Progress of Theoretical Physics, Vol. 21, No. 2, February 1959

# Radial Dependence of Imaginary Part of Nuclear Optical Potential

Kichinosuke HARADA

*Department of Nuclear Physics, Japan Atomic Energy Research Institute, Tokai-mura, Ibaraki-ken*

and

Nobuo ODA

*Department of Physics, Rikkyo University, Tokyo*

(Received October 11, 1958)

The imaginary part of tht nuclear optical potential for neutrons is calculated by means of a method of the volume direct processes, on the basis of the statistical independent particle model with a eiffuse nuclear surface. It is shown that the localization of the absorption near the surface, which was found by Bjorklund and Fernbach, can be reproduced using reasonable values for the relevant parameters. The effect of the Pauli exclusion principle, of the energy dependence of the two-body collision cross section, of the mass number dependence of the parameters, and of the correlation between nucleons are also discussed.

## § 1. Introduction

Much information on the phenomenological optical potential parameters has been obtained so far, especially from the detailed analyses of the proton-nuclei scattering cross section.[1] It was found that a fairly good fit to the experimental data could be obtained by using the diffuse-boundary shape for both the real and imaginary part of the nuclear optical potential.

On the other hand the surface absorption model had first been suggested by Feshbach, Porter, and Weisskopf[2] from a view-point that the imaginary part would be much less dependent on mass number if it is concentrated in a surface layer. Amster[3] studied the effects of the various surface absorption potential forms at zero energy and also analysed, in collaboration with Emmerich, $\sigma_t$ and $\sigma_c$ of the first few Mev neutrons using several potential forms. However, the determination of the best fit surface potential form and the relevant parameters was not aimed at in their analysis.

Recently, Bjorklund and Fernbach[4] have analysed the scattering of 4.1-, 7-, and 14-Mev neutrons by complex nuclei, using the optical potential with the following form* :

* The notation is slightly changed from that used by Bjorklund and Fernbach.

$$V(r)=V_0p(r)+iWq(r)+U(\hbar/\mu c)^2(1/r)\,(dp(r)/dr)\,(\sigma\cdot\boldsymbol{l}), \qquad (1)$$

where

$$p(r)=\{1+\exp[(r-R_0)/s]\}^{-1}, \qquad (1a)$$

$$q(r)=\exp[-(r-R_0)^2/t^2], \qquad (1b)$$

$$R_0=r_0A^{1/3}. \qquad (1c)$$

The characteristic feature of this potential is the use of a Gaussian form centered at the nuclear edge for the imaginary part ($Wq(r)$). At each energy considered, was found a single set of parameters that give a fairly good agreement with experimental data of the elastic, total, and nonelastic cross section. Further, the calculated nonelastic cross sections show better agreement with experimental data when the surface imaginary potential was used instead of the diffuse-boundary (Fermi type) imaginary potential.

Roughly speaking, two types of methods have been developed in order to derive the phenomenological potential mentioned above from the more fundamental two-body interaction; a) the quantum mechanical one[5] which directly relates the complex potential to the two-body potentials as has recently been elaborately developed by Brueckner and his collaborators, and b) the semi-classical one, first proposed by Goldberger,[6] which relates the imaginary part of the complex potential to the empirical two-body cross sections*. While the method a) is clearly more fundamental than the method b), in the former approach we must deal with all of the complications of the nuclear many-body problem, which in fact is very difficult to do at present[7]. On the other hand, the method b), notwithstanding its simplicity, could successfully give the absolute magnitude of the imaginary potential and its dependence upon the energy of the incident particle[8)9)]. However, since all the imaginary potentials so far derived were based on the Fermi gas having a constant density and a sharp boundary, they were constant over the whole nuclear volume.

In a previous paper[10], along the line of method b), we used the Thomas-Fermi gas model with a diffuse nuclear surface to explain the inelastic scattering of protons at intermediate energies by complex nuclei. The inelastic scattering was explained as the direct processes ocurring at the diffuse rim of the nucleus, and it was indicated in that paper that it would be very interesting to derive the radial dependence of the absorption (imaginary) potential from our theory, in which the Pauli principle was exactly taken into account.

We shall show in this work that by using the Thomas-Fermi gas model as in the previous paper, it is possible to reproduce the phenomenological absorption potential of Bjorklund and Fernbach. In this connection, it is to be noted that the simple Thomas-Fermi gas model as used by us is also applicable with a considerable validity to other surface problem[11], such as the derivation of nuclear surface energy.

---

* The real part of the optical potential cannot be derived on the method b).

## § 2. Calculation and results

At first we consider the case in which the incident particle is a neutron. It is assumed that the incident neutron interacts individually with the nucleons in the target with a cross section which differs from that for free nucleons only through the restrictions due to the Pauli principle. As the target nucleons are supposed to be the constituents of the Thomas-Fermi gas with a diffuse boundary, the Fermi momentum $p_F(r)$ is radially dependent and can be evaluated from the nucleon density $\rho_p(r)$ and $\rho_n(r)$ :

$$p_F^2(r)=[3\pi^2\hbar^3\rho(r)]^{2/3}. \tag{2}$$

When a Fermi type function is assumed for the actual shape of $\rho_p$ and $\rho_n$,

$$\rho(r)=a\{1+\exp[(r-c)/b]\}^{-1}, \tag{3}$$

we have

$$p_F^2(r)=[3\pi^2\hbar^3 a]^{2/3}\{1+\exp[(r-c)/b]\}^{-2/3}, \tag{4}$$

where $c$ is the half-density radius, $b$ the diffuseness parameter, and $a$ the normalization constant[10]. Then, the kinetic energy of an incident neutron in the nucleus is given by

$$E_1(r)=p_{Fn}^2(r)/2m+S_n+E, \tag{5}$$

where $p_{Fn}(r)$ is the Fermi momentum of the neutrons bound in the nucleus, $S_n$ the separation energy of the neutron, and $E$ the kinetic energy of the incident neutron outside the nucleus. Contrary to the usual theory, $E_1(r)$ is here directly given by the nucleon density distribution $\rho(r)$, without referring to the real part of the optical potential $V(r)$. The assumption that an incident neutron is absorbed from the incident channel when it collides with a target nucleon leads to an expression for the absorption coefficient $K(r)$ which is here radially dependent:

$$\begin{aligned} K(r)&=K_p(r)+K_n(r)\\ &=\rho_p(r)\bar{\sigma}_{np}(r)+\rho_n(r)\bar{\sigma}_{nn}(r), \end{aligned} \tag{6}$$

where $\bar{\sigma}_{np}(r)$ and $\bar{\sigma}_{nn}(r)$ are certain average neutron-proton and neutron-neutron cross section. The averages in $\bar{\sigma}_{np}(r)$ and $\bar{\sigma}_{nn}(r)$ are taken approximately over all possible relative collision momenta $\boldsymbol{p}(r)$, under the restrictions due to the Pauli exclusion principle.

Let the momenta of incident and target nucleons be $\boldsymbol{p}_1(r)$ and $\boldsymbol{p}_2(r)$ and the relative momenta of the initial and final states be $\boldsymbol{p}(r)$ and $\boldsymbol{p}'(r)$, respectively. Then, $\bar{\sigma}(r)$ may be expressed as

$$\bar{\sigma}(r)=\int d\boldsymbol{p}_2(r)\int d\Omega'\, 2p(r)\sigma(\boldsymbol{p}(r),\boldsymbol{p}'(r))/p_1(r)\int d\boldsymbol{p}_2(r), \tag{7}$$

where $d\Omega'$ is the solid angle element for $\boldsymbol{p}'(r)$. The two-body collision cross section $\sigma(\boldsymbol{p}(r),\boldsymbol{p}'(r))$ in the center of mass system is assumed to be isotropic and inversely

proportional to $p^2(r)$[12]. This assumption is appropriate in the present case. In eq. (7), the integral in the denominator is taken over the range of momenta at the radial coordinate $r$ of the target nucleons (i.e. from 0 to $p_F(r)$), and the integral in the numerator is performed under the restriction of the Pauli exclusion principle at the place $r$. Assuming $\sigma(\boldsymbol{p}(r), \boldsymbol{p}'(r)) = \sigma_t(p(r))/4\pi = \sigma_0/4\pi p^2(r)$, we get

$$\bar{\sigma}(r) = 6\sigma_0\, p_1(r)\, I(r)/p_F^3(r), \tag{8}$$

where

$$I(r) = \int_a^b \frac{x(1+x^2-2\alpha^2)}{\sqrt{2(1+x^2)}} \tanh^{-1} \frac{2x\sqrt{2(1+x^2)}}{1+3x^2} dx, \tag{8a}$$

$$x = p_2(r)/p_1(r), \quad \alpha = p_F(r)/p_1(r), \tag{8b}$$

and the integration limits are

$$\begin{aligned} &a=0, \ b=\alpha \qquad \text{for} \quad E_1(r) \geq 2E_F(r), \\ &a=\sqrt{2\alpha-1}, \ b=\alpha \ \text{for} \ E_1(r) \leq 2E_F(r), \end{aligned} \tag{8c}$$

respectively.

The actual numerical calculation has been carried out for Sn nucleus, where the empirical value for $S_n$ is 9.7 Mev. The parameters $b$ and $c$ in the expression for $\rho_p(r)$ can be determined fairly well from the analysis of the high energy electron scattering[13]; $b = 0.57 \times 10^{-13}$ cm and $c = 1.1 \times A^{1/3} \times 10^{-13}$ cm. Since there is no definite experimental evidence for the density of neutron $\rho_n(r)$, two sets of values of $b$ and $c$ are assumed with the same type of distribution as $\rho_p(r)$ : (A) $\rho_n$ is of the same form as $\rho_p$, and (B) $\rho_n$ is a little extended beyond $\rho_p$ but has the same surface thickness as $\rho_p$. The values for $b$ and $c$, and the corresponding Fermi energies $E_F(0)$ at the center of the nucleus are listed in Table 1.

Table 1. Values of $a$, $b$, $c$ and $E_F(0)$ for Sn

| | protons | neutrons (A) | neutrons (B) |
|---|---|---|---|
| $a$ | $0.67 \times 10^{38}$ | $0.94 \times 10^{38}$ | $0.53 \times 10^{38}$ |
| $b$ | $0.57 \times 10^{-13}$ | $0.57 \times 10^{-13}$ | $0.57 \times 10^{-13}$ |
| $c$ | $5.42 \times 10^{-13}$ | $5.42 \times 10^{-13}$ | $6.66 \times 10^{-13}$ |
| $E_F(0)$ | 32.8Mev | 41.3Mev | 28.3Mev |

The calculated absorption coefficients are shown in Figs. 1, 2, and 3 for $E=14$ Mev, 7 Mev, 4 Mev, respectively. The dotted curves are the ones calculated from the optical potential determined from the experiment[4], using the following relation between the absorption coefficient and the optical potential,

$$K^2(r) = (4m/\hbar^2)[-(E+V(r)) + \sqrt{(E+V(r))^2 + W^2(r)}]. \tag{9}$$

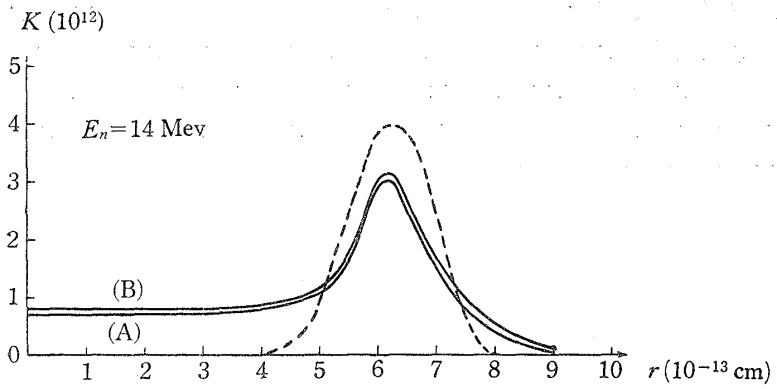


Fig. 1 Full curves are the calculated absorption coefficients for $E_n$=14 Mev. Dotted curve is the empirical one found by Bjorklund and Fernbach.

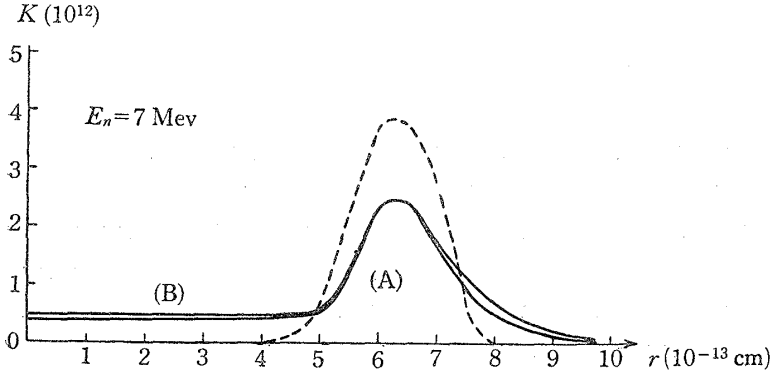


Fig. 2 Full curves are the calculated absorption coefficients for $E_n$=7 Mev. Dotted curve is the empirical one found by Bjorklund and Fernbach.

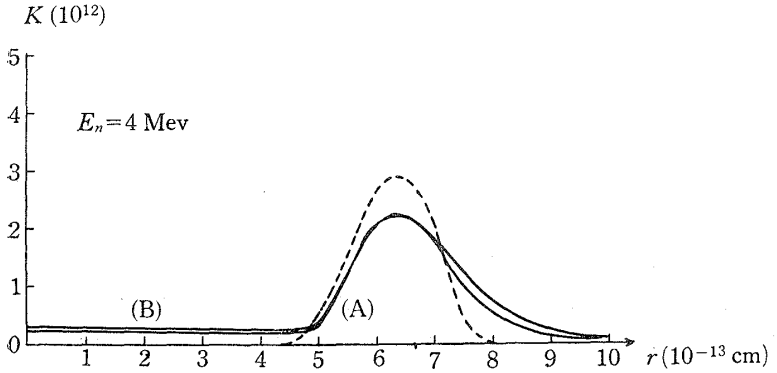


Fig. 3 Full curves are the calculated absorption coefficients for $E_n$=4 Mev. Dotted curve is the empirical one found by Bjorklund and Fernbach.

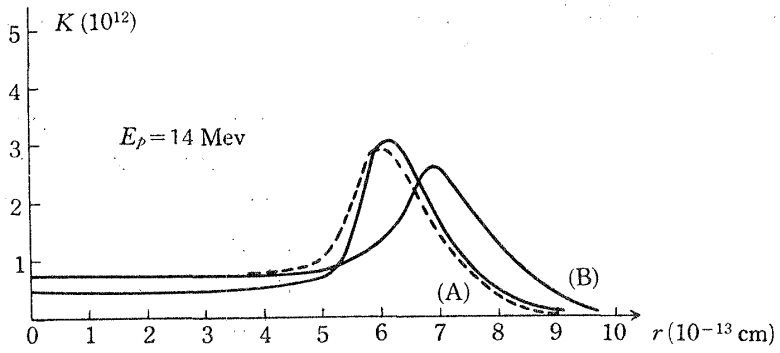


Fig. 4 Full curves are the calculated absorption coefficients for $E_p$=14 Mev. Dotted curve is the one for $E_n$=14 Mev.

One sees that the calculated absorption coefficients agree fairly well with the empirical (experimental) results, especially the localization of the absorption at the nuclear surface. It is interesting to note that $K(r)$ is almost indepent of the shape of $\rho_n$. This is due to the fact that $\sigma_{nn}$ is much smaller than $\sigma_{np}$. In the case of the incident protons it is expected that $K(r)$ will be more sensitive to the shape of $\rho_n$.

To see this $\rho_n$ dependence more quantitatively, we calculate the absorption coefficient $K(r)$ for the incident proton whose energy is 14 Mev, and show the results in Fig. 4. In this case the empirical value for $S_p$ is 6.2 Mev. One sees that with $\rho_p$ described by a type (A) $K$'s for both the incident neutron and proton have almost the same form, but with $\rho_n$ described by a type (B) the peak of $K$ for the incident proton appreciably shifts towards the outside and its width becomes slightly broader than that of $K$ for the incident neutron.

## § 3. Discussions

i) We shall here examine the problem why the absorptions were localized at the nuclear surface in our calculations on the basis of the statistical independent particle model with a diffuse nuclear surface. As one of the reasons, we may imagine that the relaxation of the Pauli exclusion principle at the surface due to the decrease of $p_F(r)$ plays an important role. To see whether this is actually the case or not, we examined the scattering of 4 Mev neutrons by target protons only. At this energy the Pauli exclusion principle is expected to be most effective. If we assume the two-body cross section to be energy-independent, $\sigma_{np}(\boldsymbol{p}(r), \boldsymbol{p}'(r)) = \sigma_t/4\pi$, we obtain (see Hayakawa et al.,[9] eq. (2.8))

$$\bar{\sigma} = \sigma_t \cdot (E.\ P.), \tag{10}$$

where

$$(E.\ P.) = 1 - \frac{7}{5} \cdot \frac{E_F(r)}{E_1(r)} \qquad \text{for } E_1(r) \geq 2E_F(r),$$

$$=1-\frac{7}{5}\cdot\frac{E_F(r)}{E_1(r)}+\frac{2}{5}\cdot\frac{F_F(r)}{E_1(r)}\left(2-\frac{E_1(r)}{E_F(r)}\right)^{5/2} \quad \text{for } E_1(r)\leq 2E_F(r). \quad (10')$$

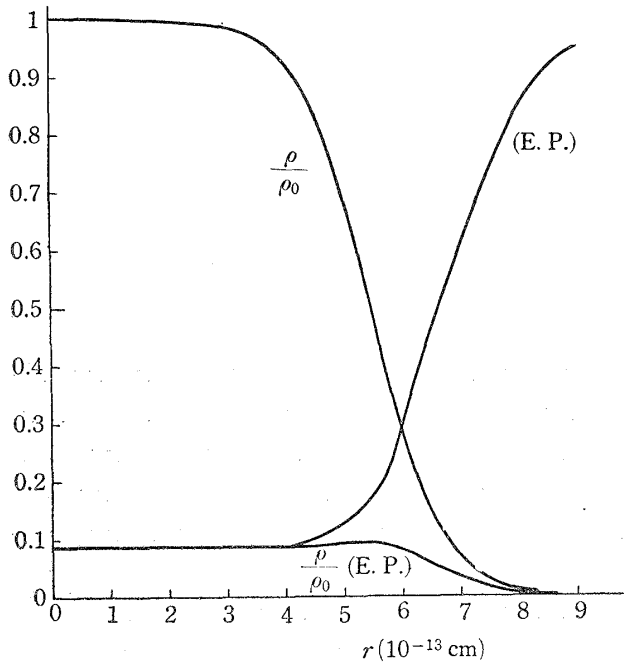


Fig. 5 $\rho/\rho_0$ is the proton density normalized to unity at the origin. (E. P.) is the factor exprssing the rate of reduction of the free collision cross section $\sigma_{np}$ due to the exclusion principle for $E_n=4$ Mev.

Here $(E.\ P.)$ is the factor expressing the rate of reduction of the free collision cross section $\sigma_{np}$ due to the exclusion principle. Then the absorption coefficient $K(r)$ is written as

$$K(r)=\rho(r)\cdot\sigma_t\cdot(E.\ P.). \quad (11)$$

$(E.\ P.)$, $\rho/\rho_0$ (the proton density normalized to unity at the origin), and $\rho/\rho_0\times(E.\ P.)$ are shown in Fig. 5. From this one sees that the effect of the relaxation of the exclusion principle at the nuclear surface is considerably cancelled out by the decrease of $\rho(r)$. Therefore, using the energy independent cross section, one can not obtain the localization of the absorption at the surface. From the above result it appears that the remarkable localization of the absorption at the nucleon surface in our theory is mainly due to the energy dependence of two-body collision cross section, although the exclusion principle considerably reduce the absorption inside the nuclear surface. The energy dependence of the two-body collision cross sections strikingly affects the results because in our model the momenta of both the target nucleons and an incident nucleon at the nuclear surface are much smaller than inside (see eqs. (2) and (5)).*

ii) On the basis of the Stanford electron scattering experiment[11], the density of protons $\rho_p$ is usually described by a Fermi type function (eq. (3)), having a central region of rather uniform density and a surface region in which the density falls from 90 to 10 per cent of the central value in a distant $D=4.39\,b$, $b$ being independent of the mass number $A$. Taking account of this fact for $\rho(r)$, we may expect that the calculated absorption coefficients $K(r)$ for nuclei with different $A$ in our theory would have nearly equal magnitude at a fixed incident energy, with the different locations of their maxima corresponding to the different values of radii. This expectation will be supported by the results of Bjorklund and Fernbach that at a fixed incident energy one can get by varying only one parameter, that is the

* The examination of this discussion (i) was done following the suggestion of Dr. Sugie.

radius $R_0$(eq.(1c)), a fairly good agreement with experimental data on the total and nonelastic cross sections as a function of $A$ (about 8 to 220).

The usual optical potential analysis assumes the same radial form[14] for the real and imaginary potential and the resulting imaginary part shows a remarkable mass number dependence, i.e. one has to decrease the imaginary part about factor two as the mass number increases, in order to obtain a fit to experimental data. The neutron excess in heavier nuclei[15] can produce this tendency, but its effect is too weak to explain the empirical evidence. On the basis of the surface absorption model, this difficulty is not the case.

iii) Since our nuclear model is a statistical independent particle model, the correlations between the nucleons are ignored. If the correlations were taken into account, the higher momentum components of the target nucleons would be increased and the nucleus would be represented by a no-degenerate Thomas-Fermi gas at some temperature. In another place[16]* we gave the detailed calculation of the absorption coefficient with a non-degenerate Fermi gas (having a constant nuclear density and a sharp boundary) in a similar way as in this paper (i.e. as a direct volume process), and the result showed that the non-degeneracy was not so effective in inceasing the absorption coefficient.** It might be, therefore, expected that in the present case with the diffuse surface also the correlation effect will not so much alter the fundamental features of the results. However, the finiteness of temperature is but one result of the correlation effect and a great complexity will be introduced in the calculation through a complete inclusion of the correlation effect, so that we think it a consistent way to work with the classical particle picture together with the Thomas-Fermi gas at zero temperature.

Finally, it may be concluded from the results and disussions given above that even with the present simple semiclassical model the physical reality underlying the nuclear surface problem can be explained successfully to a certain extent.

The authors would like to express their sincere thanks to Drs. T. Momota and A. Sugie for illuminating discussions and careful reading of the manuscript.

* This calculation was done in connection with the sticking probability in the decay process of a compound nulceus (Cohen's paradox)[17], and the residual nucleus was described by a non-degenerate Fermi gas with various excitation energies.

** An analysis of the dependence of the mean free path of nucleon on the nuclear temperature was also done by Kind and Patergnani,[18] from which the similar conclusion may be derived for this case if the nuclear temperature is not so high.

## References


1) M. A. Melkanoff, J. Nodvik, D. S. Saxon and R. D. Wood, Phys. Rev. **106** (1957), 793.
2) H. Feshbach, C. E. Porter and V. F. Weisskopf, Phys. Rev. **96** (1954), 448.
3) H. Amster, Phys. Rev. **104** (1956), 1606.
4) F. Bjorklund, S. Fernbach, and N. Sherman, Phys. Rev. **101** (1956), 1832.
   F. Bjorklund and S. Fernbach, Phys. Rev. **109** (1958), 1295.
5) M. Cini and Fubini, Nuovo Cimento **10** (1955), 75.
   K. A. Brueckner, R. J. Eden and N. C. Francis, Phys. Rev. **100** (1955), 891.
   K. A. Brueckner, Phys. Rev. **103** (1956), 172.
   N. M. Hugenholtz, Physica **23** (1957), 441.
   L. Verlet, Nuovo Cimento **7** (1958), 821.
6) M. L. Goldberger, Phys. Rev. **74** (1948), 1269.
7) R. J. Eden, Proc. Roy. Soc. (London) **A235** (1956), 408.
   H. A. Bethe, Phys. Rev. **103** (1956), 1353.
   T. H. R. Skyrme, Phil. Mag. (Eighth Ser.) L (1956), 1043.
   K. A. Brueckner, J. L. Gammel and H. Weitzner, Phys. Rev. **110** (1958), 431.
8) A. M. Lane and C. F. Wandel, Phys. Rev. **98** (1955), 1524.
   E. Clementel and C. Villi, Nuovo Cimento **11** (1955), 176.
   G. C. Morrison, H. Muirhead and P. A. B. Murdoch, Phil. Mag. **44** (1955), 795.
9) S. Hayakawa, M. Kawai and K. Kikuchi, Prog. Theor. Phys. **13** (1955), 795.
10) N. Oda and K. Harada, Nuclear Physics **7** (1958), 251.
11) see Review by L. Wilet, Revs. Mod. Phys. **30** (1958), 542.
12) Millurn, Birnbaum, Crandall and Schcter, Phys. Rev. **95** (1954), 1268.
13) R. Hofstadter, Revs. Mod. Phys. **28** (1956), 214.
14) Beyster, Salmi, Schrant, and Walt, Los Alamos Scientific Laboratory Report, LA-2099 (1957).
15) M. Kawai, M. Nagasaki M. Soga, T. Terasawa, H. Ui, and Y. Wada, Prog. Theor. Phys. **18** (1957), 66.
16) N. Oda, K. Harada, H. Natsume and T. Yamaguchi, Soryushiron Kenkyu (Mimeographed circular in Japanese) **13** (1956), 17.
17) B. L. Cohen, Phys. Rev. **92** (1953), 1245.
18) A. Kind and G. Patergnani, Nuovo Cimento **10** (1953), 1375.