Radiation Heat Transfer in Disperse Systems

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L.A. Dombrovsky
The Scientific Research Institute
of Thermal Processes
Moscow, Russia

begell house, inc., publishers
New York, Wallingford (UK)

Library of Congress Cataloging-in-Publication Data

Dombrovsky, L. A.
Radiation heat transfer in disperse systems / L.A. Dombrovsky.
p. cm.
ISBN: 1-56700-057-6
1. Heat--Radiation and absorption. 2. Heat--transmission.
1. Title.
QC331.D66 1996
536'.2--dc20 96-5544
CIP

Leonid. A. Dombrovsky
RADIATION HEAT TRANSFER IN DISPERSE SYSTEMS
1995

The manuscript gives a systematic consideration of mathematical models for the radiation heat transfer problems in disperse systems on the base of the Mie theory and approximate methods of the radiation transfer calculations. A manner of the matter presentation corresponds to the manual on applied radiative and combined heat transfer problems.

A complete set of means for the mathematical simulation of the radiation heat transfer in disperse systems is presented for the first time. It includes up-to-date physical models and concrete algorithms, which allow to provide numerical investigations of the radiative characteristics of various disperse systems as well as to obtain solutions of numerous radiative and combined heat transfer problems.

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Contents

Preface

A number of technical processes and natural phenomena are accompanied by heat transfer concerned with media thermal radiation. Generally, thermal radiation is thought to be relevant only at high temperatures. This widespread error is easily overcome if we remember, for example, that the weather and climate on our planet is mainly determined by thermal radiation of cloudy atmosphere and the Earth surface. Few people are aware, that quality rise of ordinary sleeping-bag is connected with changing of radiation transfer conditions in fibrous material.

In examples mentioned, as well as in many other cases, radiation emissivity, absorption and scattering take place in the medium, containing particles comparable with radiation wavelength. Such media are customary called disperse systems. One has to solve radiation heat transfer problems for disperse systems in highly different applications such as heat transfer in solid propellant rocket engines and magnetohydrodynamic generators, industrial technology of synthetic filaments production, propagation of microwave radiation in rain, and spacecraft thermal control through the use of droplet radiators.

In many problems the radiation heat transfer is not a sole transfer mode and it will be considered simultaneously with conduction and convective heat transfer. A rigorous mathematical statement of such problems is very complex, and therefore one is limited to an approximate description of radiative properties, as well as radiation transfer.

It should be noted, that in students teaching on heat transfer the main attention (at least in Russia) is paid to the study of the conduction and convection processes, but radiation heat transfer problems, particularly in scattering media, are beyond the scope of the majority of high-school programs. There are not enough manuals available on this subject.

All the above mentioned facts caused the appearance of this book. Data choice and their presentation manner are based on the author's experience of collaboration with students and postgraduate students of Moscow Institute of Physics and Technology. Previous publications served as the basis of the present paper.

The author thanks the esteemed colleagues for numerous discussions, which provided not only professional support, but also encouraged the development of the original intention. A particular gratitude should be expressed to Nina A. Protasova for great technical assistance in the preparation of the manuscript.

Introduction

The physical basis of all the solutions considered in this book is the notion of radiation transfer in absorbing and scattering medium as some macroscopic process, which can be described by a phenomenological transfer theory and kinetic equation for spectral radiation intensity. The question of the radiation transfer theory applicability is very complicated and is dealt with in the theoretical physics [1]. It is of great importance, that the problems, for which the radiation transfer theory can be applied, are quite numerous and contain the thermal radiation of various disperse systems.

Here one should make certain assumptions, dealing with disperse systems properties and radiation transfer process:

– radiation propagation is more rapid than any change of physical parameters, therefore radiation intensity field is quasi-stationary;

– radiative properties of the medium do not depend directly on the radiation intensity, but they vary only with the change of the temperature;

– the local thermodynamic equilibrium is preserved;

– in calculations of the thermal radiation flux the wave polarization need not be taken into account;

– radiation scattering in a medium is not accompanied by frequency variation;

– radiation absorption and scattering properties of disperse system elementary volume can determined from individual particles parameters regardless of the couple effects.

The last assumption simplifies the problem, and also gives a chance for direct investigation of the medium disperse composition influence on thermal radiation transfer. The restrictions occurring from the above assumptions are not significant, as it might seem, since the assumption about small couple effects remains valid up to high enough particle concentration [2].

The radiation transfer theory has been long studied by a number of famous scientists, working in physical optics, astrophysics, nuclear reactor theory, and heat transfer theory. The mathematical theory was created, containing up-to-date analytical and numerical methods. Numerous special publications dealt with computational methods applied to radiation transfer problems [3–9].

Together with the development of the radiation transfer theory, significant achievements took place in theoretical investigations of particle radiative properties for various disperse systems. Properties of particles, comparable to the wavelength, turned out to be diverse and complex. Many applied investigations and well-known monographs [10–13] were published on this subject. For the solution of some practical problems dealing with thermal radiation of disperse systems it is necessary to combine achievements of the scattering theory and the radiation transfer theory. In this case, a reasonable choice of method for the solution of the radiation transfer equation depends upon the scattering medium properties. At the same time, requirements for completeness and accuracy of individual particle properties calculations are determined by essential precision of radiation flux calculation. This was also reflected in the manuscript.

While solving most practical heat transfer problems one must take into account not only the thermal radiation, but also heat transfer by conduction and convection in medium. Most general problems of combined radiative-convective heat transfer are very complex, and their solution may be possible only by employing approximate computational methods for radiation transfer [14, 15]. Therefore, for orientation in mathematical description development for such problems it was not necessary to discuss all the computational methods. The description of this methods can be found, for example, in monographs [8, 9]. Contrary to the above mentioned papers, more attention was given to errors, resulting from the simple approximate methods for solution of the radiation transfer equation (Chapter 1), to analysis of some optical properties of various particles (Chapter 2), and to solution of model problems including combined heat transfer problems (Chapter 3).

The choice of material for this book and, particularly, the combination of model problems correspond to the author's field of practical work.

Taking into consideration the main questions, connected with thermal radiation of disperse systems, the author paid no attention to radiative properties of gases (see, for example, [16]) and to description of selective radiation; nor did he review radiative properties of materials, that can be presented in boundary conditions for radiation transfer equation [17, 18].

It should be noted, that this book was written as a practical handbook for radiation heat transfer calculations in disperse systems, rather than a theoretical monography. Computer codes are presented in appendices. They may be useful for studying the methods, discussed in the book. Computer modules presented may be also used for the initial investigation of some real practical problems.

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1

Approximate Methods for Solution of the Radiation Transfer Equation

Before proceeding to mathematical formulation of radiation transfer problem for scattering media it is reasonable to recall some definitions of the main physical quantities [1, 2].

Amount of the radiation energy in spectral interval $(\lambda, \lambda + d\lambda)$, passing per time dt in solid angle $d\mathbf{\Omega}$ near the direction $\mathbf{\Omega}$ through the area $d\sigma$ placed in the point $\boldsymbol{r}$ and oriented perpendicular to $\mathbf{\Omega}$, is equal to $\varphi_\lambda(\boldsymbol{r}, \mathbf{\Omega}) d\lambda dt d\sigma d\mathbf{\Omega}$, where function $\varphi_\lambda(\boldsymbol{r}, \mathbf{\Omega})$ is the spectral intensity of radiation at wavelength λ in the point $\boldsymbol{r}$ in direction $\mathbf{\Omega}$. Spectral intensity of equilibrium thermal radiation is defined by the Planck's function: $\varphi_\lambda = B_\lambda(T)$. For radiating medium a deviation of the function $\varphi_\lambda(\boldsymbol{r}, \mathbf{\Omega})$ from the intensity of the homogeneous isotropic equilibrium radiation is described by radiation transfer equation.

Spectral energy density of radiation is given by:

$$\rho_\lambda(\boldsymbol{r}) = \frac{1}{c} \int_{(4\pi)} \varphi_\lambda(\boldsymbol{r}, \mathbf{\Omega})\, d\mathbf{\Omega} \tag{1.1}$$

Further, this name for brevity will be used for the quantity

$$\varphi_0(\boldsymbol{r}) = \int_{(4\pi)} \varphi_\lambda(\boldsymbol{r}, \mathbf{\Omega})\, d\mathbf{\Omega} \tag{1.2}$$

In the case of equilibrium (black-body) radiation $\varphi_0 = 4\pi B_\lambda(T)$.

Spectral radiative flux is given by:

$$q_\lambda(\boldsymbol{r}) = \int\limits_{(4\pi)} \varphi_\lambda(\boldsymbol{r}, \boldsymbol{\Omega})\, \boldsymbol{\Omega} d\boldsymbol{\Omega} \tag{1.3}$$

On the black-body surface radiating in vacuum this quantity $\boldsymbol{q}_\lambda = \pi B_\lambda(T)\boldsymbol{n}$, where $\boldsymbol{n}$ is the external normal to this surface.

Absorption and scattering of the radiation in a medium are described by spectral coefficients Σ_a and Σ_s, respectively, by the extinction coefficient $\Sigma = \Sigma_a + \Sigma_s$, and by the scattering function $f(\boldsymbol{\Omega}'\boldsymbol{\Omega})$ (called also scattering phase function or indicatrix of scattering), presenting the angular intensity distribution for the radiation, scattered by elementary volume by one act of scattering. Scattering function satisfies the normalizing condition:

$$\int\limits_{(4\pi)} f(\boldsymbol{\Omega}'\boldsymbol{\Omega})\, d\boldsymbol{\Omega} = 1 \tag{1.4}$$

Spectral hemispherical emissivity of a radiating body ε_λ is defined as a ratio of the surface radiation flux to the black-body radiation flux.

Aside from spectral characteristics of a medium and radiation field, one can also employ the corresponding integral quantities. Integral intensity of the equilibrium thermal radiation $\varphi = \frac{\sigma}{\pi} T^4$, and integral radiation flux from black-body surface $\boldsymbol{q}_\lambda = \sigma T^4 \boldsymbol{n}$, where σ is the Stefan-Boltzmann constant.

Radiation transfer equation for unpolarized radiation, assuming that the local thermodynamic equilibrium takes place in absorbing and scattering medium, can be written as follows [3]:

$$\boldsymbol{\Omega}\nabla\varphi_\lambda(\boldsymbol{r}, \boldsymbol{\Omega}) + \Sigma\varphi_\lambda(\boldsymbol{r}, \boldsymbol{\Omega}) = \frac{\Sigma_s}{4\pi} \int\limits_{(4\pi)} \varphi_\lambda(\boldsymbol{r}, \boldsymbol{\Omega}') f(\boldsymbol{\Omega}'\boldsymbol{\Omega})\, d\boldsymbol{\Omega}' + \Sigma_a B_\lambda(T(\boldsymbol{r})) \tag{1.5}$$

The physical meaning of equation (1.5) is evident: variation of the spectral radiation intensity in direction $\boldsymbol{\Omega}$ takes place due to extinction by absorption and by scattering in other directions, as well as due to scattering from other directions (integral term) and thermal radiation of the medium. Generally, boundary conditions on the body surfaces should take into account angular characteristics of surface reflection and emission, which makes them very complex [4, 5].

Integration of the radiation transfer equation (1.5) over all values of solid angle by taking into account equations (1.2)–(1.4) gives the following expression for the divergence of the spectral radiation flux:

$$\nabla \boldsymbol{q}_\lambda = \Sigma_a [4\pi B_\lambda(T(\boldsymbol{r})) - \varphi_0(\boldsymbol{r})] \tag{1.6}$$

A similar equation for integral characteristics of the radiation field

$$\nabla \boldsymbol{q} = \int\limits_0^\infty \Sigma_a [4\pi B_\lambda(T) - \varphi_0]\, d\lambda \tag{1.7}$$

is also called the radiative energy conservation equation [5].

The exact numerical solution of the multi-dimensional radiation transfer equation is an extremely complicated mathematical problem. Therefore, as a rule, one is confined to simple geometrical configurations and approximate methods. This problem would be most easily solved, if scattering was not taken into consideration and hence the integral term was equal to zero. Unfortunately, this assumption is unacceptable for our work, since scattering by particles is one of the main features of thermal radiation transfer in disperse systems. For scattering media the radiation transfer equation (1.5) is integro-differential, but employing various approximations for scattering functions simplifies the integral term [5, 6].

In approximation of scattering function one of the following methods (or their combination) is commonly used [5–8]:

– expansion in a series on Legendre functions;

– description of the main scattering function extremums by use of several delta-functions with some weight coefficients.

The simplest approximations of each method mentioned above are well known. If one is restricted to two terms in expansion on Legendre functions, the result is the linear-anisotropic approximation:

$$f(\mu_0) = 1 + 3\bar{\mu}\mu_0 \qquad \mu_0 = \mathbf{\Omega}'\mathbf{\Omega} \tag{1.8}$$

where $\bar{\mu}$ is mean cosine of the scattering angle:

$$\bar{\mu} = \frac{1}{4\pi} \int\limits_{(4\pi)} (\mathbf{\Omega}'\mathbf{\Omega}) f(\mathbf{\Omega}'\mathbf{\Omega})\, d\mathbf{\Omega}' \tag{1.9}$$

The linear-anisotropic approximation loses any physical sense with $\bar{\mu} > 1/3$, i.e. at large scattering anisotropy, since $f(\mu_0)$ for $\mu_0 < -1/(3\bar{\mu})$ becomes negative.

If one takes into account only the backward scattering, presenting the scattering function as a linear combination of delta-functions $\delta(1+\mu_0)$ and $\delta(1-\mu_0)$, the back-scattering model will be derived. The approximation, in which the integral term in the radiation transfer equation disappears, gives good results in some one-dimensional problems [8], but cannot be applied to scattering description in multidimensional radiation transfer problems for inhomogeneous and nonisothermal disperse systems.

Transport approximation appears to be a highly successful method, according to which the scattering function is replaced by a sum of isotropic component and the item, describing "forward scattering":

$$f(\mu_0) = (1-\bar{\mu}) + 4\bar{\mu}\delta(1-\mu_0) \tag{1.10}$$

With the use of transport approximation, the radiation transfer equation (1.5) may be written in the same way as for isotropic scattering, i.e. with $f \equiv 1$, with scattering and extinction coefficients being replaced by corresponding "transport" values:

$$\Sigma_s^{tr} = \Sigma_s(1-\bar{\mu}) \qquad \Sigma_{tr} = \Sigma_a + \Sigma_s^{tr} \tag{1.11}$$

Approximate expressions (1.8) or (1.10) for the scattering function provide a satisfactory precision of the main radiation characteristics of disperse systems in the majority of practical problems [6].

It is of interest, that delta-Eddington approximation is frequently employed in current investigations [9]. This method presents the scattering function as a combination of linear function (1.8) and transport approximation (1.10). Delta-Eddington approximation may be also completed by description of backward scattering in accordance with backscattering model [10].

If the problem solved has no symmetry, the solution of equation (1.5) is very complicated even for the simplest description of the scattering function. In some cases the azimuthal symmetry of a solution takes place, i.e. radiation intensity depends only upon angular variable. Radiation transfer calculation is considerably simplified, when φ_λ is a function of only one space coordinate (one-dimensional transfer problem).

The most detailed investigation of the radiation transfer was performed in the plane-parallel layer of a medium. This model problem was used for testing various exact and approximate methods of solution of the radiation transfer equation [3–5]. Even for this simple problem, the advantages of approximate differential methods of the radiation transfer description are evident, since these methods simplify sufficiently the calculations keeping the available accuracy.

All the differential methods of solution of the radiation transfer equation are based on simple assumptions of the angular dependence of the spectral radiation intensity φ_λ. These assumptions enable us to deal with a limited number of functions $\varphi_i(\boldsymbol{r})$ instead of function $\varphi_\lambda(\boldsymbol{r}, \boldsymbol{\Omega})$ and turn to the system of the ordinary differential equations by the use of integration of the transfer equation. The same result may be obtained, if the integral term in equation (1.5) substitute for quadrature [3]. Particularly, through the use of the discrete ordinate method one may derive the same system of the ordinary differential equations, as those by expansion of φ_λ on the spherical functions.

Differential approximations are suitable for calculation of radiation transfer by arbitrary optical depth, but the descriptions of the radiation intensity angular dependence and the scattering function are approximate. Particularly, in the first approximation of the spherical harmonics method the solutions for scattering functions (1.8) and (1.10) appear to be identical.

The simplest differential approximations, brought together in this book, as in monograph [11], by the general term "diffusion approximation", give the following representation of the spectral radiation flux:

$$q_\lambda = -D\nabla\varphi_0 \tag{1.12}$$

and differ only by expressions for diffusion coefficient D. Sometimes the term "diffusion approximation" is related only to the case, when $D = 1/(3\Sigma_{tr})$ [12], that corresponds to Eddington approximation.

Substituting (1.12) in (1.6), we obtain nonhomogeneous modified Helmgoltz equation for the spectral radiation energy density:

$$-\nabla(D\nabla\varphi_0) + \Sigma_a\varphi_0 = \Sigma_a 4\pi B_\lambda(T) \tag{1.13}$$

It can be shown, that equation (1.12) may be derived by concrete assumptions of the angular dependence of radiation intensity [3–5, 12].

For internal region of optically thick volume, the following simple approximate expression may be used for the integral thermal radiation flux:

$$q = -\lambda_r \nabla T \tag{1.14}$$

Equation (1.14) is called radiative conduction approximation, or Rosseland approximation. By transport approximation for the scattering function (1.10) the radiative conductivity coefficient is defined as

$$\lambda_r = \frac{16}{3}\frac{\sigma T^3}{\Sigma_{tr}^R} \tag{1.15}$$

with Rosseland mean transport extinction coefficient given by:

$$\Sigma_{tr}^R = 4\sigma T^3 / \int_0^\infty \frac{1}{\Sigma_{tr}} \frac{\partial B_\lambda(T)}{\partial T} d\lambda \tag{1.16}$$

Along with approximate methods of solution of the transfer equation there are also exact numerical methods. A review of this kind may be found in monograph [3]. In the present paper, due to the complexity of the heat transfer problems under consideration, attention, according to general recommendations [13, 14], is mainly given to the simplest approximate methods. Exact numerical methods are employed only for accuracy control of approximate methods by solving some model problems.

Differential approximations have a long history. This is reflected in their well-known names: Eddington method, Schwarzschild-Schuster method. The progress in computer engineering and numerical methods for boundary-value problems makes it possible to obtain more accurate solutions. Nevertheless, due to simplicity and physical clarity, the differential approximations are widely used at present for the solution of the radiation transfer problems in scattering media, particularly, in combined heat transfer problems [5, 13–15].

Together with intensive independent development of the radiation transfer theory, the numerical methods for transfer equation solution have been worked out. These methods allow to obtain more and more accurate solutions of classical problems. Since 60-s, the interest to the radiation transfer calculations has got the practical basis mainly owing to the radiative gas dynamics [16] and combined heat transfer problems [17], for their successive development is the basis for the progress in rocket and spacecraft engineering and in advanced industrial technologies.

New problems differ from classical problems not only by geometrical complexity or by medium properties. The main difficulty in obtaining results of practical-value is the necessity to take into account other heat transfer modes (by conduction or convection) and volume heat sources, connected with external fields or physical and chemical conversions in the medium. The complexity of the general mathematical formulation, as well as vague knowledge of the thermodynamical and optical parameters of a medium, as a rule, make the wide use of the exact numerical methods for solution of the radiation transfer equation unreasonable. These circumstances explain "the second birth" of relatively simple differential approximations.

It is rather difficult to call all the papers, where the differential approximations, classical or modified, are employed for solution of some problems. The main references can be found in cited monographs. Nevertheless, it is worth reviewing several publications of the last 10–15 years, presenting some examples of employing differential approximations.

The most widely used are P_1 and P_3 approximations of the spherical harmonics method, as well as DP_0 and, to a smaller degree, DP_1-approximation of the double spherical harmonics method. Note, that P_1 and DP_0, according to our terminology, are classified as diffusion approximations.

In solving one-dimensional radiative transfer problems, in the majority of papers one is not limited by diffusion approximation, but employs it only for one of the solution stages: as the initial approximation [18, 19] or at the first iterative step [20]. The diffusion approximation is used in similarity analysis of radiation transfer problems in scattering media [21, 22] and in obtaining convenient analytical solutions for simple geometrical forms of radiating volume [21, 23]. In a number of papers, there are various modifications of simple differential approximations, that give a possibility to take into account, for instance, special boundary conditions [24–26].

Information on the accuracy of the diffusion approximation and of analogous modified differential approximations by solving of the model one-dimensional radiative transfer problems may be found in monographs [3–6, 14], as well as in papers [9, 10, 22, 24–28].

One of the commonly used high-order differential approximations is the P_3-approximation [15, 29]. In papers [30, 31], attention is drawn to comparatively high accuracy of DP_1-approximation, which has considerable advantages in solving one-dimensional radiative transfer problems with discontinuous boundary conditions [3, 32].

Review of the multidimensional radiative transfer problems solutions may be found in papers [33, 34]. Two- and three-dimensional problems, connected with the thermal radiation of disperse systems, are solved often through the use of the diffusion approximation, at least for the first stage of the solution [34–43]. The diffusion approximation is directly employed in the calculation of the infrared radiation transfer in clouds [36–38], in the radiative heat transfer calculations in rocket engines [39–41] and in problems dealing with image formation by observing through some scattering medium [42]. In special cases, radiative transfer problems are solved with the help of P_3-approximation [44, 45].

In papers [46, 47] it was shown, that scattering treatment with the use of the transport approximation provides a high accuracy of calculations not only for thermal radiation flux, but also for the radiation intensity of the scattering medium in different directions. This allows to develop different applications of the methods, which employ the diffusion approximation at the first stage, and then assume a solution of the transfer equation with a source function, determined in the transport approximation. This idea was suggested comparatively long ago in papers [48, 49], and was developed later in [18, 23]. It is used also at present, for example, in solving problems dealing with the thermal radiation of two-phase jets [41], in interpretation of dust plasma diagnostic results [50], and in determination of laser radiation scattering in two-phase erosion plume [43, 47, 51].

This book makes extensive use of the simplest approximate methods: variants of the diffusion approximation and DP_1-approximation of the double spherical harmonics method. Therefore, it seems natural to consider first the main relations of these methods.

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