Radiative heat transfer at the nanoscale

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Charge and heat dynamics in nano-systems
Orsay 10-12 Oct 2011

radiative heat transfer

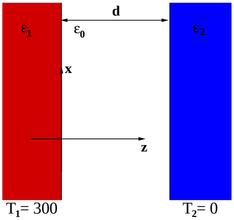
- heat transfer (emissivity ϵ)

$$\rightarrow \Phi = \epsilon\sigma(T_1^4 - T_2^4)$$

- black bodies ($\epsilon = 1$)

$$\rightarrow \Phi_{\rm BB} = \sigma(T_1^4 - T_2^4)$$

- but only for $d \gg \lambda_{\rm th}$!!!
- for $d \ll \lambda_{\rm th}$:
  - heat flux can exceed $\Phi_{\rm BB}$ by orders of magnitude! $\rightarrow$ evanescent modes



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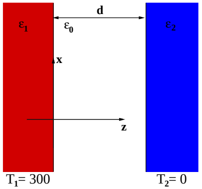
radiative heat transfer - theory



- heat flux ($T_1 = T$ and $T_2 = 0$)

$$\Phi = \int \frac{d\omega}{2\pi} \frac{\hbar\omega}{e^{\hbar\omega/(k_B T)} - 1} \int \frac{d^2\kappa}{(2\pi)^2} (\mathcal{T}_s + \mathcal{T}_p)$$

Polder and van Hove, PRB **4**, 3303 (1971)

- metamaterials Joulain et al., PRB **81**, 165119 (2010)
- anisotropy Biehs et al. Opt. Expr. **19**, A1088-A1103 (2011), APL **98** 243102 (2011)
- roughness Biehs & Greffet, PRB **82**, 245410 (2010)
- sphere-sphere Narayanaswamy & Chen, PRB **77**, 075125 (2008)
- sphere-plane Krüger et al., PRL **106**, 210404 (2011)
- arbitrary geometries Messina & Antezza PRA **84**, 042102 (2011),
- many bodies Ben-Abdallah et al., PRL **107**, 114301 (2011)
- ...

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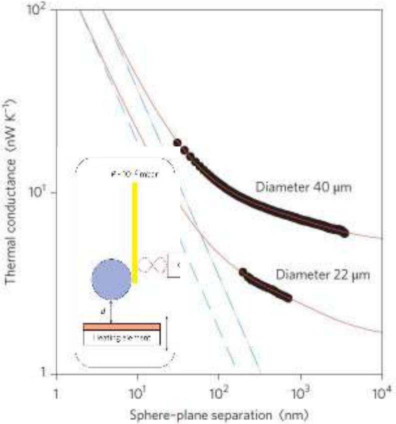
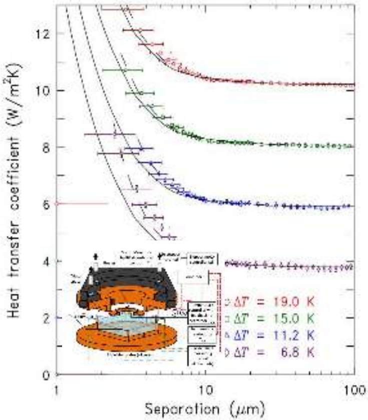
radiative heat transfer - experiments



Rousseau et al., Nature Photonics **3**, 514 (2009)



Ottens et al., PRL **107**, 014301 (2011)

radiative heat transfer

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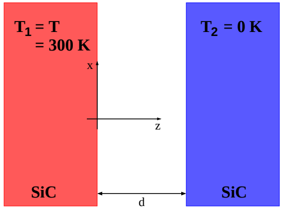
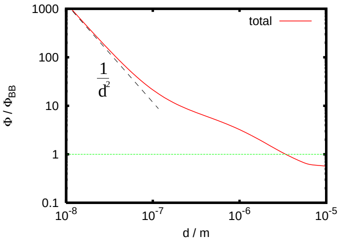
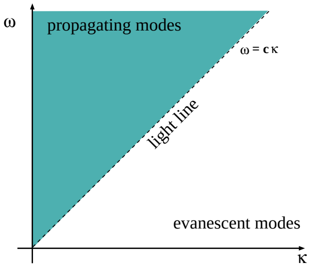


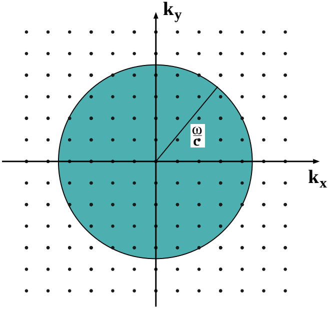


- transmission coefficient (Polder and van Hove, PRB **4**, 3303 (1971))

$$\mathcal{T}_i(\omega, \boldsymbol{\kappa}; d) = \begin{cases} \dfrac{(1-|r_i^{10}|^2)(1-|r_i^{20}|^2)}{|1-r_i^{10} r_i^{20} \exp(2\mathrm{i}k_z d)|^2}, & \kappa < \dfrac{\omega}{c} \\ \dfrac{\mathrm{Im}(r_i^{10})\mathrm{Im}(r_i^{20}) e^{-2|k_z|d}}{|1-r_i^{10} r_i^{20} \exp(2\mathrm{i}k_z d)|^2}, & \kappa > \dfrac{\omega}{c} \end{cases}$$

propagating modes





- plane wave

$$E_y = A(x, y; t)e^{ik_z z}$$

- for prop. modes $\kappa < \frac{\omega}{c}$

$$k_z = \sqrt{\frac{\omega^2}{c^2} - \kappa^2} \in \mathbb{R}$$

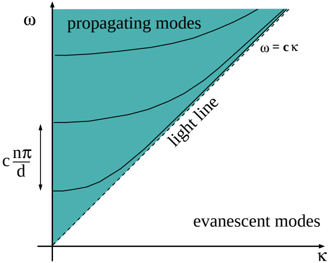
- for evan. modes $\kappa > \frac{\omega}{c}$

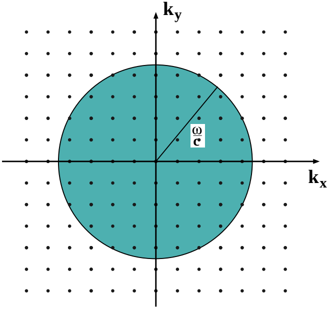
$$k_z = i\sqrt{\kappa^2 - \frac{\omega^2}{c^2}} \in \mathbb{C}$$

- resonant transmission

$$k_z \equiv \frac{n\pi}{d}, n \in \mathbb{N}$$

propagating modes





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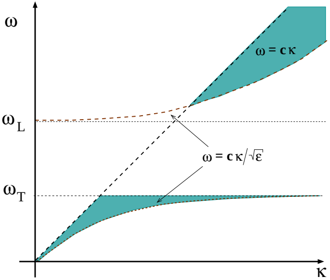
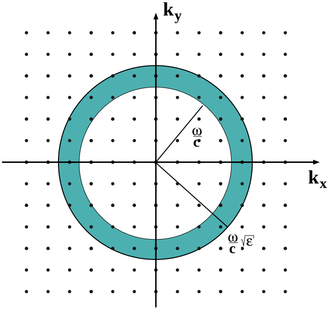
frustrated internal reflection modes

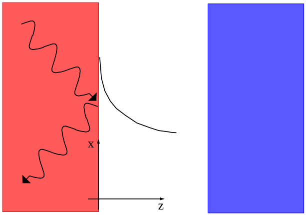




- propagating inside the bulk

$$k_{1,z} = \sqrt{\frac{\omega^2}{c^2}\epsilon - \kappa^2} \in \mathbb{R}$$

$$\Leftrightarrow \kappa < \frac{\omega}{c}\sqrt{\epsilon}$$



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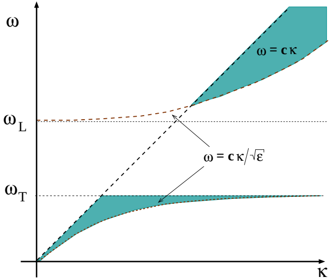

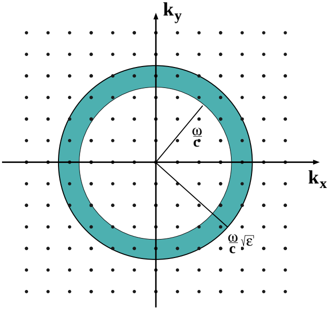

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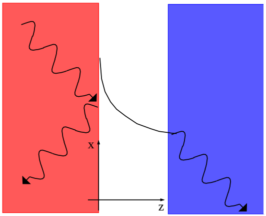

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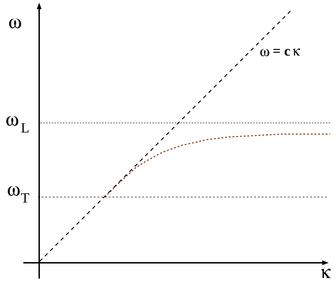
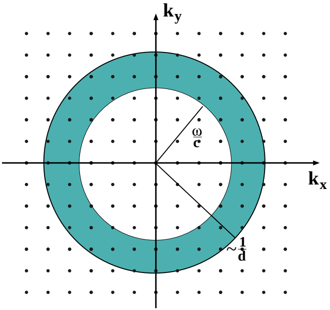
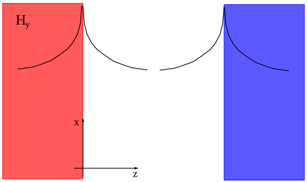
surface modes



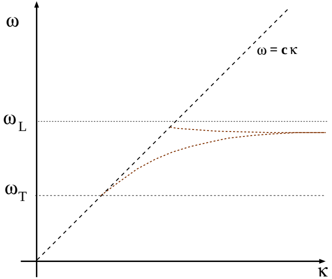


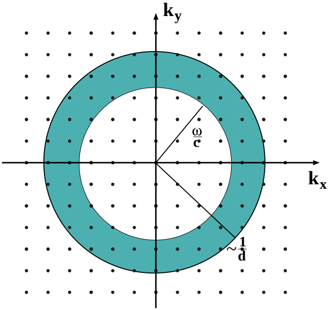
- 'bound' to the interface
- only p-polarized
- Transmission coefficient
  $\kappa \gg \omega/c$

$$\mathcal{T}_{\rm p} \approx \frac{{\rm Im}(r_{\rm p}^{10}){\rm Im}(r_{\rm p}^{20})e^{-2\kappa d}}{|1 - r_{\rm p}^{10} r_{\rm p}^{20} \exp(-2\kappa d)|^2}$$



surface modes





- 'bound' to the interface
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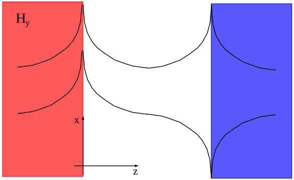
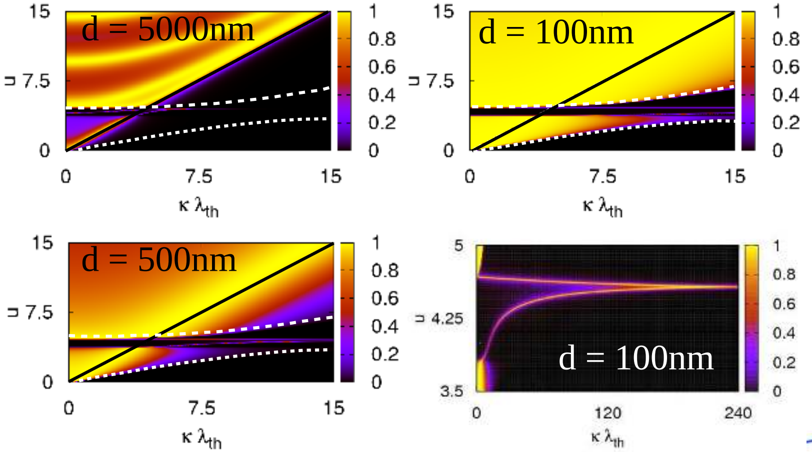
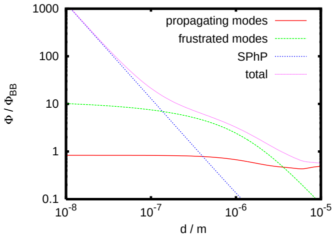
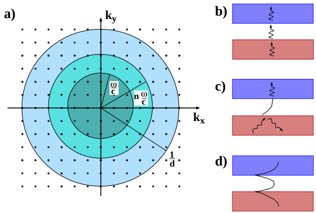


transmission coefficient (SiC)



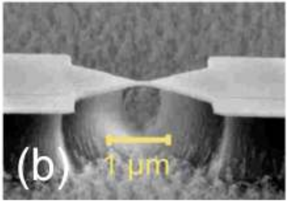
Resulting heat flux (SiC, $T_1 = 300\,\mathrm{K}$, $T_2 = 0\,\mathrm{K}$)



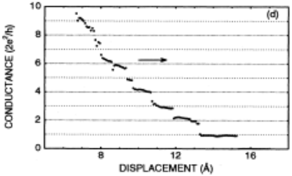


$$\Phi = \int \frac{\mathrm{d}\,\omega}{2\pi} \frac{\hbar\omega}{\mathrm{e}^{\hbar\omega/(k_\mathrm{B} T)} - 1} \int \frac{\mathrm{d}^2\kappa}{(2\pi)^2} \left(\mathcal{T}_\mathrm{s} + \mathcal{T}_\mathrm{p}\right)$$

Mesoscopic point of view



Wu et al., PRB **78**, 235421 (2008)



Brandbyge et al., PRB **52**, 8499 (1995)

- charge transport on mesoscopic scale (Landauer)

$$I = \Gamma V = \frac{2e^2}{h}\left[\sum_n T_n\right] V$$

- Landauer-like expression for heat radiation at nanoscale?

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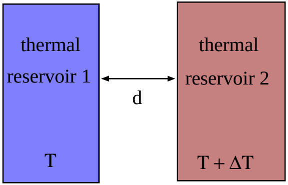
heat flux expression



- all distances, including

$$a \ll d \ll \lambda_{\mathrm{th}}$$

with lattice constant a

- resulting heat flux, $b(T) = \frac{1}{e^{(\hbar\omega)/(k_B T)} - 1}$

$$\Phi = \int \frac{d\omega}{2\pi} \hbar\omega \left[b(T + \Delta T) - b(T)\right] \sum_{i=s,p} \int \frac{d^2\kappa}{(2\pi)^2} \mathcal{T}_i$$

$$\Delta T \ll T$$

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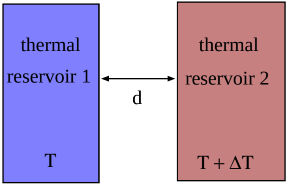
heat flux expression



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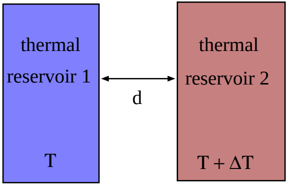
$$\Phi = \int \frac{\mathrm{d}\,\omega}{2\pi} \hbar\omega \left(\frac{\partial}{\partial T} b(T) \right) \sum_{i=\mathrm{s,p}} \int \frac{\mathrm{d}^2\kappa}{(2\pi)^2} \mathcal{T}_i \Delta T$$

$$\Delta T \ll T$$

heat flux expression



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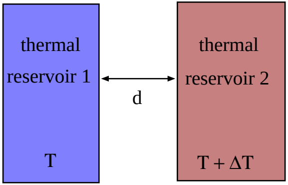
$$\Phi = \frac{k_{\mathrm{B}}^2 T}{h} \int \mathrm{d}u\, u^2 \left(-\frac{\partial}{\partial u} b(u)\right) \sum_{i=\mathrm{s,p}} \int \frac{\mathrm{d}^2\kappa}{(2\pi)^2} \mathcal{T}_i \Delta T$$

introducing $u = \hbar\omega/(k_{\mathrm{B}}T)$

heat flux expression



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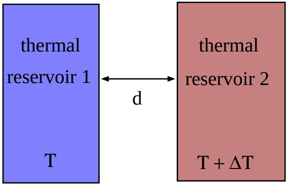
interchanging integrals

heat flux expression



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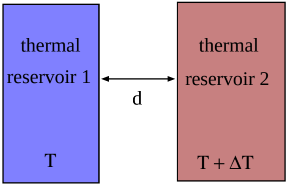
introduce $\int du\, u^2 \left(-\frac{\partial}{\partial u} b(u) \right) = \pi^2/3$

heat flux expression



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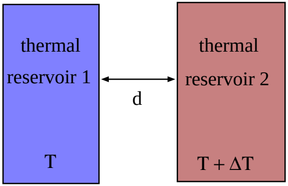

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$$\Phi = \frac{\pi^2 k_B^2 T}{3h} \sum_{i=\mathrm{s,p}} \int \frac{d^2\kappa}{(2\pi)^2} \left[\frac{\int du\, u^2 \left(-\frac{\partial}{\partial u} b(u) \right) \mathcal{T}_i}{\int du\, u^2 \left(-\frac{\partial}{\partial u} b(u) \right)} \right] \Delta T$$

defining $\overline{\mathcal{T}}_i \equiv \frac{\int du\, u^2 \left(\frac{\partial}{\partial u} b(u) \right) \mathcal{T}_i}{\int du\, u^2 \left(\frac{\partial}{\partial u} b(u) \right)}$

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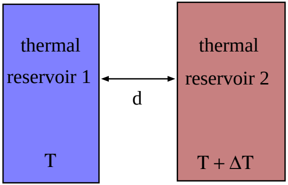
heat flux expression



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Landauer-like heat flux expression

- final result

$$\Phi = \frac{\pi^2 k_\mathrm{B}^2 T}{3h}\left[\sum_{i=\mathrm{s,p}}\int\frac{\mathrm{d}^2\kappa}{(2\pi)^2}\overline{\mathcal{T}}_i\right]\Delta T \qquad \leftrightarrow \qquad I = \Gamma V = \frac{2e^2}{h}\left[\sum_n T_n\right]V$$

- quantum of thermal conductance

$$\frac{\pi^2 k_\mathrm{B}^2 T}{3h}$$

Pendry, J. Phys. A **16**, 2161 (1983)

- mean transmission coefficient

$$\overline{\mathcal{T}}_i = \frac{\int \mathrm{d}u\, u^2 \left(\frac{\partial}{\partial u} b(u)\right)\mathcal{T}_i}{\int \mathrm{d}u\, u^2 \left(\frac{\partial}{\partial u} b(u)\right)}$$

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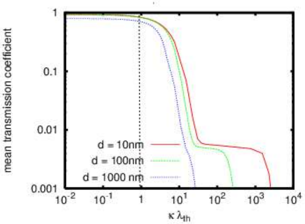
mean transmission coefficient



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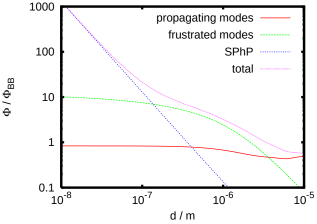
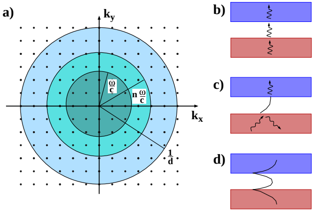
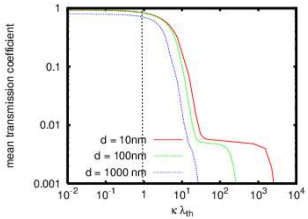


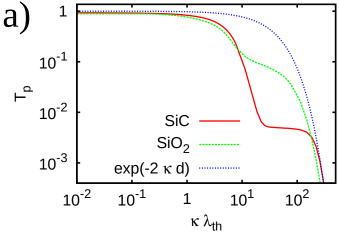


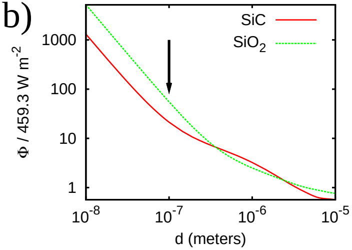
Biehs et al., PRL **105**, 234301 (2010)

mean transmission coefficient



$$\Phi = \frac{\pi^2 k_B^2 T}{3h} \sum_{i=\mathrm{s,p}} \int \frac{\mathrm{d}^2\kappa}{(2\pi)^2} \overline{\mathcal{T}}_j \Delta T$$





Biehs et al., PRL **105**, 234301 (2010)

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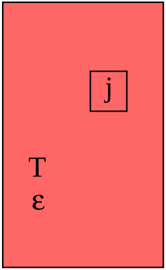
Summary/Conclusions

- Introduction to nanoscale heat transfer
  - propagating modes
  - frustrated internal reflection modes
  - coupled surface phonon polaritons
- Landauer-like expression
  - highlights number of contributing modes
  - tradeoff between NOM and transmission
  - fundamental limit of near-field heat flux

  P. Ben-Abdallah and K. Joulain, PRB **82**, 121419 (R)(2010)

Thanks for your attention !!!

Fluctuational electrodynamics (Rytov)



- Maxwell's Eqs. + fluct. currents

$$\langle \mathbf{j} \rangle = \mathbf{0}$$

- fluctuating fields

$$E_\alpha(\omega, \mathbf{r}) = \mathrm{i}\omega\mu_0 \int_V \mathrm{d}^3 r \, \mathbb{G}_{\alpha\beta}(\mathbf{r}, \mathbf{r}') j_\beta(\mathbf{r}, \omega)$$

- fluctuation dissipation theorem

$$\langle j_\alpha(\mathbf{r}) j^*_\beta(\mathbf{r}') \rangle = \frac{\hbar\omega}{\mathrm{e}^{\hbar\omega/k_\mathrm{B}T} - 1} \left[\mathrm{Im}(\epsilon) \delta_{\alpha\beta} \delta(\mathbf{r} - \mathbf{r}') \right]$$

- Correlation functions of the fields

$$\langle E_\alpha E^*_\beta \rangle, \langle H_\alpha H^*_\beta \rangle, \langle E_\alpha H^*_\beta \rangle$$

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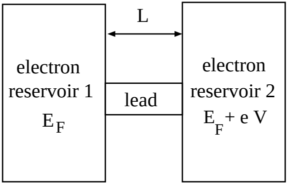
Landauer expression



- mesoscopic system

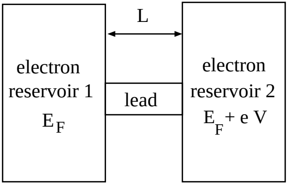
  $l_e, l_\varphi \ll$ rel. length scales

- resulting current (*env*), $f(E) = \frac{1}{\mathrm{e}^{(E-E_\mathrm{F})/(k_\mathrm{B}T)}+1}$

$$I = e \int \mathrm{d}k \, n(k) v_k \left[f(E - eV) - f(E) \right] \sum_n T_n(E)$$

$$n(k)\mathrm{d}k = 2 \frac{1}{L} \frac{\mathrm{d}k}{\frac{2\pi}{L}} = 2 \frac{\mathrm{d}k}{2\pi}$$

Landauer expression



- mesoscopic system

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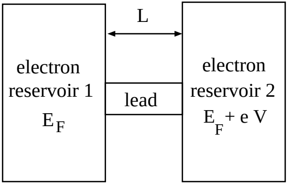
Landauer expression



- mesoscopic system

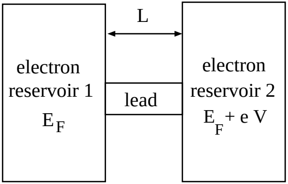
  $l_e, l_\varphi \ll$ rel. length scales

- resulting current (*env*), $f(E) = \frac{1}{\mathrm{e}^{(E-E_\mathrm{F})/(k_\mathrm{B}T)}+1}$

$$I = 2e \int \frac{\mathrm{d}k}{2\pi} v_k \left[f(E-eV) - f(E) \right] \sum_n T_n(E)$$

$$v_k = \frac{1}{\hbar} \frac{\partial E}{\partial k}$$

Landauer expression



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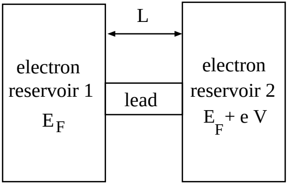


- mesoscopic system

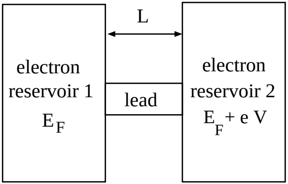
  $l_e, l_\varphi \ll$ rel. length scales

- resulting current (*env*), $f(E) = \frac{1}{\mathrm{e}^{(E-E_\mathrm{F})/(k_\mathrm{B}T)}+1}$

$$I = 2e \int \frac{\mathrm{d}k}{2\pi} \frac{1}{\hbar} \frac{\partial E}{\partial k} \left[f(E - eV) - f(E) \right] \sum_n T_n(E)$$

$$v_k = \frac{1}{\hbar} \frac{\partial E}{\partial k}$$

Landauer expression



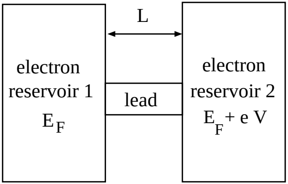
- mesoscopic system

  $l_e, l_\varphi \ll$ rel. length scales

- resulting current (*env*), $f(E) = \frac{1}{e^{(E-E_F)/(k_B T)}+1}$

$$I = \frac{2e}{h} \int dk \, \frac{\partial E}{\partial k} \left[f(E - eV) - f(E) \right] \sum_n T_n(E)$$

Landauer expression



- mesoscopic system

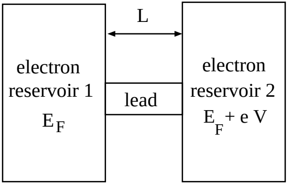
  $l_e, l_\varphi \ll$ rel. length scales

- resulting current (env), $f(E) = \frac{1}{\mathrm{e}^{(E-E_\mathrm{F})/(k_\mathrm{B}T)}+1}$

$$I = \frac{2e}{h} \int \mathrm{d}E \left[f(E - eV) - f(E) \right] \sum_n T_n(E)$$

$$eV \ll E_\mathrm{F}$$

Landauer expression



- mesoscopic system

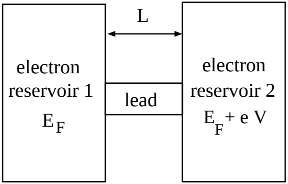
  $l_e, l_\varphi \ll$ rel. length scales

- resulting current (*env*), $f(E) = \frac{1}{e^{(E-E_{\mathrm{F}})/(k_{\mathrm{B}}T)}+1}$

$$I = \frac{2e}{h} \int \mathrm{d}E \left[-\frac{\partial}{\partial E} f(E) \right] \sum_n T_n(E) eV$$

$$k_{\mathrm{B}} T \ll E_{\mathrm{F}}$$

Landauer expression



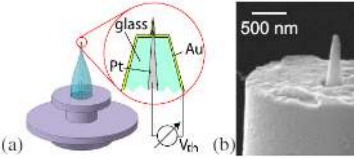
- mesoscopic system

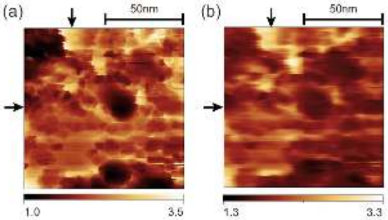
  $l_e, l_\varphi \ll$ rel. length scales

- resulting current (*env*), $f(E) = \frac{1}{e^{(E-E_\mathrm{F})/(k_\mathrm{B}T)}+1}$

$$I = \frac{2e^2}{h}\left[\sum_n T_n(E_\mathrm{F})\right] V$$

NSThM





Kittel et al., APL **93**, 193109 (2008)