Radial birefringent element and its application to laser resonator design

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We have invented a quasi-Gaussian profile-transmittance filter based on radially varying the phase retardation in a birefringent element. The radial birefringent element has been applied to resonator design and has demonstrated its usefulness in generating an improved resonator spatial-mode profile.

It has been known for more than one decade that resonators constructed with a Gaussian profile-reflector element offer the possibility of arbitrary large mode diameters,[1–4] thus overcoming the small-mode-volume limitation of TEM_{00} mode resonators. To date, experimental verification of the theory has not been demonstrated. It has also been recognized that unstable resonators[5] offer an advantage of large mode volume, but at a sacrifice of mode quality because of aperture-generated Fresnel fringes.[6,7] Furthermore, the stability of unstable resonators improves with increasing magnification and thus with higher output coupling. Thus unstable resonators have been limited to relatively high-gain laser media, such as CO_2 (Ref. 8) and Nd:YAG.[9]

We propose an elegant yet simple optical device, the radial birefringent element (RBE), as a method of realizing a Gaussian reflectance profile. We show that the RBE can be used in both stable and unstable resonators to realize the advantages predicted by theory. In particular, the RBE, in conjunction with an unstable resonator, provides a smooth spatial-mode profile and arbitrary output coupling and yet maintains the stability associated with large-magnification unstable resonators.

The RBE is shown schematically in Fig. 1 as the end reflector of a resonator. The element consists of a birefringent crystal with a radius of curvature polished on one surface situated between a polarizer and an end mirror. The reflectance at the center of the element is determined by the net phase retardation $\phi_0 = 2\pi\Delta n l_0/\lambda$, where Δn is the birefringence, l_0 is the physical thickness, and λ is the wavelength. Since the phase retardation $\phi(r)$ is a function of radial position, the reflectance also varies radially. If we assume that the reflected beam retraces the optical path of the forward beam at the element, an analysis of the mirror, polarizer, and RBE combination using Jones's matrix approach[10] shows that the reflectance and transmittance of the device are given by

$$R = \cos^2 \phi(r) + \sin^2 \phi(r)\cos^2 2\theta, \tag{1a}$$

$$T = \sin^2 \phi(r)\sin^2 2\theta, \tag{1b}$$

where θ is the angle between the polarizer and the principal axis of the birefringent plate. To a good approximation,

$$\phi(r) = \frac{2\pi\Delta n}{\lambda} l(r) = \frac{2\pi\Delta n}{\lambda}\left(l_0 \pm \frac{r^2}{2\rho}\right), \tag{2}$$

where r is the radial position and ρ is the curvature, with the upper and lower signs corresponding to convex or concave curvatures, respectively. Equation (1a) can be written in the form

$$R(r) = \cos^2 2\theta + \sin^2 2\theta \cos^2 \phi(r),$$

which explicitly shows that the reflectance is given by a constant factor depending on θ modulated by a term varying as $\cos^2 \phi(r)$ with amplitude $\sin^2 2\theta$. For $\theta = 45°$, we find that $R(r) = \cos^2 \phi(r)$, or

$$R(r) = \cos^2\left[\frac{2\pi\Delta n}{\lambda}\left(l_0 \pm \frac{r^2}{2\rho}\right)\right]. \tag{3}$$

If we want to set $R(r) = 0$ at a specific radius r_0, determined, for example, by an aperture or rod radius within the resonator, then Eq. (3) can be inverted to give

$$l_0 = \frac{\lambda}{2\pi\Delta n}\{\cos^{-1}[R(0)]^{1/2} + m\pi\},$$

$$\rho = \frac{r_0{}^2\Delta n 2\pi}{2\lambda}\{\cos^{-1}[R(r_0)]^{1/2} - \cos^{-1}[R(0)]^{1/2}\}^{-1}, \tag{4}$$

where m is an integer.

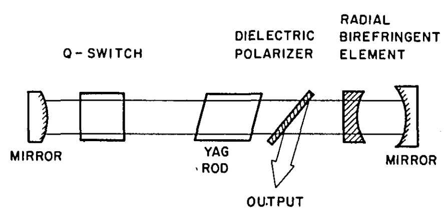

Fig. 1. The radial birefringent element consisting of a polarizer and a birefringent plate with curvature ρ, within an unstable resonator cavity.

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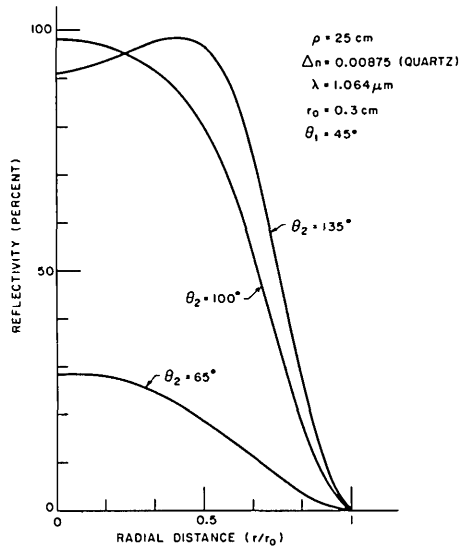

Fig. 2. Reflectance profiles of the RBE using two crystal-quartz elements. Two elements have equal but opposite curvature ρ. θ_1 and θ_2 are the angles between the principal axis of the quartz elements and the polarizer axis. The mode radius r_0 is chosen equal to the radius of the Nd:YAG rod.

It is convenient to define the equivalent Gaussian spot size of the RBE w_m as

$$w_m{}^2 = r_0{}^2 \frac{\cos^{-1}[R(0)/e]^{1/2} - \cos^{-1}[R(0)]^{1/2}}{\cos^{-1}[R(r_0)]^{1/2} - \cos^{-1}[R(0)]^{1/2}}, \qquad (5)$$

where e is the Neper number. Note that for $R(0) = 1$ and $R(r_0) = 0$, $w_m{}^2 = 0.58\, w_0{}^2$, and that, unlike for a Gaussian reflector, we can arrange $R(r_0) > R(0)$. Thus the RBE is a pseudo-Gaussian mirror with reflectance

$$\sqrt{R(0)}e^{-(r^2/w_m{}^2)}.$$

Equation (5) can be used with previous analyses of resonators[1] with Gaussian reflectance elements as the effective spot size of the Gaussian reflector.

Finally, for ABCD resonator analysis of the cavity shown in Fig. 1, the radius of curvature of the equivalent mirror is given by

$$R_e = \left(\frac{1}{R_m} + \frac{1}{f}\right)^{-1}, \qquad (6)$$

where f is the focal length of the birefringent plate with curvature ρ and R_m is the end mirror's radius of curvature. This is a good approximation when the element is a distance from the end mirror that is small compared with the mirror's curvature or the element's focal length.

We have also designed, constructed, and analyzed[11] other variations of the RBE. Of interest for resonator studies is a two-element RBE consisting of a positive-curvature element next to a negative-curvature element. By varying the angles θ_1 and θ_2, which are the angles between the polarizer and the principal axis of the birefringent elements, various reflectance profiles can be generated. Figures 2 shows some reflectance profiles for various θ_2 and fixed θ_1. Here both elements have equal refractive indices and equal but opposite curvatures ρ. This variation of the RBE was used in the resonator studies described next.

We have constructed both stable and unstable resonators using the RBE for a Nd:YAG gain medium. The resonators with the geometric magnification larger than 2 demonstrated that large TEM_{00} mode spot sizes could be supported, as expected from previous theoretical analysis.[1,3,4] However, when the mode size is big enough to extract high energy from the rod, the diffraction loss that is due to the rod causes the stability of such a resonator to be marginal.

The RBE in combination with an unstable resonator offers the advantages of good alignment stability, excellent output spatial-mode profile, and good discrimination against higher-order modes. The RBE was constructed by combining a convex and a concave crystal quartz lens with equal ρ = 25-cm curvatures. The mirror curvatures of the RBE resonator were chosen to form an unstable resonator with the thermal focusing of the rod taken into account. Best collimation at the output was achieved with the combination of a −400-cm-curvature mirror and a −200-cm-curvature mirror. The resonator length was 65 cm. In this resonator, the output coupling was changed by rotating the RBE. The optimum performance was found when $\theta_1 = 45°$ and $\theta_2 = 100°$, although the angles were relatively uncritical with $\Delta\theta_1 = \pm 5°$ and $\Delta\theta_2 = \pm 10°$.

We chose to compare the RBE resonator performance against the standard 60-cm-long, M = 3.3 Nd:YAG confocal unstable resonator.[9] Figure 3 shows the transverse-mode quality of the RBE unstable resonator and the confocal unstable resonator with a 1.8-mm-diameter polka-dot-coated output coupler. The video scans clearly show the flat-top profile of the RBE resonator compared with strongly Fresnel-modulated unstable-resonator output.[7] The RBE, with its $\cos^2 \phi(r)$ reflectance profile, effectively eliminates

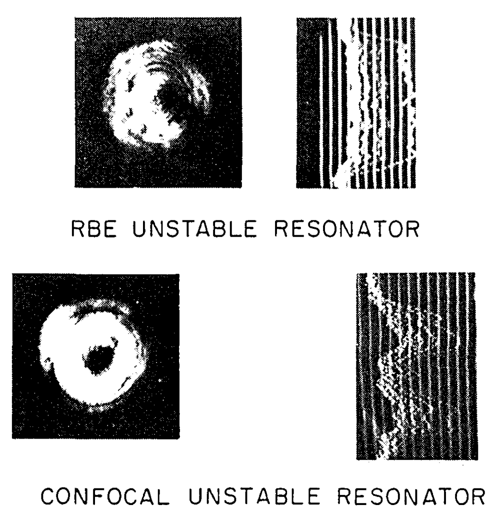

Fig. 3. Transverse-mode profiles of the RBE and standard confocal unstable resonator configurations. The Polaroid exposures of the output are at the right side and the vidicon-scanned profiles are at the left side. The slightly asymmetric patterns are due to a damage spot on one end of the Nd:YAG rod.

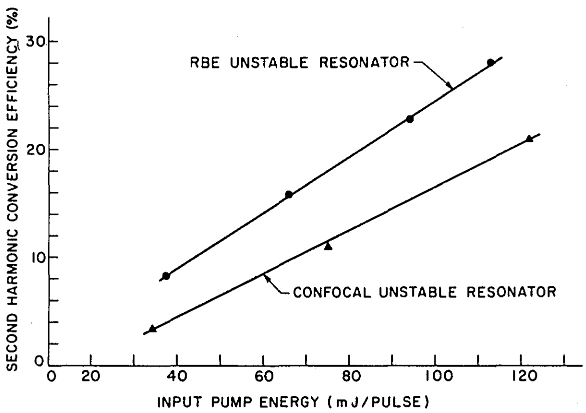

Fig. 4. Second-harmonic-generation conversion efficiency versus input pump energy for the confocal unstable resonator and for the RBE unstable resonator in Type II KD*P.

Fresnel fringes that arise from diffraction caused by hard apertures. The weak Fresnel fringes on the profile of the RBE resonator are due to the finite aperture size of the YAG rod.

As a further comparison of performance, we carried out second-harmonic-generation (SHG) conversion-efficiency measurements in a Type II angle-phase-matched KD*P crystal. Figure 4 shows the improved SHG performance of the RBE unstable resonator.

In conclusion, we have invented a new class of optical elements based on radially varying the phase retardation in a birefringent plate. In particular, the radial birefringent element has been applied to resonator design and has demonstrated its usefulness in generating an improved resonator spatial-mode profile and improved nonlinear conversion. The RBE is simple to implement and should prove useful in designing unstable resonators for large-aperture but low-gain laser media. The RBE should also find applications as a spatial filter or soft aperture.

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