

# RADIATION FROM A POINT CHARGE MOVING THROUGH TWO COAXIAL CIRCULAR APERTURES OF A CAVITY CONSISTING OF TWO PARALLEL SCREENS

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## SUMMARY

This paper deals with the problem of diffraction radiation from an electric point charge which moves, at a constant speed, through two coaxial circular apertures in two parallel plane screens. The screens are assumed to be electrically perfectly conducting. The problem is reduced to the solution of two integral equations for the unknown field functions in the two apertures. Low-frequency solutions as well as high-frequency solutions are considered. Results pertaining to the far-field behaviour of the radiated field are given. Finally, numerical results pertaining to the radiation loss of the charge are presented.

## 1. Introduction

THE energy lost by diffraction radiation when a charged particle passes a conducting structure is of considerable importance in accelerating systems. In the past, attention has been limited to a single conducting structure and to periodic structures. Bolotovskii and Voskresenskii (1, 2) and Hazeltine, Rosenbluth and Sessler (3) have reviewed the literature on this subject.

Hitherto, very little consideration has been given to the radiation of a charged particle passing a single resonating structure. Therefore, in the present paper the diffraction radiation from an electric point charge that moves, at a constant speed, through two coaxial circular apertures in two plane parallel screens is investigated. It is one of the simplest configurations which can exhibit resonating features. To gain an insight into the necessary mathematical technique for solving the present problem, we first solved the relevant problem involving one aperture in a single screen (4). In the present paper we exclude mathematical details and restrict

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ourselves to the salient points and final results in the method of solution. The complete mathematical approach can be found elsewhere (5).

## 2. Formulation of the problem

The point charge  $q$  under consideration moves with constant velocity  $\mathbf{v} = v_0 \mathbf{i}_z$  ( $v_0 < c_0$ , where  $c_0$  is the velocity of light) along the axis of a system of circular cylindrical coordinates  $(r, \phi, z)$  through coaxial circular apertures in two parallel plane screens at  $z = 0$  and  $z = l$ . The apertures occupy the regions  $z = 0$ ,  $0 \leq r \leq a$ ,  $0 \leq \phi < 2\pi$  and  $z = l$ ,  $0 \leq r \leq a$ ,  $0 \leq \phi < 2\pi$ , respectively. Since the geometrical configuration is independent of  $\phi$ , all field quantities are independent of  $\phi$ . From Maxwell's equations it follows that  $H_\phi$ ,  $E_r$  and  $E_z$  are the only non-zero components of the magnetic and electric fields and that  $E_r$  and  $E_z$  can be expressed in terms of  $H_\phi$ . Let  $H_\phi$  be Fourier analysed as

$$H_\phi(r, z, t) = \frac{1}{\pi} \operatorname{Re} \left[ \int_0^\infty H_{\phi\omega}(r, z) \exp(i\omega t) d\omega \right], \quad (2.1)$$

with  $H_{\phi\omega} = H_{\phi\omega}^i + H_{\phi\omega}^r$ . The 'incident' field  $H_{\phi\omega}^i$  pertains to the field of the moving charge in free space. The field  $H_{\phi\omega}^r$  pertains to the diffracted field and this field can give rise to radiation. For convenience, we write

$$H_{\phi\omega} = -\partial\Pi/\partial r, \quad (2.2)$$

with  $\Pi = \Pi^i + \Pi^r$ , in which

$$\Pi^i(r, z) = \frac{q}{2\pi} \exp(-ik\beta_0 z) K_0(k\Gamma r), \quad (2.3)$$

where  $k = \omega(\epsilon_0\mu_0)^{\frac{1}{2}} = \omega/c_0$  ( $\epsilon_0$  is the permittivity and  $\mu_0$  the permeability of the vacuum),  $\beta_0 = c_0/v_0$ ,  $\Gamma = (\beta_0^2 - 1)^{\frac{1}{2}}$  and  $K_0$  is the modified Bessel function of second kind and zero order (6). The function  $\Pi^r$  satisfies the homogeneous Helmholtz equation. At the electrically perfectly conducting screens, the tangential component of the total electric field vector should vanish. These boundary conditions can be formulated as

$$\partial H_{\phi\omega} / \partial z = 0 \quad \text{at } z = 0 \text{ and } z = l, \quad a < r < \infty. \quad (2.4)$$

Further, we have to satisfy the edge condition and the radiation condition.

## 3. Integral equations

In order to obtain a solution of our problem, we choose the method of integral equations. They are obtained as follows. Apply Green's theorem to the domains  $z < 0$ ,  $0 < z < l$  and  $z > l$ , respectively, with respect to the diffracted field  $\Pi^r$ . With respect to the incident field  $\Pi^i$ , apply Green's theorem to the same domains, respectively, but outside the region

$r < \epsilon$  ( $\epsilon \rightarrow 0$ ). Combine the results in such a way that the integrals over the screens cancel, and require then that in the apertures  $H_{\phi\omega}^r$  be continuous. After introducing the dimensionless variables  $\rho = r/a$ ,  $\alpha = ka$  and  $\tau = l/a$ , we obtain the system of two simultaneous integral equations

$$\left. \begin{aligned} \int_0^1 \rho f(\rho) \{K_1(\rho_0, \rho) + K_2(\rho_0, \rho)\} d\rho - \int_0^1 \rho g(\rho) K_3(\rho_0, \rho) d\rho \\ = C_1 + \Pi^l(a\rho_0, 0), \\ \int_0^1 \rho f(\rho) K_3(\rho_0, \rho) d\rho - \int_0^1 \rho g(\rho) \{K_2(\rho_0, \rho) + K_1(\rho_0, \rho)\} d\rho \\ = C_2, \quad 0 < \rho_0 < 1, \end{aligned} \right\} \quad (3.1)$$

with  $f(\rho) = -[a \partial \Pi / \partial z]_{z=0}$ ,  $g(\rho) = -[a \partial \Pi / \partial z]_{z=l}$  and

$$\left. \begin{aligned} K_1(\rho_0, \rho) &= \frac{1}{2\pi} \int_0^{2\pi} \frac{\exp\{-i\alpha(\rho_0^2 - 2\rho_0\rho \cos \phi + \rho^2)^{\frac{1}{2}}\}}{(\rho_0^2 - 2\rho_0\rho \cos \phi + \rho^2)^{\frac{1}{2}}} d\phi, \\ K_2(\rho_0, \rho) &= \frac{1}{2\pi} \sum_{n=1}^{\infty} \int_0^{2\pi} \frac{\exp\{-i\alpha(\rho_0^2 - 2\rho_0\rho \cos \phi + \rho^2 + 4n^2\tau^2)^{\frac{1}{2}}\}}{(\rho_0^2 - 2\rho_0\rho \cos \phi + \rho^2 + 4n^2\tau^2)^{\frac{1}{2}}} d\phi, \\ K_3(\rho_0, \rho) &= \frac{1}{2\pi} \sum_{n=1}^{\infty} \int_0^{2\pi} \frac{\exp\{-i\alpha(\rho_0^2 - 2\rho_0\rho \cos \phi + \rho^2 + (2n-1)^2\tau^2)^{\frac{1}{2}}\}}{(\rho_0^2 - 2\rho_0\rho \cos \phi + \rho^2 + (2n-1)^2\tau^2)^{\frac{1}{2}}} d\phi. \end{aligned} \right\} \quad (3.2)$$

$C_1$  and  $C_2$  are arbitrary constants and are determined by the edge condition that  $df/d\rho$  and  $dg/d\rho$  are of order  $(1-\rho^2)^{-\frac{1}{2}}$  as  $\rho \rightarrow 1$ . By adding and subtracting the two integral equations (3.1), we obtain two ordinary integral equations

$$\int_0^1 \rho f^{\pm}(\rho) K^{\pm}(\rho_0, \rho) d\rho = C^{\pm} + \Pi^l(a\rho_0, 0), \quad 0 < \rho_0 < 1, \quad (3.3)$$

in which

$$\begin{aligned} f^{\pm}(\rho) &= f(\rho) \pm g(\rho), \\ K^{\pm}(\rho_0, \rho) &= K_1(\rho_0, \rho) + K_2(\rho_0, \rho) \mp K_3(\rho_0, \rho), \\ C^{\pm} &= C_1 \mp C_2. \end{aligned}$$

In section 5 we discuss the solution for small  $\alpha$  and in section 6 we give some results for large  $\alpha$ .

#### 4. Far-field behaviour

We first consider the far-field behaviour of the radiation field in the domains  $z < 0$  and  $z > l$ . In the same way as in (4), we obtain the results that

$$H_{\phi\omega}^r(\mathbf{r}, z) = A(\theta) \exp(-ikR)/R + O(R^{-2}) \quad \text{as } R \rightarrow \infty, \quad (4.1)$$

and where  $\mathbf{r} = R \sin \theta$  and  $z = -R \cos \theta$ ,  $0 \leq \theta < \frac{1}{2}\pi$ ,

$$H_{\phi\omega}^r(\mathbf{r}, z) = -C(\theta) \exp(-ikR)/R + O(R^{-2}) \quad \text{as } R \rightarrow \infty, \quad (4.2)$$

where  $\mathbf{r} = R \sin \theta$  and  $z = l + R \cos \theta$ ,  $0 \leq \theta < \frac{1}{2}\pi$ ,

in which the far-field amplitudes  $A(\theta)$  and  $C(\theta)$  are given by

$$A(\theta) = -\frac{q}{2\pi} \alpha^2 \beta_0 \sin \theta \int_0^\infty \rho K_0(\alpha \Gamma \rho) J_0(\alpha \rho \sin \theta) d\rho \\ - i\alpha \sin \theta \int_0^1 \rho f(\rho) J_0(\alpha \rho \sin \theta) d\rho, \quad \alpha \Gamma \neq 0, \quad (4.3)$$

$$C(\theta) = -\frac{q}{2\pi} \alpha^2 \beta_0 \sin \theta \exp(-i\beta_0 \alpha \tau) \int_0^\infty \rho K_0(\alpha \Gamma \rho) J_0(\alpha \rho \sin \theta) d\rho \\ - i\alpha \sin \theta \int_0^1 \rho g(\rho) J_0(\alpha \rho \sin \theta) d\rho, \quad \alpha \Gamma \neq 0, \quad (4.4)$$

where  $J_0$  is the Bessel function of the first kind and zero order (6). In the domain  $0 < z < l$  however, we apply an expansion in modes as  $r \rightarrow \infty$ :

$$H_{\phi\omega}^r(r, z) = \frac{i}{4l} \sum_{m=0}^M \epsilon_m B_m \left( \frac{2}{\pi g_m r} \right)^{\frac{1}{2}} \exp(-i g_m r + \frac{1}{4} \pi i) \cos(m\pi z/l) + \\ + O(r^{-1}) + \text{exponentially vanishing terms}, \quad (4.5)$$

where  $g_m = (k^2 - (m^2 \pi^2 / l^2))^{\dagger}$ ,  $\epsilon_m = \begin{cases} 1, & m = 0, \\ 2, & m \geq 1, \end{cases}$  and

$$B_m = -q \alpha^2 \gamma_m \beta_0 (1 - \exp(-i\beta_0 \alpha \tau)) \int_0^\infty \rho K_0(\alpha \Gamma \rho) J_0(\alpha \gamma_m \rho) d\rho \\ - 2\pi i \alpha \gamma_m \int_0^1 \rho (f(\rho) - g(\rho)) J_0(\alpha \gamma_m \rho) d\rho, \quad \alpha \Gamma \neq 0, \quad (4.6)$$

with  $\gamma_m = (1 - (m^2 \pi^2 / \alpha^2 \tau^2))^{\dagger}$ . In the next sections the expressions (4.3), (4.4) and (4.6) will be considered further.

## 5. Solution for small $\alpha$

The solution of an integral equation like (3.3) can be transformed into the solution of a Fredholm integral equation of the second kind, which is very suitable for iteration for small  $\alpha$  (7, 8). After determination (4) of the constants  $C^\pm$  in such a way that the edge conditions have been satisfied we obtain the integral equations

$$W^\pm(v) = \frac{\cosh(\alpha v)}{\cosh \alpha} B(1) - B(v) + \\ + \int_0^1 W^\pm(w) \left[ \frac{\cosh(\alpha v)}{\cosh \alpha} L^\pm(1, w) - L^\pm(v, w) \right] dw, \quad 0 < v < 1, \quad (5.1)$$

in which

$$\begin{aligned}
 W^\pm(v) &= \int_v^1 \rho f^\pm(\rho) \frac{\cos\{\alpha(\rho^2 - v^2)^\dagger\}}{(\rho^2 - v^2)^\dagger} d\rho, \\
 B(v) &= -\frac{q}{2\pi} \frac{d}{dv} \int_0^v w K_0(\alpha\Gamma w) \frac{\cosh\{\alpha(v^2 - w^2)^\dagger\}}{(v^2 - w^2)^\dagger} dw, \\
 L^\pm(v, w) &= L_1(v, w) + L_2(v, w) \mp L_3(v, w), \\
 L_1(v, w) &= -\frac{i}{\pi} \left[ \frac{\sinh\{\alpha(v+w)\}}{v+w} + \frac{\sinh\{\alpha(v-w)\}}{v-w} \right], \\
 L_2(v, w) &= \frac{2}{\pi} \sum_{n=1}^{\infty} \int_0^{\infty} \cos\{(\beta + i\alpha)v\} \cos\{(\beta + i\alpha)w\} \exp\{-(\beta + i\alpha)2n\tau\} d\beta, \\
 L_3(v, w) &= \frac{2}{\pi} \sum_{n=1}^{\infty} \int_0^{\infty} \cos\{(\beta + i\alpha)v\} \cos\{(\beta + i\alpha)w\} \times \\
 &\quad \times \exp\{-(\beta + i\alpha)(2n-1)\tau\} d\beta.
 \end{aligned}$$

Collins (7) and Thomas (8) have carried out the summations in  $L_2$  and  $L_3$  after an interchange of the summation and integration, but it does not simplify the calculations. We remark that  $L_2$  and  $L_3$  are singular at  $\alpha\tau = m\pi$ , where  $m$  is an integer. However, the choice of the constants  $C^\pm$  ensures that the kernel of the integral equation (5.1) is finite for all values of  $\alpha\tau$ . Physically, it means that the cavity formed by the two screens is not excited in a resonating behaviour. This is in contrast with the problem of the diffraction of a uniform plane wave through the circular apertures in two parallel screens, where resonances indeed occur (7, 8). When  $\alpha$  is small,  $\tau$  large and  $\beta_0$  not too large, say  $1 < \beta_0 < 2$ , an iterative solution of equation (5.1) can be obtained. Our next step is to calculate the far-field amplitudes from (4.3), (4.4) and (4.6). The final results are

$$\begin{aligned}
 A(\theta) &= -\frac{q}{2\pi} \sin\theta \left[ \beta_0 \frac{1}{\Gamma^2 + \sin^2\theta} + \frac{i}{\pi} \alpha p(\alpha) + \frac{i}{9\pi} \alpha^3 \cos^2\theta - \frac{8}{9\pi^2} \alpha^4 + \right. \\
 &\quad \left. + o(\alpha^4) + \frac{E_2}{3\pi^2\tau^2} \left\{ \alpha^2 p(\alpha) + \frac{1}{5} \alpha^4 \cos^2\theta - \frac{22}{45} \alpha^4 + O(\alpha^5) \right\} \right. \\
 &\quad \left. - \frac{iE_3}{6\pi^2\tau^3} \left\{ \alpha p(\alpha) + \frac{1}{5} \alpha^3 \cos^2\theta + \frac{34}{45} \alpha^3 + \frac{8i}{9\pi} \alpha^4 + o(\alpha^4) \right\} + O(\tau^{-4}) \right], \quad (5.2)
 \end{aligned}$$

$$\begin{aligned}
 C(\theta) &= -\frac{q}{2\pi} \sin\theta \left[ \beta_0 \frac{\exp(-i\beta_0\alpha\tau)}{\Gamma^2 + \sin^2\theta} \right. \\
 &\quad \left. - \frac{F_2}{3\pi^2\tau^2} \left\{ \alpha^2 p(\alpha) + \frac{1}{5} \alpha^4 \cos^2\theta - \frac{22}{45} \alpha^4 + O(\alpha^5) \right\} + \right. \\
 &\quad \left. + \frac{iF_3}{6\pi^2\tau^3} \left\{ \alpha p(\alpha) + \frac{1}{5} \alpha^3 \cos^2\theta + \frac{34}{45} \alpha^3 + \frac{8i}{9\pi} \alpha^4 + o(\alpha^4) \right\} + O(\tau^{-4}) \right], \quad (5.3)
 \end{aligned}$$

$$\begin{aligned}
 B_m = & -q\gamma_m \left[ \beta_0 \frac{1 - \exp(-i\beta_0\alpha\tau)}{\Gamma^2 + \gamma_m^2} + \frac{i}{\pi} \alpha p(\alpha) + \frac{i\alpha m^2\pi}{9\tau^2} - \frac{8}{9\pi^2} \alpha^4 + o(\alpha^4) + \right. \\
 & + \frac{(E_2 + F_2)}{3\pi^2\tau^2} \left\{ \alpha^2 p(\alpha) + \frac{\alpha^2 m^2\pi^2}{5\tau^2} - \frac{22}{45} \alpha^4 + O(\alpha^5) \right\} \\
 & \left. - \frac{i(E_3 + F_3)}{6\pi^2\tau^3} \left\{ \alpha p(\alpha) + \frac{\alpha m^2\pi^2}{5\tau^2} + \frac{34}{45} \alpha^3 + \frac{8i}{9\pi} \alpha^4 + o(\alpha^4) \right\} + O(\tau^{-4}) \right]; \quad (5.4)
 \end{aligned}$$

where  $p(\alpha) = 2 + \frac{2}{3}\alpha^2\Gamma^2 \ln(\alpha\Gamma) - \frac{1}{3}\alpha^2\{\Gamma^2(8 - 6\gamma) + 5\}$ ,  $\gamma = 0.577216 \dots$  is Euler's constant, and

$$\left. \begin{aligned}
 E_s &= \sum_{n=1}^{\infty} n^{-s} \exp(-2in\alpha\tau) = Li_s\{\exp(-2i\alpha\tau)\}, \\
 F_s &= \sum_{n=1}^{\infty} (n - \frac{1}{2})^{-s} \exp\{-(2n-1)i\alpha\tau\} \\
 &= 2^{s-1}[Li_s\{\exp(-i\alpha\tau)\} - Li_s\{\exp(-i\alpha\tau + i\pi)\}],
 \end{aligned} \right\} \quad (5.5)$$

where  $Li_s$  is the polylogarithm of order  $s$  (see (9)). We observe that the singular terms with  $E_1$  and  $F_1$  have indeed vanished in the field expressions. We note in (5.4) that  $m\pi/\tau = O(\alpha)$ .

## 6. Solution for large $\alpha$

To obtain a solution of integral equation (3.3) for large  $\alpha$  is rather difficult. In the relevant problem involving one screen, a closed solution for large  $\alpha$  has been obtained when we neglect the interaction between opposite points at the edge of the aperture (4). As we have seen from the considerations in section 5, there is no singular behaviour of the resonator formed by the two screens in our present problem. Hence, to the same degree of approximation, we can also neglect the interaction between the two aperture fields when  $l > a$  ( $\tau > 1$ ). Then, in the aperture at  $z = 0$ , the field distribution is given by equation (6.19) of (4) and in the aperture at  $z = l$ , the field distribution is given by  $g(w) \simeq \exp(-i\beta_0\alpha\tau)f(w)$ . In the same manner as in (4), we obtain the far-field amplitude as

$$\begin{aligned}
 A(\theta) \simeq \frac{q}{2\pi} \beta_0\alpha \left[ \sin \theta \frac{\sin \theta J_1(\alpha \sin \theta) K_0(\alpha\Gamma) - \Gamma K_1(\alpha\Gamma) J_0(\alpha \sin \theta)}{\Gamma^2 + \sin^2 \theta} \right. \\
 \left. - \frac{J_1(\alpha \sin \theta) K_0(\alpha\Gamma)}{(1 - i\Gamma)^{\frac{1}{2}}} \right], \quad (6.1)
 \end{aligned}$$

$$C(\theta) \simeq \exp(-i\beta_0\alpha\tau)A(\theta), \quad (6.2)$$

$$B_m \simeq q\beta_0\alpha[1 - \exp(-i\beta_0\alpha\tau)] \times \left[ \gamma_m \frac{\gamma_m J_1(\alpha\gamma_m) K_0(\alpha\Gamma) - \Gamma K_1(\alpha\Gamma) J_0(\alpha\gamma_m)}{\Gamma^2 + \gamma_m^2} - \frac{J_1(\alpha\gamma_m) K_0(\alpha\Gamma)}{(1 - i\Gamma)^{\dagger}} \right], \quad (6.3)$$

in which  $J_1$  is the Bessel function of the first kind and first order and  $K_1$  is the modified Bessel function of the second kind and first order.

**7. Radiation loss of the point charge. Numerical results**

The total radiated energy is the sum of the energies radiated through hemispheres of large radius  $R$  with centres at  $r = 0, z = 0$  and  $r = 0, z = l$  bounding the regions  $z < 0$  and  $z > l$ , respectively, and through the cylindrical surface of large radius with axis the  $z$ -axis bounding the region  $0 < z < l$ ; we obtain

$$Q_{\text{tot}} = \int_0^\infty [P_1(\omega) + P_2(\omega) + P_3(\omega)] d\omega, \quad (7.1)$$

with  $P_1(\omega) = 2(\mu_0/\epsilon_0)^{\dagger} \int_0^{\frac{1}{2}\pi} |A(\theta)|^2 \sin \theta d\theta,$

$$P_2(\omega) = \frac{1}{4\pi} (\mu_0/\epsilon_0)^{\dagger} (\alpha\tau)^{-1} \sum_{m=0}^M \epsilon_m |B_m|^2,$$

$$P_3(\omega) = 2(\mu_0/\epsilon_0)^{\dagger} \int_0^{\frac{1}{2}\pi} |C(\theta)|^2 \sin \theta d\theta,$$

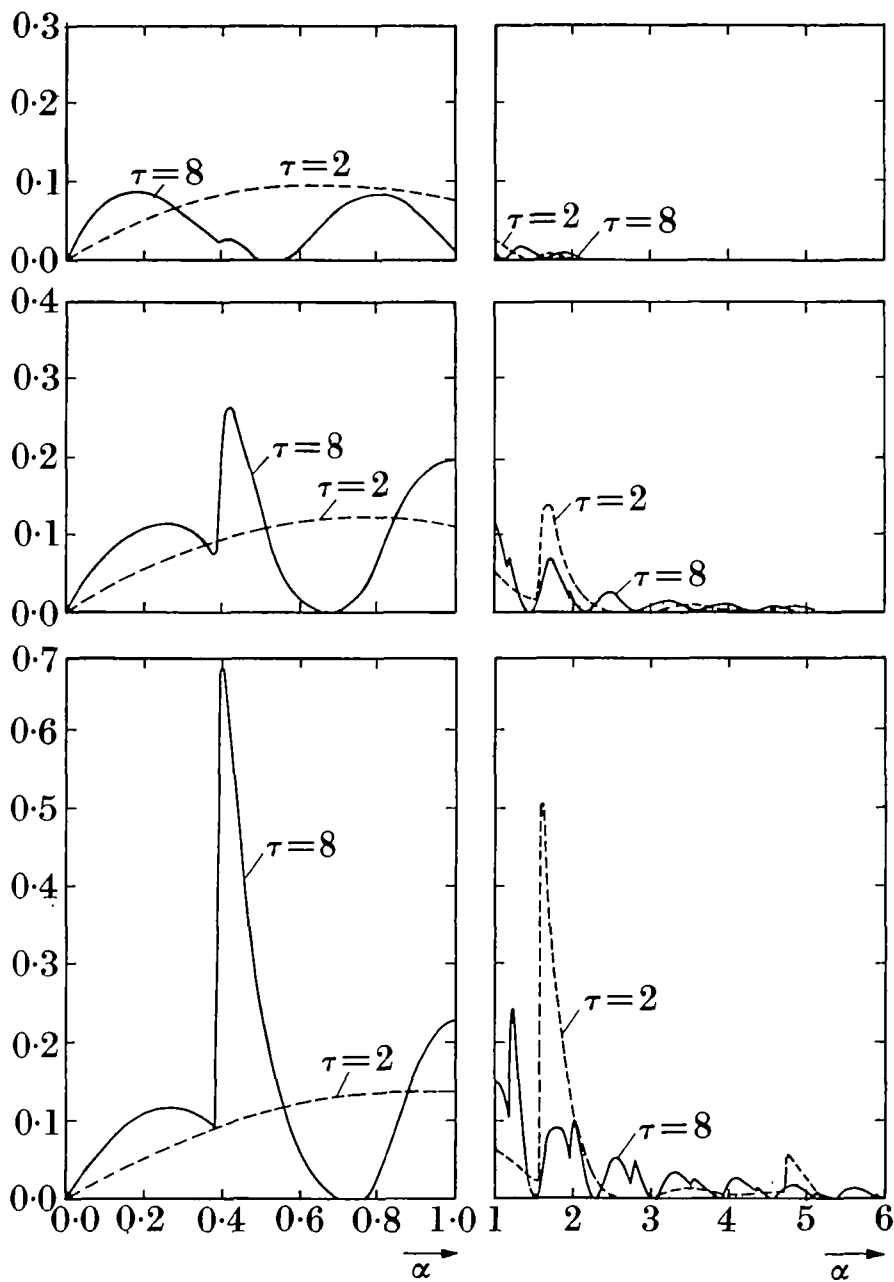
in which  $(\mu_0/\epsilon_0)^{\dagger} \simeq 120\pi$  is the wave impedance of the vacuum. Let us now introduce

$$Q_{\text{diff}} = \int_0^\infty P_{\text{diff}}(\omega) d\omega, \quad (7.2)$$

with  $P_{\text{diff}}(\omega) = P_1(\omega) + P_2(\omega) + P_3(\omega) - P(\omega),$

in which  $P(\omega)$  is the relevant radiation loss in the problem of a single screen (4). We note that  $P_{\text{diff}}(\omega) d\omega$  is the difference in radiation loss between that in the present problem of two screens and that in the former problem involving a single screen, caused by radiation between  $\omega$  and  $\omega + d\omega$ .

Numerical results concerning the radiation loss  $q^{-2}(\epsilon_0/\mu_0)^{\dagger} P_{\text{diff}}(\omega)$  are presented in Fig. 1 for small  $\alpha$  ( $\alpha < 1$ ) and for large  $\alpha$  ( $\alpha > 1$ ) with  $\alpha = \omega a/c_0$ . We observe that the extra radiation loss  $P_{\text{diff}}(\omega)$ , due to the presence of the second screen at  $z = l$ , exhibits a fluctuating behaviour with decreasing amplitude for increasing  $\alpha > 1$ . Peaks in the curves occur at values of  $\alpha$  a little larger than  $m\pi/\tau$  ( $m = 1, 3, 5, \dots$ ). This peaked character increases with increasing velocity of the charge. The radiation loss  $P_{\text{tot}}(\omega) = P_1(\omega) + P_2(\omega) + P_3(\omega)$  caused by the presence of the two



FIGS 1(a),  $v_0/c_0 = 0.70$ , 1(b),  $v_0/c_0 = 0.90$ , 1(c),  $v_0/c_0 = 0.96$ .



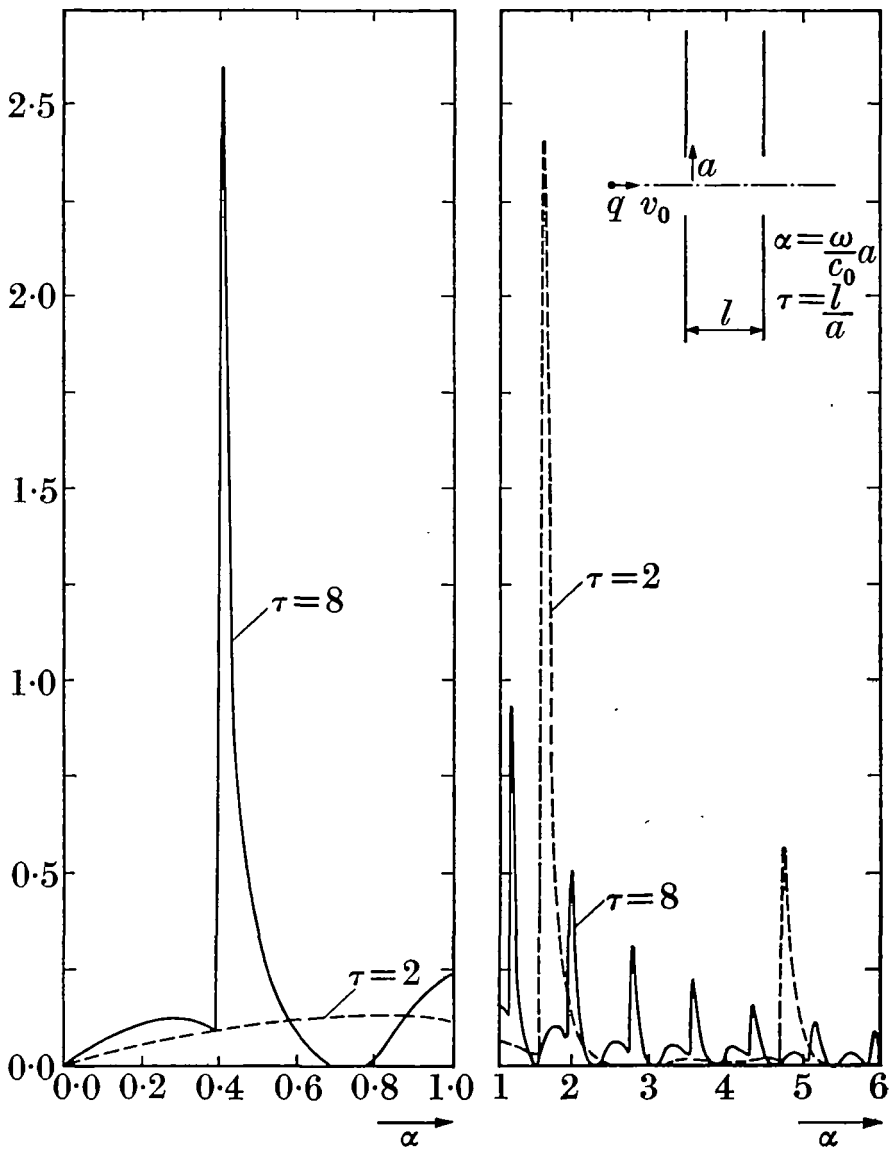


FIG. 1(d),  $v_0/c_0 = 0.99$ .

screens can be obtained by summation of the results of Fig. 1 of this paper and those of Fig. 1 of the paper on the problem of a single screen (4).

In Table 1, numerical results for  $\epsilon_0 a q^{-2} Q_{\text{diff}}$  are presented. The numeri-

TABLE 1. Numerical values of  $\epsilon_0 a q^{-2} Q_{\text{diff}}$ ;  
the last line of the Table gives the corresponding value of  $(\beta_0 \Gamma)^{-1}$

$\tau$	$v_0/c_0$ = 0.70	$v_0/c_0$ = 0.80	$v_0/c_0$ = 0.90	$v_0/c_0$ = 0.96	$v_0/c_0$ = 0.98	$v_0/c_0$ = 0.99
1.5	0.08	0.09	0.14	0.25	0.38	0.57
2	0.08	0.10	0.16	0.28	0.42	0.62
4	0.06	0.09	0.16	0.29	0.43	0.63
8	0.05	0.09	0.15	0.28	0.42	0.62
$\infty$	0.05	0.09	0.16	0.30	0.44	0.64
$(\beta_0 \Gamma)^{-1}$	0.69	1.07	1.86	3.29	4.83	6.95

cal values are obtained from a numerical integration of the values of  $P_{\text{diff}}(\omega)$ , switching from the results for small  $\alpha$  to the ones for large  $\alpha$  at  $\alpha = 1$ . The values on the penultimate line ( $\tau = \infty$ ) are obtained in a different manner. When  $\tau = \infty$  ( $l = \infty$ ), the total extra radiation loss  $Q_{\text{diff}}$  in the presence of the second screen has to be the same as the total radiation loss in the absence of the second screen. In the case  $\tau = \infty$ , we have then reproduced the values obtained in the problem of a single screen. For the sake of completeness we remark that the latter values have been obtained from a numerical integration of the values of  $P(\omega)$ , switching from the results for small  $\alpha$  to the ones for large  $\alpha$  at the crossover point of the relevant curves. We observe that already for  $\tau = 4$  the results coincide with the ones for  $\tau = \infty$ . From an extrapolation of the results of the problems involving one and two screens to the problem involving a certain *small* number of screens, it seems likely that the total radiation loss  $Q_{\text{tot}}$  of a charge moving through a certain small number  $N$  of coaxial circular apertures in parallel screens can be written as

$$Q_{\text{tot}} \simeq 0.1 N q^2 / (\epsilon_0 a \beta_0 \Gamma). \quad (7.3)$$

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## REFERENCES

1. B. M. BOLOTOVSKII and G. V. VOSKRESENSKII, *Soviet Phys.-Usp.* **9** (1966) 73-96.
2. ——— *ibid.* **11** (1968) 143-162.
3. R. D. HAZELTINE, M. N. ROSENBLUTH and A. M. SESSLER, *J. math. Phys.* **12** (1971) 502-514.
4. P. M. VAN DEN BERG, *J. Inst. Maths. Applies* **17** (1976).
5. ——— *Radiation from a Point Charge Moving through two Coaxial Circular Apertures in Two Parallel Screens* (Unpublished report, available on request).
6. G. N. WATSON, *Theory of Bessel Functions* (Cambridge University Press, 1966).
7. W. D. COLLINS, *Proc. Lond. math. Soc.* **15** (1965) 167-192.
8. D. P. THOMAS, *Q. Jl Mech. appl. Math.* **18** (1965) 107-120.
9. L. LEWIN, *Dilogarithms and Associated Functions* (Macdonald, London, 1958).