Radiative Entropy Production

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Abstract

THE radiative tensor obtained from the specular moments of the transfer equation is considered. The radiative entropy production is expressed in terms of this tensor.

Contents

The theory of gas radiation dates back to studies Rayleigh made over a century ago on the illumination and polarization of the sunlit sky. Since then, the theory has rapidly grown because of the efforts of astrophysicists and later of applied scientists and engineers. However, the entropy production associated with radiation apparently remains untreated and is the motivation of this study.

As is well known, the entropy production results from dissipative processes (involving mass, species, momentum, and/or heat transfer and electromagnetic or nuclear transport). Less known is the fact that the dissipation may have a diffusive or hysteretic origin, the diffusion being directional and the hysteresis being cyclic. However, except for a few cases (such as strain hardening and the magnetic saturation), the majority of dissipative processes (including the dissipation of radiation) are of diffusive nature. A recent study by Arpaci[1] shows, in terms of the radiative stress obtained from the specular (kinetic) moments of the transfer equation, the diffusive nature of radiation for any optical thickness. Accordingly, the expression to be developed for entropy production is in terms of this stress and includes also the dissipation resulting from the conduction of heat and viscous friction. First, some remarks on the radiative stress are needed. These will be made in terms of spectrally averaged radiation because of its simplicity. A monochromatic approach, which may be needed for a quantitative study, is not essential here because of the conceptual nature of the intended study.

As pointed out by Felske and Tien,[2] there are a variety of practical situations in which scattering is not important. For these situations, consider the spectrally averaged transfer equation,

$$l_i \frac{\partial I}{\partial x_i} = \kappa (I_0 - I) \tag{1}$$

where I denotes the intensity, I_0 its equilibrium, κ the absorption coefficient, l_i the direction of optical path, and x_i the Cartesian coordinate. The usual definitions of the radiative internal energy, heat flux, and stress in terms of the intensity are

$$u^R = \frac{1}{c}\int_\Omega I \mathrm{d}\Omega = \frac{1}{c} J \tag{2}$$

$$q_i^R = \int_\Omega I l_i \mathrm{d}\Omega \tag{3}$$

$$\tau_{ij}^R = \frac{1}{c}\int_\Omega I l_i l_j \mathrm{d}\Omega = \frac{1}{c}\Pi_{ij} \tag{4}$$

where the J scalar and the Π_{ij} tensor are introduced for notational convenience, c is the velocity of light, and Ω is the solid angle. In terms of these definitions, the first three specular moments of the transfer equation are

$$\frac{\partial q_i^R}{\partial x_i} = \kappa_P (B - J) \tag{5}$$

$$\frac{\partial \Pi_{ij}}{\partial x_j} = -\kappa_R q_i^R \tag{6}$$

$$\Pi_{ij} = \frac{1}{3} B \delta_{ij} - \frac{1}{\kappa_M} \frac{\partial}{\partial x_k}\int_\Omega I l_i l_j l_k \mathrm{d}\Omega \tag{7}$$

where $B = 4E_b$; $E_b = \sigma T^4$, the Stefan-Boltzmann law for the blackbody emissive power; κ_P and κ_R the Planck and Rosseland means of the absorption coefficient, respectively; and $\kappa_M = (\kappa_P \kappa_R)^{1/2}$, the geometric mean of these coefficients. The incorporation of κ_P and κ_R into the foregoing equations is discussed in Refs. 3-11. Clearly, Eq. (5) denotes the thermal balance, Eq. (6) the momentum balance associated with radiation, and Eq. (7) the definition of the Π_{ij} tensor. A procedure for the evaluation of this tensor in terms of the Wallis integrals is described in Unno and Spiegel.[12] This procedure leads to

$$\Pi_{ij} = \sum_{n=0}^{\infty} \frac{\nabla^{2n-2}(2n\partial_i\partial_j + \nabla^2 \delta_{ij})B}{\kappa_M^{2n}(2n+1)(2n+3)} \tag{8}$$

where $\partial_i \equiv \partial/\partial x_i$ and $\partial_j \equiv \partial/\partial x_j$ are used for notational convenience. The same result may be found also in earlier works (see, for example, Ref. 13). The formal similarity of Eq. (8) to the Hookean constitution for elastic solids should be noted.

An alternate from for this stress may be given in terms of the isotropic radiative pressure. First, invoking the assumption of isotropy, Eqs. (2) and (4) are related as

$$\tau_{ij}^R = \tfrac{1}{3} u^R \delta_{ij} \tag{9}$$

or, equivalently, as

$$\tau_{ij}^R = \tfrac{1}{3}\Pi_{kk}\delta_{ij} = \tfrac{1}{3} J \delta_{ij} \tag{10}$$

where

$$\tfrac{1}{3}\Pi_{kk} = -p \tag{11}$$

is the (isotropic) pressure of radiation. Then, from the trace of Π_{ij}, noting that $l_k l_k = 1$,

$$\Pi_{kk} = \sum_{n=0}^{\infty} \left(\frac{\nabla^2}{\kappa_M^2}\right)^n \frac{B}{(2n+1)} \tag{12}$$

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Now, in a manner similar to the inclusion of the isotropic pressure to the development of viscous stress from elastic stress (e.g., see Ref. 14), adding the identity

$$\tfrac{1}{3}J\delta_{ij} - \tfrac{1}{3}\Pi_{kk}\delta_{ij} = 0 \tag{13}$$

to Eq. (8), the Π_{ij} tensor may be rearranged in terms of the radiative pressure,

$$\Pi_{ij} = \frac{1}{3}J\delta_{ij} + \sum_{n=0}^{\infty} \frac{2n\nabla^{2n-2}(\partial_i\partial_j - \tfrac{1}{3}\nabla^2\delta_{ij})B}{\kappa_M^{2n}(2n+1)(2n+3)} \tag{14}$$

The operational similarity of Eq. (14) to the viscous (Stokesean) stress and the electromagnetic (Maxwell) stress should be noted. The use of the first term of Eq. (14) in place of Eq. (8) is the well-known Eddington approximation. The rest of the study develops an expression for the radiative entropy production in terms of Π_{ij} given by Eqs. (8) and (14).

As is well known, the major contribution of thermal radiation to thermomechanics is the radiation heat flux q_i^R (e.g., see Refs. 15-19). Accordingly, with the addition of this flux, the thermal energy becomes, in terms of the usual notation,

$$\rho\frac{\mathrm{D}u}{\mathrm{D}t} + p\left(\frac{\partial v_i}{\partial x_i}\right) = -\frac{\partial}{\partial x_i}(q_i^K + q_i^R) + \tau_{ij}s_{ij} \tag{15}$$

and the balance of entropy becomes

$$\rho\frac{\mathrm{D}s}{\mathrm{D}t} = -\frac{\partial}{\partial x_i}\left(\frac{q_i^K + q_i^R}{T}\right) + s''' \tag{16}$$

Now, consider the Gibbs (thermodynamic) relation

$$\rho\frac{\mathrm{D}u}{\mathrm{D}t} = \rho T\frac{\mathrm{D}s}{\mathrm{D}t} + \frac{p}{\rho}\frac{\mathrm{D}\rho}{\mathrm{D}t} \tag{17}$$

and rearrange it in terms of the conservation of mass to get

$$\rho\frac{\mathrm{D}u}{\mathrm{D}t} = \rho T\frac{\mathrm{D}s}{\mathrm{D}t} - p\left(\frac{\partial v_i}{\partial x_i}\right) \tag{18}$$

Eliminating the internal energy and the entropy among Eqs. (15), (16), and (18) yields the expression for entropy production

$$s''' = \frac{1}{T}\left[-\frac{1}{T}(q_i^K + q_i^R)\left(\frac{\partial T}{\partial x_i}\right) + \tau_{ij}s_{ij}\right] \tag{19}$$

Inserting the usual conductive and viscous constitutions

$$q_i^K = -k\left(\frac{\partial T}{\partial x_i}\right), \quad \tau_{ij} = 2\mu s_{ij} \tag{20}$$

and the radiative constitution from Eq. (6)

$$q_i^R = -\frac{1}{\kappa_R}\left(\frac{\partial \Pi_{ij}}{\partial x_j}\right)$$

into this expression gives

$$s''' = \frac{1}{T}\left\{\frac{1}{T}\left[k\left(\frac{\partial T}{\partial x_i}\right) + \frac{1}{\kappa_R}\left(\frac{\partial \Pi_{ij}}{\partial x_j}\right)\right]\left(\frac{\partial T}{\partial x_i}\right) + 2\mu s_{ij}s_{ij}\right\} \tag{21}$$

Explicitly, in terms of Eq. (8),

$$s''' = \frac{1}{T}\left\{\frac{1}{T}\left[k\left(\frac{\partial T}{\partial x_i}\right) + \frac{4}{\kappa_R}\sum_{n=0}^{\infty}\left(\frac{\nabla^2}{\kappa_M^2}\right)^n\left(\frac{\partial E_b/\partial x_i}{2n+3}\right)\right] \times \left(\frac{\partial T}{\partial x_i}\right) + 2\mu s_{ij}s_{ij}\right\} \tag{22}$$

or, in terms of Eq. (14),

$$s''' = \frac{1}{T}\left\{\frac{1}{T}\left[k\left(\frac{\partial T}{\partial x_i}\right) + \frac{1}{3\kappa_R}\left(\frac{\partial J}{\partial x_i}\right) + \frac{4}{3\kappa_R}\sum_{n=0}^{\infty}\frac{4n}{(2n+1)(2n+3)}\left(\frac{\nabla^2}{\kappa_M^2}\right)^n\left(\frac{\partial E_b}{\partial x_i}\right)\right] \times \left(\frac{\partial T}{\partial x_i}\right) + 2\mu s_{ij}s_{ij}\right\} \tag{23}$$

Conclusions

The radiative stress obtained from the specular (kinetic) moments of the transfer equation is considered. In terms of an isotropic radiative pressure, this stress is given an alternate form by following a development similar to that of the viscous (Stokesean) stress from the elastic (Hookean) stress.

An expression for local entropy production, including the effect of radiation as well as that of conduction and (viscous) friction, is developed. The radiative contribution to this production is expressed in terms of the radiative stress. Alternate forms of the entropy production are stated by considering the "elastic" and "viscous" equivalents of the radiative stress. These forms include also the usual contributions of conduction and (viscous) friction.

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