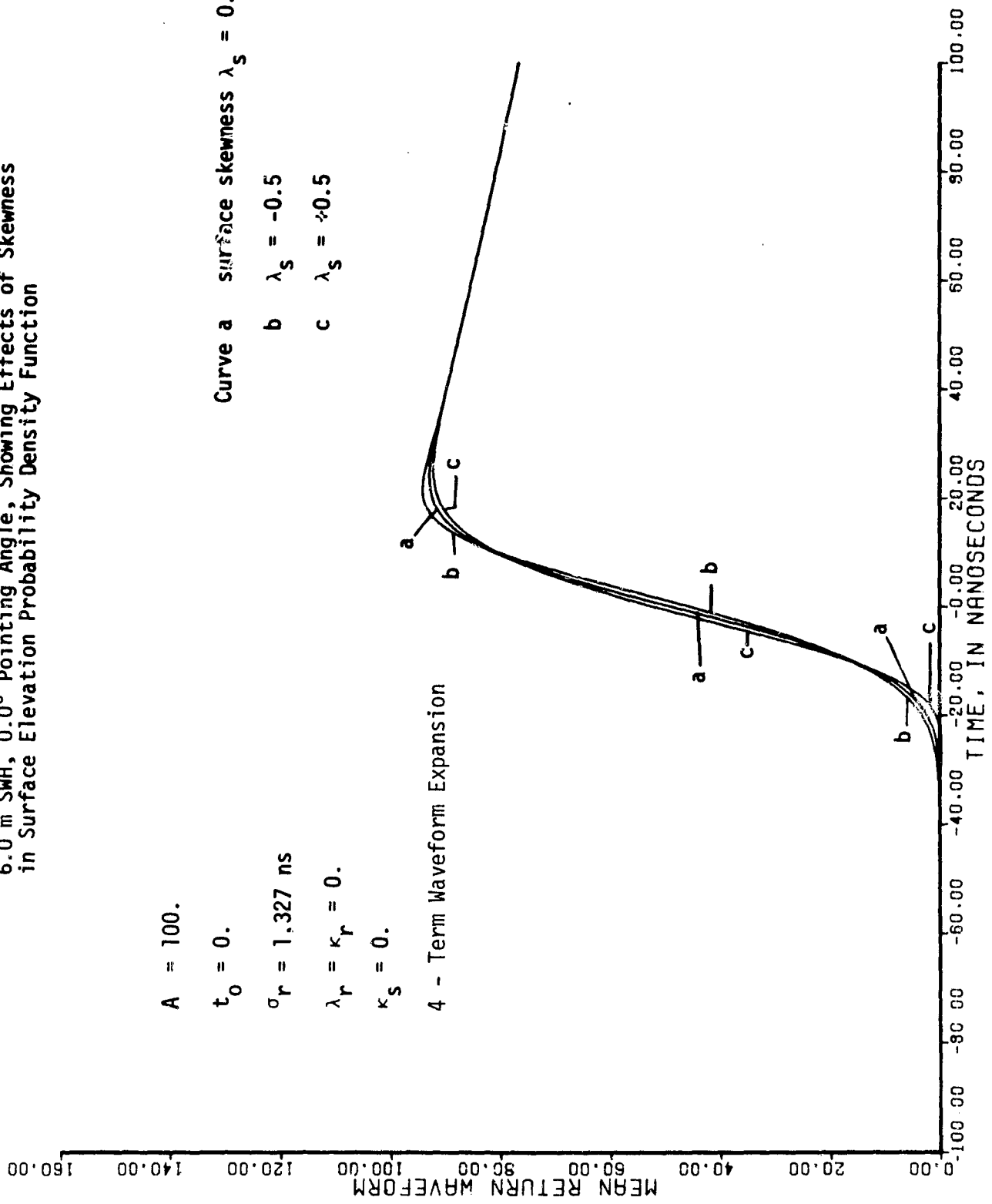


Figure 6. Mean Return Waveforms in Idealized SEASAT-1 Radar Altimeter at 6.0 m SWH, 0.0° Pointing Angle, Showing Effects of Including Skewness Squared Terms in Surface Elevation Probability Density Function

$A = 100.$
 $t_0 = 0.$
 $\sigma_r = 1.327 \text{ ns}$
 $\lambda_r = \kappa_r = 0.$
 $\kappa_s = 0.$

4 - Term Waveform Expansion

