Ref.TH.3439-CERN

RADIATIVE CORRECTIONS TO PSEUDOSCALAR MESON DECAYS

L. Bergström
CERN -- Geneva

ABSTRACT

QED radiative corrections to the decay of a pseudoscalar meson into a lepton pair have been estimated. The results are applied to the decay of the neutral pion into electron and positron. Formulas are also given for a pseudoscalar Higgs-like particle decaying to a lepton or quark pair. The results are also of relevance for QCD sum rules for the pseudoscalar current, where conflicting calculations exist in the literature.

Ref.TH.3439-CERN
7 October 1982

1. - INTRODUCTION

The rare decay $\pi^0 \to e^+e^-$ has recently been given renewed interest. This is due to the fact that the electromagnetic contribution to this decay can be rather unambiguously calculated. Any experimentally measured deviation from this contribution then has the possibility of being attributed to some "exotic", direct quark-lepton coupling. Theories where such extra contributions may be expected include technicolour theories, composite models of quarks and leptons and models with axions. The reader is referred to Ref. 1) for a discussion and a summary of the situation and earlier references. Very recently, a new measurement has been published[2),3)], giving an unexpectedly large value for the probability of this decay. However, the experimental errors prevent one from drawing any firm conclusions from that experiment or from a previous one[4)] also giving a high value. The need for a new, high precision measurement is apparently urgent.

However, increasing the precision of the measurement will also increase the sensitivity to radiative corrections. This is so, since to avoid the large background from single Dalitz pairs, $\pi^0 \to e^+e^-\gamma$, and in experiments of the type in Refs. 2),3) also from internal conversions, $\pi^-p \to e^+e^-n$, the experiment must have a very narrow acceptance of the lepton pair around the invariant mass of the pion. Due to the ability of the light leptons to radiate photons, this means that many lepton pairs will have reduced their invariant mass to escape detection. A similar phenomenon has long been known in the case of charged pion decay [e,g, Refs. 5) and 6)] $\pi \to e\nu$, where QED radiative corrections may easily amount to 15-20% for realistic experimental set-ups. The aim of this paper is to estimate these radiative corrections as well as the interference between the single Dalitz pair diagrams and the bremsstrahlung diagrams. Although the interference term complicates the analysis, in principle it could have the virtue of giving an opportunity to experimentally determine the otherwise unknown sign of the dispersive part of the lowest order decay amplitude (since it may, depending on the sign, interfere constructively or destructively with the calculable lowest order Dalitz pair diagrams). However, as we shall see, the interference term is much too small to be measurable in practice. To facilitate calculations, the problem will be formulated as a one-loop correction to the coupling of a general pseudoscalar current to a fermion pair. In this way, contact is made with one-loop gluonic corrections to the pseudoscalar current in QCD, which are of importance for the duality calculations of resonance parameters[7)]. Unfortunately, here two groups which have performed these calculations, Shifman et al.[8)] and Reinders et al.[9)], do not agree on the result. As we will see, with a specific choice of a finite renormalization constant (which is of little significance in the QCD duality applications), our calculation favours the results of Ref. 9). In the case of a Higgs-like particle (with the Yukawa coupling proportional to the bare fermion mass), the renormalization constant can be explicitly calculated, and we will give results also for this case.

2. - CALCULATION OF BREMSSTRAHLUNG CORRECTIONS

We consider first the lowest order diagram for $\pi^0 \to e^+e^-$. Assuming CP invariance, the coupling between the pion and the lepton pair can be parametrized in terms of an effective coupling constant g: $M = g\,\bar{u}(p_-)\gamma^5 v(p_+)$ (Fig. 1). This gives the width in lowest order in α and g:

$$\Gamma^0_{e^+e^-} = \frac{|g|^2}{8\pi} m_\pi \sqrt{1 - 4m_e^2/m_\pi^2} \quad . \tag{1}$$

Using the results of Ref. 1), we find the electromagnetic contribution (from the two-photon loop)

$$g_{e.m.} = -\frac{m_e}{2f_\pi}\left(\frac{\alpha}{\pi}\right)^2 \times R \;, \tag{2}$$

where $f_\pi \approx 93$ MeV is the pion decay constant, and R is a dimensionless constant, the imaginary part of which is model independent:

$$\mathrm{Im}(R) = \frac{\pi}{2v_0} \ln\left(\frac{1 - v_0}{1 + v_0}\right) \;, \tag{3}$$

where $v_0 = \sqrt{1 - 4m_e^2/m_\pi^2}$.

For the soft bremsstrahlung contribution, which is our main interest here, the variation of g with the invariant mass of the lepton pair will be neglected. Our task is thus reduced to calculating the diagrams in Fig. 2a, b. If the coupling is mediated, e.g., by a Higgs-like particle, this is obviously no approximation in this order. However, for the two-photon mediated process, we should really introduce radiative photons in all possible ways in Fig. 1a, which would lead to very complicated two-loop calculations. For the bremsstrahlung contribution we would have to calculate diagrams 2a,b with coupling strength g depending on the off-shellness of the lepton propagators. In addition, we should calculate diagram 1a with an external photon attached to the lepton propagator. However, we believe that our approximation is reasonable also in this case since, firstly, soft photons are our main concern and the probability of their emission should not depend strongly on the structure of the vertex.

Secondly, as usual we have reasons to expect that bremsstrahlung from the internal lepton propagator should be strongly suppressed since it is prominent only for particles near their mass shell. (Indeed, the dominant electromagnetic contribution is determined by the absorptive part of the two-photon contribution which is given by the region of loop momentum where both photons are on their mass shell. In this case, the lepton propagator is far off-shell.) With these assumptions, we can calculate the differential distribution $d\Gamma/ds$ of lepton pairs resulting from bremsstrahlung (to lowest order, $d\Gamma/ds$ is a δ function at $s = m_\pi^2$). To get results for the experimentally interesting quantity of the fraction of lepton pairs that is missed if the experiment only detects pairs within an invariant mass interval of $m_\pi^2(1-\delta) < s \leq m_\pi^2$, we also need an estimate of the total radiative correction including virtual corrections. This is not crucial, however, since for the total correction we expect some typical QED result of at most a few percent. As we shall see, the differential corrections we will be concerned with are much larger; 15-25%. In fact, on general grounds [see, e.g., the discussion in Ref. 9)] we expect the total radiative corrections to behave, after renormalization, as

$$\frac{\alpha}{\pi}\left(\frac{3}{2}\ln\left(\frac{1-v_0}{1+v_0}\right) + \text{const.}\right) , \tag{4}$$

where $3/2$ is the anomalous dimension of the pseudoscalar current. If the constant is not anomalously large, this only amounts to a few percent. Anyhow, to estimate this constant we will also calculate the virtual correction of Fig. 2c.

For the diagrams of Fig. 2a,b we have the matrix element

$$M_{(2a,b)} = -eg\bar{u}(p_-)\left\{\not{\epsilon}\frac{i}{\not{p}_- + \not{k} - m_e + i\varepsilon}\gamma^5 + \gamma^5\frac{i}{-\not{p}_+ - \not{k} - m_e + i\varepsilon}\not{\epsilon}\right\}v(p_+). \tag{5}$$

For our purpose, terms of relative order $(m_e/m_\pi)^2$ may be neglected in the probability distribution $d\Gamma/ds$, which is easily obtained from (5). The only technical point worth noting is that terms proportional to m_e^2 should not be dropped too early since the products of denominators occasionally give rise to factors of $1/m_e^2$. The result is

$$\frac{1}{\Gamma^0_{e^+e^-}}\frac{d\Gamma^{brems}}{ds} = \frac{1}{m_\pi^4}\frac{\alpha}{\pi}\left\{\frac{s^2+m_\pi^4}{m_\pi^2 - s}\ln\left(\frac{1+v}{1-v}\right) - \frac{2m_\pi^2 s v}{m_\pi^2 - s}\right\} , \tag{6}$$

where $s = (p_+ + p_-)^2$; $v = \sqrt{1 - 4m_e^2/s}$.

To determine how large a fraction of the lepton pairs is missed if the experiment only detects pairs within $(1-\delta)m_\pi^2 < s \leq m_\pi^2$, we first integrate (6) to obtain

$$\frac{1}{\Gamma^0_{e^+e^-}} \int_{4m_e^2}^{(1-\delta)m_\pi^2} \frac{d\Gamma^{brems}}{ds}\, ds = \frac{\alpha}{\pi} \left\{ 2\ln(\delta)\left(\ln\left(\frac{1-v_0}{1+v_0}\right)+1\right) + \right.$$

$$\left. + \left(\frac{3}{2} - 2\delta\right) \ln\left(\frac{1-v_0}{1+v_0}\right) + \frac{13}{4} - \frac{\pi^2}{3} - 2\delta + O\left(\frac{m_e^2}{m_\pi^2}; \delta^2\right) \right\}, \tag{7}$$

where $v_0 = \sqrt{1 - 4m_e^2/m_\pi^2}$.

To obtain the total bremsstrahlung correction, we have to perform the modifications appropriate for a fictitious photon mass $\lambda \ll m_e$. The result is

$$\frac{\Gamma^{brems}}{\Gamma^0_{e^+e^-}} = \frac{\alpha}{\pi} \left\{ \frac{13}{4} - \frac{2\pi^2}{3} + \frac{3}{2} \ln\left(\frac{1-v_0}{1+v_0}\right) + \frac{1}{2}\ln^2\left(\frac{1-v_0}{1+v_0}\right) + \right.$$

$$\left. + \ln\left(\frac{\lambda}{m_e}\right)\left(2\ln\left(\frac{1-v_0}{1+v_0}\right)+2\right) + O\left(\frac{m_e^2}{m_\pi^2}\right) \right\}. \tag{8}$$

In the model of Fig. 2, the total radiative correction is then obtained by adding to this result the virtual contribution from the interference of diagram 2c with the lowest order diagram, Fig. 1b.

3. - VIRTUAL CORRECTION

When writing the diagram 2c we have assumed that mass renormalization has already been carried out so that the lepton self-energy diagrams and wave function renormalization only contribute a multiplicative factor Z_2 to the vertex. Since we know that the γ^5 vertex in QED is multiplicatively renormalizable, we may write a subtracted dispersion relation; $g\gamma^5 \to g f_P(s)\gamma^5$:

$$f_P(s) - f_P(0) = \frac{s}{\pi} \int_{4m_e^2}^{\infty} \frac{\mathrm{Im}\,(f_P(s'))}{s'(s'-s-i\varepsilon)} \quad . \tag{9}$$

Using the Cutcosky rule, we find the imaginary part of diagram 2c:

$$\mathrm{Im}\left(f_P(s)\right) = \frac{\alpha}{2}\,\frac{s-2m_e^2}{\sqrt{s(s-4m_e^2)}}\,\ln\left(\frac{s-4m_e^2}{\lambda^2}\right)\theta\left(s-4m_e^2\right). \tag{10}$$

To calculate the interference with diagram 1b we only need the real part of $f_P(s)$. Introducing $v = \sqrt{1-4m_e^2/s}$ as a new integration variable one finds, for $s > 4m_e^2$,

$$\begin{aligned}\mathrm{Re}\, f_P(s) - f_P(0) = \frac{\alpha}{2\pi}\Big\{ &\ln\left(\frac{4m_e^2}{\lambda^2}\right)\left(\frac{1+v^2}{2v}\ln\left(\frac{1-v}{1+v}\right)+1\right) + \\ &\frac{1+v^2}{2v}\Big[\pi^2 - 4\ell(v) + \ell(v^2) - \ln 2\,\ln\left(\frac{1-v}{1+v}\right) - \\ &-\frac{1}{2}\left(\ln^2(1-v) - \ln^2(1+v)\right) - \ell\left(\frac{1-v}{2}\right) + \ell\left(\frac{1+v}{2}\right)\Big] - 2\ln 2\Big\},\end{aligned} \tag{11}$$

where $\ell(v) = \int_0^v t^{-1}\ln(1-t)^{-1}dt$.

To leading order in m_e^2/m_π^2, the result is

$$\begin{aligned}\mathrm{Re}\, f_P(s) - f(0) = \frac{\alpha}{2\pi}\Big\{ &-2\ln\left(\frac{\lambda}{m_e}\right)\ln\left(\frac{1-v}{1+v}\right) - 2\ln\left(\frac{\lambda}{m_e}\right) - \\ &-\frac{1}{2}\ln^2\left(\frac{1-v}{1+v}\right) + \frac{2\pi^2}{3}\Big\}.\end{aligned} \tag{12}$$

The subtraction constant $f_P(0)$ remains to be determined. To compare with Refs. 8) and 9), we first follow their example and simply put $f_P(0) = 1$. Converting the problem to QCD by changing $\alpha \to 4/3 \times \alpha_s$, the total radiative correction from diagrams 2a-c is (for a hypothetical decay into quarks, $P \to q\bar{q}$)

$$\left(\Gamma^{brems} + \Gamma^{virt}\right)/\Gamma^0_{q\bar{q}} = \frac{4\alpha_s}{3\pi}\left\{\frac{3}{2}\ln\left(\frac{1-v_0}{1+v_0}\right) + \frac{13}{4} + \mathcal{O}\left(\frac{m_q^2}{m_P^2}\right)\right\}. \tag{13}$$

In this limit ($v_0 = \sqrt{1-4m_q^2/m_P^2}$ close to one), this coincides with the result of Reinders et al.[9)], whereas in Ref. 8) the constant term was found to be $-5/4$ *).

---

*) Note added: the two groups now agree on the correctness of the result of Ref. 9) (L.J. Reinders, private communication).

Without knowing anything more about the lowest order coupling g, the ambiguity of the subtraction constant persists. However, in the interesting case where a large contribution to $\pi^0 \to e^+e^-$ comes from a Higgs-like particle, $f_P(0)$ can actually be calculated. This is because in that case we can write the coupling $g = g' \cdot m_0$, where m_0 is the bare electron mass. In the case of the axion, for instance, we have[1)]

$$g' = F_\pi \cdot 2^{\frac{1}{4}} \sqrt{G_F} / x \;, \tag{14}$$

where F_π is the coupling at the pion vertex and x is a parameter depending on the Higgs sector of the theory. Working to lowest order in g' and first order in α, the tree level amplitude $g' m_0 \, \bar{u}(p)\gamma^5 u(p)$ at zero momentum transfer is modified to

$$g' m_0 \frac{Z_2}{Z_P} \bar{u}(p) \gamma^5 u(p) \, , \tag{15}$$

where we have to express the bare mass m_0 in terms of the physical, renormalized mass m_e to first order in α. From the diagram 2c evaluated at zero momentum transfer we can calculate $Z_P^{-1} - 1$, e.g., by using a UV cut-off Λ and a small photon mass λ (we avoid dimensional regularization due to the known difficulties of continuing γ^5 to $4-\varepsilon$ dimensions). A simple calculation gives

$$Z_P^{-1} - 1 = \frac{\alpha}{\pi} \left\{ 2 \ln\left(\frac{\Lambda}{m_e}\right) + 1 + \ln\left(\frac{\lambda}{m_e}\right) \right\} . \tag{16}$$

Using the known first-order results

$$\delta m = \frac{3\alpha}{4\pi} m_e \left\{ 2 \ln\left(\frac{\Lambda}{m_e}\right) + \frac{1}{2} \right\} ; \quad Z_2^{-1} - 1 = \frac{\alpha}{2\pi} \left\{ \ln\left(\frac{\Lambda}{m_e}\right) + 2\ln\left(\frac{\lambda}{m_e}\right) + \frac{9}{4} \right\} ,$$

we get

$$m_0 \frac{Z_2}{Z_P} \equiv m_e \cdot f_P(0) = m_e \left(1 - \frac{\alpha}{2\pi}\right) , \tag{17}$$

i.e., we find a non-trivial subtraction constant $f_P(0) = 1 - \alpha/2\pi$. We note that this is the same as the finite renormalization factor for the axial vertex[10)] meaning that we have the axial Ward identity preserved to this order. Using this result, our expression for the total radiative correction is

$$\left(\Gamma^{brems} + \Gamma^{virt}\right)/\Gamma^{\circ}_{e^+e^-} = \frac{\alpha}{\pi}\left\{\frac{3}{2}\ln\left(\frac{1-v_0}{1+v_0}\right) + \frac{9}{4} + \mathcal{O}\left(\frac{m_e^2}{m_\pi^2}\right)\right\}. \tag{18}$$

We prefer to use this form also for the correction to the two-photon electromagnetic contribution to g. As we have argued before, the uncertainty is numerically insignificant (and it is at least not inconsistent to do so, since $g_{e.m.}$ [Eq. (2)] also contains an explicit factor of m_e). Now we can also make the simple replacement $\alpha \to 4\alpha_s/3$ in Eq. (17) to obtain the QCD corrected width for a pseudoscalar Higgs-like particle decaying into a $q\bar{q}$ pair:

$$\Gamma^{(1)}_{P\to q\bar{q}} / \Gamma^{\circ}_{P\to q\bar{q}} \simeq 1 + \frac{4\alpha_s}{3\pi}\left\{3\ln\left(\frac{m_q}{m_P}\right) + \frac{9}{4}\right\}. \tag{19}$$

The logarithmic term coincides with the corresponding result for a scalar Higgs[11), which is expected since the anomalous dimensions of the scalar and pseudoscalar currents are equal. It is interesting to note, however, that also the constant term agrees with the scalar case[11). [In Ref. 9), on the other hand, the constant term for the scalar case is 21/4. This is because in that work all subtraction constants are put equal to one. Repeating the calculation leading to Eq. (17), however, we find $f_s(0) = 1 - 3\alpha/2\pi$ for the scalar vertex, which explains the discrepancy. This result can also be derived from an unsubtracted dispersion relation for the convergent quantity $f_s(s) - f_p(s)$ (which assumes asymptotic γ^5 invariance), giving $f_s(0) - f_p(0) = -\alpha/\pi$.]

For any realistic value of α_s, the correction implied by (19) is too large to be trusted. Since the leading log terms are equal to those in the scalar case, however, one can immediately take over the analysis in Ref. 11) and sum up the leading logs to obtain the simple formula for the hadronic rate:

$$\Gamma \simeq \frac{|g'|^2 m_P}{8\pi}\left(\tilde{m}_q(m_P)\right)^2, \tag{20}$$

where $\tilde{m}_q(m_P)$ is the running mass of the quark, evaluated at the mass of the Higgs-like boson.

4. - INTERFERENCE WITH THE DALITZ PAIR DIAGRAM

In the case of pion decay we also have to study the possible interference from the Dalitz decay process, $\pi^0 \to e^+e^-\gamma$. Most radiative corrections to Dalitz pair decay (Fig. 3) have been calculated by Mikaelian and Smith[12]. However, they did not include the interference between the lowest order Dalitz pair diagram (Fig. 3a) and diagrams 2a,b. In our case, this may give a significant contribution since diagram 3a is one order in α lower than 2a,b. In other words, since $\Gamma^{Dalitz}/\Gamma_{e^+e^-} \sim 10^5$, what is a small correction to the Dalitz pair spectrum may be a big correction to the e^+e^- bremsstrahlung spectrum. (It is only because the Dalitz pair spectrum is sharply peaked at very low invariant mass that the situation is not hopeless.) The expression for diagram 3a can be taken from Ref. 13). Taking the Dalitz pair form factor to be a constant (which should be a good approximation for the pion), we find the interference term

$$2\,\mathrm{Re}\{M_{(3a)}M^*_{(2a,b)}\} = \frac{2e^2}{s}\,\mathrm{Re}\left\{\frac{i}{4\pi^2 f_\pi}\, g^* \epsilon_{\mu\nu\alpha\beta}\, k_1^\alpha k_2^\beta\, \epsilon^\nu(\lambda)\, \times\right.$$

$$\left.\times\,\mathrm{Tr}\left[(\not p_- + m_e)\gamma^\mu(\not p_+ - m_e)\left(\frac{\gamma^5(\not p_- + \not k_2 + m_e)\epsilon^*(\lambda)}{2p_-\cdot k_2} - \frac{\epsilon^*(\lambda)(\not p_+ + \not k_2 - m_e)\gamma^5}{2p_+\cdot k_2}\right)\right]\right\}, \qquad (21)$$

where $\epsilon^\sigma(\lambda)$ is the polarization vector of the photon. A straightforward calculation then gives for the interference contribution

$$\frac{1}{\Gamma^0_{e^+e^-}}\frac{d\Gamma^{int}}{ds} = \frac{-2\,\mathrm{Re}(R)}{m_\pi^4\,|R|^2} \times \frac{(m_\pi^2 - s)^2}{s}\,\ln\left(\frac{1+v}{1-v}\right) + \mathcal{O}\left(\frac{m_e^2}{m_\pi^2}\right), \qquad (22)$$

where R is related to g according to (2). In the purely electromagnetic case, $\mathrm{Re}(R)$ is rather model-independent, positive and fulfilling[1] $\mathrm{Re}(R) \approx 0.6|\mathrm{Im}(R)|$.

In the general case, both the magnitude and sign of $\mathrm{Re}(R)$ are unknown. However, we can use the model-independent value (3) for $\mathrm{Im}(R)$ to derive an upper limit for the modulus of the interference contribution:

$$\left|\frac{1}{\Gamma^0_{e^+e^-}}\frac{d\Gamma^{int}}{ds}\right| \leq \frac{2}{\pi\ln\left(\frac{1+v_0}{1-v_0}\right)}\,\frac{(s-m_\pi^2)^2}{s\,m_\pi^2}\,\ln\left(\frac{1+v}{1-v}\right), \qquad (23)$$

where equality is achieved for $|\mathrm{Im}(R)| = |\mathrm{Re}(R)|$. This expression can be integrated to give the maximal total contribution to the lepton pair yield from the interference term. We find

$$\frac{|\Gamma^{int}|}{\Gamma^0_{e^+e^-}} \lesssim \frac{1}{\pi \ln(m_\pi/m_e)} \left\{ 2\ln^2\left(\frac{m_\pi}{m_e}\right) - 3\ln\left(\frac{m_\pi}{m_e}\right) + \frac{7}{4} - \frac{\pi^2}{6} + \mathcal{O}\left(\frac{m_e^2}{m_\pi^2}\right) \right\}. \quad (24)$$

Thus, we find $|\Gamma^{int}|/\Gamma^0_{e^+e^-} \lesssim 2.6$, i.e., the interference term can be as large as $\sim$2.6 times the lowest order e^+e^- contribution. However, due to the $(m_\pi^2 - s)^2$ distribution [see Eq. (22)], the yield is concentrated at low invariant mass where it is completely drowned in the Dalitz pair spectrum. Indeed, we find that even for an experimental resolution as large as $\sim$10%, the interference term only contributes a fraction of a percent and can be totally neglected.

5. - DISCUSSION

With the aid of our results and those calculated in Ref. 12), we are now in a position to present the leading contributions from diagrams 1, 2a-c and 3a-d as a function of the experimental resolution δ.

The correction coming from the bremsstrahlung and virtual contributions is given by the total correction, Eq. (18) minus the fraction that is missed with a given resolution, Eq. (7). A good approximation for $\delta \lesssim 10\%$ is given by

$$\frac{\Delta\Gamma^{virt+brems}}{\Gamma^0_{e^+e^-}} = \frac{\alpha}{\pi} \left\{ 2.3 + 2(\delta - \ln\delta)\left(2\ln\left(\frac{m_e}{m_\pi}\right) + 1\right) \right\}. \quad (25)$$

This correction is independent of the value of the lowest order width, $\Gamma^0_{e^+e^-}$. The contribution from single Dalitz pairs (including radiative corrections) is given by (assuming a constant form factor)

$$\frac{\Delta\Gamma^{Dalitz}}{\Gamma^0_{e^+e^-}} = \frac{1}{\rho} \frac{m_\pi^2}{3\pi m_e^2 \alpha \ln^2\left(\frac{m_\pi}{m_e}\right)} \int_{1-\delta}^{1} \frac{dx}{x} (1-x)^3 (1 + \delta^{rad}(x)) , \quad (26)$$

where $x = s/m_\pi^2$ and ρ is the lowest order branching ratio for $\pi^0 \to e^+e^-$ in units of the unitarity lower bound [which is given by setting $\mathrm{Re}(R) = 0$ in Eq. (2)]. An analytic expression for the quite complicated function $\delta^{rad}(x)$ is given in Ref. 12), and we have integrated (26) numerically using that expression.

To be definite, we first assume that only the electromagnetic part as calculated in Ref. 1) contributes to $\pi^0 \to e^+e^-$. Under this assumption, the situation is as summarized in Fig. 4. As can be seen, the total correction is rather large and negative (around -20%) for values of δ in the range of 1-5 %, which should be typical values for most experiments. For larger values of δ the background from single Dalitz pairs rapidly becomes dominant. In Fig. 5, the total correction is shown for both a small ("standard") and a rather large value of ρ, the lowest order branching ratio. It can be seen that for a resolution δ better than around four per cent the corrections are not very sensitive to the value of ρ. It is also in this region that the approximations made in this paper can be most trusted (since the emission of soft photons should not depend too much on the structure of the vertex). However, if the precision is increased to much better than 1%, then the corrections become so big that a higher order calculation is needed. We finally note that the formulas in this paper can be trivially modified to other decays with a large mass ratio, $\eta \to e^+e^-$, $\eta_c \to \mu^+\mu^-$, etc. For $\eta \to \mu^+\mu^-$ we expect the radiative corrections to be small since the mass ratio is small.

ACKNOWLEDGEMENTS

The author wishes to thank J.S. Bell for many useful remarks and criticisms, and J.D. Davies, N.W. Tanner and A.G. Zephat for many discussions.

REFERENCES

1) L. Bergström, Z. Phys. C14 (1982) 129.

2) R.E. Mischke et al., Phys. Rev. Lett. 48 (1982) 1153.

3) J.S. Frank et al., Los Alamos preprint LA-UR 82-1954 (1982), unpublished.

4) J. Fischer et al., Phys. Lett. 73B (1978) 364.

5) T. Kinoshita, Phys. Rev. Lett. 2 (1959) 477.

6) A. Garcia, Phys. Rev. D25 (1982) 1985.

7) V.A. Novikov et al., Phys. Rep. 41C (1978) 1.

8) M.A. Shifman et al., Phys. Lett. 77B (1978) 80.

9) L.J. Reinders, H.R. Rubinstein and S. Yazaki, Nucl. Phys. B186 (1981) 109.

10) P. Langacker and H. Pagels, Phys. Rev. D9 (1974) 3413.

11) E. Braaten and J.P. Leveille, Phys. Rev. D22 (1980) 715.

12) K.O. Mikaelian and J. Smith, Phys. Rev. D5 (1972) 1763.

13) L. Bergström and H. Snellman, Z. Phys. C8 (1981) 363.

FIGURE CAPTIONS

Fig. 1 : a) Basic QED two-photon diagram for the decay $\pi^0 \to e^+e^-$. Here the blob represents the form factor for the $\pi^0\gamma\gamma$ vertex.

b) Simplified equivalent point-like γ^5 coupling.

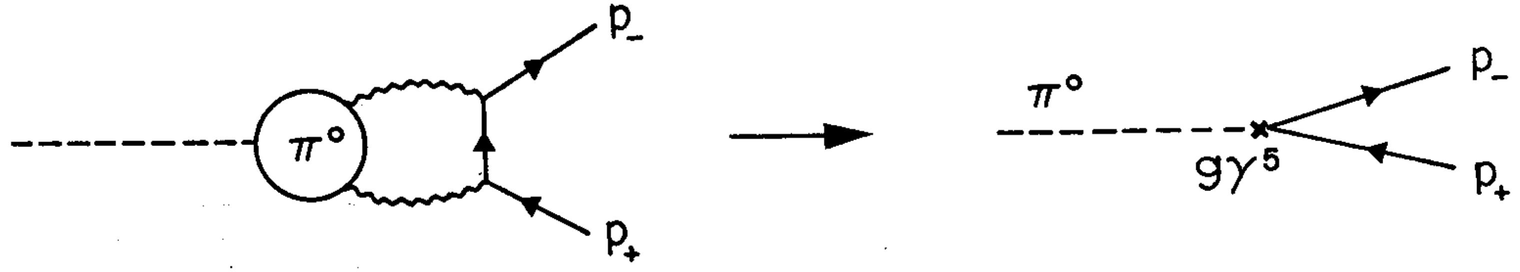
Fig. 2 : First order radiative corrections to the lowest order diagram, Fig. 1b.

Fig. 3 : a) Lowest order diagram for single Dalitz pair decay, $\pi^0 \to e^+e^-\gamma$.

b-d) First order radiative corrections as calculated in Ref. 12).

Fig. 4 : Radiative corrections normalized to the lowest order width $\Gamma^0_{e^+e^-}$ as a function of δ, where the acceptance for the invariant mass of the lepton pair is $(1-\delta)m_\pi^2 \le s \le m_\pi^2$. Here only the two-photon electromagnetic contribution to $\Gamma^0_{e^+e^-}$ as calculated in Ref. 1) is included.

Fig. 5 : Total radiative corrections as a function of δ for two different values of ρ, which is the lowest order branching ratio in units of the unitary lower bound.



(a) (b)

Fig. 1

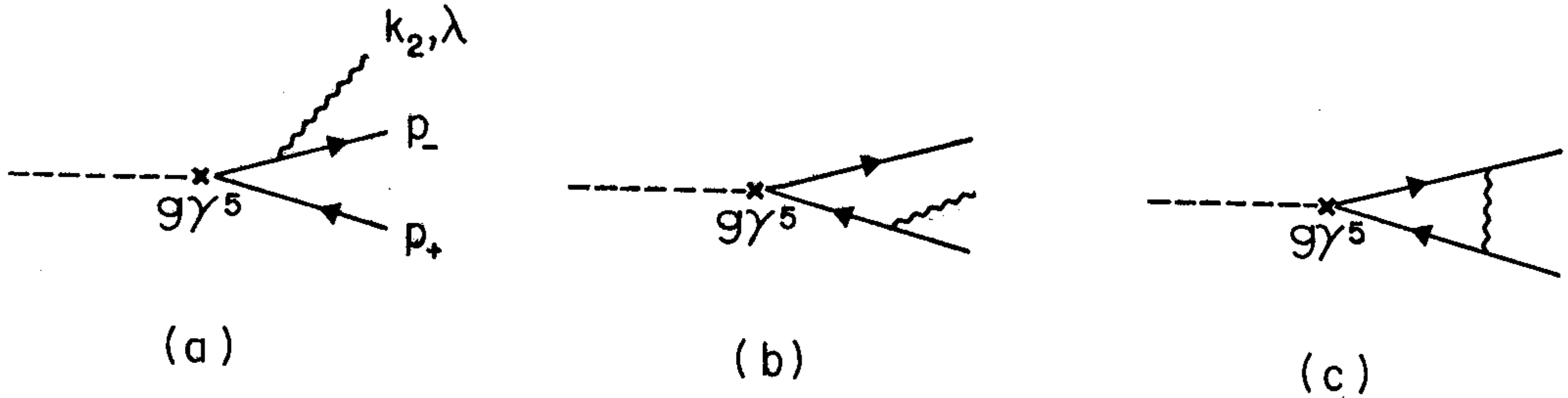


Fig. 2

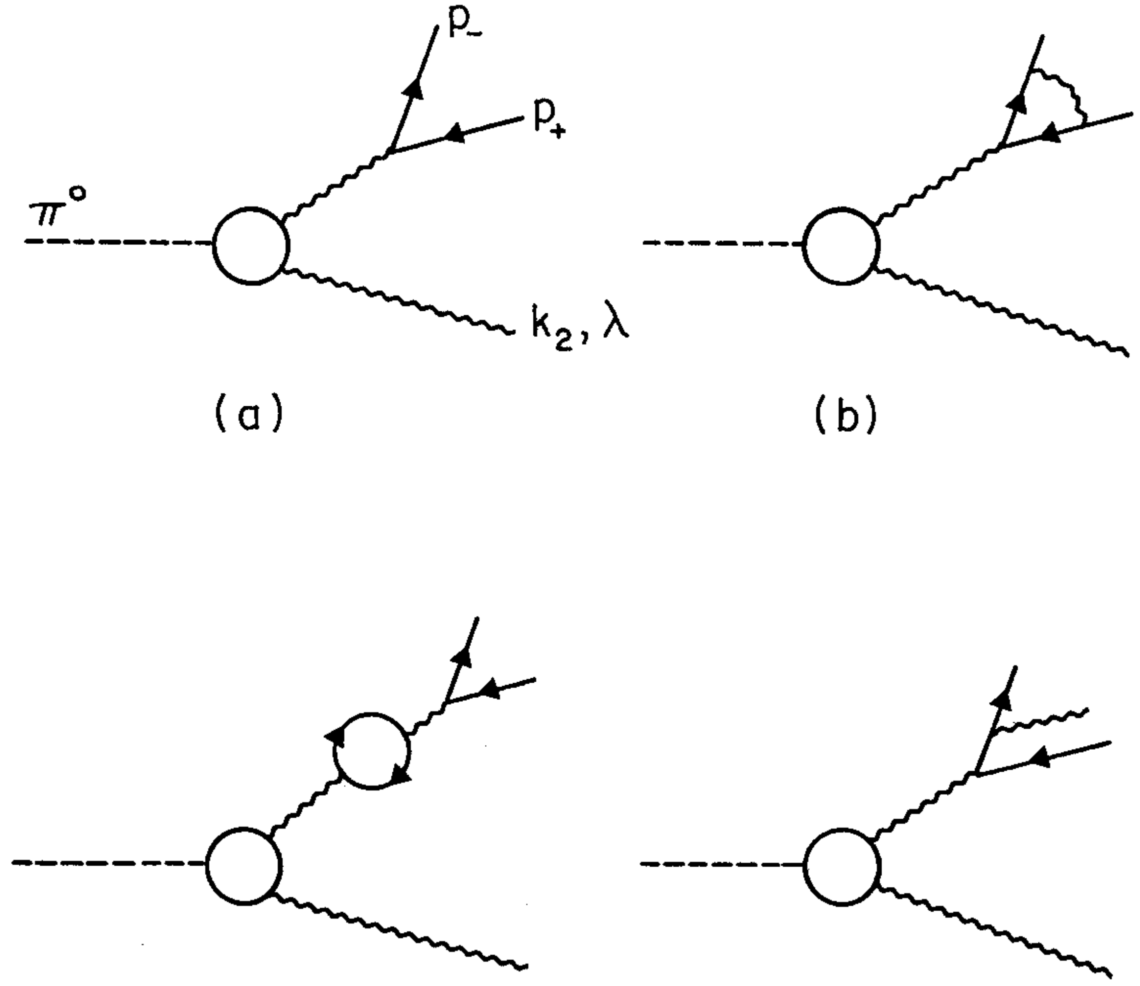


Fig. 3

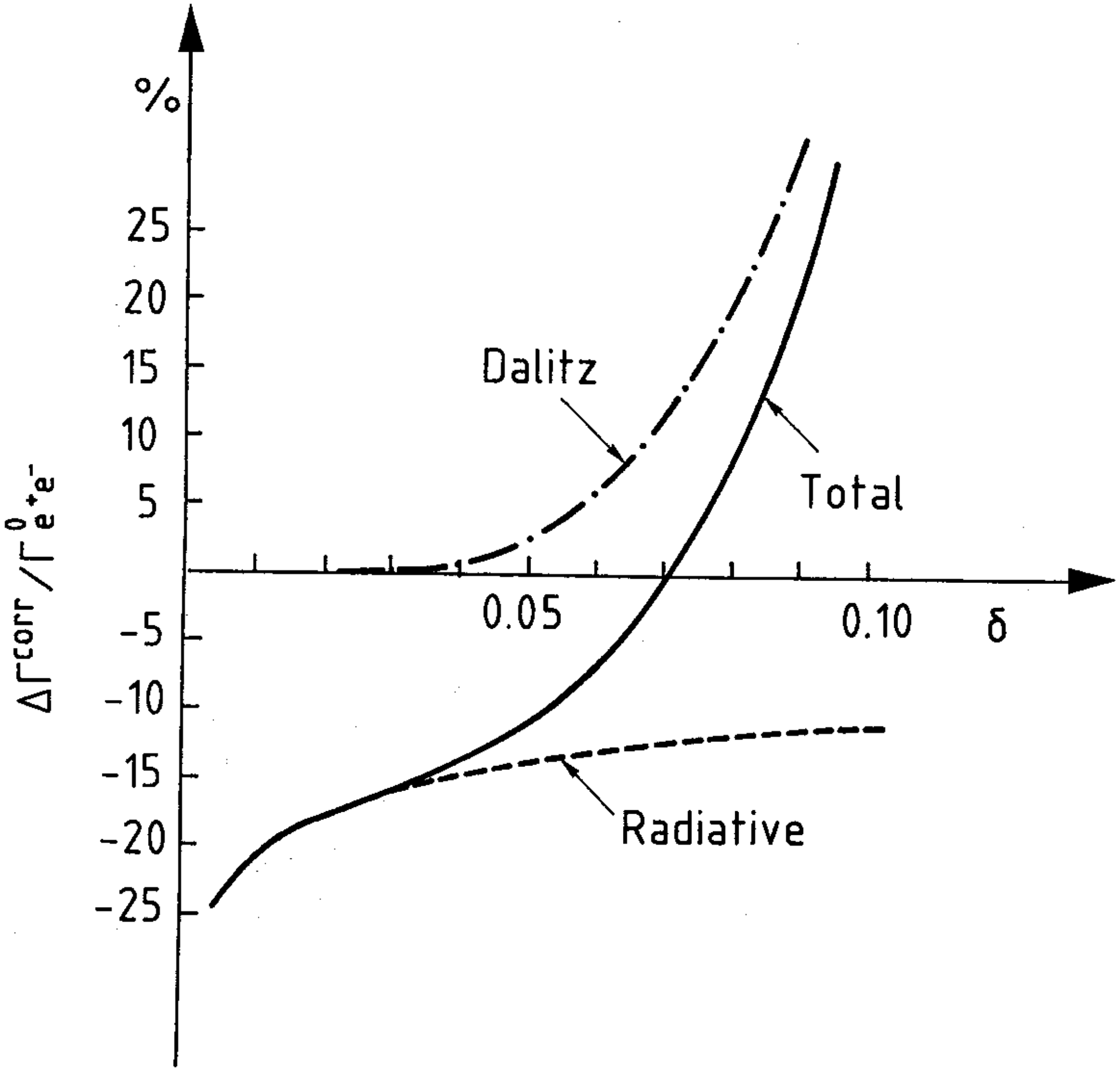

%
25
20
15
10
5
-5
-10
-15
-20
-25
$\Delta\Gamma^{corr}/\Gamma^{0}_{e^+e^-}$
0.05
0.10
δ
Dalitz
Total
Radiative

Fig.4

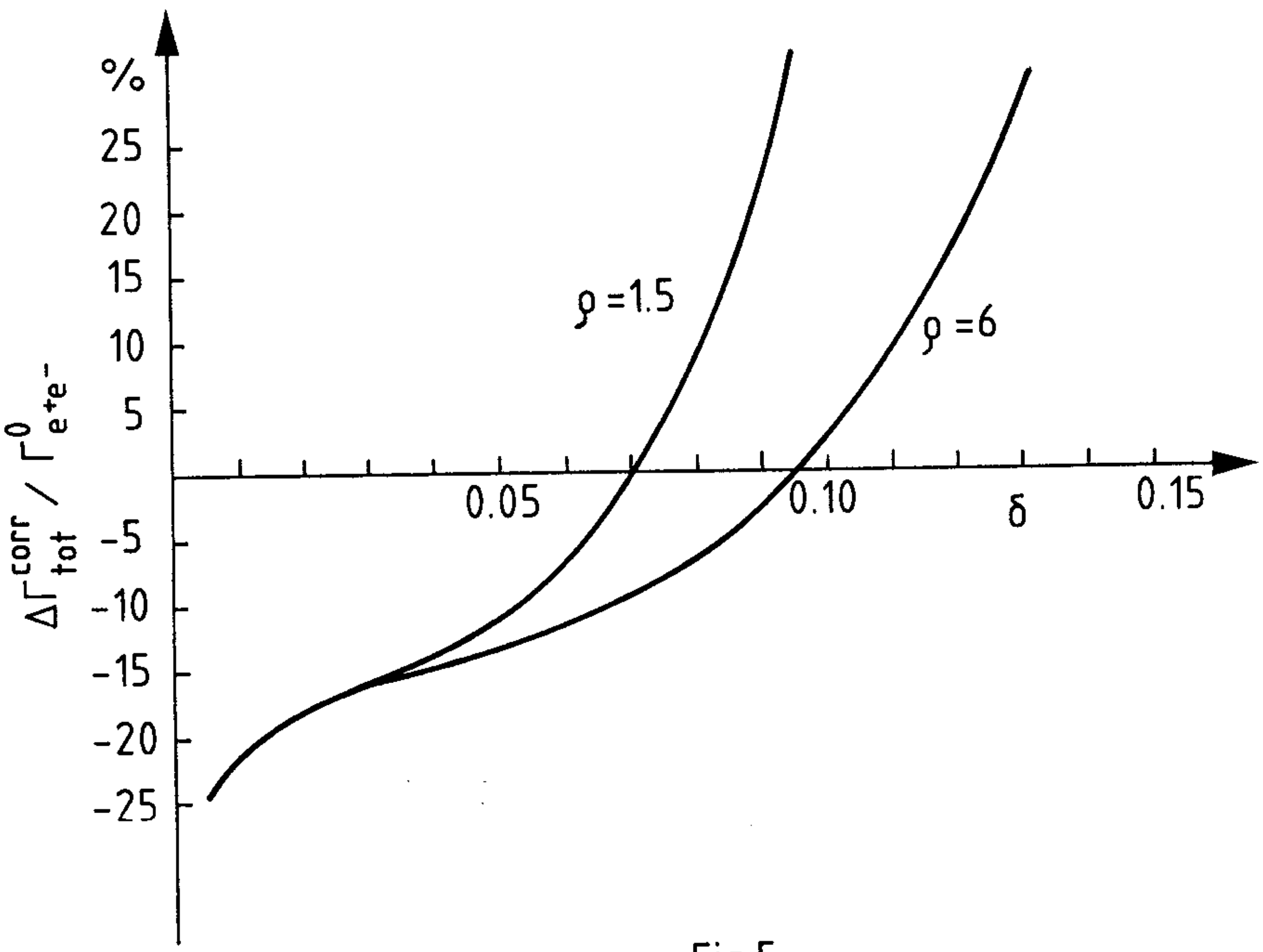

%
25
20
15
10
5
-5
-10
-15
-20
-25
$\Delta\Gamma^{corr}_{tot} / \Gamma^{0}_{e^+e^-}$
0.05
0.10
0.15
δ
ρ =1.5
ρ =6

Fig.5