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RADIATION FROM SURFACE WITH PERIODIC BOUNDARY OF METAMATERIALS EXCITED BY A CURRENT

P. Melezhik, A. Poyedinchuk, and N. Yashina

Institute of Radiophysics and Electronics
National Academy of Sciences of Ukraine
12, Ak. Proskury Street, Kharkov, 61085, Ukraine

G. Granet

LASMEA, Université Blaise Pascal
Av. Des Landais, 24, 63177, Aubière Cedex, France

M. Ney

Laboratoire d'Electronique et des Systèmes de Télécommunications
CS 83818, 29238 BREST, France

Abstract—The rigorous modeling and analysis of electromagnetic wave transformation and radiation from the periodic boundary of metamaterial are presented. The nature of the phenomenon of resonant radiation and the influence of various parameters on it are investigated.

The study is carried out with the objective of potential applications to antenna design. Simulated results show that very high directivity can be obtained and that beam steering can be achieved by adjusting proper parameters.

1. INTRODUCTION

The idea to design composite materials (metamaterials) that are able to acquire negative values of their effective (measured or calculated) constitutive parameters in the microwave and millimeter wave ranges, has received some intensive development. It is caused by the rather unusual phenomena that may appear in metamaterials. Possibility developments include the creation of a sub-wavelength resolution "lens", inversion of Doppler frequency shift, change of the Cherenkov emission direction, etc. [1]. Due to such extraordinary properties, these metamaterials may find applications in antenna design, microwave engineering and techniques and others areas. Various aspects of possible metamaterial efficient applications in microwave region are nowadays widely studied both in the US and Europe; see for example [2–4]. The majority of papers are focused on electromagnetic modeling of composite metamaterials aimed at determining their constitutive parameters and on the design of microwave and terahertz range devices exploiting the new properties of the fabricated materials [5–7].

The present paper is devoted to the rigorous modeling and treatment of electromagnetic waves transformation by periodic boundary of metamaterials that are supposed to be characterized by certain given effective constitutive parameters. On the basis of the results presented, we clearly demonstrate potential applications of the modeled and studied phenomena.

2. PROBLEM FORMULATION

We consider the problem of radiation diffraction by periodic boundary (with period d and maximal deviation h) of certain metamaterial with constitutive parameters ε_2, μ_2 (see Fig. 1) within the given current approximation model. This two-dimensional model has been successfully applied to modeling surface wave's antenna with plane metal strip grating located on conventional dielectric with $\varepsilon_2 > 0,\ \mu_2 > 0$ [8]. Here we suppose that we use a planar waveguide, exited by certain electromagnetic source, that can be replaced in our case with

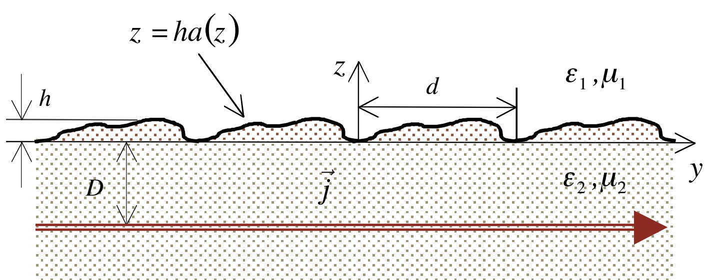

Figure 1. Problem geometry: $a(y)$ is the profile of periodic boundary, h – its depth, D – distance to the excitation current $\vec{j}$, d – period of the boundary, ε_1, μ_1 are the constitutive parameters of the free space, ε_2, μ_2 – effective constitutive parameters of the medium, in which $\vec{j}$ flows.

an impressed current $\vec{j}$

We have to find the scattered electromagnetic field $\vec{E}, \vec{H}$ that is excited by the given current $\vec{j}$, placed below the periodic boundary $\overline{z} = ha(\overline{y})$ at the distance D, from the equations

$$\begin{aligned} rot\vec{H} &= -ik\varepsilon(\overline{z})\vec{E} + \frac{4\pi}{c}\vec{j} \\ rot\vec{E} &= ik\mu(\overline{z})\vec{H} \\ \varepsilon(z) &= \begin{cases} \varepsilon_1 & \overline{z} > ha(\overline{y}) \\ \varepsilon_2 & \overline{z} < ha(\overline{y}) \end{cases} \quad \mu(z) = \begin{cases} \mu_1 & \overline{z} > ha(\overline{y}) \\ \mu_2 & \overline{z} < ha(\overline{y}) \end{cases} \end{aligned}$$

Suppose that in the domain $\overline{z} < ha(\overline{y})$ the current $\vec{j}$ is given as

$$j_y = j_0 \exp\left(i\gamma\overline{y} - i\omega t\right)\delta(\overline{z} + D), \qquad j_x = j_z = 0$$

Here γ is a given parameter of the current and ω is a given angular frequency.

We also suppose that $\partial/\partial\overline{x} \equiv 0$ and $\mathrm{Re}(\varepsilon_2) < -1$, $\mathrm{Im}(\varepsilon_2) \geq 0$, $\varepsilon_1 = \mu_1 = \mu_2 = 1$, $\gamma > 0$.

First, we consider the case when the permittivity of metamaterial is frequency independent.

For further convenience we shall introduce the notations of normalized coordinates $z = 2\pi\overline{z}/d$, $y = 2\pi\overline{y}/d$.

The equation defining the boundary shape is given as $z = A_0 a(y), a(y)$ is a periodic function, $A_0 = 2\pi h/d$.

Now the given problem can be reduced to the following one,

$$\begin{aligned} \Delta H_x^{(1)} + \kappa^2\varepsilon_1\mu_1 H_x^{(1)} &= 0, & z > A_0 a(y) \\ \Delta H_x^{(2)} + \kappa^2\varepsilon_2\mu_2 H_x^{(2)} &= \frac{2d}{c}\frac{\partial j_y}{\partial z}, & z < A_0 a(y) \end{aligned}, \tag{1}$$

here $\kappa = \omega d/2\pi c$, c is a light velocity in vacuum.

Functions $H_x^{(1)}$ and $H_x^{(2)}$ have to meet the continuity conditions on the boundary $z = A_0 a(y)$:

$$H_x^{(1)} = H_x^{(2)}; \qquad \frac{\partial H_x^{(1)}}{\partial z} - A_0\dot{a}\frac{\partial H_x^{(1)}}{\partial y} = \frac{\varepsilon_1}{\varepsilon_2}\left(\frac{\partial H_x^{(2)}}{\partial z} - A_0\dot{a}\frac{\partial H_x^{(2)}}{\partial y}\right) \tag{2}$$

and the radiation condition

$$\begin{aligned} H_x^{(1)} &= \sum_n T_n e^{i\Phi_n y} e^{i\rho_{n1}(z - A_0)} & z \geq A_0 \\ H_x^{(2)} - H_x^{(i)} &= \sum_n R_n e^{i\Phi_n y} e^{-i\rho_{n2} z} & z \leq 0 \end{aligned} \tag{3}$$

$$
\begin{aligned}
\rho_{n1} &= \sqrt{\varepsilon_1\mu_1\kappa^2 - \Phi_n^2}, \qquad \rho_{n2} = \sqrt{\varepsilon_2\mu_2\kappa^2 - \Phi_n^2} \\
\Phi_n &= n + \frac{\gamma d}{2\pi}, \qquad \dot{a} = da(y)/dy, \\
\mathrm{Re}(\rho_{n1}) &\geq 0, \quad \mathrm{Re}(\rho_{n2}) \geq 0, \quad \mathrm{Im}(\rho)_{n1} \geq 0, \quad \mathrm{Im}(\rho)_{n2} \geq 0.
\end{aligned}
$$

Since we consider the dielectric in general may be replaced by metamaterial, then we have to take care about the proper choice of propagation constants [9, 10]. So if we have $\mathrm{Re}(\varepsilon_2) < 0$ and $\mathrm{Re}(\mu_2) < 0$, then $\mathrm{Re}(\sqrt{\varepsilon_2\mu_2}) \leq 0$ and $\mathrm{Im}(\sqrt{\varepsilon_2\mu_2}) \geq 0$. In order to meet boundary conditions, we have to choose $\mathrm{Re}(\rho_{n2}) \leq 0$ and $\mathrm{Ime}(\rho)_{n2} \geq 0$.

The value of $H_x^{(i)}$ is the magnetic component of electromagnetic field created by the current $\vec{j}$ in unbounded space with constitutive parameters ε_2, μ_2. It is easy to show that $H_x^{(i)}$ can be presented in the form

$$
H_x^{(i)} = \frac{2d}{c} j_0 \frac{|z + B_0|}{z + B_0} e^{i\overline{\gamma}y} e^{-\rho|z+B_0|} \tag{4}
$$

where $B_0 = \frac{2\pi D}{d}$, $\overline{\gamma} = \frac{\gamma d}{2\pi}$, $\rho = \sqrt{\overline{\gamma}^2 - \varepsilon_2\mu_2\kappa^2}$.

As one can see from (4) the field $H_x^{(i)}$ decays exponentially when $|z| \to \infty$.

The electrical components of electromagnetic field can be expressed via $H_x^{(1)}$ and $H_x^{(2)}$

$$
\begin{aligned}
E_y^{(2)} &= -\frac{1}{i\kappa\varepsilon_2}\left(\frac{\partial H_x^{(2)}}{\partial z} - \frac{4\pi}{c} j_y\right) \qquad E_z^{(2)} = \frac{1}{i\kappa\varepsilon_2}\frac{\partial H_x^{(2)}}{\partial y} \\
E_y^{(1)} &= -\frac{1}{i\kappa\varepsilon_1}\frac{\partial H_x^{(1)}}{\partial z} \qquad E_z^{(1)} = \frac{1}{i\kappa\varepsilon_1}\frac{\partial H_x^{(1)}}{\partial y}
\end{aligned}
$$

The solution to the boundary value problem (1)–(4) has been carried out within the frame of the regularized C-method [9, 10]. The investigation of the solution and capability of the corresponding numerical algorithm and its implementation have been discussed in details in [10].

In the case when $\mathrm{Re}(\varepsilon_2) < -1$, in the domain $z < A_0 a(y)$ the electromagnetic field decays when $z \to -\infty$. The electromagnetic field in the domain $z > A_0 a(y)$ is the superposition of a finite number of plane H-polarized propagating waves and of an infinite number of non-homogeneous plane waves, exponentially decaying with $z \to +\infty$. It is rather clear that under the condition $\overline{\gamma} > \kappa$, all space harmonics of positive values of n are decaying with $z \to +\infty$.

Therefore the radiation field (in the domain $z > A_0 a(y)$) is composed of the space harmonics with negative value of n (see

Equation (3). The radiation condition for the -1 harmonic is defined by inequality $\kappa > |\overline{\gamma} - 1|$.

The angle of the radiation θ_{-1} (defined from the axis y) can be calculated from the expression

$$\theta_{-1} = \begin{cases} \operatorname{arctg}\left(\dfrac{\sqrt{\kappa^2 - (\overline{\gamma} - 1)^2}}{\overline{\gamma} - 1}\right) & \overline{\gamma} - 1 > 0 \\ \pi - \operatorname{arctg}\left(\dfrac{\sqrt{\kappa^2 - (\overline{\gamma} - 1)^2}}{\overline{\gamma} - 1}\right) & \overline{\gamma} - 1 < 0. \end{cases} \tag{5}$$

From (5), in the case $\overline{\gamma} \cong 1$, the radiation of electromagnetic waves in the direction $\theta_{-1} \cong 90°$ becomes possible. We shall study here the radiation of the -1 spatial harmonic that proved to be the most interesting from point of view of applications to surface and leaky wave antennas [8].

3. NUMERICAL RESULTS

Our numerical experiments focused on the study of the problem depicted in Fig. 1 with its possible incorporation into antenna design. Therefore, we studied the possibility of radiation from the periodic surface. It is known that the most interesting case is the radiation from the surface of the -1 harmonic, as it appears at the smallest frequency parameter values in the frequency range when no other spatial waves can propagate.

So we investigated the influence of the structure parameters $\kappa, \varepsilon_2, \overline{\gamma}, h$ and D on the magnitude of the amplitude of -1 spatial harmonic $|T_{-1}|$and the direction of its radiation angle θ_{-1} within the frequency range $|\overline{\gamma} - 1| < \kappa < \overline{\gamma}$.

Naturally we started our numerical experiments with the study of conventional materials with positive real parts for the constitutive parameters. The corresponding results for H-polarizations are presented in Fig. 2. These results show that for these types of dielectrics, the radiation from periodic boundary is inefficient. The amplitude of the radiation harmonic for the parameters studied does not exceed the value $|T_{-1}| < 0.3$. Such "regular" behavior can be predicted, on the diffraction properties of grating, using conventional dielectrics that are rather smooth without any pronounced resonant effects, see e.g., Figure 4 in [10].

Absorption resonances appear in electromagnetic wave diffraction by periodic boundaries of materials with single or double negative constitutive parameters (see, for example [10], Figures 10, 11).

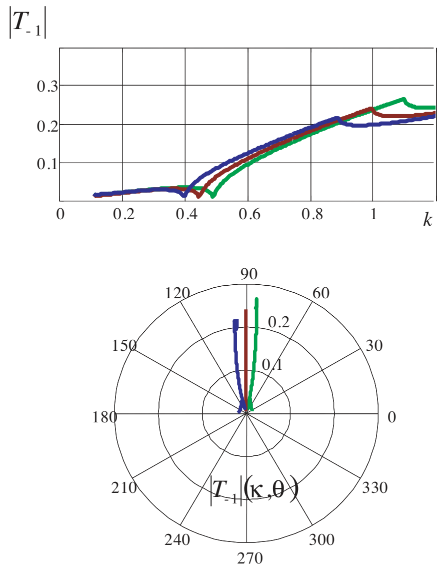

Figure 2. Radiation of -1 harmonic from periodic boundary $a(y) = 0.5(1 + \cos(y))$ of dielectric with constitutive parameters $\varepsilon_2 = 5.1 + i0.01$; $\mu_2 = 1$; with depth of grooves $A_0 = 0.4$; $B_0 = 0.1$, $\overline{\gamma} = 0.89$, blue lines, $\overline{\gamma} = 0.9999$; red lines and $\overline{\gamma} = 1.11$; green lines, H-polarization.

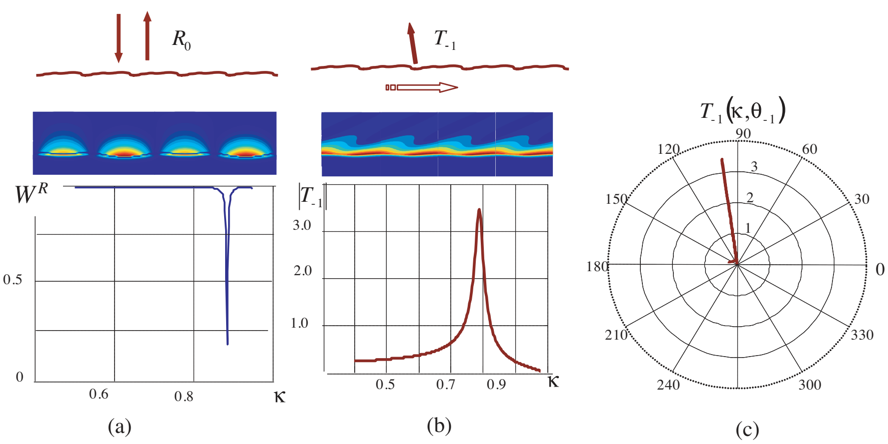

Figure 3. The diffraction, (a) and radiation (b), (c) characteristics of periodic boundary of the profile $a(y) = 0.5(1 + \cos(y))$, $\varepsilon_2 = -5.1 + i0.01$; $A_0 = 0.4$; $\mu_2 = 1$; $\overline{\gamma} = 0.89$, $B_0 = 0.4$. The absorption resonance in (a) corresponds to the eigen frequency $\kappa = 0.89 + i0$.

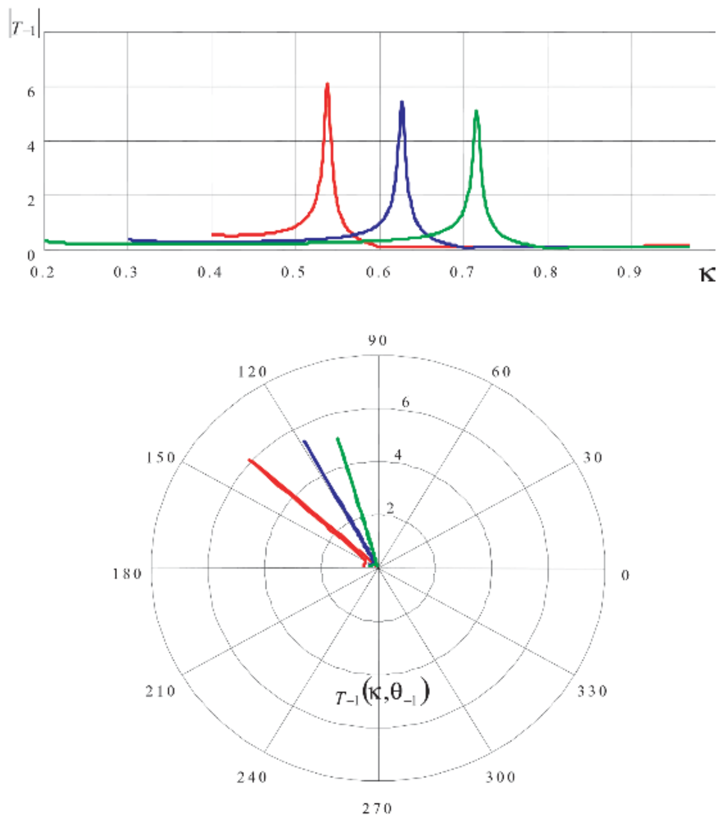

Figure 4. Radiation from the periodic surface of metamaterial with profile $a(y) = 0.5(1 + \cos(y))$, $\varepsilon_2 = -5.1 + i0.01$; $A_0 = 0.4$; $\mu_2 = 1$; $B_0 = 0.1$, for various parameter s: $\overline{\gamma} = 0.6$ – red lines, $\overline{\gamma} = 0.7$ – blue lines, and $\overline{\gamma} = 0.8$ – green lines.

In Figure 3 we present the absorption resonance for the diffraction of H-polarized waves by the structure we are studying. Such resonances can exist if the parameters meet the requirement: $\varepsilon_2 < -\varepsilon_1$. For rather smooth surfaces $A_0 \ll 1$, $\kappa < 1$ the location of absorption resonances can be approximated by the formula,

$$\kappa = n\mathrm{Re}\left(\sqrt{\frac{\varepsilon_2^2 - \varepsilon_1^2}{\varepsilon_1\varepsilon_2(\varepsilon_2\mu_1 - \varepsilon_1\mu_2)}}\right)(1 + O(A_0))\,, n = 1, 2\ldots$$

From the picture of field pattern presented in upper part of Figure 3(a), one can suggest that the resonances occur at frequencies when eigenmodes or eigenwaves of the periodic structure are excited [9–11]. To prove this statement we present the value of eigenfrequency that is the result of the solution to eigenvalue problem considered in [10]. It is clearly seen from Figure 3(a) that the real part of eigenfrequency corresponds to the absorption resonance.

The existence of such resonances provides the possibility of efficient transformation of spatial harmonics to surface ones. This motivates the modeling of radiation from periodic surfaces of metamaterial when excited by the current $\vec{j}$ propagating in the metamaterial at a distance D from its boundary. For our modeling, we have chosen the metamaterial with $\varepsilon_2 = -5.1 + i0.01$; and $\mu_2 = 1$; the periodic surface of the shape $z = 0.5A_0(1 + \cos(y))$ with period d and deviation h. The current is located at the normalized distance $B_0 = 0.4$ from the boundary. Radiation of the -1 harmonic as a function of frequency parameter κ are presented in Figure 3(b). In Figure 3(c), the curve for $T_{-1}(\kappa, \theta_{-1})$ is presented. One can observe the pronounced resonance in curve $T_{-1}(\kappa)$. Figure 3(c) shows that the radiation is concentrated within a rather narrow sector of angles θ_{-1} found from (5). So here we can expect efficient radiation from the periodic boundary of metamaterials.

We have studied the radiation for different current (or waveguide) parameters $\overline{\gamma} = 0.6; 0.7; 0.8$. For a chosen structure, we have sharp resonances in amplitude of the radiating -1 spatial harmonic (Figure 4(a)). Peaks of amplitude appear at different values of frequency parameter κ for various magnitudes of $\overline{\gamma}$. Their frequency position can be approximately estimated by,

$$\kappa = \overline{\gamma}\mathrm{Re}\left(\sqrt{\frac{\varepsilon_2^2 - \varepsilon_1^2}{\varepsilon_1\varepsilon_2\left(\mu_1\varepsilon_2 - \varepsilon_1\mu_2\right)}}\right). \tag{6}$$

The radiation peaks are concentrated within the narrow sector of angle θ_{-1} and the position of the maximum varies with $\overline{\gamma}$ as shown by Figure 4(b). The maximum values of amplitude at resonance frequency decrease when $\overline{\gamma}$ increases. From the results presented in Figure 4, we can conclude that $\overline{\gamma}$ may serve as one of the parameters to control the frequency of the main maximum of the radiating mode and its direction. (Note, in our model $\overline{\gamma}$ is the characteristic of excitation current, which may be changed electrically by various means).

We also carried out the study of the influence of the magnitude of effective ε_2 on the radiation from the periodic surface. Several examples from our numerical experiments performed with metamaterials with effective permittivity values $\varepsilon_2 = -2.55 + i0.01, \varepsilon_2 = -5.1 + i0.01$ and $\varepsilon_2 = -7.65 + i0.01$ are presented in

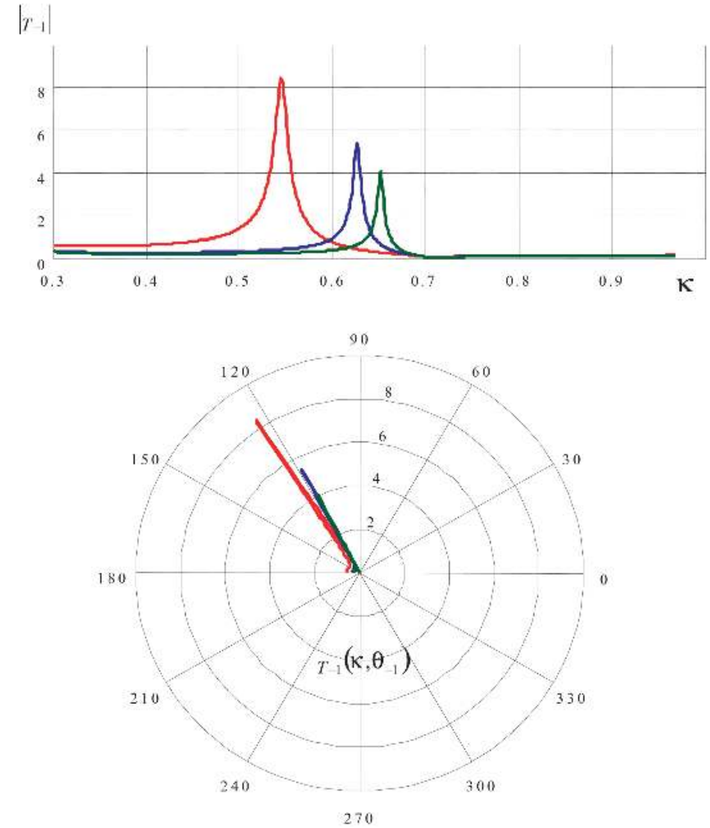

Figure 5. Radiation from the periodic surface of metamaterial with profile $a(y) = 0.5(1+\cos(y))$, $A_0 = 0.4$; $\mu_2 = 1$; current parameter $\overline{\gamma} = 0.7$, $B_0 = 0.1$, for various effective parameters ε_2: $\varepsilon_2 = -2.55 + i0.01$; red lines, $\varepsilon_2 = -5.1 + i0.01$; blue lines, and $\varepsilon_2 = -7.65 + i0.01$; green lines.

Figure 5. Here we have to point out that influence of the effective constitutive parameter ε_2 on the resonant frequency radiation and especially on the angle of radiation θ_{-1} is smaller than the influence of parameter $\overline{\gamma}$. The resonant amplitude $T_{-1}(\kappa)$ decreases considerably when ε_2 increases, and resonance values are closer (in terms of the frequency). From (6) one can see that increasing ε_2 produces the frequency of resonant radiation that tend to $\overline{\gamma}$. The influence of variation of ε_2 is much less significant for the radiation angle. Thus θ_{-1} is nearly the same for materials with $\varepsilon_2 = -5.1 + i0.01$ and $\varepsilon_2 = -7.65 + i0.01$ (see blue and green curves in Figure 5(b). Nevertheless,

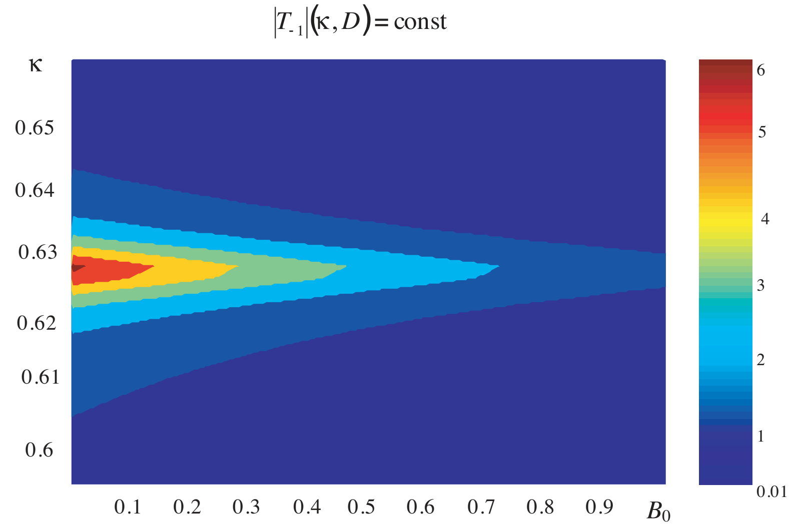

Figure 6. Illustration of the behavior of the amplitude of the radiation -1 harmonic $|T_{-1}|(\kappa, D)$ around the resonant frequency; $\overline{\gamma} = 0.7$ and $\varepsilon_2 = -5.1 + i0.01$.

there is the possibility to change the frequency of radiation resonance by varying ε_2.

The increase of the distance B_0 between the current and the surface of the boundary results in the decrease of the resonant radiation amplitude. As a result, this parameter may serve also as a tuning parameter in certain microwave application. In Figure 6, we present the picture of $|T_{-1}(\kappa, B_0)| = \text{const}$ for $\overline{\gamma} = 0.7$. and $\varepsilon_2 = -5.1 + i0.01$ It shows the decrease of the radiation amplitude around the resonance frequency when the distance B_0 between excitation current $\vec{j}$ and periodic boundary $z = A_0 a(y)$ increases. There is a distance limit B_0 (for certain parameters of the material ε_2 and current $\vec{j}$) for which the surface wave on the boundary can not be excited and the radiation of -1 harmonic does not occur. For example with $\overline{\gamma} = 0.7$. and $\varepsilon_2 = -5.1 + i0.01$ and $B_0 = 1$, the resonant amplitude is only $|T_{-1}| = 0.01$.

The efficient radiation of -1 spatial harmonic is also present in the case of double negative materials (see Figure 7). The introduction of negative μ_2 reduces the value of the resonant amplitude, moves its frequency position, and changes the radiation angle. For double negative metamaterials the evaluation of absorption and radiation

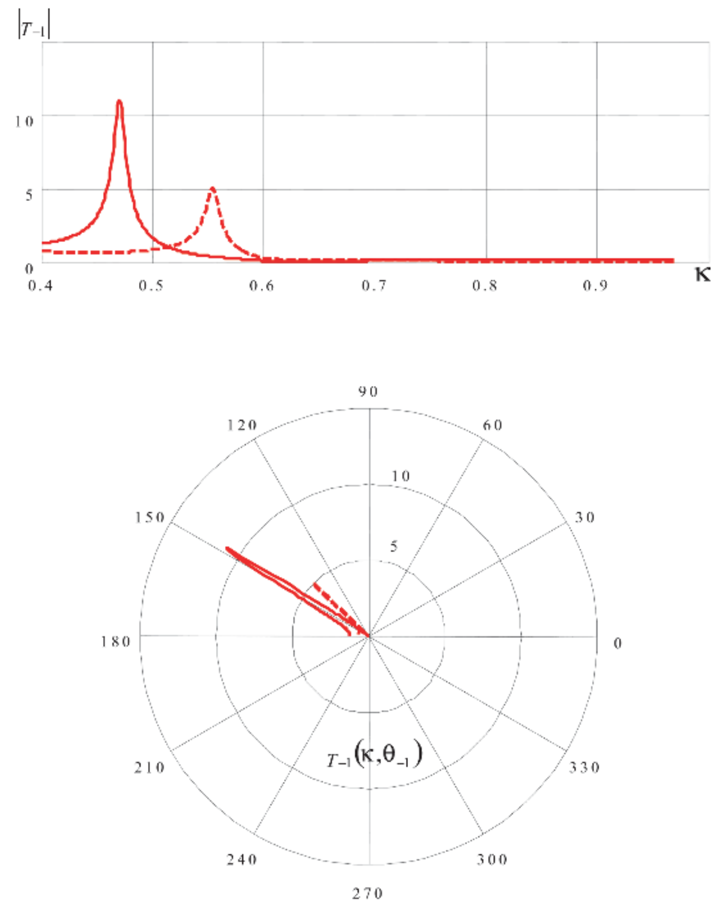

Figure 7. Radiation from double negative material profile $a(y) = 0.5(1 + \cos(y))$, current parameter $\overline{\gamma} = 0.6$, $B_0 = 0.1$, $\varepsilon_2 = -2.55 + i0.01$; $A_0 = 0.4$; $\mu_2 = -0.02 + i0.0$ – dashed line; and from the same structure with $\mu_2 = 1$.

resonances is more complicated. The following relations between ε_2 and μ_2 has to be hold:

If $\mathrm{Re}(\varepsilon_2) < -1$ then $\mathrm{Re}(\varepsilon_2)\mathrm{Re}(\mu_2) < 1$;

If $\mathrm{Re}(\varepsilon_2) > -1$ then $\mathrm{Re}(\varepsilon_2)\mathrm{Re}(\mu_2) > 1$ ($\varepsilon_1 = \mu_1 = 1$). The resonant frequency in the case $A_0 \ll 1$ can be calculated according the formula,

$$\kappa = \overline{\gamma}\mathrm{Re}\left(\sqrt{\frac{\varepsilon_2^2 - 1}{\varepsilon_2^2 - \varepsilon_2\mu_2}}\right). \tag{7}$$

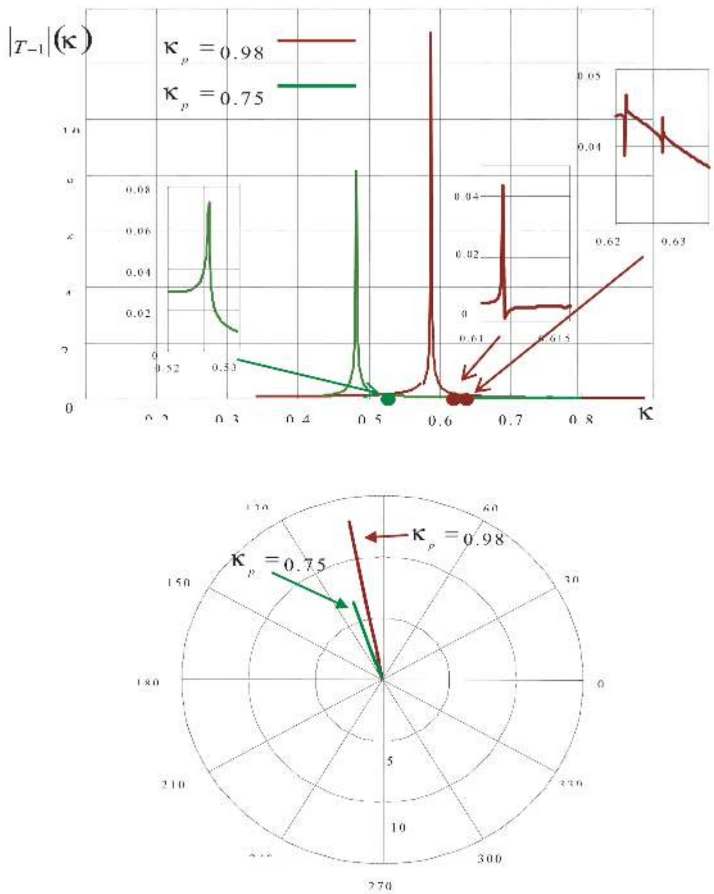

Figure 8. Radiation from the periodic boundary of dispersive metamaterial with different parameters $a(y) = 0.5(1 + \cos(y))$; $B_0 = 0.1$; $\overline{\gamma} = 0.89$; $v = 0.001$; $\mu_2 = 1$; $A_0 = 0.4$.

This case has to be studied in depth.

The essential question for all modeling of metamaterials, is the presence of dispersion that is a typical feature of artificial materials. The preliminary study of the present problem accounts for dispersion and assumes

$$\varepsilon_2 = 1 - \frac{\kappa_p^2}{\kappa^2 + iv\kappa} \tag{8}$$

where κ_p and v are characteristic values of the material considered. Figure 8 shows that the effect of resonant radiation is even enhanced in this case, and new high-Q resonances appear. Additional parameter of the structure, κ_p, appears to influence and to control radiation beam angle. This has some potential interest for beam steering applications and deserves some further investigation.

As mentioned above, the nature of the radiation resonance of the -1 harmonic is related to the excitation and transformation of eigen surface waves of the periodic boundary of metamaterials excited by the current $\vec{j}$. From numerical experiments, we can observe the rise in amplitudes of higher (evanescent) spatial harmonics that appear around the resonant frequency.

4. CONCLUSION

The numerical modeling of the radiation from the periodic boundary of metamaterial showed that the transformation of surface waves—excited on the boundary by means of properly chosen current $\vec{j}$ (the magnitude of $\overline{\gamma}$ has to be adjusted to the required frequency range)—can be very efficient.

The frequency range and the direction of radiation can be controlled by the parameters of the current $\vec{j}$ and effective constitutive parameters of metamaterials ε_2, μ_2. In addition, the resonant radiation is present in the case of metamaterials with dispersion and the additional parameter κ_p may serve for control of the radiation angle. Also, the described effects of resonant transformation are not present in the case of periodic boundaries of conventional dielectrics.

Finally, numerical results suggest that the use of periodic boundary of metamaterial with relevant excitation leads to a super directed surface wave antenna [10].

REFERENCES

1. Pendry, J. B., "Negative refraction," *Contemporary Physics,* Vol. 45, No. 3, 191–203, 2004.
2. Engheta, N., "An idea for thin subwave length cavity resonators using metamaterials with negative permittivity and permeability," *IEEE Antennas and Wireless Propagation Letters,* Vol. 1, No. 1, 10–13, 2002.
3. Antoniades, M. A. and G. V. Eleftheriades, "Compact, linear, lead/lag metamaterial phase shifters for broadband applications," *IEEE Antennas and Wireless Propagation Letters,* Vol. 2, No. 7, 103–106, 2003.

4. Lai, A., C. Caloz, and T. Itoh, "Composite right/left-handed transmission line metamaterials," *IEEE Microwave Mgazine,* Vol. 5, No. 3, 34–50, 2004.
5. Lim, S., C. Caloz, and T. Itoh, "Electronically scanned composite right/left handed microstrip leaky-wave antenna," *IEEE Microwave and Wireless Components Letters,* Vol. 14, No. 6, 277–279, 2004.
6. Bilotti, F., A. Alù, N. Engheta, and L. Vegni, "Anomalous properties of scattering from cavities partially loaded with double-negative or single-negative metamaterials," *Progress In Electromagnetics Research,* PIER 51, 49–63, 2005.
7. Caloz, C., C.-C. Chang, and T. Itoh, "Full-wave verification of the fundamental properties of lefthanded materials in waveguide configurations," *Journal of Applied Physics,* Vol. 90, No. 11, 5483–5486, 2001.
8. Shestopalov, V. P., "Physical principles of mm and sub mm engineering," *Kiev. Naukova Dumka,* 256, 1985.
9. Chandezon, J., G. Granet, A. Ye. Poyedinchuk, and N. P. Yashina, "Rigorous model for the materials with negative permittivity and permeability electromagnetic study," *PIERS*, Pisa, March 28–31, 2004.
10. Poyedinchuk, A. Ye., Yu. A. Tuchkin, N. P. Yashina, J. Chandezon, and G. Granet, "C-method: several aspects of spectral theory of gratings," *Progress In Electromagnetics Research,* PIER 59, 113–149, 2006.
11. Granet, G., A. Ye. Poyedinchuk, and N. P. Yashina, "Mode coupling at the periodic boundary of metamaterial," *PIERS*, Cambridge, USA, March 26–29, 2006.