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Radar Measurement of Rainfall With and Without Polarimetry

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1. INTRODUCTION

In two prior papers [Atlas and Ulbrich (2006) and Ulbrich and Atlas (2007), (AU06 and UA 07, respectively)] we introduced the Gamma Parameter Diagram as a means of characterizing the drop size distribution (DSD) in convective storms. Such storms are typically comprised of convective (C), transition (T) and stratiform (S) stages. Stage C commonly features nearly uniform or equilibrium DSD with narrow width with near constant median volume diameter (D_0). A notable feature of an equilibrium DSD is that the number concentration $N(D)=Rf(D)$ where R is rain rate and $f(D)$ is the form of the spectrum. This means that all the moments of $f(D)$ are constant and independent of R. One of the main consequences is that the radar reflectivity factor Z, which is proportional to the 6th moment, is linearly related to R; i.e $Z=AR^b$ where b=1. So too are other integral parameters linearly related to one another.

Oddly enough the large collection of Z-R relations in the literature [Battan (1973), Rosenfeld and Ulbrich (2003)] show only a rare value of b=1. Only Fujiwara (1965) shows a few cases of $b\approx1$ in convective storms. This is due to the failure of most investigators to divide convective rains by their C, T, and S stages.

Using data gathered during TOGA COARE, Tokay et al (1999) did classify the stages into C and S. However they included the transition stage within C, or within a “mixed convective-stratiform class” thus failing to find equilibrium DSDs. Their average values of Z and D_0 (median volume or mass diameter) and μ (the shape parameter in the gamma DSD) are also considerably smaller than those found by UA07. Subsequently, Atlas et al (1999) used the C, T, S classification in the same experiment and found three days in which $b\approx1$ during the C stages. They also showed that the coefficient A is proportional to $(D_0)^{2.33}$. Well formed equilibrium DSDs were found on 01/17/93 **(**Atlas and Ulbrich, 2000)**.**

The primary thrusts of this paper are: 1) to demonstrate that the rain rate can be determined accurately from measurements of Z_H and Z_{DR} not only for equilibrium DSDs, but for any DSD; and more particularly, that there is a distinct difference between the $R/Z\text{-}Z_{DR}$ relations for stage C rains in tropical continental and maritime storms according to the breadth of the DSD and/or number concentration; 2) because of the latter DSD differences in convective storms and their sharp contrast with those of stratiform storms [Bringi et al. (2003), hereafter BAL03; UA 07], it is possible to estimate rainfall from the climatological values of number concentration and Z alone, thus providing a capability to use a conventional radar without polarimetric capability until the latter are more broadly available.

2. BACKGROUND

In what follows we shall refer to the gamma function fit to the DSD given by

$$N(D)=N_0D^{\mu}exp\text{-}(\Lambda D) \qquad (1)$$

where D is drop diameter, μ is the shape parameter (inversely related to the normalized breadth of the DSD) and Λ is the slope of the tail. Also,

$$D_0 = (3.67 + \mu)\Lambda \qquad (2)$$

and

$$N_w=[\,(4)^4/(\pi\rho_w)]\;[W/(D_m)^4] \qquad (3)$$

where N_W is the generalized number concentration of an exponential DSD having the same liquid water content W and mass weighted diameter as the actual DSD (Testud et al, 2001).

This work uses the theoretical relation between the differential reflectivity Zdr and the median volume diameter D_0 derived by Bringi

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and Chandrasekar, 2001 (hereafter BC] which can be expressed in the form

$$Z_{DR} = 0.295(D_o)^{2.058} \qquad (4)$$

Z_{DR} has been calculated for each of the size spectra for the four storms considered here and empirical Z_{DR}-D_0 fits have been performed for each of them. The results (not shown) demonstrate that the latter equation is very well satisfied for each of the four storms over the full range of observed D_0. A dependence on the parameter μ is implicit as shown in Fig. 1 where Z_{DR} is plotted vs D_0 with μ as a parameter. Eq. (4) (dash) is shown in Fig. 2. It is seen that as D_0 increases Eq.(4) crosses lines of constant μ toward smaller values. In the range $1<D_0<2$ mm Z_{DR} initially follows the curve for μ=4; for $2.0<D_0<3.0$, Z_{DR} lies between curves for μ=3 and $\mu\approx1$. Although these ranges of μ may not seem appreciable, the changes in Z_{DR} are large.

It will be shown in Sect. 3 that the dependence of *R, Z_H,* and Z_{DR} on both D_0 and μ produces a plot of Z_{DR} versus *R/Z* which involves isopleths of μ. Nevertheless it will be seen that for certain classes of storms, data for Z_{DR} and R/Z_H (from disdrometer measurements) lie along just two distinct isopleths. This implies that knowledge of μ (from climatology, say), permits accurate measurement of *R* from only two measurables, viz., Z_{DR} and Z_H.

2. OBSERVATIONS.

For the convenience of the reader we repeat observations of the time history of *Z*, *R*, D_0 and N_W made at Arecibo on 10/15/98 as shown in Fig. 2 (UA07). The data are based on 1 min samples with the J-W disdrometer. This storm was classified as continental. It was comprised of two convective cells followed by transition and stratiform stages.

Table 1 presents key statistics corresponding to Fig. 2. Note that stages C1 and C2 with the largest *Z* and D_0 values are responsible for a total accumulation of 45.3 mm or 96% of the total rain while they occur in only 45% of the time. Because of the high 0°C level and the absence of a bright band, we have identified the generation mechanism as a mix of warm and cold processes. Stage C1 is characterized by equilibrium DSDs of large D_0 and maximum *Z*. Stage C2 has slightly smaller and more variable *Z* and D_0, but the DSDs are also close to equilibrium.

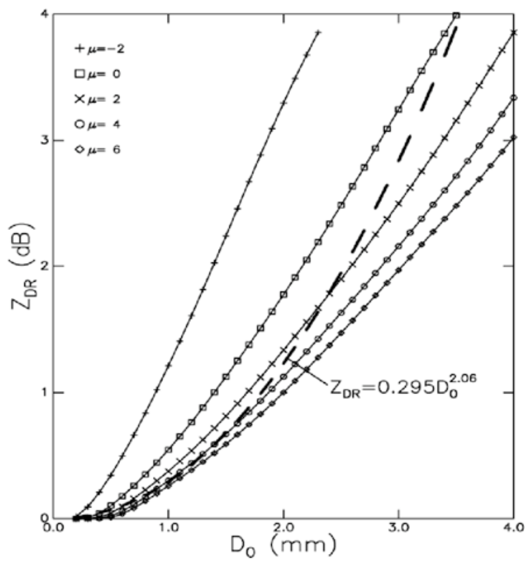

Fig. 1. Z_{DR} vs D_o with m as parameter. Dashed curve from Bring and Chandrasekar, 2001.

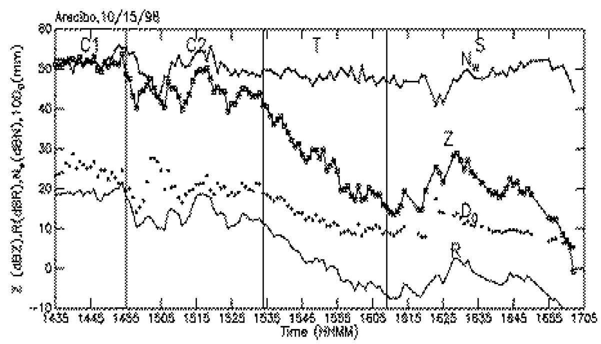

Fig. 2. Time dependence of *R* (dBR), *Z* (dBZ), D_0 (mm), and N_W (dBN) at Arecibo, Puerto Rico on 15 October 1998 in local time. Note ordinate scales for each parameter. The record has been divided into the large drop, large *Z* region of convective segments (C1, C2), the progressively decreasing *Z* in the transition (T), and the stratiform (S) rains. $\{N_W(\text{dBN})=10\times\log_{10}[N_W(\text{m}^{-3}\ \text{cm}^{-1})]\}$

Table 1. $<D_0>$, $Z=AR^b$ relations & total accumulations at Arecibo

Stage	$<D_0>$ (mm)	A	b	H (mm)	%
C1	2.45	2750	0.90	28.5	60.4
C2	2.04	1470	0.96	16.8	35.6
T	1.24	310	1.46	1.36	2.98
S	0.97	280	1.46	0.48	1.02

3. RELATION OF R/Z TO Z_{DR}

Appendix A presents the derivation of the ratio R/Z as a function of Z_{DR}. A similar form was derived by BC. The difference is that they used a fall velocity $v(D)$ versus drop diameter relation of the form $v(D)=16.7D^{0.67}$ which produces a $(D_0)^{2.33}$ dependence. Instead we used the more accurate fall speed relation of Atlas et al. (1973) which results in a $(D_0)^3$ dependence.

Figs. 3 a,b illustrate the relationship of Z_{DR} (left ordinate) and D_0 (right ordinate) to R/Z with μ as a parameter. At constant Z_{DR} the ratio of R/Z varies by only 2.5 db between $0<\mu<12$. We have also superimposed the observations of D_0 and R/Z in the C stages of the convective storms in maritime (TOGA COARE) and continental (Arecibo and LBA Brazil regimes).

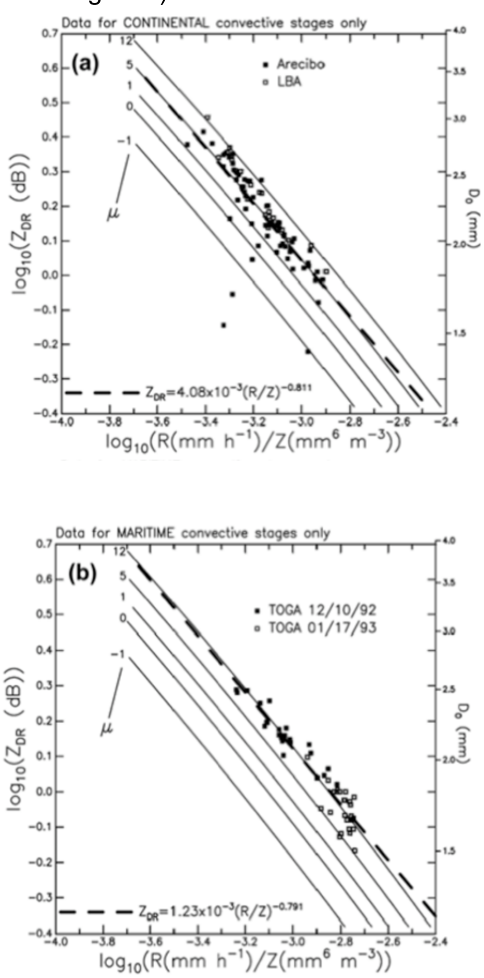

Fig. 3a. $\text{Log}_{10}(Z_{DR}(dB))$ versus $\text{Log}_{10}(R/Z)$ for continental convective stage C at Arecibo, P.R. b). As in (a) two convective storms during TOGA COARE. See text.

The regression relations are

$$Z_{DR} = 1.23\times10^{-3}(R/Z)^{-0.791} \text{ (Maritime) } (\mu=12) \quad (4a)$$

$$Z_{DR} = 4.08\times10^{-3}(R/Z)^{-0.811} \text{ (Continent) } (\mu=5) \quad (4b)$$

The average values $<\mu>=12$ and 5, respectively, are consistent with the finding that tropical continental C rains produce larger (D_0) in smaller concentrations than do maritime C rains. [BAL03; UA07, Fig. 9].

In addition note that maritime C stages are characterized by both weaker updrafts and smaller D_0 than the continental C stages. Moreover, for the same liquid water content, W, the distribution of water is concentrated near the larger D_0 in maritime storms; i.e. in narrower DSDs with smaller normalized distribution of water mass $[(\sigma_m)/D_m]$ and larger μ (BAL03, Fig. 5). The three outliers in the Arecibo observations of Fig. 3a correspond to $\mu<-1$, thus approximating exponential DSDs; these also correspond to the three smallest values of Z and D_0 following the end of stage C1 in Fig. 2 (1458 to 1500 LT). They occur in the tail end of the precipitation streamer emanating from the convective cell C1.

The implication of the finding that R/Z is dependent only upon D_0 for distinct values of μ confirms what we have known from the start, i.e., there is a linear $Z=AR$ relation for each D_0 and corresponding Z_{DR}. Fig. 4 shows a plot of the coefficient A vs D_0 with μ as a parameter. This has also been shown more recently by BC (Eqs.7.69 and Fig. 7.19). Tentatively we may use $\mu=5$ or 12 for continental or maritime C stage rain respectively.

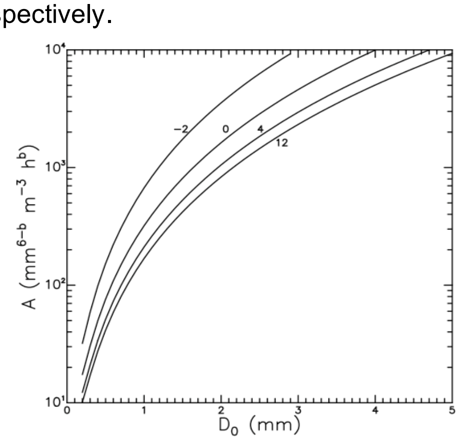

Fig. 4. A in the relation $Z=AR$ for equilibrium DSDs or the average D_0 for a selected time period or area.

However because of the variability in $\boldsymbol{D_0}$ and $\boldsymbol{N_W}$ from storm to storm (BAL03) further study is required to determine whether or not it is necessary to consider such variability.

4. TESTING THE *R=Z/A* RELATION.

In order to test the utility of the *R=Z/A* relation we have used Fig. 4 to select the values of *A* corresponding to the individual values of $\boldsymbol{D_0}$ for each stage of the Arecibo storm (Table 1) and calculated *R* minute by minute. These are compared to the actual values of R_{DISD} found from the disdrometer data in Fig. 5(a), (b), (c), and (d). The agreement between $\boldsymbol{R_{CALC}}$ and R_{DISD} is excellent in all four stages. However, during the T stage where $\boldsymbol{D_0}$ decreases systematically with time (Fig. 2) both $\boldsymbol{R_{CALC}}$ and $\boldsymbol{R_{DISD}}$ overestimate the rain at the beginning and underestimate it toward the end of that stage. It is only near the middle of this period where $\boldsymbol{D_0}$=<$\boldsymbol{D_0}$> that the correct average *R* is measured properly. When one is concerned with excessive variability during any one stage, we suggest dividing that period into smaller segments of ΔD_0 to enhance accuracy. Of course, the total rain during any period will be accurate if one simply measures <*R*> and the time T for that average.

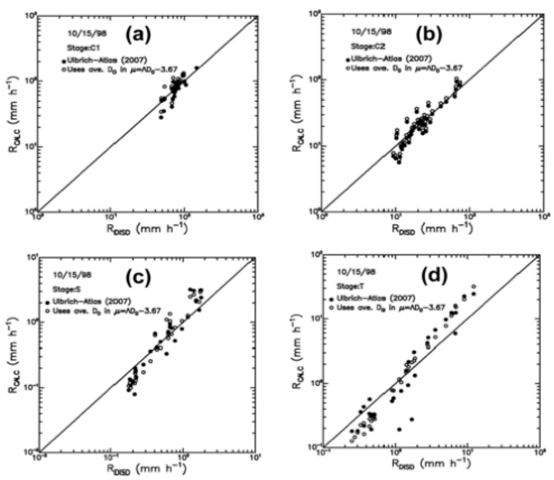

Fig. 5. R_{CALC} vs R_{DISD} for periods C1, C2, T, and S in Fig. 2.

5. CONVENTIONAL RADAR

Until now it has been assumed that it is necessary to use either a polarimetric radar to measure Z_{DR} and D_0, or another dual parameter method. We now explore the idea of deducing D_0 from the physical and climatological conditions and the associated geometric features of the echo patterns.

This work and its predecessors have emphasized the importance of D_0 and N_W or μ in the measurement of rainfall. We now have abundant evidence of the physical and climatological factors which control the nature of the DSD and the value of D_0. This subject has been covered in considerable depth by Rosenfeld and Ulbrich (2003). They have shown the essential features of DSDs resulting from coalescence, breakup, and evaporation. They have also ordered the variations in liquid water content and D_0 as a function of continental, intermediate, maritime, and orographic classes in general agreement with the findings of BAL03 and UA07. Lightning assures us that the precipitation occurs above the 0°C level; its frequency of occurrence is a rough proxy for the updraft strength. These factors and the structural features of clouds and storms as seen visually, from space, or radar provide a means of identifying D_0 and N_W in real time. They also set the boundary conditions for modelers to predict the latter parameters. For example, we discern convective cells and stratiform rain with ease. From Doppler velocity measurements one may measure winds and convergence, and thereby estimate the updrafts that determine D_0. Drop number concentrations alone can be approximated both climatologically and physically in the sense that a region of very large *Z*, near the asymptotic value of 50 dBZ, implies very large D_0 near its asymptote of 3.0 mm (UA07, Fig.7). In short, we suggest that the stage is set to make reasonable estimates and/or predictions of D_0 without dual polarization radar, and thus to estimate rainfall with conventional weather radars. This is a challenge to the next generation of scientists in the field.

These ideas are illustrated in Fig. 6 showing a new *Z-R* rain parameter diagram with isopleths of D_0 and N_W. Of course, Z_{DR} is a proxy for D_0.This graph applies strictly to a value of μ =5; changes in μ cause only small changes in *Z* and *R* (Appendix). Superimposed in Fig. 6 are the regions of maritime and continental convective, and stratiform rains as reported by BAL03 and UA07. Transient zones of small N_W and large D_0 such as occur at the start of an intense convective rain occupy the region of about 25<*Z*<40 dBZ and *R*<10 mm h^{-1}. Unusually large concentrations of small drops will be found below the stratiform area. The reader may readily experiment with this diagram to find the properties of any DSD from *Z*-*R* relations or the converse.

6. CONCLUSIONS

The six-decade rarity of radar reflectivity-rainfall (Z-R) relations characteristic of equilibrium DSDs is due largely to the failure to subdivide convective rains into convective (C), transition (T) and stratiform (S) stages. Both didrometer and polarization measurements have suffered from this deficiency. When properly classified, one finds that the convective stage commonly features essentially constant median volume diameter (D_0). The result is a linear relation *Z=AR* where *A* is essentially proportional to $(D_0)^3$. This permits the definition of *A* as a function of Z_{DR} or D_0, or its average for all stages (C,T, and S). A test case demonstrates the method. Also, the finding that maritime and continental convective storms occupy different domains in D_0 and number concentration space, and do not overlap the stratiform domain, suggests that it is possible to estimate number concentration and D_0 from physical and climatological considerations; and along with *Z,* to estimate rain rate. This approach is necessary for use with conventional radars until polarimetric systems become broadly available.

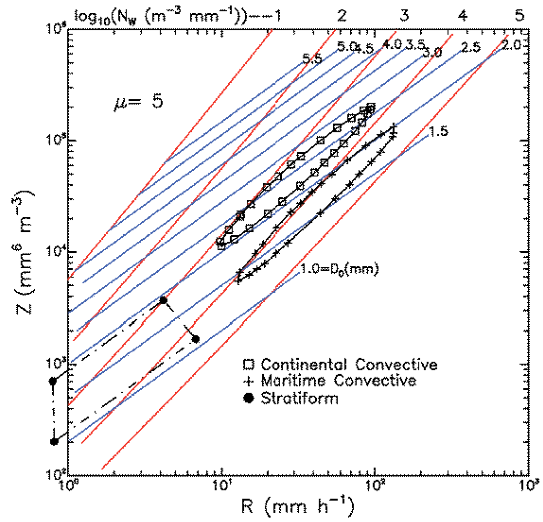

Fig. 6. Advanced rain parameter diagram of Z versus R with isopleths of median volume diameter D_0 and generalized number concentration N_W for a value of μ =5. Areas of stratiform (dashed), convective maritime and continental rains are outlined.

APPENDIX

Using the Atlas, et al (1973) approximation to the drop fall speeds, V(D)=9.65-10.3exp(-6D) (*D* is in cm), and the gamma DSD [Eqs.1 and 2], it may be shown that the ratio of rainfall rate *R(mm* h^{-1}*)* to reflectivity factor *Z(mm*^{6} *m*$^{-3}$*)* that

$$\frac{R}{Z} = \frac{0.6\pi\Gamma(4+\mu)(3.67+\mu)^3}{10^6\Gamma(7+\mu)D_0^3}\left[9.65 - 10.3\left(1+\frac{6D_0}{3.67+\mu}\right)^{-(4+\mu)}\right]$$

The inverse of this relation can be written in the form $Z/R = F(\mu, D_0)D_0^3$ so that, for a given value of μ, when D_0 is constant (as in the convective stages of the storms considered here), then. *Z=AR*, where *A* is a function of μ and D_0 as shown in Fig. 4.

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