

Radiative neutrino masses, lepton flavor mixing and muon $g - 2$ in a leptoquark model

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ABSTRACT: We propose a leptoquark model with two scalar leptoquarks $S_1 \left(\bar{3}, 1, \frac{1}{3} \right)$ and $\tilde{R}_2 \left(3, 2, \frac{1}{6} \right)$ to give a combined explanation of neutrino masses, lepton flavor mixing and the anomaly of muon $g - 2$, satisfying the constraints from the radiative decays of charged leptons. The neutrino masses are generated via one-loop corrections resulting from a mixing between S_1 and \tilde{R}_2 . With a set of specific textures for the leptoquark Yukawa coupling matrices, the neutrino mass matrix possesses an approximate μ - τ reflection symmetry with $(M_\nu)_{ee} = 0$ only in favor of the normal neutrino mass ordering. We show that this model can successfully explain the anomaly of muon $g - 2$ and current experimental neutrino oscillation data under the constraints from the radiative decays of charged leptons.

KEYWORDS: Neutrino Physics, Beyond Standard Model

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1 Introduction

As a quantum field theory with $SU(3)_c \times SU(2)_L \times U(1)_Y$ gauge symmetry, the Standard Model (SM) has successfully described the behaviors of strong, weak and electromagnetic interactions of all the known fundamental particles. However, there exist some compelling evidences that the SM is incomplete and only an effective field theory at low energy scales at least in its leptonic flavor sector [1]. In the neutrino sector, a number of successful neutrino oscillation experiments have firmly proved that neutrinos have tiny masses and the lepton flavor mixing exists [2]. These facts are in contradiction with the SM and strongly call for new physics beyond the SM. Usually, some new heavy particles are introduced to generate the tiny neutrino masses at the tree level, such as right-handed neutrinos, $SU(2)_L$ -triplet Higgs and $SU(2)_L$ -triplet fermions in the type-I [3–7], type-II [8–13] and type-III [14, 15] seesaw mechanisms, respectively. An alternative and interesting way to naturally generate the tiny neutrino masses is via radiative corrections [16–18] (see. e.g., refs. [19, 20], for a recent review). In the charged-lepton sector, there is a longstanding anomaly of the muon anomalous magnetic moment, namely the discrepancy between the theoretical computations and its measured value, which motivates extensions of the SM with new couplings to leptons. Very recently, the Muon ($g - 2$) Collaboration at Fermi National Laboratory has announced their new result [21], which combined with the previous result from the E821 experiment at Brookhaven National Laboratory [22], leads to a 4.2σ discrepancy with the SM prediction [23], that is $\Delta a_\mu = a_\mu(\text{Exp}) - a_\mu(\text{SM}) = (251 \pm 59) \times 10^{-11}$ with 1σ error.¹ Inspired by this new result, there have been plenty of theoretical works gushing out to explain this discrepancy [25–88]. A systematic summary of various new-physics scenarios and a great deal of earlier references can be found in refs. [60, 89].

¹Note that the recent result from lattice QCD is compatible with the experimental value [24].

Motivated by above facts, we attempt to extend the SM to give a combined explanation of the tiny neutrino masses, lepton flavor mixing and muon $g - 2$. Unfortunately, the most popular way to generate neutrino masses via the three typical seesaw mechanisms can not account for the anomaly of muon $g - 2$ [71, 90]. Thus in this work, we focus on an interesting scenario of radiative neutrino masses, where two scalar leptoquarks (LQs), $S_1 \left(\bar{3}, 1, \frac{1}{3} \right)$ and $\tilde{R}_2 \left(3, 2, \frac{1}{6} \right)$ with the numbers in parenthesis denoting $SU(3)_c \times SU(2)_L \times U(1)_Y$ quantum numbers are introduced into the SM. LQ extensions of the SM have been discussed extensively due to its possible origination from the grand unification framework [91–94] and its ability to explain neutrino masses via radiative corrections, the muon $g - 2$ and lepton flavor universality violations in semi-leptonic B -meson decays [26, 31, 39, 60, 69, 70, 72, 82, 87, 95–121] (see, e.g., ref. [122], for a recent review of LQs). In this work, we only focus on the combined explanation of neutrino masses, lepton flavor mixing and the anomaly of muon $g - 2$, as well as constraints from the lepton-flavor-violation (LFV) radiative decays of charged leptons, by means of S_1 and \tilde{R}_2 . It will show that our model works well and differs from the previous works at least in following three aspects:

- To our best knowledge, it is the first time that the combination of S_1 and \tilde{R}_2 is used to simultaneously interpret the tiny neutrino masses and the anomaly of muon $g - 2$, including the constraints from radiative decays of charged leptons, though the radiative neutrino masses generated by S_1 and \tilde{R}_2 and contributions from S_1 to the muon $g - 2$ have been separately discussed before.
- It is well-known that the mixing between S_1 and \tilde{R}_2 induced by a LQ-Higgs interaction plays a greatly important role in radiatively generating neutrino masses [100, 101]. This mixing also has significant effects on the branching ratios of radiative charged-lepton decays and then the muon $g - 2$. Thus we recalculate all contributions from the physical S_1 and \tilde{R}_2 to the branching ratios of radiative charged-lepton decays and the muon $g - 2$ in this work, after the mixing between non-physical S_1 and \tilde{R}_2 is absorbed into the redefinitions of physical fields.
- Motivated by the current experimental data for neutrino oscillations, a set of specific structures for the leptoquark Yukawa coupling matrices are taken to make the neutrino mass matrix approximately possess the μ - τ reflection symmetry [123, 124]. Given the Yukawa coupling matrices in eq. (3.1), the neutrino mass matrix additionally has the texture with one zero element, i.e., $(M_\nu)_{ee} = 0$, which rules out the inverted neutrino mass ordering, and in the limit of μ - τ reflection symmetry leads to the lightest neutrino mass m_1 determined fully by values of θ_{12} and θ_{13} , besides $\theta_{23} = 45^\circ$, $\delta_{CP} = \pm 90^\circ$ and $\rho, \sigma = 0$ or 90° . We analytically discussed the corresponding neutrino masses and lepton flavor mixing induced by this novel neutrino mass matrix with both the exact and approximate μ - τ reflection symmetry in detail.

The rest of this paper is organized as follows. In section 2, we construct the LQ model with S_1 and \tilde{R}_2 , and calculate the radiative neutrino mass matrix at the one-loop level, the branching ratios of radiative decays of charged leptons and the muon anomalous magnetic

moment. With specific textures of the leptoquark Yukawa coupling matrices, the neutrino masses and lepton flavor mixing with an approximate μ - τ reflection symmetry are discussed in detail in section 3. The numerical calculations are carried out in section 4. Finally, we make a summary in section 5.

2 The LQ model

Enforcing baryon number conservation (e.g., assigning baryon number $B = -\frac{1}{3}$ to S_1 and \tilde{R}_2^* under a global $U(1)_B$ symmetry) to avoid potentially dangerous proton decay,² the Lagrangian associated with S_1 ($\bar{3}, 1, \frac{1}{3}$) and \tilde{R}_2 ($3, 2, \frac{1}{6}$) is given by

$$\begin{aligned} \mathcal{L}_{\text{LQ}} = & \lambda_{i\alpha}^L \overline{Q}_{iL}^c \epsilon \ell_{\alpha L} S_1 + \lambda_{i\alpha}^R \overline{u}_{iR}^c E_{\alpha R} S_1 + \lambda_{i\alpha} \overline{d}_{iR} \tilde{R}_2^T \epsilon \ell_{\alpha L} + \text{h.c.}, \\ & + (D_\mu S_1)^\dagger (D^\mu S_1) + (D_\mu \tilde{R}_2)^\dagger (D^\mu \tilde{R}_2) - V_{\text{LQ}}, \end{aligned} \quad (2.1)$$

where as usual, Q_L and ℓ_L are the left-handed quark and lepton doublets of $SU(2)_L$, and u_R , d_R and E_R are the right-handed up-type quark, down-type quark and charged lepton singlets of $SU(2)_L$, and the Latin and Greek subscripts running over $(1, 2, 3)$ and (e, μ, τ) are quark and lepton flavor indices, respectively. In eq. (2.1), ϵ is the $SU(2)_L$ Levi-Civita tensor, the superscript “c” stands for charge conjugation (i.e., $\psi^c = C\overline{\psi}^T$ with C being the charge-conjugation matrix), D_μ is the gauge covariant derivative. The most general renormalizable potential involving S_1 and \tilde{R}_2 is

$$\begin{aligned} V_{\text{LQ}} = & \mu_S^2 S_1^\dagger S_1 + \mu_R^2 \tilde{R}_2^\dagger \tilde{R}_2 + (\lambda_{\text{mix}} S_1^* \tilde{R}_2^\dagger H + \text{h.c.}) + \lambda_{HS} (H^\dagger H) (S_1^\dagger S_1) + \lambda_{HR}^{(1)} (H^\dagger H) (\tilde{R}_2^\dagger \tilde{R}_2) \\ & + \lambda_{HR}^{(3)} (H^\dagger \tau^I H) (\tilde{R}_2^\dagger \tau^I \tilde{R}_2) + \lambda_S (S_1^\dagger S_1)^2 + \lambda_R^{(1)} (\tilde{R}_2^\dagger \tilde{R}_2)^2 + \lambda_R^{(8)} (\tilde{R}_2^\dagger T^A \tilde{R}_2)^2 \\ & + \lambda_{SR}^{(1)} (S_1^\dagger S_1) (\tilde{R}_2^\dagger \tilde{R}_2) + \lambda_{SR}^{(8)} (S_1^\dagger T^A S_1) (\tilde{R}_2^\dagger T^A \tilde{R}_2), \end{aligned} \quad (2.2)$$

in which H is the SM Higgs doublet, and τ^I (for $I = 1, 2, 3$) and T^A (for $A = 1, \dots, 8$) are the Pauli and Gell-Mann matrices, respectively. As we will see later, the LQ-Higgs interaction $\lambda_{\text{mix}} S_1^* \tilde{R}_2^\dagger H$ provides a mixing between S_1 and $\tilde{R}_2^{-\frac{1}{3}*}$ after spontaneous gauge symmetry breaking (SSB), which is required to realize lepton number violation and hence radiatively generate neutrino masses. After the neutral Higgs field obtains its vacuum expectation value, i.e., $\langle H^0 \rangle = v/\sqrt{2}$, the mass matrix for S_1 and $\tilde{R}_2^{-\frac{1}{3}*}$ in the basis $(S_1, \tilde{R}_2^{-\frac{1}{3}*})$ is found to be

$$M_{\text{mix}}^2 = \begin{pmatrix} m_S^2 & \frac{v}{\sqrt{2}} \lambda_{\text{mix}} \\ \frac{v}{\sqrt{2}} \lambda_{\text{mix}}^* & m_R^2 \end{pmatrix} \quad (2.3)$$

²At the renormalizable level, the baryon-number-violating diquark couplings (i.e., $\overline{Q}_{iL}^c \epsilon Q_{jL} S_1^\dagger$ and $\overline{u}_{iR}^c d_{jR} S_1^\dagger$ with i, j being flavor indices) are forbidden by the global $U(1)_B$ symmetry. Note that the dimension five operators involving the leptoquark \tilde{R}_2 (i.e., $\epsilon_{abe} u_{iR}^a d_{jR}^b (H^\dagger \tilde{R}_2^e)/\Lambda$ and $\epsilon_{abe} u_{iR}^a e_{\alpha R} (\tilde{R}_2^b \tilde{R}_2^e)/\Lambda$ with i, j, α and a, b, e being flavor indices and color indices, respectively) also lead to proton decay, but such dimension five operators can be forbidden by embedding the SM in a larger gauge symmetry group with the right symmetry properties [125, 126].

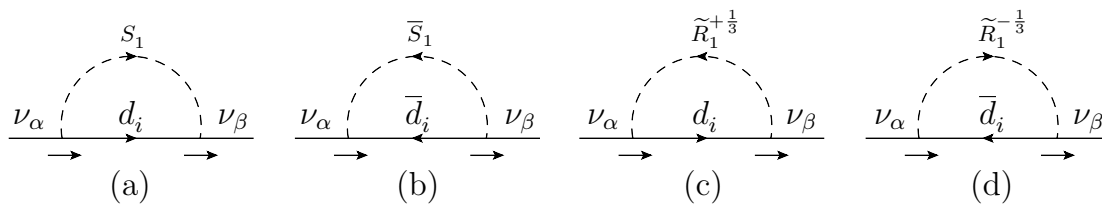


Figure 1. Self-energy diagrams for Majorana neutrinos. The arrows below vertices represent the orientations of fermion flow [127, 128].

with

$$m_S^2 = \mu_S^2 + \frac{v^2}{2} \lambda_{HS}, \quad m_R^2 = \mu_R^2 + \frac{v^2}{2} (\lambda_{HR}^{(1)} + \lambda_{HR}^{(3)}). \quad (2.4)$$

For simplicity, we take all parameters in eqs. (2.3) and (2.4) to be real, thus we can make the following transformations

$$S_1 \rightarrow \cos \theta S_1 + \sin \theta \tilde{R}_2^{-\frac{1}{3}*}, \quad \tilde{R}_2^{-\frac{1}{3}*} \rightarrow -\sin \theta S_1 + \cos \theta \tilde{R}_2^{-\frac{1}{3}*} \quad (2.5)$$

with $\tan 2\theta = \sqrt{2} \lambda_{\text{mix}} v / (m_R^2 - m_S^2)$ to work in the basis of the physical LQs which are named in the same way as the non-physical leptoquarks. To avoid misunderstandings, we will make it clear when referring to the physical fields. The masses of the physical LQs are given by

$$M_{1,2}^2 = \frac{1}{2} \left[m_S^2 + m_R^2 \pm \sqrt{(m_S^2 - m_R^2)^2 + 2\lambda_{\text{mix}}^2 v^2} \right], \quad M_3^2 = \mu_R^2 + \frac{v^2}{2} (\lambda_{HR}^{(1)} - \lambda_{HR}^{(3)}), \quad (2.6)$$

where M_1, M_2 and M_3 are the masses of the physical $S_1, \tilde{R}_2^{-\frac{1}{3}}$ and $\tilde{R}_2^{+\frac{2}{3}}$, respectively.

Given the transformations in eq. (2.5), the Yukawa couplings involving the physical LQs after SSB are

$$\begin{aligned} \mathcal{L}_Y = & \bar{\nu}_\alpha \left(\lambda_{i\alpha}^* \sin \theta P_R - \lambda_{i\alpha}^L \cos \theta P_L \right) d_i S_1 + \bar{l}_\alpha^c \left(\lambda_{i\alpha}^L P_L + \lambda_{i\alpha}^R P_R \right) \cos \theta u_i S_1 \\ & - \bar{\nu}_\alpha \left(\lambda_{i\alpha}^* \cos \theta P_R + \lambda_{i\alpha}^L \sin \theta P_L \right) d_i \tilde{R}_2^{-\frac{1}{3}*} + \bar{l}_\alpha^c \left(\lambda_{i\alpha}^L P_L + \lambda_{i\alpha}^R P_R \right) \sin \theta u_i \tilde{R}_2^{-\frac{1}{3}*} \\ & + \lambda_{i\alpha} \bar{d}_i P_L l_\alpha \tilde{R}_2^{+\frac{2}{3}} + \text{h.c.}, \end{aligned} \quad (2.7)$$

with $\lambda^L = V^T \lambda^L$, where we work in the down-type quark and charged-lepton mass eigenstate bases (i.e., d_i and l_α), and the up-type quark fields have been transformed into their mass eigenstates u_i by the Cabibbo-Kobayashi-Maskawa (CKM) matrix V .

2.1 Radiative neutrino masses

In the considered LQ model described by eqs. (2.1) and (2.2), neutrinos are exactly massless at the tree level, but they can acquire masses radiatively via the one loop diagrams in figure 1.³ The general structure of the Majorana neutrino self-energy is given by [130]

$$\Sigma_{\alpha\beta}(\not{p}) = \not{p} P_L \Sigma_{\alpha\beta}^{R*}(p^2) + \not{p} P_R \Sigma_{\alpha\beta}^R(p^2) + P_L \Sigma_{\alpha\beta}^{M*}(p^2) + P_R \Sigma_{\alpha\beta}^M(p^2), \quad (2.8)$$

³All Feynman diagrams in this work are produced by using JaxoDraw [129].

where only the last two terms will contribute to neutrino masses. From the Feynman diagrams in figure 1, one can obtain

$$\begin{aligned}
 -i\Sigma_{\alpha\beta}(\not{p}) = N_c \int \frac{d^4k}{(2\pi)^4} & \left[\left(\lambda_{i\beta}^* \sin\theta P_R - \lambda_{i\beta}^{L*} \cos\theta P_L \right) \frac{1}{\not{k} - \hat{m}_i} \left(\lambda_{i\alpha} \sin\theta P_L - \lambda_{i\alpha}^{L*} \cos\theta P_R \right) \right. \\
 & \times \frac{1}{(k-p)^2 - M_1^2} + \left(\lambda_{i\beta} \sin\theta P_L - \lambda_{i\beta}^{L*} \cos\theta P_R \right) \frac{1}{\not{k} - \hat{m}_i} \left(\lambda_{i\alpha}^* \sin\theta P_R - \lambda_{i\alpha}^L \cos\theta P_L \right) \\
 & \times \frac{1}{(k-p)^2 - M_1^2} + \left(\lambda_{i\beta}^* \cos\theta P_R - \lambda_{i\beta}^L \sin\theta P_L \right) \frac{1}{\not{k} - \hat{m}_i} \left(\lambda_{i\alpha} \cos\theta P_L - \lambda_{i\alpha}^{L*} \sin\theta P_R \right) \\
 & \times \frac{1}{(k-p)^2 - M_2^2} + \left(\lambda_{i\beta} \cos\theta P_L - \lambda_{i\beta}^{L*} \sin\theta P_R \right) \frac{1}{\not{k} - \hat{m}_i} \left(\lambda_{i\alpha}^* \cos\theta P_R - \lambda_{i\alpha}^L \sin\theta P_L \right) \\
 & \left. \times \frac{1}{(k-p)^2 - M_2^2} \right], \tag{2.9}
 \end{aligned}$$

then radiative neutrino mass matrix M_ν can be extracted from eq. (2.9), that is⁴

$$\begin{aligned}
 (M_\nu)_{\alpha\beta} = \Sigma_{\alpha\beta}^M(0) & = \frac{i}{2} N_c \left(\lambda_{i\alpha}^* \lambda_{i\beta}^{L*} + \lambda_{i\alpha}^{L*} \lambda_{i\beta}^* \right) \sin 2\theta \int \frac{d^4k}{(2\pi)^4} \frac{\hat{m}_i}{k^2 - \hat{m}_i^2} \\
 & \times \left[\frac{1}{(k-p)^2 - M_2^2} - \frac{1}{(k-p)^2 - M_1^2} \right] \\
 & = -\frac{N_c}{2(4\pi)^2} \left(\lambda_{i\alpha}^* \lambda_{i\beta}^{L*} + \lambda_{i\alpha}^{L*} \lambda_{i\beta}^* \right) \hat{m}_i \sin 2\theta \left[\frac{M_2^2 \ln \frac{\hat{m}_i^2}{M_2^2}}{M_2^2 - \hat{m}_i^2} - \frac{M_1^2 \ln \frac{\hat{m}_i^2}{M_1^2}}{M_1^2 - \hat{m}_i^2} \right] \\
 & \simeq \frac{3 \sin 2\theta}{32\pi^2} \ln \frac{M_2^2}{M_1^2} \left[\left(\lambda^\dagger \right)_{\alpha i} \hat{m}_i \left(\lambda^{L*} \right)_{i\beta} + \left(\lambda^{L\dagger} \right)_{\alpha i} \hat{m}_i \left(\lambda^* \right)_{i\beta} \right], \tag{2.10}
 \end{aligned}$$

where $N_c = 3$ is the color number and $\hat{m} = (m_d, m_s, m_b)$ denotes the masses of down-type quarks. Note that the mixing between S_1 and $\tilde{R}_2^{-\frac{1}{3}}$ is important for radiatively generating neutrino masses, and if the masses of the physical S_1 and $\tilde{R}_2^{-\frac{1}{3}}$ are exactly degenerate, three neutrinos will remain massless at the one-loop level.

2.2 LFV decays and muon $g - 2$

Induced by the Yukawa couplings of the physical LQs in eq. (2.7), the LFV decays of charged leptons, i.e., $l_\alpha^- \rightarrow l_\beta^- + \gamma$ with $(\alpha, \beta) = (\tau, \mu), (\tau, e)$ and (μ, e) occur at the one-loop level, as shown in figure 2. The muon anomalous magnetic moment can be easily extracted from the total amplitude of the muon radiative decay. Note that all physical LQs will contribute to the total amplitude and lepton anomalous magnetic moment, but the dominant contributions only come from the physical S_1 and $\tilde{R}_2^{-\frac{1}{3}}$ due to the chiral enhancement.

⁴All one-loop integrals in this work are calculated with the help of Package-X [131, 132].

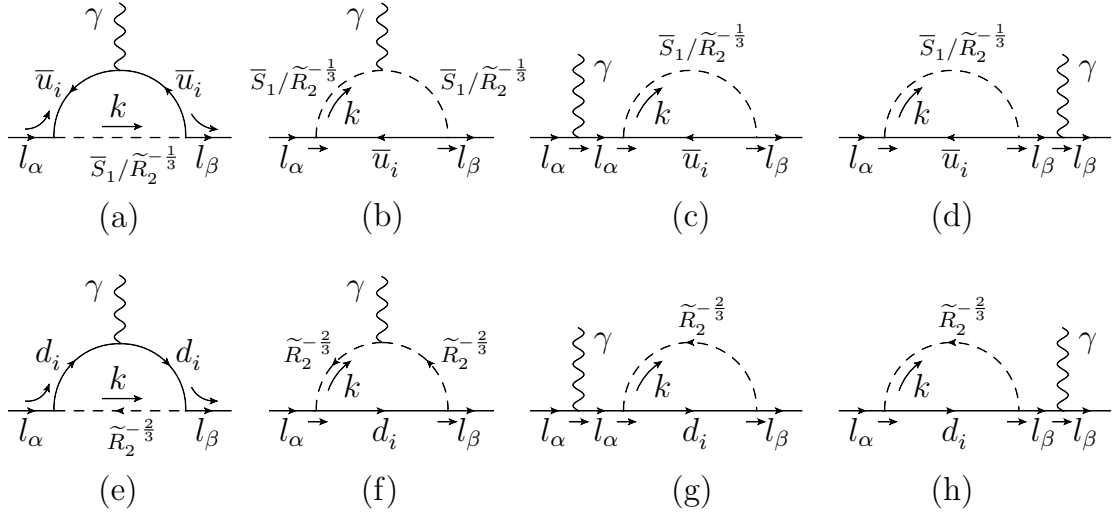


Figure 2. Diagrams for the radiative decays of charged leptons, i.e., $l_\alpha^- \rightarrow l_\beta^- + \gamma$ with $(\alpha, \beta) = (\tau, \mu)$, (τ, e) and (μ, e) . The arrows around vertices represent the orientations of fermion flow [127, 128].

The amplitudes for the radiative decays of charged leptons mediated by the physical S_1 as shown by diagrams (a)–(d) in figure 2 are given by

$$\begin{aligned}
 i\mathcal{M}_{(a)}^{S_1} &= \frac{2}{3} N_c e \cos^2 \theta \epsilon_\mu^*(q) \bar{u}(p-q) \left\{ \int \frac{d^4 k}{(2\pi)^4} \left(\lambda_{i\beta}^{L*} P_R + \lambda_{i\beta}^{R*} P_L \right) \frac{(\not{p} - \not{k} - \not{q} + \tilde{m}_i)}{(k+q-p)^2 - \tilde{m}_i^2} \right. \\
 &\quad \left. \times \gamma^\mu \frac{(\not{p} - \not{k} + \tilde{m}_i)}{(k-p)^2 - \tilde{m}_i^2} \left(\lambda_{i\alpha}^L P_L + \lambda_{i\alpha}^R P_R \right) \frac{1}{k^2 - M_1^2} \right\} u(p), \\
 i\mathcal{M}_{(b)}^{S_1} &= \frac{1}{3} N_c e \cos^2 \theta \epsilon_\mu^*(q) \bar{u}(p-q) \left\{ \int \frac{d^4 k}{(2\pi)^4} \left(\lambda_{i\beta}^{L*} P_R + \lambda_{i\beta}^{R*} P_L \right) \frac{(\not{p} - \not{k} + \tilde{m}_i)}{(k-p)^2 - \tilde{m}_i^2} \right. \\
 &\quad \left. \times \left(\lambda_{i\alpha}^L P_L + \lambda_{i\alpha}^R P_R \right) \frac{1}{(k-q)^2 - M_1^2} (2k-q)^\mu \frac{1}{k^2 - M_1^2} \right\} u(p), \\
 i\mathcal{M}_{(c)}^{S_1} &= -\frac{N_c e \cos^2 \theta}{m_\alpha^2 - m_\beta^2} \epsilon_\mu^*(q) \bar{u}(p-q) \left\{ \int \frac{d^4 k}{(2\pi)^4} \left(\lambda_{i\beta}^{L*} P_R + \lambda_{i\beta}^{R*} P_L \right) \frac{(\not{p} - \not{k} - \not{q} + \tilde{m}_i)}{(k+q-p)^2 - \tilde{m}_i^2} \right. \\
 &\quad \left. \times \left(\lambda_{i\alpha}^L P_L + \lambda_{i\alpha}^R P_R \right) (\not{p} - \not{q} + m_\alpha) \gamma^\mu \frac{1}{k^2 - M_1^2} \right\} u(p), \\
 i\mathcal{M}_{(d)}^{S_1} &= \frac{N_c e \cos^2 \theta}{m_\alpha^2 - m_\beta^2} \epsilon_\mu^*(q) \bar{u}(p-q) \left\{ \int \frac{d^4 k}{(2\pi)^4} \gamma^\mu (\not{p} + m_\beta) \left(\lambda_{i\beta}^{L*} P_R + \lambda_{i\beta}^{R*} P_L \right) \right. \\
 &\quad \left. \times \frac{(\not{p} - \not{k} + \tilde{m}_i)}{(k-p)^2 - \tilde{m}_i^2} \left(\lambda_{i\alpha}^L P_L + \lambda_{i\alpha}^R P_R \right) \frac{1}{k^2 - M_1^2} \right\} u(p), \tag{2.11}
 \end{aligned}$$

in which $\tilde{m} = (m_u, m_c, m_t)$ and $m = (m_e, m_\mu, m_\tau)$ denote the masses of up-type quarks and charged leptons, respectively. Then the total amplitude mediated by the physical S_1 is found to be

$$i\mathcal{M}^{S_1} = -\frac{ie}{(4\pi)^2} \epsilon_\mu^*(q) \bar{u}(p-q) \left(\mathcal{A}_L^{S_1} P_L + \mathcal{A}_R^{S_1} P_R \right) i\sigma^{\mu\nu} q_\nu u(p), \quad (2.12)$$

where

$$\begin{aligned} \mathcal{A}_L^{S_1} &= \frac{N_c \cos^2 \theta}{12M_1^2} \left[2\tilde{m}_i \lambda_{i\alpha}^L \lambda_{i\beta}^{R*} \mathcal{F} \left(\frac{\tilde{m}_i^2}{M_1^2} \right) - (m_\beta \lambda_{i\alpha}^L \lambda_{i\beta}^{L*} + m_\alpha \lambda_{i\alpha}^R \lambda_{i\beta}^{R*}) \mathcal{G} \left(\frac{\tilde{m}_i^2}{M_1^2} \right) \right], \\ \mathcal{A}_R^{S_1} &= \frac{N_c \cos^2 \theta}{12M_1^2} \left[2\tilde{m}_i \lambda_{i\alpha}^R \lambda_{i\beta}^{L*} \mathcal{F} \left(\frac{\tilde{m}_i^2}{M_1^2} \right) - (m_\alpha \lambda_{i\alpha}^L \lambda_{i\beta}^{L*} + m_\beta \lambda_{i\alpha}^R \lambda_{i\beta}^{R*}) \mathcal{G} \left(\frac{\tilde{m}_i^2}{M_1^2} \right) \right], \end{aligned} \quad (2.13)$$

with

$$\mathcal{F}(x) = \frac{7 - 8x + x^2 + 2(2+x) \ln x}{(1-x)^3}, \quad \mathcal{G}(x) = \frac{1 + 4x - 5x^2 + 2x(2+x) \ln x}{(1-x)^4}. \quad (2.14)$$

If $x \ll 1$ is satisfied, $\mathcal{F}(x)$ and $\mathcal{G}(x)$ may be expanded into $\mathcal{F}(x) \simeq 7 + 4 \ln x$ and $\mathcal{G}(x) \simeq 1$ at the leading order, respectively.

The contributions from the physical $\tilde{R}_2^{-\frac{1}{3}}$ can be easily obtained by making the replacements $\cos^2 \theta \rightarrow \sin^2 \theta$ and $M_1 \rightarrow M_2$ in eqs. (2.12)–(2.13), that is

$$i\mathcal{M}^{\tilde{R}_2^{-\frac{1}{3}}} = i\mathcal{M}^{S_1}(M_1 \rightarrow M_2) \tan^2 \theta, \quad (2.15)$$

with $\mathcal{A}_L^{\tilde{R}_2^{-\frac{1}{3}}} = \mathcal{A}_L^{S_1}(M_1 \rightarrow M_2) \tan^2 \theta$ and $\mathcal{A}_R^{\tilde{R}_2^{-\frac{1}{3}}} = \mathcal{A}_R^{S_1}(M_1 \rightarrow M_2) \tan^2 \theta$.

The contributions from $\tilde{R}_2^{+\frac{2}{3}}$ can be achieved by calculating diagrams (e)–(h) in figure 2, namely

$$\begin{aligned} i\mathcal{M}_{(e)}^{\tilde{R}_2^{+\frac{2}{3}}} &= \frac{1}{3} N_c e \lambda_{i\alpha} \lambda_{i\beta}^* \epsilon_\mu^*(q) \bar{u}(p-q) \left\{ \int \frac{d^4 k}{(2\pi)^4} P_R \frac{\not{p} - \not{k} - \not{q} + \hat{m}_i}{(k+q-p)^2 - \hat{m}_i^2} \gamma^\mu \frac{\not{p} - \not{k} + \hat{m}_i}{(k-p)^2 - \hat{m}_i^2} P_L \frac{1}{k^2 - M_3^2} \right\} u(p), \\ i\mathcal{M}_{(f)}^{\tilde{R}_2^{+\frac{2}{3}}} &= \frac{2}{3} N_c e \lambda_{i\alpha} \lambda_{i\beta}^* \epsilon_\mu^*(q) \bar{u}(p-q) \left\{ \int \frac{d^4 k}{(2\pi)^4} P_R \frac{\not{p} - \not{k} + \hat{m}_i}{(k-p)^2 - \hat{m}_i^2} P_L \frac{1}{(k-q)^2 - M_3^2} \frac{1}{k^2 - M_3^2} \right\} u(p), \\ i\mathcal{M}_{(g)}^{\tilde{R}_2^{+\frac{2}{3}}} &= -\frac{N_c e \lambda_{i\alpha} \lambda_{i\beta}^*}{m_\alpha^2 - m_\beta^2} \epsilon_\mu^*(q) \bar{u}(p-q) \left\{ \int \frac{d^4 k}{(2\pi)^4} P_R \frac{\not{p} - \not{k} - \not{q} + \hat{m}_i}{(k+q-p)^2 - \hat{m}_i^2} P_L (\not{p} - \not{q} + m_\alpha) \gamma^\mu \frac{1}{k^2 - M_3^2} \right\} u(p), \\ i\mathcal{M}_{(h)}^{\tilde{R}_2^{+\frac{2}{3}}} &= \frac{N_c e \lambda_{i\alpha} \lambda_{i\beta}^*}{m_\alpha^2 - m_\beta^2} \epsilon_\mu^*(q) \bar{u}(p-q) \left\{ \int \frac{d^4 k}{(2\pi)^4} \gamma^\mu (\not{p} + m_\beta) P_R \frac{\not{p} - \not{k} + \hat{m}_i}{(k-p)^2 - \hat{m}_i^2} P_L \frac{1}{k^2 - M_3^2} \right\} u(p), \end{aligned} \quad (2.16)$$

then the total amplitude mediated by $\tilde{R}_2^{+\frac{2}{3}}$ is given by

$$i\mathcal{M}^{\tilde{R}_2^{+\frac{2}{3}}} = -\frac{ie}{(4\pi)^2} \epsilon_\mu^*(q) \bar{u}(p-q) \left(\mathcal{A}_L^{\tilde{R}_2^{+\frac{2}{3}}} P_L + \mathcal{A}_R^{\tilde{R}_2^{+\frac{2}{3}}} P_R \right) i\sigma^{\mu\nu} q_\nu u(p), \quad (2.17)$$

where

$$\mathcal{A}_L^{\tilde{R}_2^{+\frac{2}{3}}} = -\frac{N_c m_\beta}{12M_3^2} \lambda_{i\alpha} \lambda_{i\beta}^* \mathcal{I} \left(\frac{\hat{m}_i^2}{M_3^2} \right), \quad \mathcal{A}_R^{\tilde{R}_2^{+\frac{2}{3}}} = -\frac{N_c m_\alpha}{12M_3^2} \lambda_{i\alpha} \lambda_{i\beta}^* \mathcal{I} \left(\frac{\hat{m}_i^2}{M_3^2} \right), \quad (2.18)$$

with

$$\mathcal{I}(x) = \frac{x [5 - 4x - x^2 + (2 + 4x) \ln x]}{(1 - x)^4}, \quad (2.19)$$

and $\mathcal{I}(x) \simeq x(5 + 2 \ln x)$ for $x \ll 1$.

Therefore, the total amplitude for the radiative decays of charged leptons is found to be

$$i\mathcal{M} = -\frac{ie}{(4\pi)^2} \epsilon_\mu^*(q) \bar{u}(p - q) (\mathcal{A}_L P_L + \mathcal{A}_R P_R) i\sigma^{\mu\nu} q_\nu u(p), \quad (2.20)$$

with

$$\mathcal{A}_L = \mathcal{A}_L^{S_1} + \mathcal{A}_L^{\tilde{R}_2^{-\frac{1}{3}}} + \mathcal{A}_L^{\tilde{R}_2^{+\frac{2}{3}}}, \quad \mathcal{A}_R = \mathcal{A}_R^{S_1} + \mathcal{A}_R^{\tilde{R}_2^{-\frac{1}{3}}} + \mathcal{A}_R^{\tilde{R}_2^{+\frac{2}{3}}}. \quad (2.21)$$

Note that all divergences in the amplitude have been cancelled out, which must be satisfied since there is no corresponding counterterm for charged-lepton radiative decays. With the help of eq. (2.20), one can obtain the branching ratio of $l_\alpha^- \rightarrow l_\beta^- + \gamma$,

$$\mathcal{B}(l_\alpha^- \rightarrow l_\beta^- + \gamma) = \frac{\alpha_{\text{em}} (m_\alpha^2 - m_\beta^2)^3}{4(4\pi)^4 m_\alpha^3 \Gamma_\alpha} (|\mathcal{A}_L|^2 + |\mathcal{A}_R|^2) \quad (2.22)$$

with $\alpha_{\text{em}} = e^2/(4\pi)$ and Γ_α being the fine-structure constant and the total decay width of the charged lepton l_α^- , respectively. Then the contributions from the LQs to the muon $g - 2$ can be easily extracted from eq. (2.20), i.e.,

$$\Delta a_\mu = \Delta a_\mu^{S_1} + \Delta a_\mu^{\tilde{R}_2^{-\frac{1}{3}}} + \Delta a_\mu^{\tilde{R}_2^{+\frac{2}{3}}}, \quad (2.23)$$

with

$$\begin{aligned} \Delta a_\mu^{S_1} &= -\frac{N_c m_\mu \cos^2 \theta}{6(4\pi)^2 M_1^2} \left[2\tilde{m}_i \text{Re}(\lambda_{i\mu}^L \lambda_{i\mu}^{R*}) \mathcal{F}\left(\frac{\tilde{m}_i^2}{M_1^2}\right) - m_\mu \left(|\lambda_{i\mu}^L|^2 + |\lambda_{i\mu}^R|^2 \right) \mathcal{G}\left(\frac{\tilde{m}_i^2}{M_1^2}\right) \right], \\ \Delta a_\mu^{\tilde{R}_2^{-\frac{1}{3}}} &= -\frac{N_c m_\mu \sin^2 \theta}{6(4\pi)^2 M_2^2} \left[2\tilde{m}_i \text{Re}(\lambda_{i\mu}^L \lambda_{i\mu}^{R*}) \mathcal{F}\left(\frac{\tilde{m}_i^2}{M_2^2}\right) - m_\mu \left(|\lambda_{i\mu}^L|^2 + |\lambda_{i\mu}^R|^2 \right) \mathcal{G}\left(\frac{\tilde{m}_i^2}{M_2^2}\right) \right], \\ \Delta a_\mu^{\tilde{R}_2^{+\frac{2}{3}}} &= \frac{N_c m_\mu^2 |\lambda_{i\mu}|^2}{6(4\pi)^2 M_3^2} \mathcal{I}\left(\frac{\hat{m}_i^2}{M_3^2}\right). \end{aligned} \quad (2.24)$$

Considering $\tilde{m}_i, \hat{m}_j \ll M_k$ for $i, j, k = 1, 2, 3$ and $m_\beta \ll m_\alpha$ for $(\alpha, \beta) = (\tau, \mu), (\tau, e), (\mu, e)$, the branching ratio in eq. (2.22) and the muon $g - 2$ in eq. (2.23) are approximately given by

$$\begin{aligned} \mathcal{B}(l_\alpha^- \rightarrow l_\beta^- + \gamma) &\simeq \frac{\alpha_{\text{em}} m_\alpha^3}{16(4\pi)^4 \Gamma_\alpha} \left\{ \left| \tilde{m}_i \lambda_{i\alpha}^L \lambda_{i\beta}^{R*} \left[\frac{\cos^2 \theta}{M_1^2} \left(7 + 4 \ln \frac{\tilde{m}_i^2}{M_1^2} \right) + \frac{\sin^2 \theta}{M_2^2} \left(7 + 4 \ln \frac{\tilde{m}_i^2}{M_2^2} \right) \right] \right|^2 \right. \\ &\quad \left. + \left| \tilde{m}_i \lambda_{i\alpha}^R \lambda_{i\beta}^{L*} \left[\frac{\cos^2 \theta}{M_1^2} \left(7 + 4 \ln \frac{\tilde{m}_i^2}{M_1^2} \right) + \frac{\sin^2 \theta}{M_2^2} \left(7 + 4 \ln \frac{\tilde{m}_i^2}{M_2^2} \right) \right] \right|^2 \right\}, \end{aligned} \quad (2.25)$$

and

$$\Delta a_\mu \simeq \frac{4m_\mu \tilde{m}_i}{(4\pi)^2} \text{Re} \left(\lambda_{i\mu}^L \lambda_{i\mu}^{R*} \right) \left[\frac{\cos^2 \theta}{M_1^2} \left(\ln \frac{M_1^2}{\tilde{m}_i^2} - \frac{7}{4} \right) + \frac{\sin^2 \theta}{M_2^2} \left(\ln \frac{M_2^2}{\tilde{m}_i^2} - \frac{7}{4} \right) \right], \quad (2.26)$$

respectively. It is obvious that both the branching ratio of $l_\alpha^- \rightarrow l_\beta^- + \gamma$ and the muon $g-2$ are enhanced by the masses of up-type quarks, especially that of top quark, and if there is no mixing between S_1 and $\tilde{R}_2^{-\frac{1}{3}}$ (i.e., $\theta = 0$) or the masses of the physical S_1 and $\tilde{R}_2^{-\frac{1}{3}}$ are exactly degenerate (i.e., $M_1 = M_2$), the results in eqs. (2.25) and (2.26) will recover the results caused by S_1 all alone [102].

3 Neutrino masses and lepton flavor mixing

As can be seen in section 2, the Yukawa coupling matrices λ and λ^L contribute to the neutrino mass matrix, and both the radiative decays of charged leptons and the muon $g-2$ involve the Yukawa coupling matrices λ^L and λ^R . In order to explain the neutrino oscillation observables and the anomaly of muon $g-2$, meanwhile satisfy the constraints from the radiative decays of charged leptons, we assume that the Yukawa coupling matrices have the following textures:

$$\lambda = \begin{pmatrix} 0 & 0 & 0 \\ b(1+\varepsilon) & 0 & c(1+\varepsilon) \\ \frac{m_s b^*}{m_b} & \frac{m_s c^*}{m_b} & d \end{pmatrix}, \quad \lambda^L = a \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad \lambda^R = a \begin{pmatrix} 0 & 0 & 0 \\ 0 & x & 0 \\ 0 & 1 & 0 \end{pmatrix}, \quad (3.1)$$

with ε being a small and real quantity, as well as a and d being real. The textures of λ and λ^L are motivated by the fact that the lepton flavor mixing approximately has the μ - τ symmetry [1, 124]. The (3,2) elements of λ^L and λ^R are responsible for explaining the muon $g-2$, while the (2,2) element of λ^R is used to suppress the constraint from the LFV decay of tauon (i.e., $\tau \rightarrow \mu + \gamma$), which has been largely enhanced.

By means of eqs. (2.10) and (3.1), one can obtain the neutrino mass matrix, that is

$$M_\nu = \frac{3 \sin 2\theta}{32\pi^2} \ln \left(\frac{M_2^2}{M_1^2} \right) a m_b \begin{pmatrix} 0 & \frac{m_s b}{m_b} & \frac{m_s b^* (1+\varepsilon)}{m_b} \\ \frac{m_s b}{m_b} & 2 \frac{m_s c}{m_b} & d \\ \frac{m_s b^* (1+\varepsilon)}{m_b} & d & 2 \frac{m_s c^* (1+\varepsilon)}{m_b} \end{pmatrix}, \quad (3.2)$$

whose (1,1) element vanishes (i.e., $(M_\nu)_{ee} = 0$) thanks to the vanishing elements in the first line and the first column of λ^L given in eq. (3.1). It is worth emphasizing that this novel feature is only in favor of the normal neutrino mass ordering. In the basis where the charged-lepton mass matrix is diagonal, M_ν can be diagonalized by the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix U [133–135], namely, $U^\dagger M_\nu U^* = D_\nu$ with $D_\nu \equiv \text{Diag}\{m_1, m_2, m_3\}$. In the standard parametrization [2], the PMNS matrix U

can be decomposed as

$$U = P_l \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{\text{CP}}} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta_{\text{CP}}} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta_{\text{CP}}} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta_{\text{CP}}} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta_{\text{CP}}} & c_{13}c_{23} \end{pmatrix} P_\nu, \quad (3.3)$$

where $c_{ij} = \cos \theta_{ij}$ and $s_{ij} = \sin \theta_{ij}$ (for $ij = 12, 13, 23$), $P_l = \text{Diag}\{e^{i\phi_e}, e^{i\phi_\mu}, e^{i\phi_\tau}\}$ contains three unphysical phases, and $P_\nu = \text{Diag}\{e^{i\rho}, e^{i\sigma}, 1\}$ is the Majorana phase matrix.

If $\varepsilon = 0$, the neutrino mass matrix M_ν in eq. (3.2) will preserve the μ - τ reflection symmetry [123, 124]. For simplicity, we rewrite M_ν into

$$M_\nu^{\mu-\tau} = \begin{pmatrix} 0 & B & B^* \\ B & C & D \\ B^* & D & C^* \end{pmatrix}, \quad (3.4)$$

in the limit of the μ - τ reflection symmetry, where $B = 3 \sin 2\theta / (32\pi^2) \ln(M_2^2/M_1^2) m_s ab$, $C = 6 \sin 2\theta / (32\pi^2) \ln(M_2^2/M_1^2) m_s ac$ and $D = 3 \sin 2\theta / (32\pi^2) \ln(M_2^2/M_1^2) m_b ad$. It is well-known that the μ - τ reflection structure leads us to [136, 137]

$$\theta_{23} = \frac{\pi}{4}, \quad \delta_{\text{CP}} = \pm \frac{\pi}{2}, \quad \rho, \sigma = 0 \text{ or } \frac{\pi}{2}, \quad \phi_e = 0 \text{ or } \frac{\pi}{2}, \quad \phi_\mu + \phi_\tau = 2\phi_e \pm \pi. \quad (3.5)$$

In addition, we can obtain simple relationships between the other two mixing angles together with three neutrino masses and the elements of $M_\nu^{\mu-\tau}$, namely [137–140]

$$\left\{ \begin{array}{l} \tan \theta_{13} = ie^{i\delta} \frac{\text{Im}(C')}{\sqrt{2}\text{Re}(B')} \\ \tan 2\theta_{12} = \frac{\sqrt{2}\text{Re}(B')\text{Im}(C')}{c_{13}[\text{Re}(C')\text{Im}(C') - \text{Re}(B')\text{Im}(B')]} \\ m_1^\rho = -D' + \frac{\text{Re}(B')\text{Im}(B')}{\text{Im}(C')} - \frac{\sqrt{2}\text{Re}(B)}{c_{13}\sin 2\theta_{12}} \\ m_2^\sigma = -D' + \frac{\text{Re}(B')\text{Im}(B')}{\text{Im}(C')} + \frac{\sqrt{2}\text{Re}(B)}{c_{13}\sin 2\theta_{12}} \\ m_3 = \frac{2\text{Re}(B')\text{Im}(B')}{\text{Im}(C')} \end{array} \right\} \iff \left\{ \begin{array}{l} \text{Re}(B') = \frac{(m_2^\sigma - m_1^\rho)c_{13}\sin 2\theta_{12}}{2\sqrt{2}} \\ \text{Im}(B') = ie^{i\delta} \frac{m_3 \tan \theta_{13}}{\sqrt{2}} \\ \text{Re}(C') = \frac{(m_2^\sigma - m_1^\rho)\cos 2\theta_{12} + m_3}{2} \\ \text{Im}(C') = ie^{i\delta} \frac{(m_2^\sigma - m_1^\rho)s_{13}\sin 2\theta_{12}}{2} \\ D' = \frac{m_3 - m_2^\sigma - m_1^\rho}{2} \end{array} \right., \quad (3.6)$$

as well as

$$m_1^\rho \cos^2 \theta_{12} + m_2^\sigma \sin^2 \theta_{12} - m_3 \tan^2 \theta_{13} = 0, \quad (3.7)$$

$$\text{Re}(B')\text{Im}(C')[\text{Re}(C') + D'] - \text{Im}(B')\left[2(\text{Re}(B'))^2 - (\text{Im}(C'))^2\right] = 0,$$

where $m_1^\rho = m_1 \exp(2i\rho)$, $m_2^\sigma = m_2 \exp(2i\sigma)$, $B' = B \exp[-i(\phi_e + \phi_\mu)]$, $C' = C \exp(-2i\phi_\mu)$ and $D' = D \exp[-i(\phi_\mu + \phi_\tau)]$. It is interesting that there exists a relation between the mixing parameters and neutrino masses or between the elements of $M_\nu^{\mu-\tau}$ and unphysical phases, resulting from the absence of (0,0) element of $M_\nu^{\mu-\tau}$, as shown in eq. (3.7). Thus the mixing angles and neutrino masses are not independent. If the experimental values of Δm_{21}^2 , Δm_{31}^2 or Δm_{32}^2 , θ_{12} and θ_{13} are taken into account, one will find that only the normal neutrino mass ordering (i.e., $m_1 < m_2 < m_3$) can satisfy the first equation in eq. (3.7) with

two possible values of m_1 corresponding to $(\rho, \sigma) = (\pi/2, 0)$ and $(0, \pi/2)$, respectively, and the inverted neutrino mass ordering (i.e., $m_3 < m_1 < m_2$) is ruled out. By the way, the second equation in eq. (3.7) can be used to determinate the unphysical phase ϕ_μ or ϕ_τ , only one of which is independent.

Now we consider ε as a small quantity to break the μ - τ reflection symmetry, then the neutrino masses and mixing parameters will gain corresponding corrections, namely, $\Delta m_i = m'_i - m_i$ (for $i = 1, 2, 3$), $\Delta \theta_{ij} = \theta'_{ij} - \theta_{ij}$ (for $ij = 12, 13, 23$), $\Delta \delta = \delta'_{\text{CP}} - \delta_{\text{CP}}$, $\Delta \rho = \rho' - \rho$ and $\Delta \sigma = \sigma' - \sigma$ where the parameters with superscripts “ \prime ” are the neutrino masses and mixing parameters for $\varepsilon \neq 0$. The PMNS matrix U' to diagonalize the mass matrix with $\varepsilon \neq 0$ in eq. (3.2) consists of those new mixing parameters and three new unphysical phases ϕ'_α (for $\alpha = e, \mu, \tau$). Following the method described in ref. [141], one can obtain

$$\begin{aligned}
 \Delta m_i &\simeq \text{Re} \left(U^\dagger \Delta M_\nu U^* \right)_{ii}, \\
 \text{Im} \left(\Delta U^\dagger U \right)_{ii} &\simeq - \frac{\text{Im} \left(U^\dagger \Delta M_\nu U^* \right)_{ii}}{2m_i}, \\
 \text{Re} \left(\Delta U^\dagger U \right)_{jk} &\simeq \frac{\text{Re} \left(U^\dagger \Delta M_\nu U^* \right)_{jk}}{m_j - m_k}, \\
 \text{Im} \left(\Delta U^\dagger U \right)_{jk} &\simeq - \frac{\text{Im} \left(U^\dagger \Delta M_\nu U^* \right)_{jk}}{m_j + m_k},
 \end{aligned} \tag{3.8}$$

for $i = 1, 2, 3$ and $jk = 12, 13, 23$, where $\Delta M_\mu \equiv M_\nu - M_\nu^{\mu-\tau}$ and $\Delta U \equiv U' - U$. Solving above linear equations leads us to

$$\begin{aligned}
 \Delta m_1 &\simeq \frac{\eta}{2} \varepsilon \left\{ \text{Re} (C') s_{12}^2 - \sqrt{2} \text{Re} (B') \sin 2\theta_{12} c_{13} + s_{13} \left[\eta_\delta \text{Im} (C') \sin 2\theta_{12} - \text{Re} (C') c_{12}^2 s_{13} \right. \right. \\
 &\quad \left. \left. - 2\sqrt{2} \eta_\delta \text{Im} (B') c_{12}^2 c_{13} \right] \right\}, \\
 \Delta m_2 &\simeq - \frac{\eta}{2} \varepsilon \left\{ \text{Re} (C') c_{12}^2 + \sqrt{2} \text{Re} (B') \sin 2\theta_{12} c_{13} - s_{13} \left[\eta_\delta \text{Im} (C') \sin 2\theta_{12} + \text{Re} (C') s_{12}^2 s_{13} \right. \right. \\
 &\quad \left. \left. + 2\sqrt{2} \eta_\delta \text{Im} (B') s_{12}^2 c_{13} \right] \right\}, \\
 \Delta m_3 &\simeq \frac{1}{2} \varepsilon c_{13} \left[\text{Re} (C') c_{13} - 2\sqrt{2} \eta_\delta \text{Im} (B') s_{13} \right],
 \end{aligned} \tag{3.9}$$

and

$$\begin{aligned}
 \Delta \theta_{12} &\simeq \frac{\varepsilon}{2} \left\{ \frac{1}{2} \sin 2\theta_{12} s_{13} \left(\text{Re} (C') s_{13} + \sqrt{2} \eta_\delta \text{Im} (B') \cos 2\theta_{13} \sec \theta_{13} \right) \left(\frac{1}{m_3 - \eta m_2} - \frac{1}{m_3 + \eta m_1} \right) \right. \\
 &\quad \left. + s_{13} \left(\eta_\delta \text{Im} (C') + \sqrt{2} \text{Re} (B') \tan \theta_{13} \right) \left(\frac{s_{12}^2}{m_3 + \eta m_1} + \frac{c_{12}^2}{m_3 - \eta m_2} \right) \right. \\
 &\quad \left. + \left[\frac{1}{2} \sin 2\theta_{12} \left(\text{Re} (C') (1 + s_{13}^2) + \sqrt{2} \eta_\delta \text{Im} (B') \sin 2\theta_{13} \right) + \cos 2\theta_{12} \right. \right. \\
 &\quad \left. \left. \times \left(\eta_\delta \text{Im} (C') s_{13} - \sqrt{2} \text{Re} (B') c_{13} \right) \right] \frac{\eta}{m_1 + m_2} \right\},
 \end{aligned}$$

$$\begin{aligned}
 \Delta\theta_{13} &\simeq \frac{\varepsilon}{4} \left[\sin 2\theta_{12} \left(\sqrt{2}\text{Re}(B') s_{13} + \eta_\delta \text{Im}(C') c_{13} \right) \left(\frac{1}{m_3 + \eta m_1} - \frac{1}{m_3 - \eta m_2} \right) \right. \\
 &\quad \left. - \left(\text{Re}(C') \sin 2\theta_{13} + 2\sqrt{2}\eta_\delta \text{Im}(B') \cos 2\theta_{13} \right) \left(\frac{c_{12}^2}{m_3 + \eta m_1} + \frac{s_{12}^2}{m_3 - \eta m_2} \right) \right], \\
 \Delta\theta_{23} &\simeq \frac{\varepsilon}{2} \left[\frac{1}{2} \sin 2\theta_{12} \left(\sqrt{2}\text{Re}(B') \cos 2\theta_{13} \sec \theta_{13} - \eta_\delta \text{Im}(C') s_{13} \right) \left(\frac{1}{m_3 - \eta m_1} - \frac{1}{m_3 + \eta m_2} \right) \right. \\
 &\quad \left. + \left(\sqrt{2}\eta_\delta \text{Im}(B') \tan \theta_{13} - \text{Re}(C') \right) \left(\frac{c_{12}^2}{m_3 + \eta m_2} + \frac{s_{12}^2}{m_3 - \eta m_1} \right) \right], \\
 \Delta\delta_{\text{CP}} &\simeq -\frac{\varepsilon}{2} \left\{ \left(2\sqrt{2}\eta_\delta \text{Re}(B') \cot 2\theta_{13} - \text{Im}(C') \right) \left(\frac{\cos 2\theta_{12} - c_{13}^2 c_{12}^2}{m_3 + \eta m_2} - \frac{\cos 2\theta_{12} + c_{13}^2 s_{12}^2}{m_3 - \eta m_1} \right) \right. \\
 &\quad \left. + \left(\eta_\delta \text{Re}(C') \csc \theta_{13} - \sqrt{2}\text{Im}(B') \sec \theta_{13} \right) \left[\frac{\cot \theta_{12} (\cos 2\theta_{12} - c_{13}^2 c_{12}^2)}{m_3 + \eta m_2} \right. \right. \\
 &\quad \left. \left. + \frac{\tan \theta_{12} (\cos 2\theta_{12} + c_{13}^2 s_{12}^2)}{m_3 - \eta m_1} \right] - \left[2 \cot 2\theta_{12} \left(\sqrt{2}\text{Im}(B') c_{13} + \eta_\delta \text{Re}(C') s_{13} \right) \right. \right. \\
 &\quad \left. \left. + \sqrt{2}\eta_\delta \text{Re}(B') \sin 2\theta_{13} - \text{Im}(C') (1 + s_{13}^2) \right] \frac{\eta}{m_2 - m_1} \right\}, \\
 \Delta\rho &\simeq \frac{\varepsilon}{4} \left[2s_{13} \left(\eta_\delta \text{Re}(C') - \sqrt{2}\text{Im}(B') \tan \theta_{13} \right) \left(\frac{\cos 2\theta_{12} \cot \theta_{12}}{m_3 + \eta m_2} + \frac{\sin 2\theta_{12}}{m_3 - \eta m_1} \right) \right. \\
 &\quad - 2 \left(\sqrt{2}\eta_\delta \text{Re}(B') \cos 2\theta_{13} \tan \theta_{13} - \text{Im}(C') s_{13}^2 \right) \left(\frac{2c_{12}^2}{m_3 - \eta m_1} - \frac{\cos 2\theta_{12}}{m_3 + \eta m_2} \right) \\
 &\quad - \eta \left(\sqrt{2}\text{Im}(B') c_{13} + \eta_\delta \text{Re}(C') s_{13} \right) \left(\frac{2 \cot \theta_{12} \cos 2\theta_{12}}{m_2 - m_1} - \frac{\sin 2\theta_{12}}{m_1} \right) \\
 &\quad - \left(\text{Im}(C') c_{13}^2 + \sqrt{2}\eta_\delta \text{Re}(B') \sin 2\theta_{13} \right) \left(\eta \frac{c_{12}^2}{m_1} + \eta \frac{2c_{12}^2}{m_2 - m_1} - \frac{1}{m_3} \right) \\
 &\quad \left. + \eta \text{Im}(C') \left(\frac{4c_{12}^2}{m_2 - m_1} + \frac{\cos 2\theta_{12}}{m_1} \right) \right], \\
 \Delta\sigma &\simeq \frac{\varepsilon}{4} \left[2s_{13} \left(\sqrt{2}\text{Im}(B') \tan \theta_{13} - \eta_\delta \text{Re}(C') \right) \left(\frac{\sin 2\theta_{12}}{m_3 + \eta m_2} - \frac{\cos 2\theta_{12} \tan \theta_{12}}{m_3 - \eta m_1} \right) \right. \\
 &\quad - 2 \left(\sqrt{2}\eta_\delta \text{Re}(B') \cos 2\theta_{13} \tan \theta_{13} - \text{Im}(C') s_{13}^2 \right) \left(\frac{\cos 2\theta_{12}}{m_3 - \eta m_1} + \frac{2s_{12}^2}{m_3 + \eta m_2} \right) \\
 &\quad + \eta \left(\sqrt{2}\text{Im}(B') c_{13} + \eta_\delta \text{Re}(C') s_{13} \right) \left(\frac{\sin 2\theta_{12}}{m_2} - \frac{2 \tan \theta_{12} \cos 2\theta_{12}}{m_2 - m_1} \right) \\
 &\quad + \left(\text{Im}(C') c_{13}^2 + \sqrt{2}\eta_\delta \text{Re}(B') \sin 2\theta_{13} \right) \left(\frac{1}{m_3} + \eta \frac{s_{12}^2}{m_2} - \eta \frac{2s_{12}^2}{m_2 - m_1} \right) \\
 &\quad \left. + \eta \text{Im}(C') \left(\frac{\cos 2\theta_{12}}{m_2} + \frac{4s_{12}^2}{m_2 - m_1} \right) \right], \tag{3.10}
 \end{aligned}$$

where $\eta_\delta = \pm 1$ for $\delta_{\text{CP}} = \pm\pi/2$ and $\eta = \pm 1$ for $(\rho, \sigma) = (0, \pi/2), (\pi/2, 0)$. It is worth pointing out that with the help of eqs. (3.6) and (3.7), one can achieve all the deviations

given in eqs. (3.9) and (3.10) as functions either only of the neutrino masses and mixing angles or only of the elements of M_ν . Here we focus on the deviations of θ_{23} and δ_{CP} , approximately given by

$$\begin{aligned} \Delta\theta_{23} &\simeq -\frac{\varepsilon}{4}, \\ \Delta\delta_{\text{CP}} &\simeq \frac{\eta_\delta \varepsilon [1 - r_{12} - (1 + r_{12}) \cos 2\theta_{12}] \cot 2\theta_{12}}{4(1 - r_{12})\theta_{13}}, \end{aligned} \quad (3.11)$$

at the leading order with $r_{12} = m_1/m_2$, where eqs. (3.6) and (3.7) have been taken into account. Note that the deviation of θ_{23} is simply proportional to ε and that of δ_{CP} is enhanced by the smallness of θ_{13} .

4 Numerical calculations

There are thirteen relevant real parameters $\{M_i, \theta, a, \text{Re}(b), \text{Im}(b), \text{Re}(c), \text{Im}(c), d, \text{Re}(x), \text{Im}(x), \varepsilon\}$ (for $i = 1, 2, 3$) involved in our model with the chosen Yukawa coupling matrices in eq. (3.1) to confront with the muon $g-2$, constraints from LFV decays of charged leptons, neutrino masses and neutrino mixing parameters. For simplicity, we assume that the mixing between S_1 and $\tilde{R}_2^{-\frac{1}{3}*}$ is maximal, namely $\theta = \pi/4$, and masses of the physical LQs satisfy $M_1 = M_2/(1+r) = M_3 = 2 \text{ TeV}$ with r defined as $r \equiv (M_2 - M_1)/M_1$. We take r as a small quantity, which means that masses of the physical S_1 and $\tilde{R}_2^{-\frac{1}{3}*}$ are nearly degenerate and is required to give the correct scale of neutrino masses. As we will see, the parameter x can be taken to be a complex constant and used to suppress the constraint from $\tau^- \rightarrow \mu^- + \gamma$. Thus, only eight parameters $\{r, a, \text{Re}(b), \text{Im}(b), \text{Re}(c), \text{Im}(c), d, \varepsilon\}$ are left.

The masses of charged fermions used in the numerical calculations are their central values at M_Z [142]:⁵

$$\begin{aligned} m_u &= 1.23 \text{ MeV}, & m_c &= 0.620 \text{ GeV}, & m_t &= 168.26 \text{ GeV}, \\ m_d &= 2.67 \text{ MeV}, & m_s &= 53.16 \text{ MeV}, & m_b &= 2.839 \text{ GeV}, \\ m_e &= 0.48307 \text{ MeV}, & m_\mu &= 0.101766 \text{ MeV}, & m_\tau &= 1.72856 \text{ GeV}, \end{aligned} \quad (4.1)$$

and the quark mixing parameters under the standard parametrization also take their central values [2]:

$$\sin \theta_{12}^q = 0.22650, \quad \sin \theta_{13}^q = 0.00361, \quad \sin \theta_{23}^q = 0.04053, \quad \delta^q = 1.196. \quad (4.2)$$

The difference between the SM value and the combined experimental average value of the muon anomalous magnetic moment with 1σ error is [21]

$$\Delta a_\mu = (251 \pm 59) \times 10^{-11}, \quad (4.3)$$

⁵In principle, one should take the pole masses of charged fermions for precision calculations. Here for illustration, we naively make use of their running masses at M_Z to show that our model can well accommodate the low-energy observables we are concerned about. The differences between pole masses and running masses at M_Z which come from high-order corrections in the perturbative calculations have very limited effects on the numerical analyses.

and the current experimental upper bounds on the branching ratios of radiative $l_\alpha^- \rightarrow l_\beta^- + \gamma$ decays are

$$\begin{aligned} \mathcal{B}(\mu^- \rightarrow e^- + \gamma) &< 4.2 \times 10^{-13}, \\ \mathcal{B}(\tau^- \rightarrow e^- + \gamma) &< 3.3 \times 10^{-8}, \\ \mathcal{B}(\tau^- \rightarrow \mu^- + \gamma) &< 4.4 \times 10^{-8}, \end{aligned} \tag{4.4}$$

at the 90% confidence level [2]. The best-fit values with 1σ errors of two neutrino mass-squared differences and neutrino mixing parameters in the normal neutrino mass ordering are [143]

$$\begin{aligned} \sin^2 \theta_{12} &= 0.304_{-0.012}^{+0.013}, & \sin^2 \theta_{13} &= 0.02221_{-0.00062}^{+0.00068}, \\ \sin^2 \theta_{23} &= 0.570_{-0.024}^{+0.018}, & \delta_{\text{CP}} &= \left(195_{-25}^{+51}\right)^\circ, \\ \Delta m_{21}^2 &= \left(7.42_{-0.20}^{+0.21}\right) \times 10^{-5} \text{ eV}^2, & \Delta m_{31}^2 &= \left(2.514_{-0.027}^{+0.028}\right) \times 10^{-3} \text{ eV}^2. \end{aligned} \tag{4.5}$$

To find out the allowed regions of model parameters, with which the predictions for low-energy observables can be compatible with the experimental data shown in eqs. (4.3)–(4.5), we define the Gaussian- χ^2 function as

$$\chi^2 = \sum_j \left(\frac{q_j(p_i) - q_j^{\text{bf}}}{\sigma_j} \right)^2, \tag{4.6}$$

where $p = \{r, a, \text{Re}(b), \text{Im}(b), \text{Re}(c), \text{Im}(c), d, \varepsilon\}$ are model parameters, $q = \{\Delta m_{21}^2, \Delta m_{31}^2, \sin^2 \theta_{12}, \sin^2 \theta_{13}, \sin^2 \theta_{23}, \Delta a_\mu\}$ are low-energy observables, $q_j(p_i)$ represent the model predictions for these observables, q_j^{bf} stand for the best-fit values of these observables given in eqs. (4.3) and (4.5), and σ_j are the symmetrized 1σ errors. Here we do not include the information of δ_{CP} in the χ^2 -function as a result of the weak constraint on it from the current global-fit results [143]. Then minimizing the χ^2 -function defined in eq. (4.6), we can obtain the best-fit values of the model parameters p_i , and we shall directly use $N\sigma = \sqrt{\chi^2}$ to derive the allowed parameter ranges at N standard deviations.

Before carrying out the numerical calculations, we first discuss some features of the muon $g - 2$, the radiative decays of charged leptons and the flavor structure of neutrinos with the chosen Yukawa coupling matrices in eq. (3.1), which will give guidance on the numerical analyses.

- Given the Yukawa coupling matrices in eq. (3.1), only the branching ratio of $\tau^- \rightarrow \mu^- + \gamma$ is largely enhanced and has the possibility to exceed the experimental constraint given in eq. (4.4). To avoid this excess, we take the (2, 2) element of λ^{R} to be

$$x = -\frac{m_t V_{\text{cb}}^* \mathcal{F}\left(\frac{m_t^2}{M_1^2}\right)}{m_c V_{\text{cs}}^* \mathcal{F}\left(\frac{m_c^2}{M_1^2}\right)} \simeq -2.59 + 8.2 \times 10^{-5}i, \tag{4.7}$$

with the inputs shown in eqs. (4.1) and (4.2). Then by means of eq. (2.25), the branching ratio of $\tau^- \rightarrow \mu^- + \gamma$ is approximately given by

$$\mathcal{B}(\tau^- \rightarrow \mu^- + \gamma) \simeq \frac{\alpha_{\text{em}} m_\tau^3 m_t^2 |V_{\text{cb}}|^2 a^4 r^2}{16\pi^4 M_1^4 \Gamma_\tau} \left(\frac{\ln \frac{m_t^2}{m_c^2}}{7 + 4 \ln \frac{m_c^2}{M_1^2}} \right)^2 \sim 1 \times 10^{-6} a^4 r^2, \quad (4.8)$$

which can easily satisfy the experimental constraint with small a and r .

- With the help of eqs. (2.26), (3.1) and (4.7), one may obtain

$$\Delta a_\mu \simeq \frac{m_\mu m_t a^2}{4\pi^2 M_1^2} \text{Re} \left(V_{\text{tb}} - \frac{V_{\text{cb}} V_{\text{ts}}}{V_{\text{cs}}} \right) \left(\ln \frac{M_1^2}{m_t^2} - \frac{7}{4} \right) \sim 251 \times 10^{-11} \left(\frac{a}{0.085} \right)^2, \quad (4.9)$$

which means that $a \sim 0.085$ can accommodate the experimental value of Δa_μ in eq. (4.3).

- The (1, 1) element of M_ν given in eq. (3.2) is vanishing, i.e., $(M_\nu)_{ee} = 0$, resulting in the elimination of the inverted neutrino mass ordering in this model. This point also can be seen from eq. (3.7) when ε is small. If we substitute the best-fit values of Δm_{21}^2 , Δm_{31}^2 , θ_{12} and θ_{13} into eq. (3.7) in the limit of μ - τ reflection symmetry, we may obtain two solutions for m_1 : $m_1 \simeq 6.3$ meV for $(\rho, \sigma) = (0, \pi/2)$ and $m_1 \simeq 2.2$ meV for $(\rho, \sigma) = (\pi/2, 0)$. This infers that there will be two parameter ranges for m_1 around these two solutions when M_ν with a small ε is considered.
- As can be seen in eq. (3.2), the mass scale of neutrinos is largely enhanced by m_b . Since $a \sim 0.085$ is necessary to explain Δa_μ , the masses M_1 and M_2 need to be highly but not exactly degenerate to get the right mass scale of neutrinos. Making use of the trace of $M_\nu M_\nu^\dagger$, we have

$$r \sim \frac{16\pi^2}{3am_b} \sqrt{\frac{1}{2d^2} (3m_1^2 + \Delta m_{21}^2 + \Delta m_{31}^2)} \sim 8 \times 10^{-7}, \quad (4.10)$$

for $a \sim 0.085$, $d \sim \mathcal{O}(0.01)$ and $m_1 \sim \mathcal{O}(0.01 \text{ eV})$.

Now carrying out the numerical calculations with all inputs, we find that the minimum of χ^2 -function is $\chi_{\text{min}}^2 = 6.7 \times 10^{-6}$ and the corresponding best-fit values of the model parameters and the predicted low-energy observables are listed in the third column of table 1. In addition, the 1σ and 3σ ranges of both the model parameters and the predicted low-energy observables are listed in the last two columns of table 1, respectively. To make results clearer, we plot correlations of some pairs of the predicted low-energy observables and also those between the predicted low-energy observables and model parameters in figures 3 and 4. Some comments on the numerical results are in order.

- With the best-fit values of the model parameters listed in the third column of table 1, our model can successfully reproduce the experimental values of Δm_{21}^2 ,

		Best-fit value	1σ range	3σ range
Model parameter	$a/10^{-2}$	8.37	7.37 \rightarrow 9.24	4.60 \rightarrow 10.9
	$\text{Re}(b)$	-0.000473	(-0.00245 \rightarrow 0.0000687) \cup (0.0961 \rightarrow 0.608)	(-0.00260 \rightarrow 0.000613) \cup (0.0748 \rightarrow 0.682)
	$\text{Im}(b)$	0.0246	-0.00410 \rightarrow 0.214	-0.0192 \rightarrow 0.241
	$\text{Re}(c)$	0.000280	-0.0278 \rightarrow 0.196	-0.0940 \rightarrow 0.237
	$\text{Im}(c)$	0.0367	0.0243 \rightarrow 1.28	0.0146 \rightarrow 1.43
	d	0.00147	0.000990 \rightarrow 0.0338	0.000579 \rightarrow 0.0366
	$r/10^{-6}$	4.10	0.136 \rightarrow 5.89	0.108 \rightarrow 10.1
	ε	-0.276	-0.342 \rightarrow -0.143	-0.470 \rightarrow -0.0249
Low-energy observable	m_1/meV	5.73	(2.12 \rightarrow 2.58) \cup (5.28 \rightarrow 6.17)	(1.70 \rightarrow 3.17) \cup (4.44 \rightarrow 7.24)
	$\Delta m_{21}^2/10^{-5} \text{eV}^2$	7.42	7.22 \rightarrow 7.61	6.81 \rightarrow 8.03
	$\Delta m_{31}^2/10^{-3} \text{eV}^2$	2.51	2.49 \rightarrow 2.54	2.43 \rightarrow 2.60
	$\theta_{12}/^\circ$	33.46	32.71 \rightarrow 34.19	31.13 \rightarrow 35.73
	$\theta_{13}/^\circ$	8.57	8.45 \rightarrow 8.69	8.19 \rightarrow 8.94
	$\theta_{23}/^\circ$	49.03	47.89 \rightarrow 50.17	45.44 \rightarrow 52.65
	$\delta_{\text{CP}}/^\circ$	242.99	(236.52 \rightarrow 249.95) \cup (275.00 \rightarrow 279.01)	(224.14 \rightarrow 266.88) \cup (270.82 \rightarrow 283.67)
	$\rho/^\circ$	8.58	(6.47 \rightarrow 10.43) \cup (94.02 \rightarrow 96.92)	(1.03 \rightarrow 13.50) \cup (90.66 \rightarrow 99.91)
	$\sigma/^\circ$	92.27	(0.29 \rightarrow 0.55) \cup (91.73 \rightarrow 92.72)	(0.05 \rightarrow 0.91) \cup (90.28 \rightarrow 93.55)
	$\Delta a_\mu/10^{-11}$	251.10	194.74 \rightarrow 305.97	75.94 \rightarrow 426.33
	$\mathcal{B}(\tau^- \rightarrow \mu^- + \gamma)$	1.17×10^{-18}	$(0.71 \rightarrow 1.75) \times 10^{-18}$	$(0.11 \rightarrow 3.39) \times 10^{-18}$

Table 1. The best-fit values (with $\chi_{\text{min}}^2 = 6.7 \times 10^{-6}$) along with the 1σ and 3σ ranges of the model parameters and the predicted low-energy observables. The symbol “ \cup ” stands for the union of two sets.

Δm_{31}^2 , $\sin^2 \theta_{12}$, $\sin^2 \theta_{13}$ and $\sin^2 \theta_{23}$, and explain the muon anomalous magnetic moment Δa_μ , as well as satisfy the constraints from the LFVs decays of charged leptons, with a quite small value of χ^2 -function, namely $\chi_{\text{min}}^2 = 6.7 \times 10^{-6}$. Meanwhile, the lightest neutrino mass and three CP-violating phases are predicted to be $m_1 = 5.73$ meV, $\delta_{\text{CP}} = 242.99^\circ$, $\rho = 8.58^\circ$ and $\sigma = 92.27^\circ$, respectively.

- As can seen from table 1 and also figure 3, there are two disconnected ranges for m_1 , δ_{CP} , ρ and σ . The two disconnected ranges for m_1 , ρ and σ can be easily understood in the limit of the μ - τ reflection symmetry (i.e., $\varepsilon = 0$), where two possible values of m_1 respectively associated with two specific combinational values of ρ and σ can be worked out by eq. (3.7), that is $m_1 = 2.2$ meV for $(\rho, \sigma) = (\pi/2, 0)$

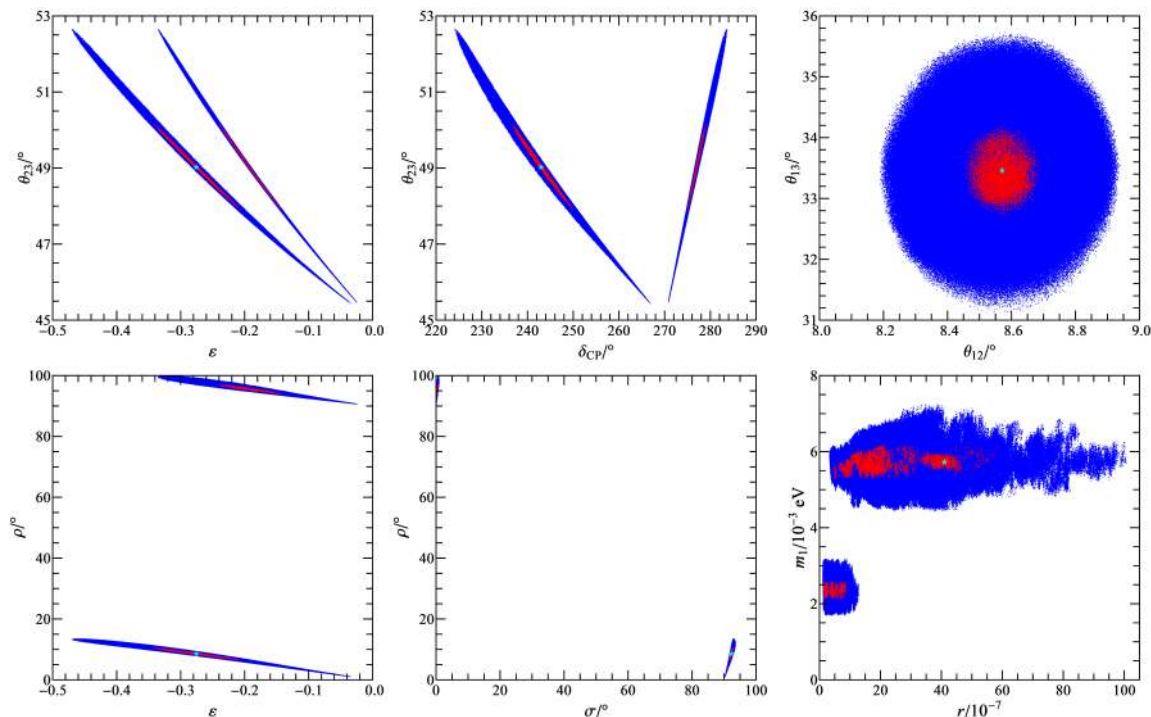


Figure 3. Correlations of the pairs $(\theta_{23}, \varepsilon)$, $(\theta_{23}, \delta_{\text{CP}})$, $(\theta_{13}, \theta_{12})$, (ρ, ε) , (ρ, σ) and (m_1, r) , where the cyan stars denote the best-fit values, and the red and blue regions correspond to parameter spaces at 1σ and 3σ levels, respectively.

and $m_1 = 6.3$ meV for $(\rho, \sigma) = (0, \pi/2)$, as discussed above. Thus when the μ - τ reflection symmetry is slightly broken, (in other words, ε is small but not zero), the values of m_1 , ρ and σ will be around these two possible solutions in the limit of μ - τ reflection symmetry, which is exactly the case considered here. One may understand the two disconnected ranges for δ_{CP} with the help of eq. (3.11). It can be easily checked that when the best-fit values of Δm_{21}^2 and θ_{12} in eq. (4.5) are taken into consideration, $1 - r_{12} - (1 + r_{12}) \cos 2\theta_{12}$ is monotonically decreasing as a function of m_1 and $m_1 \simeq 4$ meV is a zero of it. This leads to $\delta_{\text{CP}} > 270^\circ$ for $m_1 \lesssim 4$ meV and $\delta_{\text{CP}} < 270^\circ$ for $m_1 \gtrsim 4$ meV with initially $\delta_{\text{CP}} = 270^\circ$ in the limit of μ - τ reflection symmetry, which is consistent with the obtained numerical results for δ_{CP} .

- The upper-left panel of figure 3 shows that the value of $\Delta\theta_{23}$ is approximately proportional to $-\varepsilon$, and θ_{23} is in the second octant for $\varepsilon < 0$. These behaviors of θ_{23} coincide with the approximate analytical formula for the deviation of θ_{23} given in eq. (3.11), but in order to explain the two parameter spaces for θ_{23} - ε , higher order terms involving r_{12} in the analytical result for $\Delta\theta_{23}$ need to be restored. The deviation of δ_{CP} shown in the upper-middle panel of figure 3 is much larger than other mixing parameters, which is actually enhanced by $1/\theta_{13}$ as revealed in eq. (3.11). From the lower-right panel of figure 3, one can find that the maximal value r can take for $m_1 \sim 2.5$ meV is smaller than that for $m_1 \sim 5.5$ meV. This may be roughly understood with the help of eq. (4.10).

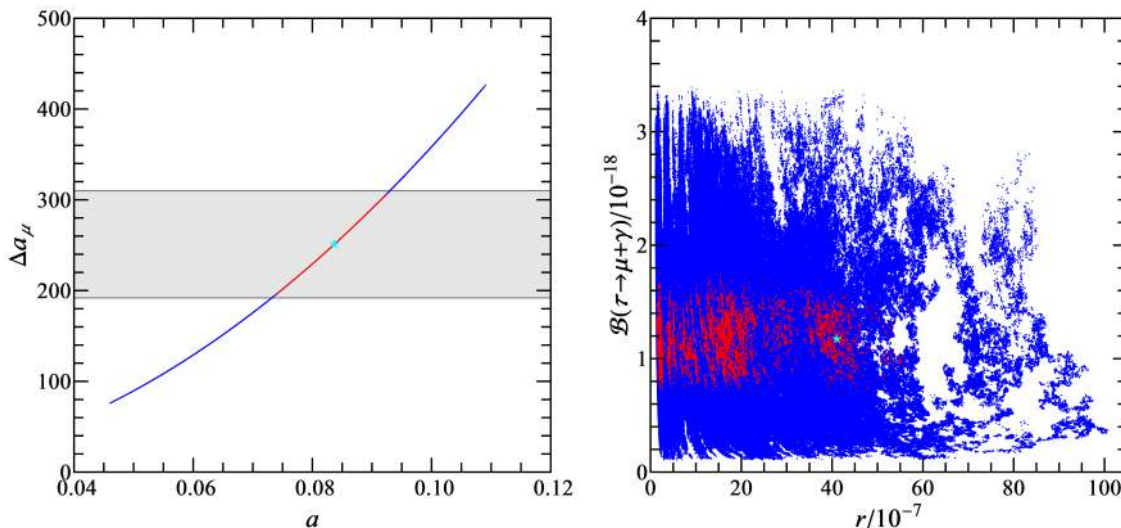


Figure 4. Correlation between Δa_μ and a , and that between $\mathcal{B}(\tau^- \rightarrow \mu^- + \gamma)$ and r , where the cyan stars denote the best-fit values, the gray region in the left panel is the experimental 1σ range of Δa_μ , and the red and blue regions correspond to parameter spaces at 1σ and 3σ levels, respectively.

- The approximate result for Δa_μ in eq. (4.9) where Δa_μ is a quadratic function of the model parameter a , perfectly describes the behavior of Δa_μ against a shown in the left panel of figure 4. However, the approximate formula for the branching ratio $\mathcal{B}(\tau^- \rightarrow \mu^- + \gamma)$ given in eq. (4.8) predicts smaller values than those shown in the right panel of figure 4. The reason is that for $r \sim 10^{-7}$, the masses of the physical S_1 and $\tilde{R}_2^{-\frac{1}{3}}$ are highly degenerate, leading to severe cancellation between the chirally enhanced contributions to $\mathcal{B}(\tau^- \rightarrow \mu^- + \gamma)$ from the physical S_1 and $\tilde{R}_2^{-\frac{1}{3}}$ with x given in eq. (4.7), and then the contributions not enhanced (e.g., the second terms in the square brackets in eq. (2.13)) dominate the value of $\mathcal{B}(\tau^- \rightarrow \mu^- + \gamma)$ in the present case.

5 Summary

For the purpose of giving a combined explanation of the tiny neutrino masses, lepton flavor mixing and muon $g-2$, we have extended the SM with two TeV-scale scalar leptoquarks S_1 and \tilde{R}_2 . After constructing the complete Lagrangian with baryon number conservation, we calculate the neutrino mass matrix generated via one-loop quantum corrections, where the mixing between S_1 and \tilde{R}_2 resulting from the lepton-number-violating LQ-Higgs interaction plays a greatly significant role. We also recalculate all the contributions to the radiative decays of charged leptons and the muon $g-2$ from all leptoquarks. The contributions from the physical S_1 and $\tilde{R}_2^{-\frac{1}{3}}$ are quite similar and both largely enhanced by the up-type quark masses, which dominate the branching ratios of charged-lepton radiative decays and the size of the muon $g-2$. Given the textures of the leptoquark Yukawa coupling matrices shown in eq. (3.1), the neutrino mass matrix possesses an approximate μ - τ reflection symmetry, and

moreover its $(1, 1)$ element vanishes, i.e., $(M_\nu)_{ee} = 0$, which is only in favor of the normal neutrino mass ordering. Considering this novel texture of the neutrino mass matrix, we have analytically discussed its main features and the associated neutrino masses and lepton flavor mixing in detail. Finally, the numerical calculations show that our model can successfully explain the anomaly of the muon $g - 2$ and accommodate the latest global-fit results of neutrino oscillation data, as well as satisfy the constraints from the radiative decays of charged leptons. More specifically, given the best-fit values of the model parameters with $\chi_{\min}^2 = 6.7 \times 10^{-6}$, the central value of Δa_μ and the best-fit values of Δm_{21}^2 , Δm_{31}^2 , θ_{12} , θ_{13} and θ_{23} respectively given in eqs. (4.3) and (4.5) are reproduced pretty well, and additionally the lightest neutrino mass and three CP-violating phases are predicted to be $m_1 = 5.73$ meV, $\delta_{\text{CP}} = 242.99^\circ$, $\rho = 8.58^\circ$ and $\sigma = 92.27^\circ$. An interesting feature of our model is that the allowed ranges of m_1 , ρ and σ are highly correlated, i.e., $m_1 \sim 2.2$ meV for $(\rho, \sigma) \sim (\pi/2, 0)$ and $m_1 \sim 6.3$ meV for $(\rho, \sigma) \sim (0, \pi/2)$, as a result of the approximate μ - τ reflection symmetry and one-zero texture of M_ν .

It is worth remarking that leptiquarks may naturally originate from the grand unification framework, and the leptiquark extensions of the SM can not only give a combined explanation of neutrino masses, lepton flavor mixing and the muon anomalous magnetic moment, as focused on in this work, but also have the potential to address flavor anomalies in the quark sector. Therefore, as appealing and promising extensions of the SM, leptiquark models deserve more attentions and further investigations.

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