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RADIO RELAY SYSTEM PERFORMANCE IN AN INTERFERENCE ENVIRONMENT

E. E. Reinhart

PREPARED FOR:
NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

The RAND Corporation
SANTA MONICA • CALIFORNIA

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PREFACE

This Memorandum provides part of the technical background for a continuing RAND study which is described in a companion publication, RM-5785-NASA, *The Technology Potentials for Satellite Spacing and Frequency Sharing*. Specifically, it examines, for several message types and modulation methods, the relationship between message quality at the single channel outputs of a radio relay system and the relative level of rf interference and noise at the inputs to the repeaters and the terminal receiver of the system. Relationships of this type are fundamental to a determination of how message quality objectives constrain design choices for satellite and terrestrial radio relay systems which share a common spectrum band.

The material on analog modulation represents an extension to relay systems and to arbitrary rf noise of the analysis reported in RM-5117-NASA, *Multiple-Access Techniques for Communication Satellites: Analog Modulation, Frequency-Division Multiplexing, and Related Signal Processing Methods*. Similarly, the treatment of digital modulation may be viewed as an extension of the work in RM-4997-NASA, *Multiple-Access Techniques for Communication Satellites: Digital Modulation, Time-Division Multiplexing, and Related Signal Processing*.

Certain features of the analytic approach are believed to be novel. For example, the effect on signal quality of the demodulators of the system is described separately from that of the terminal signal processing and demultiplexing equipment, making it possible to write a general functional equation applicable to all radio relay systems employing a given type of repeater. Another new feature is the use of white thermal noise as a reference case for the description of the interference reduction capability of a receiver. This permits a considerable simplification in determining this capability for arbitrary interference when a number of modulation and signal processing combinations are to be compared.

SUMMARY

This Memorandum deals with the overall effect on output message quality of radio frequency (rf) interference and noise at the inputs to the repeaters and the terminal receiver of a radio relay system. For a given system, these effects may be described by an equation which expresses the output message quality in terms of the ratios of wanted to unwanted signal power at the rf inputs of the system. The functional nature of the equation depends on the type of message (analog or digital), the type of modulation (analog or digital), and on whether the carrier is demodulated at the repeaters or not.

When both the messages and the modulation are of the analog type, the equation is linear with coefficients whose numerical values depend both on details of the modulation and message recovery processes and on the characteristics of the interference environment. Two types of coefficients are distinguished. The first is called the demodulator transfer characteristic (DTC) because it is the ratio of output to input signal-to-noise ratio for the demodulators of the system. The other plays a similar role for the circuits following the demodulator at the terminal receiver and is called the channelizing transfer characteristic (CTC). The product of the DTC and the CTC also occurs in the system equation and is known in the literature as the receiver transfer characteristic (RTC) or interference reduction factor.

Numerical values of these coefficients are presented for telephone and for television channels transmitted by various analog modulation methods in the presence of an unwanted signal having the characteristics of white thermal noise. The modulation methods include the double and single sideband suppressed carrier versions of amplitude modulation (SSB and DSB), phase modulation (PM), and frequency modulation (FM), both with and without preemphasis. With PM and FM, particular attention is given to the dependence of the RTC on the modulation index which determines the amount of rf bandwidth needed to transmit a given information bandwidth using these methods.

For the general case of an arbitrary unwanted signal an equation is given for the ratio of the RTC to the value it would have with white

noise interference. An important parameter in this equation is the difference between the wanted and unwanted carrier frequencies. Numerical results are presented as a function of this parameter for interference between high index FM systems. It is concluded that for carrier frequency separations greater than one-quarter the rf bandwidth of such wideband FM signals, the applicable RTC is larger (better) than that for the white noise reference case.

When digital modulation is used, or when digital messages are carried by analog-modulated systems, the equation describing the relation between output channel message quality and the input wanted-to-unwanted-signal ratios is highly nonlinear. In this case, the input-output relations of the demodulators and of the terminal equipment at the final receiver can be described by two functions, respectively called the demodulator and channelizing functions. These functions are presented graphically for PCM-encoded telephone channels and for various digital modulation methods and white-noise-like interference. The digital methods include m-level amplitude, phase, and frequency shift keying (ASK, PSK, and FSK) demodulated both coherently and noncoherently. Equations describing the cost in rf bandwidth of using these digital methods to transmit either digital data, or digitally-encoded analog messages are also presented.

The relative efficiency of analog and digital transmission links for transmitting telephone channels is compared by plotting the difference in dB between the worst channel output signal-to-noise ratio and the effective input wanted-to-unwanted-signal ratio against the ratio of rf bandwidth to information bandwidth.. This comparison includes SSB, preemphasized FM, and both binary and four-phase coherent PSK modulation carrying PCM-encoded channels using from five to ten bits per sample.

Message quality objectives for telephone and television channels subject to fading are presented in some detail for both terrestrial and satellite radio relay systems. Finally, these objectives are substituted in the previously developed system equations to determine the wanted-to-unwanted signal ratios that must be maintained in such systems in order to meet the quality objectives.

Necessary background data on the characteristics of analog messages, analog modulated carriers and noise weighting and preemphasis networks are given in Appendices.

ACKNOWLEDGMENTS

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I. INTRODUCTION

As its title implies, the primary concern of this Memorandum is with the effects of the interference that occurs when elements of one or more radio relay communication systems operate on the same radio frequencies. In some systems of this type there will be only a single repeater and it will be located in an orbiting satellite. In others, the message path will be purely terrestrial and will contain many repeaters located atop buildings or on towers or mountain tops. In still other systems, the path will involve a tandem combination of the two. The terminals themselves may be fixed on land or they may be located on land vehicles, ships, or aircraft.

In all such systems, a fundamental requirement is to maintain a specified fidelity of transmission as reflected by the quality of the messages at the system output. For analog messages, such as voice and television signals, the fidelity criterion is usually the ratio of average signal power to average noise power at the channel output. For digital messages, such as teletype and computer data, the appropriate measure of fidelity is the probability of error at the channel output. With either type of message, standards or objectives for output message quality are usually based on extensive user tests.

The output message quality actually delivered by a radio relay system will depend of course on the technical choices and equipment parameters which characterize the system and on the electromagnetic environment in which it operates. Among these are the following:

a. The way the message is multiplexed with other messages to form a baseband.

b. The way the message or the baseband is processed or encoded prior to modulation.

c. The modulation method and modulation index used to impress the baseband on a carrier.

d. The type and number of repeaters and the characteristics of the terminal receiver.

e. The strength of the message-bearing carrier relative to noise and unwanted signals at the inputs to the repeaters and the terminal receiver.

f. The statistical nature of the unwanted signals.

The dependence of output message quality on these factors is conveniently described by giving the relationship between output signal quality and the input wanted-to-unwanted signal ratios (item e). In such a relationship, the other factors (items a, b, c, d, and f) act as parameters. Once the minimum acceptable output signal quality is specified, this relationship permits a determination of the so-called "protection ratios" (i.e., the wanted-to-unwanted signal ratios that must be maintained in order to achieve specified output quality).

The functional nature of the relationship between message quality at the output of a radio relay system and the signal environment at its repeater and terminal inputs depends primarily on the modulation method used. The type of modulation, analog or digital, not only determines the form of the relationship but also determines the amount of rf spectrum required to transmit information at a specified rate.

The approach to deriving the desired relationships is much the same for all modulation methods, however. For both analog and digital types, an n-link radio relay chain will be represented schematically by the block diagram shown in Fig. 1. Here attention is focused on a single one-way message channel and the multiplexed baseband in which it may be embedded. The blocks labeled sending terminal, repeater, and receiving terminal are intended to represent only those parts of the system which carry the selected baseband. At the terminals, account is taken of the processing or encoding that may be applied to the messages, either individually or in baseband form. At a given repeater, the unwanted signals in question include just that fraction of the total noise and extraneous signal environment which lies in the repeater passband. Since successive repeaters normally receive on different carrier frequencies, this means that the unwanted signals at one repeater will not be cochannel with those at the next.

A common notation is also used insofar as possible. The symbol C_i denotes the average power of the wanted rf signal at the input

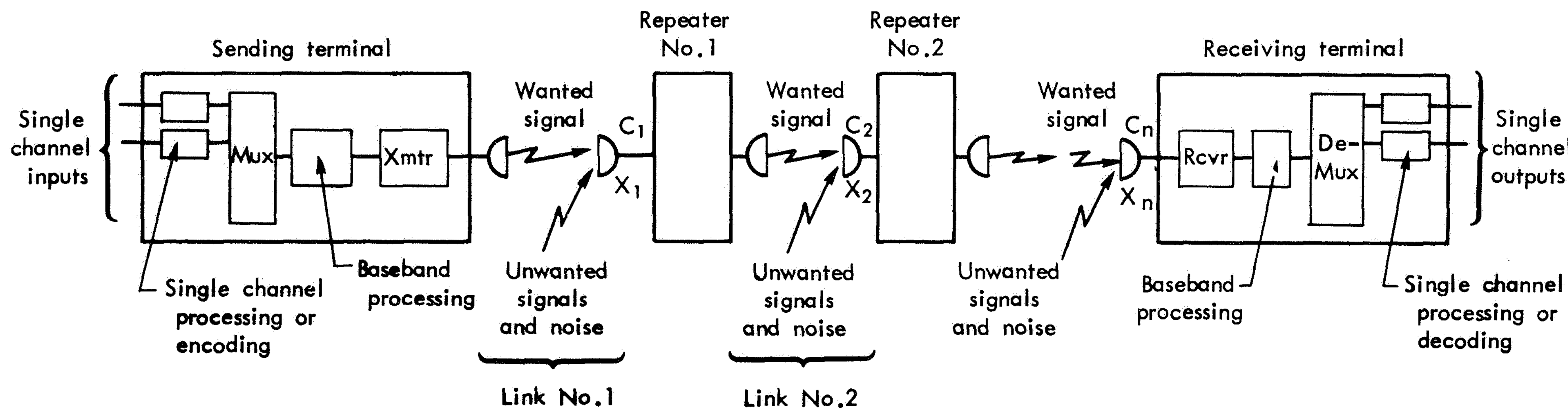


Fig.1—Radio relay chain for one direction of transmission

terminals of the i-th repeater--i.e., the modulated carrier power due to transmission from the (i-1)th repeater in the chain. Similarly, X_i denotes the total average power at the i-th repeater input due to all other sources of signal and noise within the passband of the i-th repeater. These sources include repeaters other than the (i-1)th in the system being examined, as well as the transmitters and repeaters of external systems. They also include the i-th repeater itself whose contributions to noise, distortion, and crosstalk are represented as unwanted signals at the input terminals of an equivalent noise- and distortion-free device. For brevity, X_i will often be referred to as the unwanted signal power. At the terminal receiver input, the wanted and unwanted signals are denoted by C_n and X_n respectively.

With some types of repeater, however, C_i and X_i do not represent the division of total input power between "pure" signal on the one hand and noise and interference on the other. For example, the wanted signal power C_i may include the contribution from noise and unwanted signals originating on earlier links. In these cases, the pure signal portion of C_i will be denoted by C_i' and the corresponding total noise and interference power by X_i'. Thus the sum of signal, interference, and noise powers at the i-th repeater input may be written in two equivalent ways:

$$C_i + X_i = C_i' + X_i' \tag{1}$$

The ratio C_i/X_i will be called the apparent wanted-to-unwanted signal ratio, since C_i and X_i are the directly measurable quantities whose values can be predicted from a knowledge of the transmitter, antenna, and receiver parameters and the propagation path loss. The ratio C_i'/X_i' will be called the effective wanted-to-unwanted signal ratio, since it reflects the true division between signal and noise powers. In all that follows, it will be assumed that both the apparent and the effective wanted-to-unwanted signal ratios are much greater than unity.

In terms of these quantities, the problem becomes one of expressing the output channel quality in terms of the apparent wanted-to-unwanted signal ratios C_i/X_i. If output quality is variable as a result of variations in C_i/X_i, its probability distribution can then be expressed in terms of the distributions of the C_i/X_i. For both analog and digital systems, the approach is first to introduce functions which describe the input-output transfer characteristics of each block in Fig. 1, and then, using these, to follow the message channel through the chain from its input at the sending terminal to its output at the receiving terminal.

The relationships between output message quality and the wanted-to-unwanted signal ratios will be derived for analog modulation methods in Section II. Specifically included are conventional amplitude modulation (AM), the double- and single-sideband suppressed carrier forms of amplitude modulation (DSB and SSB respectively), phase modulation (PM), and frequency modulation (FM).

Corresponding relationships will be developed in Section III for the digital modulation methods of amplitude-shift keying (ASK), phase-shift keying (PSK), and frequency-shift keying (FSK).

Finally, in Section IV, the relationships developed in Sections II and III will be applied to determine the constraints that must be imposed on the interference environment in order to achieve specified message quality objectives. The impact of such constraints on the parameters of satellite and terrestrial radio relay systems operating in a common spectral band is investigated in a companion Memorandum.(1)

Required background information on the characteristics of analog message basebands, analog modulated carriers, and noise weighting and preemphasis is given in Appendices A, B, and C respectively.

II. ANALOG MODULATED SYSTEMS

Radio relay systems employing analog modulation methods such as AM, SSB, PM, and FM are designed primarily to carry analog messages such as voice and television. As previously noted, the output quality of an analog message is normally measured by the signal-to-noise power ratio S_{ch}/N_{ch} at the output of the message channel. In this ratio, S_{ch} is an appropriate measure of the power of the message signal or its standard representation, and N_{ch} is the total effective noise power in the channel due to all sources.

DEFINITION OF TRANSFER CHARACTERISTICS

The numerical specification of output message quality S_{ch}/N_{ch} in analog systems will be discussed in detail later. For the present, the important thing to note is that, with all analog modulation methods, S_{ch}/N_{ch} is directly proportional to the effective wanted-to-unwanted signal-to-noise ratio C'_n/X'_n at the terminal receiver input, provided only that C'_n/X'_n exceeds a specified threshold value. If, as is usually the case, the message in question is part of a baseband formed by multiplexing a number of similar messages, the constant of proportionality will depend not only on the parameters which characterize the analog modulation method but also on the details of the demultiplexing operation and any associated signal processing that takes place between the receiver output and the channel output (see Fig. 1).

For purposes of analysis, it is sometimes convenient to deal with the signal-to-noise characteristics of the receiver which recovers the baseband from the modulated carrier separately from those of the circuits which isolate the single-channel outputs from the baseband. Towards this end, the signal-to-noise transfer characteristics of the terminal receiver will be described by noting that the ratio of average baseband power S_n to average noise power N_n at the demodulator output is directly proportional to C'_n/X'_n. The constant of proportionality

$$R_n = \frac{S_n/N_n}{C_n'/X_n'} \qquad (2)$$

will be called the demodulator transfer characteristic (DTC) of the terminal receiver. Its numerical value will depend on the statistical nature of the unwanted signals and noise as well as on the method of modulation and the modulation index, which jointly determine the ratio of rf bandwidth to baseband bandwidth.

The signal-to-noise transfer characteristics of the circuits following the terminal receiver can then be described by a second constant of proportionality

$$R_{ch} = \frac{S_{ch}/N_{ch}}{S_n'/N_n'} \qquad (3)$$

which will be called the channelizing transfer characteristic (CTC). In this definition, S_n' and N_n' are, respectively, the pure signal and total noise portions of the combined signal-plus-noise power at the demodulator input. They will differ from S_n and N_n in systems in which the baseband has been recovered and retransmitted at one or more repeaters. Where a distinction is necessary, S_n/N_n will be called the apparent baseband signal-to-noise ratio, and S_n'/N_n' the effective baseband signal-to-noise ratio.

In general, the numerical value of R_{ch} will depend on the position of the channel in the baseband, the nature of the signal processing (if any), the number of channels in the multiplex, and the details of the noise and interference power spectra at the demodulator output. In computing the CTC, one must also take into account the relationship between the test signal conventionally used to represent the baseband and the single-channel test signal used in specifying channel output quality, since the former is not a multiplex of the latter. However, the value of R_{ch} does not depend on the details of the modulation process and the rf bandwidth of the modulated carrier. It will be noted in passing that under certain conditions the product $R_n R_{ch}$ yields the "receiver transfer characteristic" or "interference reduction factor" as defined by other authors.

GENERAL RELATION BETWEEN SIGNAL ENVIRONMENT AND OUTPUT MESSAGE QUALITY

In discussing the overall signal-to-noise performance of analog modulated radio relay systems, two types of repeater should be distinguished according to whether or not the carrier is demodulated at the repeater. The i-f repeater, a form of which is used extensively in long-haul terrestrial microwave relay systems, is an example of the type in which no demodulation takes place. The received carrier is merely translated in frequency to an intermediate frequency, amplified in an i-f amplifier, and translated back up to a new radio frequency for additional amplification and transmission. The wideband rf repeater, as used in certain satellite repeaters, is another example of this type. It differs principally from the i-f repeater in that there is no conversion to intermediate frequency, and because of the limited availability of primary power, the output amplifier is normally operated at or close to saturation to improve its efficiency.

The message-quality - signal-environment relationship for a chain of either i-f or rf repeaters is easily derived from the fact that since noise and distortion internal to the repeater have been referred to its input, the proportion of wanted and unwanted signal power at the output of any given repeater is the same as at its input. Thus, barring distortion in the propagation medium, the signal portion of the wanted signal power at the i-th repeater input is given by

$$C_i' = \frac{C_{i-1}'}{C_{i-1}' + X_{i-1}'} C_i \qquad (4)$$

and the total noise power at this point is

$$X_i' = \frac{X_{i-1}'}{C_{i-1}' + X_{i-1}'} C_i + X_i \qquad (5)$$

The effective unwanted-to-wanted signal ratio at the i-th repeater is thus

$$\frac{X'_i}{C'_i} = \frac{X'_{i-1}}{C'_{i-1}} + \frac{X_i}{C_i} + \frac{X'_{i-1}}{C'_{i-1}} \frac{X_i}{C_i} \tag{6}$$

Since, at every repeater input, the effective wanted-to-unwanted signal ratio has been assumed to be large compared to unity, the third term of the sum in Eq. (6) may be neglected. Noting that $C'_1/X'_1 = C_1/X_1$, repeated application of the resultant recursion formula then yields an expression for the effective wanted-to-unwanted signal ratio at the terminal receiver input

$$\frac{C'_n}{X'_n} = \frac{1}{\sum_{i=1}^{n} X_i/C_i} \qquad \text{rf or i-f repeaters} \tag{7}$$

Since no demodulation takes place until the terminal receiver, there is no distinction between the apparent and effective baseband signal-to-noise ratios, S_n/N_n and S'_n/N'_n, and Eqs. (2) and (3) may be combined with Eq. (7) to yield the desired general expression for output message quality in terms of the apparent input wanted-to-unwanted signal ratios

$$\frac{S_{ch}}{N_{ch}} = \frac{R_{ch} R_n}{\sum_{i=1}^{n} (C_i/X_i)^{-1}} \qquad \text{rf or i-f repeaters} \tag{8}$$

When all of the wanted-to-unwanted signal ratios are the same, the result is even simpler:

$$\frac{S_{ch}}{N_{ch}} = \frac{1}{n} R_{ch} R_n \frac{C}{X} \qquad \begin{array}{l}\text{rf or i-f repeaters} \\ C_i/X_i = C/X\end{array} \tag{9}$$

The other basic type of repeater is called a baseband repeater because it demodulates the rf signal to baseband and then uses it to modulate a new carrier. By analogy with the terminal receiver, the signal-to-noise performance of the demodulator in the i-th repeater can be represented by the demodulator transfer characteristic

$$R_i = \frac{S_i/N_i}{C_i'/X_i'} = \frac{S_i/N_i}{C_i/X_i} \tag{10}$$

where the second equality reflects the fact that in a relay system employing baseband repeaters, there is no distinction between the apparent and effective wanted-to-unwanted signal ratios. However, the apparent baseband power S_i will contain noise from the basebands recovered at earlier repeaters. In particular, at the demodulator output in the i-th repeater, the effective baseband signal and noise powers are respectively

$$S_i' = \frac{S_{i-1}'}{S_{i-1}' + N_{i-1}'} S_i \tag{11}$$

and

$$N_i' = \frac{N_{i-1}'}{S_{i-1}' + N_{i-1}'} S_i + N_i \tag{12}$$

Hence the effective baseband noise-to-signal ratio at this point is

$$\frac{N_i'}{S_i'} = \frac{N_{i-1}'}{S_{i-1}'} + \frac{N_i}{S_i} + \frac{N_{i-1}'}{S_{i-1}'} \frac{N_i}{S_i} \tag{13}$$

If, at every repeater, the apparent baseband signal-to-noise ratio is large compared with unity, the third term may be neglected, and it follows that the effective baseband signal-to-noise ratio at the terminal receiver output is

$$\frac{S_n'}{N_n'} = \frac{1}{\sum\limits_{i=1}^{n} N_i/S_i} \qquad \text{Baseband repeaters} \tag{14}$$

Combining this with Eqs. (3) and (10) yields the desired general expression for output signal quality in terms of the apparent input wanted-to-unwanted signal ratios

$$\frac{S_{ch}}{N_{ch}} = \frac{R_{ch}}{\sum_{i=1}^{n} \left(R_i \frac{C_i}{X_i} \right)^{-1}} \qquad \text{Baseband repeaters} \tag{15}$$

If the demodulator transfer characteristic is the same for all repeaters and the terminal receiver, this becomes

$$\frac{S_{ch}}{N_{ch}} = \frac{R_{ch} R}{\sum_{i=1}^{n} (C_i/X_i)^{-1}} \qquad \begin{array}{l}\text{Baseband repeaters} \\ R_i = R; \quad i=1,\ldots,n\end{array} \tag{16}$$

which is formally identical to Eq. (8) for a system employing rf or i-f repeaters. But note that each of the transfer characteristics, R_{ch}, R_i, $i = 1, \ldots, n$, depends on the statistical properties of the various unwanted signals at the input to the circuits they represent as well as on the design of these circuits. To say that all of the repeater demodulators have the same transfer characteristic R is to imply not only identical repeaters but statistically equivalent unwanted signals at their inputs. To assume further that the repeaters of a baseband repeater chain have the same demodulator transfer characteristics as the terminal receiver of an i-f repeater system would be to imply that the statistics of the effective unwanted signal at the terminal receiver input are no different from those at each repeater input in the baseband system. Since the former signal comprises the weighted sum of the unwanted signals incident at each of the repeaters, its statistics will in general be different from those of its components. Hence even with identical receiving circuits and the same signal environment at the repeaters, it is not to be expected that a baseband repeater system will in general deliver the same performance as an i-f repeater system.

PARTITION OF OUTPUT CHANNEL NOISE

It is common practice with both types of radio relay systems to view N_{ch}, the total noise at the channel output, as the sum of contributions arising from the individual links of the system. However, since the channel is not actually isolated until the final stages of the receiving terminal, it is more accurate to regard the "noise contribution of a link" as the incremental increase in channel noise that would be observed if the channel were to be recovered at successive repeaters using equipment with the same transfer characteristics as that of the receiving terminal.

With this interpretation, it is easy to show that when the general Eqs. (8) and (15) are solved for N_{ch}, each term in the sum is in fact just the contribution from a single link. For this purpose, note that the noise that would be observed in the selected channel at a point in the output of the i-th repeater where the signal has a reference value S_{ch} is, by analogy with Eq. (3),

$$\left(N_{ch}\right)_i = \frac{S_{ch}}{R_{ch}} \frac{N_i'}{S_i'} \tag{17}$$

When this equation is applied to a system of baseband repeaters, N_i'/S_i' is the effective noise-to-signal ratio that is actually observed at the i-th repeater.

For an rf or i-f repeater system, it is to be interpreted as the ratio that would be observed if the baseband were to be recovered at this point using a receiver having the demodulator transfer characteristic of the terminal receiver. In either case, it follows from Eq. (17) that the incremental contribution of the i-th link to the total channel noise is

$$\left(\Delta N_{ch}\right)_i = \left(N_{ch}\right)_i - \left(N_{ch}\right)_{i-1} = \frac{S_{ch}}{R_{ch}} \left(\frac{N_i'}{S_i'} - \frac{N_{i-1}'}{S_{i-1}'}\right) \tag{18}$$

For systems employing baseband repeaters, Eq. (13) applies and leads directly to the result

$$\left(\Delta N_{ch}\right)_i = \frac{S_{ch}}{R_{ch}} \frac{N_i}{S_i} = \frac{S_{ch}}{R_{ch} R_i} \frac{X_i}{C_i} \qquad \text{Baseband repeaters} \tag{19}$$

where the second equality is based on Eq. (10). Comparing this with the general expression for the total channel noise in a baseband system given by Eq. (15), it is seen that, as anticipated, the i-th term in the sum is just the incremental contribution from the i-th link.

$$N_{ch} = \frac{S_{ch}}{R_{ch}} \sum_{i=1}^{n} \frac{X_i}{R_i C_i} = \sum_{i=1}^{n} \left(\Delta N_{ch}\right)_i \qquad \text{Baseband repeaters}$$

In the case of i-f repeaters, the hypothetical demodulation at the i-th repeater is characterized by the transfer characteristic R_n of the terminal receiver, and Eq. (18) gives

$$\left(\Delta N_{ch}\right)_i = \frac{S_{ch}}{R_{ch} R_{ch}} \left(\frac{X_i'}{C_i'} - \frac{X_{i-1}'}{C_{i-1}} \right)$$

whence, using Eq. (6),

$$\left(\Delta N_{ch}\right)_i = \frac{S_{ch}}{R_{ch} R_n} \frac{X_i}{C_i} \qquad \text{rf or i-f repeaters} \qquad (20)$$

This differs from Eq. (19) for baseband repeaters only in that the demodulator transfer characteristic for the terminal receiver appears in place of that for the i-th baseband repeater. Solving Eq. (8) for the total channel noise in a system of i-f repeaters, and comparing the result with Eq. (20), it is seen that the i-th term is again the contribution from the i-th link:

$$N_{ch} = \frac{S_{ch}}{R_{ch} R_n} \sum_{i=1}^{n} \frac{X_i}{C_i} = \sum_{i=1}^{n} \left(\Delta N_{ch}\right)_i \qquad \text{rf or i-f repeaters}$$

In using either Eq. (19) or (20) to calculate the noise contribution of the i-th link, it is important to remember that R_{ch} is calculated for the baseband noise spectrum at the terminal receiver and not the spectrum that is or would be observed at the i-th repeater. Also, as already mentioned, the demodulator transfer characteristic used to calculate the noise due to the i-th link of an i-f repeater system is that of the terminal receiver. Thus it reflects the statistics of the wanted and unwanted signals at the terminal receiver input and not those of the signals at the i-th repeater input.

Just as it is possible to decompose the total output channel noise into contributions from the individual links of a radio relay chain, it is also possible to distinguish the contributions arising from different types of sources, such as thermal noise, distortion, intermodulation, and, of particular interest in this report, unwanted rf signals. Assuming all sources to be statistically independent, the procedure is to consider separately the power contributed by each type of source, and for each of these to define transfer characteristics as if they were the only source of input noise or unwanted signal. These transfer characteristics may then be substituted in Eq. (8) or Eq. (15) to compute the contribution of this type of source to the total noise in the output channel. Alternatively, they may be used with Eq. (19) or (20) to find the portion represented by this source in the contribution from a particular link.

THE EFFECTS OF SIGNAL FADING

If the wanted-to-unwanted signal ratio changes at the input to any repeater or at the terminal receiver because of variations in propagation or for any other reason, the output channel quality will change in a manner governed by Eqs. (8) and (15). Unless all of the input ratios change by the same amount, however, the change in the output quality will be smaller than the change in wanted-to-unwanted signal ratio for the most severely faded individual link.

For example, consider a 20-link chain of i-f repeaters where, in the absence of fading, all wanted-to-unwanted signal ratios are equal to 50 dB. Then from Eq. (9), with $10 \log n = 13$ and $10 \log C/X = 50$, the output quality with no fading is

$$10 \log \frac{S_{ch}}{N_{ch}} = 10 \log R_{ch}R_{20} + 37$$

If, on the other hand, there is a 30-dB fade in the wanted-to-unwanted signal ratio at the first repeater input only (i.e., if 10 log C_1/X_1 = 20 and 10 log C_i/X_i = 50, i = 2,...,20), then Eq. (8) gives

$$10 \log \frac{S_{ch}}{N_{ch}} = 10 \log R_{ch}R_{20} - 10 \log\left[\frac{1}{10^2} + 19 \frac{1}{10^5}\right]$$

$$= 10 \log R_{ch}R_{20} + 19.93$$

Thus, although there is a 30-dB fade on one link, the output signal-to-noise ratio changes by only slightly more than 17 dB (i.e., 37 dB versus 19.93 dB).

The example also illustrates another general point. When only one of a series of n normally identical links fades by F dB, the net fade of the output can be shown to be about (F - 10 log n)dB. Moreover, if F is significantly greater than 10 log n, the wanted-to-unwanted signal ratio at the terminal receiver input will be very nearly equal to that at the input to the repeater on the faded link.

From the viewpoint of where the noise arises, the situation can be described by saying that in the absence of fading, each link contributes the same amount of noise--namely, 1/n-th of the total. But, as may be seen from Eqs. (19) and (20), when one link fades by F dB, the noise contribution of that link increases by the same amount and, depending on the magnitude of F, it can easily dominate the contributions from the other links.

When several links can fade simultaneously, it is better to regard the output signal quality and the input wanted-to-unwanted signal ratios as random variables whose functional relationship is given by Eq. (8) or (15). If it can be assumed that the fading distribution is the same for each link and that there is no correlation between the fading on different links, the fading distribution at the input of the terminal receiver can be computed for an n-link relay chain as the n-fold convolution of the single-link distribution. An example is shown in Fig. 2, where the experimentally determined fading of the carrier-to-noise ratio for a single terrestrial relay link is used to predict the

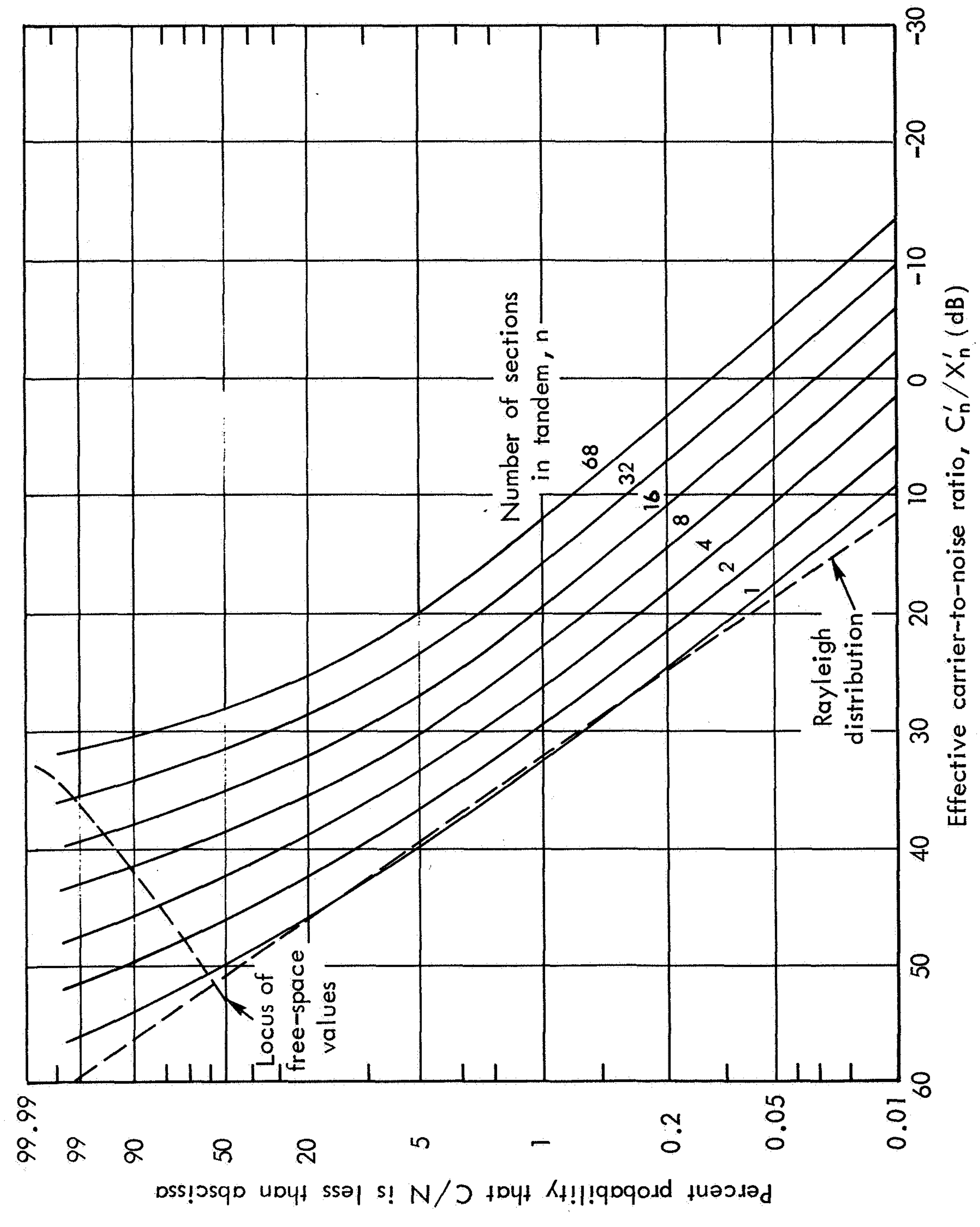


Fig.2—Example of fading in a terrestrial relay system

fading to be expected after 2, 4, 8, 16, 32, and 68 links, assuming no correlation of fading among the links.[2]

In a relay system subject to fading, the desired output signal quality is normally specified statistically (e.g., by giving the fraction of time that the output noise can be permitted to exceed certain levels). Carrier-to-noise distributions such as those in Fig. 2 are then essential tools for determining the single-link carrier-to-noise ratios required to meet output quality specifications. The ratio in dB between the median value of the single-link carrier-to-noise ratio and that value for which the output signal quality reaches its lowest acceptable value is sometimes called the fading margin. Signal fading will be discussed in further detail in Section IV.

DEMODULATOR TRANSFER CHARACTERISTICS AND OUTPUT NOISE SPECTRA

The demodulator transfer characteristic (DTC), defined in Eqs. (2) and (10), is simply the factor by which the baseband signal-to-noise ratio at the demodulator output exceeds the effective wanted-to-unwanted signal ratio at the receiver input. Alternatively, it may be regarded as the factor by which the output noise-to-signal objective must be multiplied to determine the maximum permissible unwanted-to-wanted signal ratio. As previously noted, the numerical value of the DTC depends not only on the nature of the demodulator, but also on the characteristics of the wanted and unwanted signals. This dependence will be described by choosing representative types of unwanted signal and by giving for each of them the DTC for particular combinations of wanted signal and demodulator. With these data it will then be possible to synthesize, for each of the latter combinations, the DTC corresponding to complex unwanted signals which are sums of the representative types.

The power spectrum of the output noise due to each type of unwanted rf signal will also be given, since it is needed to determine the channelizing transfer characteristic defined in Eq. (3) and discussed in detail in the next subsection.

Noise-Like Unwanted Signals

Undoubtedly the most widely analyzed type of unwanted signal is one that can be represented mathematically by additive white gaussian noise. The most familiar practical example is the thermal noise arising in the "front end" of a receiver, and indeed, an unwanted signal of this type is always present. But the analysis can be applied equally well to the case where the unwanted signal is the sum of signals received from a number of transmitters other than the one to which the receiver is "tuned." It is only necessary that the composite unwanted signal have a gaussian amplitude distribution and a more or less flat power spectrum across the passband of the receiver.

Expressions showing the way in which the DTC depends on the parameters of the wanted signal when the unwanted signal is white noise with a power density $N_o/2$ are given in column 1 of Table 1.* As in Appendix B, the parameter Λ describes the peak-to-average power ratio of the baseband that modulates the wanted signal. M, Φ, and D are the (peak) modulation indices for AM, PM, and FM respectively, and W/B is the bandwidth expansion ratio--i.e., the ratio of the bandwidth of the wanted rf signal to the bandwidth of the baseband. Expressions for the bandwidth expansion ratios developed in Appendix B are given in column 2 of the table.

For some types of modulation, the tabulated expressions for the DTC are valid only if the effective wanted-to-unwanted signal ratio exceeds the characteristic threshold value $(C'/X')_{min}$ shown in column 3 of Table 1. For values of C'/X' less than threshold, the actual output signal-to-noise ratio will be significantly worse than that implied by the DTC. The threshold values shown are those for which the actual S/N is only 0.5 dB smaller than they would be using the DTC.

The expressions in column 4 of the table give the output noise power spectral density $W_n(f)$ for above-threshold operation. The demodulators are assumed to have unit sensitivity and to be operating into a unit load--e.g., an FM discriminator with a sensitivity of 1 V/Hz

*This is the density of the "two-sided" noise spectrum. The total noise power in an rf bandwidth W is N_oW.

Table 1

DEMODULATOR TRANSFER CHARACTERISTIC
FOR WHITE NOISE OR NOISE-LIKE INTERFERENCE

Modulation Method	1 Demodulator Transfer Characteristic $\frac{S/N}{C'/X'}$	2 Bandwidth Expansion Ratio W/B	3 Threshold Condition $(C'/X')_{min}$	4 Output Noise Power Spectrum $W_n(f)$	5 Output Signal Power S
AM	$\frac{2}{1 + \Lambda/M^2}$	2		N_o	$C' - C_o'$
DSB	2	2		N_o	C'
SSB	1	1		$\frac{1}{2} N_o$	$\frac{1}{2} C'$
PM	$\frac{\Phi^2}{\Lambda} \frac{W}{B}$	$2(\Phi/\sqrt{3} + 1)$ (gaussian baseband)	$\frac{36\Phi^{\frac{1}{2}}}{W/B}$	N_o/C'	Φ_r^2
FM	$3 \frac{D^2}{\Lambda} \frac{W}{B}$	$2(D + 1)$	$\frac{2\sqrt{10}(\frac{D}{F} + 1)^{3/2}}{W/B}$ [a]	$f^2 N_o/C'$	f_r^2

[a] With maximum feedback, the expression for the FM threshold may be approximated by $3.5\ D^{-0.45}$.

and a 1-Ω load. For comparison, column 5 gives the output signal power for the same demodulator sensitivities.

A quick inspection of the entries in Table 1 will confirm that the results for the amplitude modulation methods (AM, DSB, SSB) are much simpler than those for the angle modulation methods (PM, FM). Indeed, for the former, all entries are constant except in the case of the DTC for AM, where the term Λ/M^2 represents the contribution of the spectral line at carrier frequency to the total average power of the modulated carrier.

The entries for the angle modulation methods are somewhat more interesting. To begin with, observe that the threshold levels shown in column 3 are functions of modulation index. In particular, the expression for PM applies to a phase-lock loop receiver, while that for FM may be applied either to a feedback FM receiver with feedback factor F or to a conventional FM receiver (F = 1). With a feedback FM receiver, however, there are theoretical and practical upper limits to the amount of feedback that can be used. But since the maximum value of F itself depends on the FM modulation index D, the ultimate threshold for FM can be given as function of D alone. The result shown in the footnote to Table 1 was obtained[3] using Enloe's two-threshold theory[4] with both thresholds dependent on modulation index. Another result, based on Enloe's theory but with the simplifying assumption of constant 10-dB thresholds, is[5]

$$\left(\frac{C'}{X'}\right)_{min} = 5\pi \frac{D^{1/3} + 1}{D + 1} \qquad \text{FM threshold with maximum feedback} \qquad (21)$$

The variable-threshold expression yields feedback FM thresholds which range from about -1 dB for D = 10 to -3.5 dB for D = 100. For the same values of D, the expression in Eq. (21) gives thresholds of about +6.5 dB and -1.5 dB respectively.

Providing that the threshold condition is met, the DTC for PM and FM is seen from columns 1 and 2 in Table 1 to be given by a cubic in the modulation index. In the case of FM, for example, the DTC is

$$R = \frac{6}{\Lambda} D^2(D + 1) \qquad \text{FM} \tag{22}$$

This relation is plotted in Fig. 3, where it should be noted that the maximum usable value of the DTC depends on the desired value of output signal-to-noise ratio and on the amount of feedback used, as indicated by the scales on the curve. In labeling the scale for the DTC, a baseband peak-to-average power ratio Λ of 10 was assumed. For larger values of Λ, the quantity 10 log Λ - 10 dB should be subtracted from the DTC indicated by the curve.

To illustrate the way in which Fig. 3 is used, consider conventional FM with D = 8.5. As long as C'/X' exceeds the conventional threshold of 9.5 dB, S/N will exceed C'/X' by 26.5 dB for all values of S/N higher than 36 dB, the output signal-to-noise ratio corresponding to threshold operation at this modulation index. With maximum feedback, a DTC of 26.5 dB can be achieved for an S/N as low as 33 dB, corresponding to a 6.5 dB threshold.

For a given value of S/N, it is clear that the minimum permissible wanted-to-unwanted signal ratio C'/X' can be decreased (DTC increased) to a point determined by the threshold by merely increasing the modulation index and hence the bandwidth expansion ratio. The nature of this tradeoff between rf bandwidth and the wanted-to-unwanted signal ratio required to achieve a given S/N is displayed in a different fashion by Figs. 4 and 5. These figures differ from each other only in that Fig. 4 shows C'/X' versus <u>rms</u> modulation index for constant values of <u>average</u> baseband signal-to-noise ratio S/N, whereas Fig. 5 gives C'/X' versus <u>peak</u> modulation index for constant values of the <u>peak</u> baseband signal-to-noise ratio ΛS/N. The latter curves are better suited to planning FM systems for TV transmission, since TV signal quality objectives are usually specified in terms of peak baseband power. In both figures, the auxiliary bandwidth scale was constructed using Carson's rule; for Fig. 5, a value Λ = 10 was again assumed.

The dashed curves in Figs. 4 and 5 show the performance possible through the use of threshold-extending feedback FM receivers. The

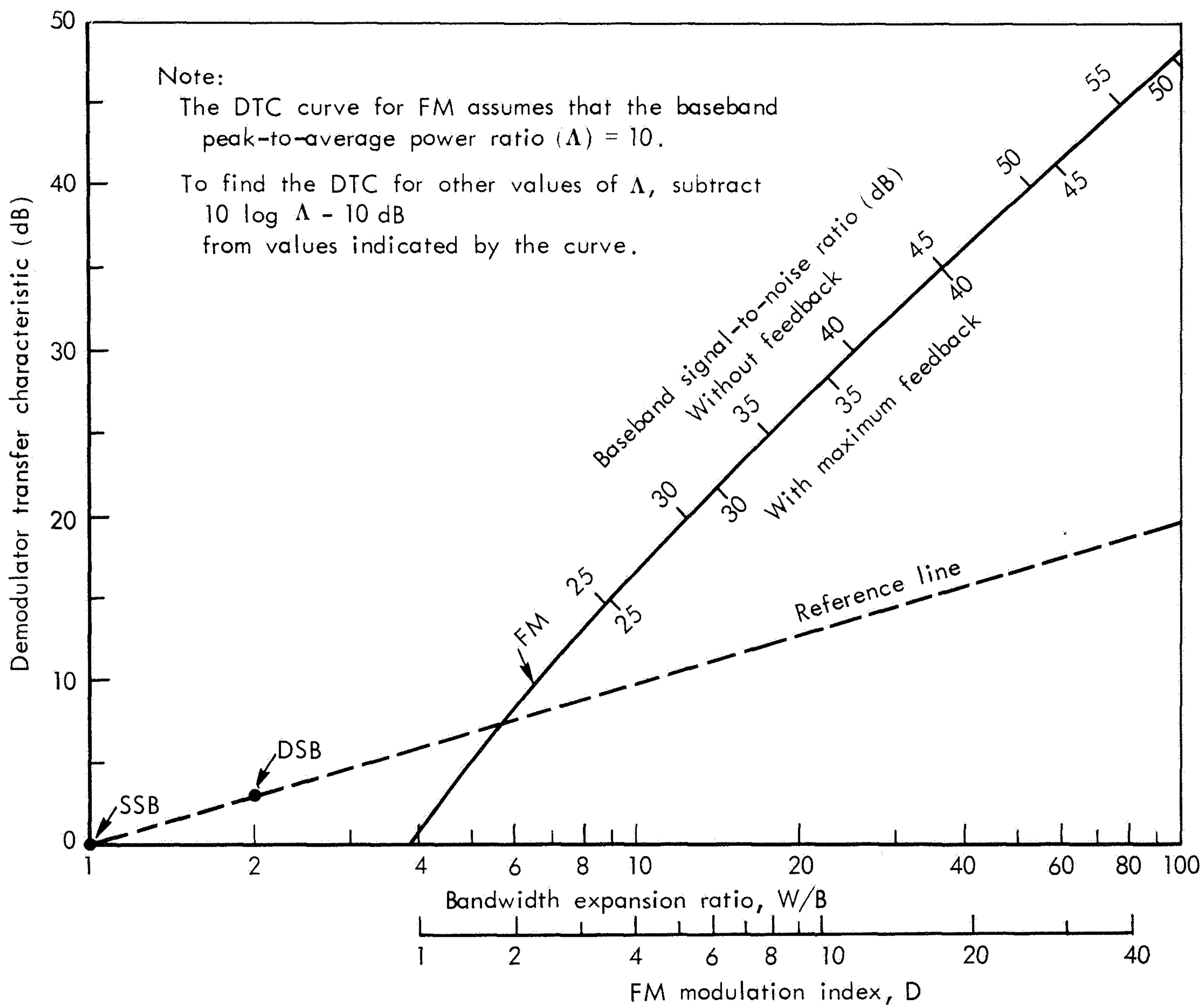


Fig.3 — Demodulator transfer characteristic for white noise unwanted signal

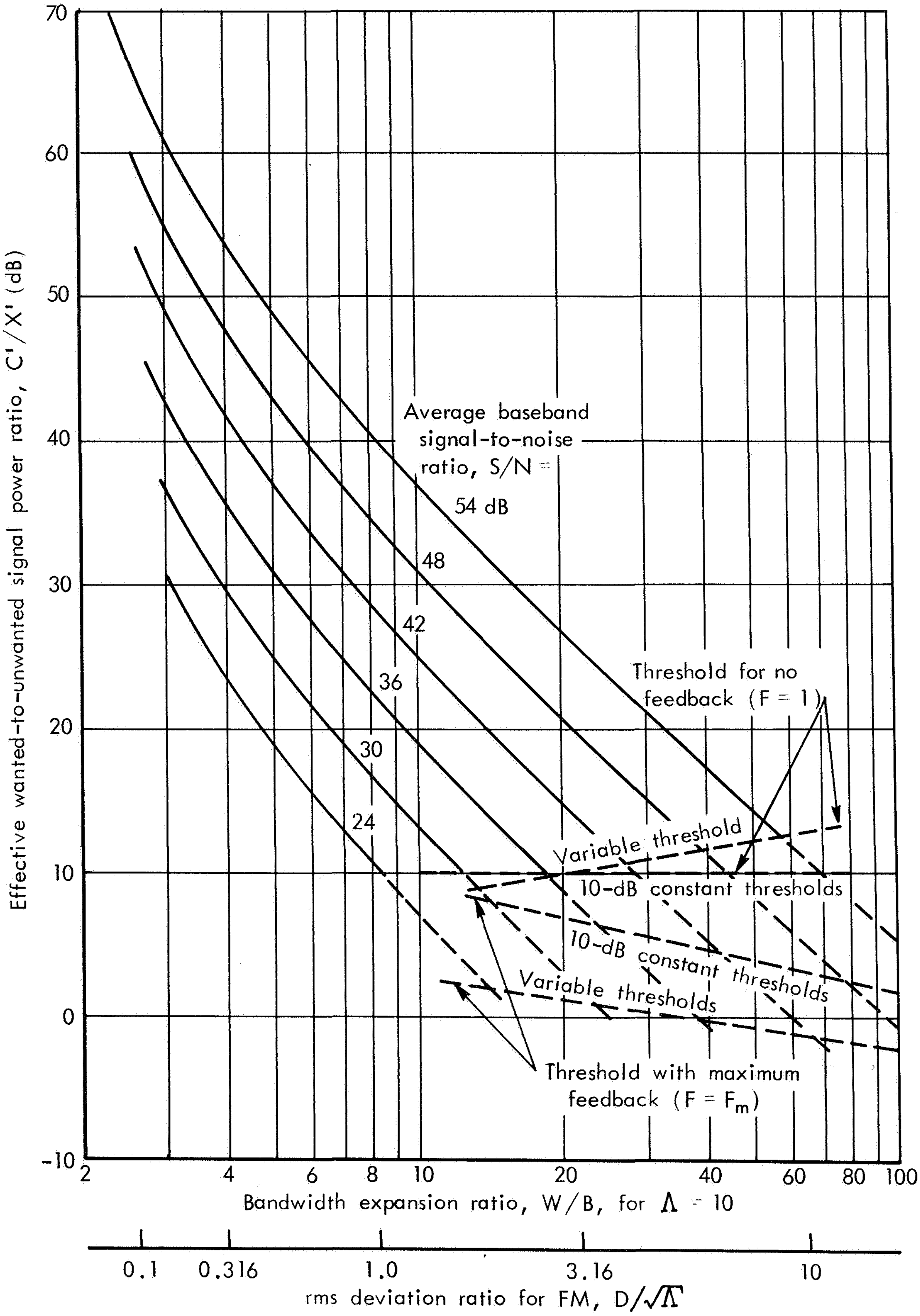


Fig. 4—FM power/bandwidth tradeoffs for specified average baseband signal-to-noise ratios

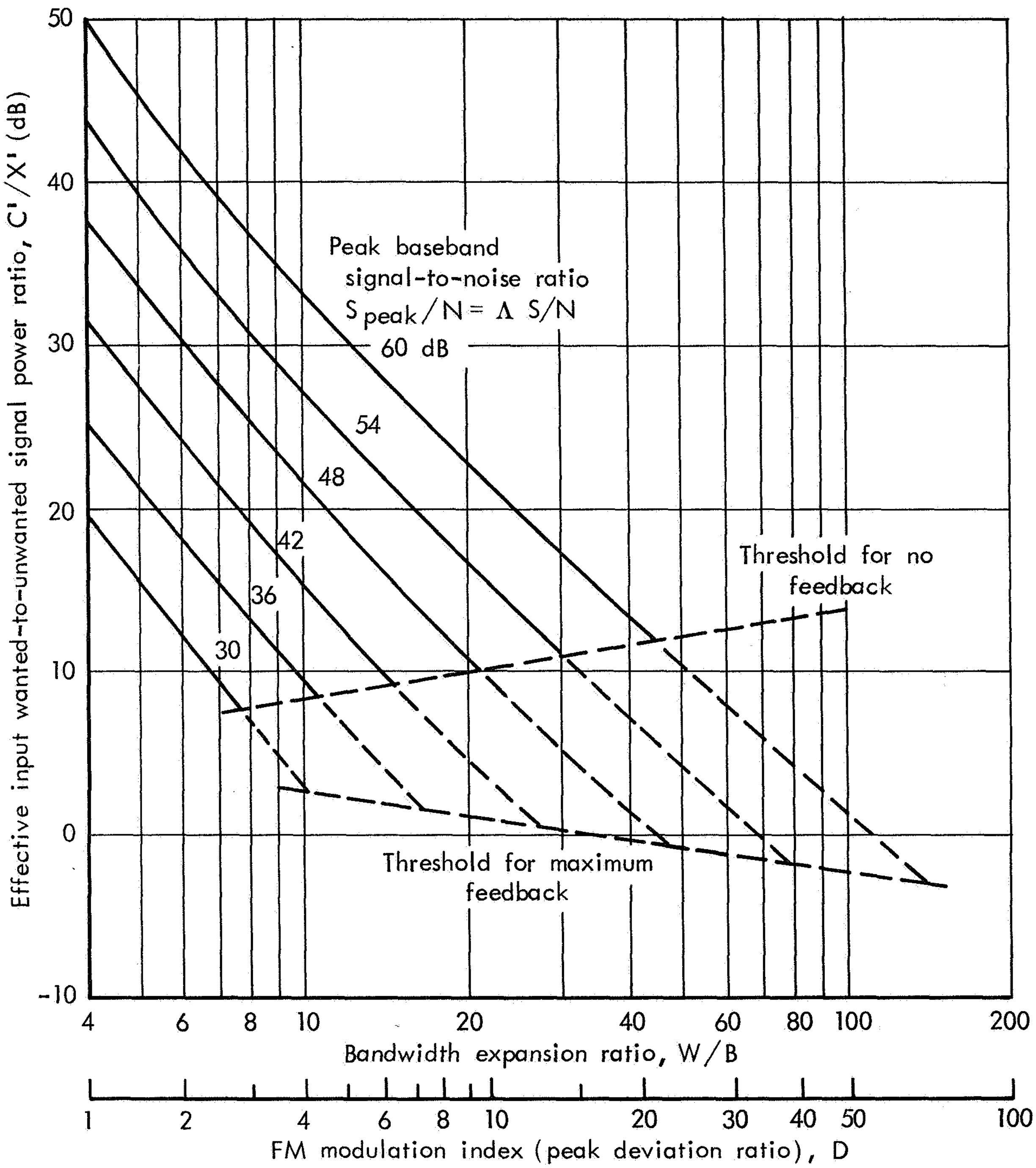


Fig.5—FM power/bandwidth tradeoffs for specified peak baseband signal-to-noise ratios

amount of feedback is indicated by the feedback factor F, where $F = 1$ corresponds to zero feedback or conventional FM operation and $F = F_m$ to the use of the maximum practical amount of feedback. Two curves are given in Fig. 4 for each of these limiting cases: one corresponds to the threshold values shown in Table 1, the other to the assumption of a constant 10-dB threshold for zero feedback and to the threshold given by Eq. (21) for maximum feedback.

For later reference, it will be noted from column 4 of Table 1 that the output noise power spectrum $W_n(f)$ is flat (independent of f) for PM but is parabolic (proportional to f^2) for FM.

Unmodulated Unwanted Signal

Probably the simplest type of interference to analyze is that originating from a single unmodulated sinusoidal carrier. Let the wanted and unwanted signals, each normalized to the amplitude of the unmodulated wanted signal, be respectively

$$c(t) = a(t)\cos[\omega_c t + \varphi(t)] \tag{23}$$

$$x(t) = r \cos \omega_x t = r \cos(\omega_c + \omega_D)t \tag{24}$$

where $r \ll 1$ is the relative amplitude of the unwanted carrier, $\omega_D = \omega_x - \omega_c$ is the difference between the unwanted and wanted carrier frequencies, and the dependence of $a(t)$ and $\varphi(t)$ on the baseband $b(t)$ is determined by the modulation method used with the wanted signal as shown in Table B-1. Note that, due to the normalization of $x(t)$, $r = \sqrt{X_o/C_o}$, where X_o and C_o are respectively the powers of the unnormalized unwanted and wanted carriers in the absence of modulation.

For all of the amplitude modulation methods, the waveform at the demodulator output is easily shown to be

$$y(t) = Mb(t) + n(t) \tag{25}$$

where M is the modulation index, and

$$n(t) = r \cos \omega_D t \qquad \text{AM, DSB, SSB} \tag{26}$$

is an interference noise term due to the presence of the unmodulated rf carrier at the receiver input. The power spectrum of the output interference noise is thus simply a line at the carrier difference frequency $f_D = \omega_D/2\pi$

$$W_n(f) = \frac{1}{4} r^2 \left[\delta(f_D) + \delta(-f_D)\right] \qquad \text{AM, DSB, SSB} \tag{27}$$

If f_D is less than the baseband bandwidth B, all of the interference will fall in the baseband and, for multichannel basebands, will be concentrated in the channel which includes the frequency f_D. The effect on this channel will depend on the magnitude of r and on the fraction of wanted signal power that falls in the affected channel, but unless r is very small the effect is likely to be severe. If f_D changes in time due to carrier frequency drift, other channels can be affected one at a time.

If the wanted signal uses PM with modulation index Φ, the waveform at the demodulator output is

$$y(t) = \Phi\, b(t) + n(t) \qquad \text{PM} \tag{28}$$

where now the interference noise waveform is given by

$$n(t) = r \sin[\omega_D t - \varphi(t)] \qquad \text{PM} \tag{29}$$

Comparison of Eq. (23) with Eq. (29) after straightforward Fourier expansion of the phase modulation $\varphi(t) = \Phi\, b(t)$ permits the power spectrum of the interference to be expressed in terms of the normalized low-pass version of the one-sided power spectrum* $w_c(f)$ of the wanted carrier[6,7]

*This concept is defined in Appendix B.

$$W_n(f) = \frac{1}{4} r^2 \left[w_c(f_D + f) + w_c(f_D - f) \right] \qquad \text{PM} \qquad (30)$$

Thus, in contrast to the amplitude modulation methods, the effect of a small unmodulated unwanted carrier on a phase modulated signal is to create an output interference spectrum whose shape and magnitude are determined not only by the unwanted-to-wanted signal ratio $X/C = X_o/C_o = r^2$ and the carrier frequency difference f_D, but also by the nature of the wanted carrier spectrum within the rf frequency band $f_x \pm B$ as expressed by its low-pass version $w_c(f)$ in the range $f_D \pm B$. If the wanted carrier spectrum is continuous, the interference spectrum will be also. If there is an appreciable carrier component in the wanted carrier spectrum, as there will be for low index PM, and if $f_D < B$, there will be a corresponding line in the interference spectrum at baseband frequency f_D. For a given wanted-to-unwanted signal ratio $C/X = r^{-2}$, it is apparent from Eq. (30) that the output interference $W_n(f)$ will in general be reduced by increasing the modulation index and hence the rf bandwidth of the wanted signal, since this of necessity reduces $w_c(f)$.

This conclusion may be stated quantitatively by writing an expression for the demodulator transfer characteristic. Noting from Eq. (28) that the output baseband signal power is given by

$$S = \Phi^2 \overline{b^2(t)} = \Phi^2/\Lambda \equiv \Phi_r^2$$

and from Eq. (30) that

$$N = 2 \int_0^B W_n(f) = \frac{1}{2} r^2 \int_0^B \left[w_c(f_D + f) + w_c(f_D - f) \right] df$$

the desired expression is

$$R = r^2 \, S/N = \frac{2\Phi_r^2}{\int_0^B [w_c(f_D + f) + w_c(f_D - f)] df} \qquad \text{PM} \qquad (31)$$

Referring to the expressions for $W_c(f)$ in Table B-3, it is obvious that increasing the PM modulation index increases the DTC both by increasing the numerator and by reducing the denominator in Eq. (31).

When the wanted signal is frequency modulated, the interference spectrum originating from an unmodulated rf carrier can be obtained from Eq. (30) for PM by multiplying the right side by f^2.

$$W_n(f) = \frac{1}{4} r^2 f^2 \left[w_c(f_D + f) + w_c(f_D - f) \right] \qquad \text{FM} \tag{32}$$

With this modification, all other remarks about the nature of the interference spectrum and the effect on interference and the DTC of increasing the modulation index apply with equal force to FM.

Arbitrary Unwanted Signal

Expressions for the output interference power spectrum and the demodulator transfer characteristic for the case in which the unwanted signal is modulated may be derived from the results just presented for interference by an unmodulated carrier. The wanted and unwanted signals are assumed to be uncorrelated and the latter is regarded as a discrete spectrum of unmodulated carriers whose amplitudes depend on frequency in such a way as to approximate the power spectrum of the unwanted signal. For example, the portion of the unwanted signal whose power lies in the infinitesimal frequency range f to f + df is represented by a sinusoid of frequency f and amplitude A such that

$$A^2/2 = W_x(f)df \tag{33}$$

where $W_x(f)$ is the power spectrum of the unwanted signal. The total output interference is then obtained by integrating the contributions from each of the sinusoidal components.

When the wanted signal is amplitude modulated, the demodulator merely translates both wanted and unwanted frequency components towards zero frequency by an amount equal to the wanted carrier frequency f_c. As a result, the interference spectrum is simply

$$W_n(f) = \frac{1}{4} r^2 \frac{X}{X_o} \left[w_x(f - f_D) + w_x(-f - f_D) \right] \qquad \text{AM, DSB, SSB} \qquad (34)$$

where $w_x(f)$ is the normalized low-pass version of the one-sided power spectrum of the unwanted signal (truncated to include only those components which lie within the wanted signal passband) and $r^2 = X_o/C_o$ is the ratio of unwanted to wanted signal power in the absence of modulation.

It is apparent that for a given wanted-to-unwanted signal ratio, the DTC will depend on the difference f_D between the carrier frequencies of the wanted and unwanted signals and on the relative extent of their spectra. In particular, if the unwanted signal is angle modulated, the overall interference it causes will decrease as its modulation index is increased.

When the wanted signal is phase modulated, each spectral component of the unwanted signal will give rise to an output spectrum of the form given by Eq. (30). In that equation, r^2 represented the power of the unmodulated unwanted carrier relative to that of the wanted carrier. Thus, in applying it to compute the interference contribution of the modulated unwanted carrier components in an infinitesimal frequency interval dp at frequency $f_c + p$, r^2 should be replaced by the ratio of the power represented by the unwanted components in that interval to the wanted carrier power--viz.,

$$r^2 \frac{X}{X_o} w_x(p - f_D) dp$$

Here again, $w_x(f)$ is the normalized low-pass version of the truncated one-sided power spectrum of the unwanted signal and $r^2 = X_o/C_o$ is the ratio of the total unwanted signal power to that of the wanted signal. With this replacement, the output interference noise at baseband frequency f due to the components of the unwanted signal in the band of width dp at frequency $f_c + p$ is

$$dW_n(f) = \frac{1}{4} \frac{X}{C_o} w_x(p - f_D) \left[w_c(p + f) + w_c(p - f) \right] dp$$

The total interference power spectrum at frequency f is then obtained by integrating over all frequencies within the rf passband $f_c \pm W/2$ of the wanted signal. This is equivalent to integrating the expression for $dW_n(f)$ over the range $-W/2 \leq p \leq W/2$

$$W_n(f) = \frac{1}{4}\frac{X}{C_o}\int_{-W/2}^{W/2}\left[w_c(p + f) + w_c(p - f)\right]w_x(p - f_D)\,dp \qquad \text{PM} \qquad (35)$$

From this, it is clear that an alternative expression for the output interference noise spectrum is

$$W_n(f) = \frac{1}{4}\frac{X}{C_o}\left[W(f - f_D) + W(f + f_D)\right] \qquad \text{PM} \qquad (36)$$

where W(f) is the convolution* of the normalized low-pass versions of the one-sided power spectra of the unwanted and wanted signals, the latter reflected about zero frequency and the former truncated to include only components within the wanted signal passband

$$W(f) = w_x(f) * w_c(-f) \qquad (37)$$

Since no assumption was made about the modulation used with the unwanted signal, Eq. (35) or (36) applies equally well to both amplitude and angle modulated unwanted signals. In particular, when applied to an unwanted signal whose power spectrum is constant over the wanted signal passband, the formulas yield the results for white gaussian noise displayed for PM in Table 1.

If neither the wanted nor the unwanted signal has an appreciable carrier frequency component, the interference noise spectrum $W_n(f)$ will be continuous. If either or both do have significant carrier components, the effect may be seen by substituting

---

*The convolution of two functions f(x) and g(x) is defined as the integral

$$f(x) * g(x) = \int_{-\infty}^{\infty} f(u)g(x - u)\,du$$

$$w_c(f) = k_1\delta(f) + w_1(f)$$

$$w_x(f) = k_2\delta(f) + w_2(f) \tag{38}$$

into Eq. (35). The result is*

$$W_n(f) = \frac{1}{4}\frac{X}{C_o}\left\{\int_{-W/2}^{W/2}\left[w_1(p + f) + w_1(p - f)\right]w_2(p - f_D)dp\right.$$

$$+ k_1\left[w_2(f - f_D) + w_2(f + f_D)\right] + k_2\left[w_1(f - f_D) + w_1(f + f_D)\right]$$

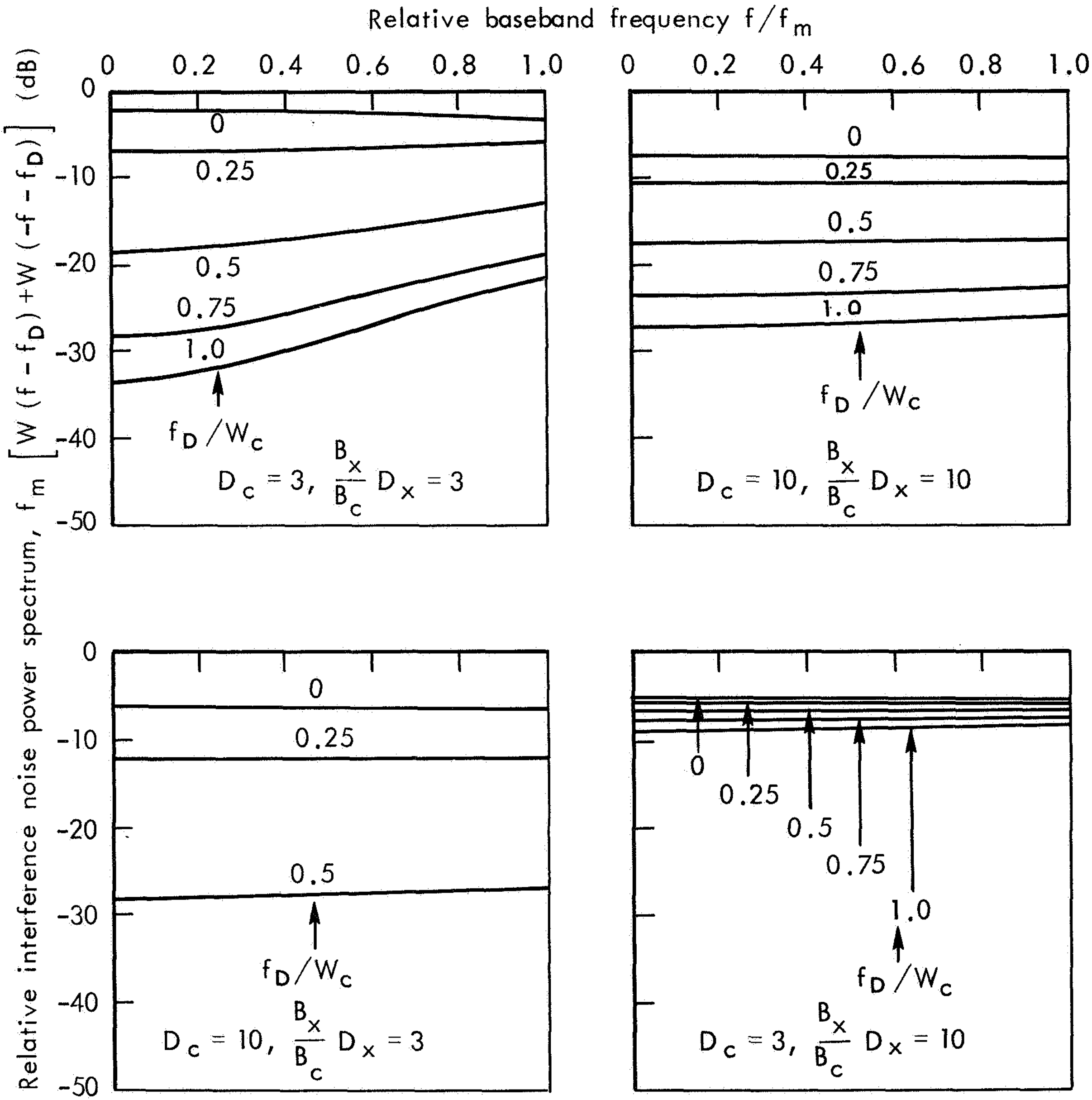
$$\left. + k_1k_2\delta(f - f_D)\right\} \tag{39}$$

where the constants k_1 and k_2 indicate the fractions of wanted and unwanted signal power residing in the carriers, respectively. Of the three terms following the integral, the last two exist only if the carrier frequency difference f_D is less than W/2, the half bandwidth of the wanted rf signal. All terms but the last represent continuous spectra; the last represents a line at frequency f_D and will affect the baseband only if $f_D < B$, the baseband bandwidth.

Equations (35), (36), and (39) describe the interference noise spectrum at the output of a phase demodulator caused by an arbitrary unwanted signal at its input. Corresponding expressions for a frequency demodulator may be obtained by simply multiplying the right sides of these equations by f^2.

Values of the convolution expressed by Eq. (37) have been computed and plotted for carriers which have been frequency modulated by FDM equivalent noise basebands using modulation indices characteristic of present terrestrial and satellite relay system practice.[9,10] Examples of interference noise power spectra obtained by direct numerical integration of Eq. (35) are shown in Fig. 6 for wideband FM systems. However,

*This differs from Medhurst's "general formula"[8] by a factor of 2 since Medhurst considers a one-sided or trigonometric spectrum in place of the two-sided exponential spectrum considered here.



B_c, B_x = baseband bandwidths of wanted and unwanted signal

D_c, D_x = FM modulation indices of wanted and unwanted signal

f_m = highest baseband frequency of wanted signal

W_c = rf bandwidth of wanted signal

$W(f) = w_x(f) * w_c(-f)$

Fig. 6—Examples of output noise power spectra for interference between wideband FM signals

since the shapes of these spectra are dependent on the shapes of the wanted and unwanted rf signal spectra as well as to the carrier frequency difference, no attempt will be made to compute demodulator transfer characteristics as such. Instead, the output interference spectra will be used directly to compute values for the overall receiver transfer characteristic to be discussed next.

CHANNELIZING AND OVERALL RECEIVER TRANSFER CHARACTERISTICS

The CTC was defined in Eq. (3) as the factor by which the signal-to-noise ratio at the output of a selected message channel exceeds the effective baseband signal-to-noise ratio at the demodulator output. For that channel, it takes into account not only the demultiplexing or channelizing operation, but also the effect of any signal processing to which either the baseband or the single-channel message may have been subjected following demodulation. In addition, the CTC must allow for the fact that in the channel output signal-to-noise ratio which appears in its definition, the signal power may refer to a special test signal whose power level and statistical characteristics are quite different from the message signals that comprise the baseband, and that the noise power is usually weighted to account for the subjective difference in the annoying effect of noise at different frequencies within the channel.

For these reasons, the CTC usually differs from unity even in those cases in which the baseband consists of only a single channel, as it often does with television. In the more common situation of a multichannel baseband, the CTC is normally different from one channel to another--i.e., its value is dependent on the baseband frequency at which the channel is located.

If the baseband signal recovered by the terminal receiver has not been corrupted by noise due to previous demodulation in baseband-type repeaters, the overall signal-to-noise performance of the receiving terminal is given by the product of the DTC and the CTC. This product is the factor by which the signal-to-noise ratio at the output of a selected channel exceeds the wanted-to-unwanted signal ratio at the receiver input. It is usually called the receiver transfer characteristic (RTC), or interference reduction factor. Like the CTC, its value will normally be different for different channels. In this connection, it should be noted that some authors choose to define "the" RTC as the overall transfer characteristic for the "worst" channel of the baseband--i.e., the channel for which the RTC has its smallest value. In some cases, such as those in which the unwanted signal is a single modulated carrier, no special purpose is served by computing the DTC and the CTC separately; in these cases the RTC will be computed directly from the output interference noise spectrum, $W_n(f)$.

Telephone Basebands

With the background presented in Appendix C, it is easy to derive expressions for the CTC and RTC corresponding to any channel in an n-channel FDM telephone baseband transmitted by any given analog modulation method in the face of a small but arbitrary unwanted signal. From the definition given in Eq. (3), the CTC in dB can be written

$$\text{CTC} = 10 \log R_{ch} = 10 \log(S_{ch}/S) - 10 \log(N_{ch}/N) \tag{40}$$

The test-tone power S_{ch} and the busy hour average baseband power S, each expressed in mW at a point of zero relative level, are given by Eqs. (C-8) and (A-2), respectively. Combining these equations,

$$10 \log \frac{S_{ch}}{S} = \begin{cases} 15 - 10 \log n & n \geq 240 \\ 1 - 4 \log n & 12 \leq n < 240 \end{cases} \tag{41}$$

The weighted channel noise N_{ch} is given by Eq. (C-10), and the total noise in the baseband bandwidth at the demodulator output by

$$N = 2 \int_{f_\ell}^{f_m} W_n(f) df = 2 \overline{W}_n B \tag{42}$$

where f_ℓ and f_m are the frequency limits of the baseband and $\overline{W}_n$ is the value of the output noise power spectrum $W_n(f)$ averaged over the baseband bandwidth $B = f_m - f_\ell$. Combining Eqs. (C-10) and (42) gives for a preemphasized baseband,

$$10 \log \frac{N_{ch}}{N} = 10 \log \frac{W_n(f_{ch})}{\overline{W}_n} + 10 \log \frac{B_{ch}}{B}$$

$$+ 10 \log \overline{G}_N + 10 \log G_{Do}(f_{ch}) \tag{43}$$

Except for the deletion of the term involving $G_{Do}(f_{ch})$, the same expression applies when preemphasis is not used. To express this in terms of the number of channels n, note from Eq. (A-2) that, since B_{ch} = 3.1 kHz,

$$10 \log \frac{B_{ch}}{B} \cong - 1.6 - 10 \log n \tag{44}$$

With this substitution, Eq. (43) and Eq. (41) combine with Eq. (40) to yield general expressions for the channelizing transfer characteristic for a preemphasized n-channel FDM telephone baseband

$$\mathrm{CTC} = \begin{cases} 16.6 - 10 \log \overline{G}_N - 10 \log \dfrac{W_n(f_{ch})}{\overline{W}_n} \\ \qquad + 10 \log G_{Do}(f_{ch}) & n > 240 \\ 2.6 - 10 \log \overline{G}_N - 10 \log \dfrac{W_n(f_{ch})}{\overline{W}_n} + 6 \log n \\ \qquad + 10 \log G_{Do}(f) & 12 < n < 240 \end{cases} \tag{45}$$

subject only to the condition that $W_n(f)$ varies slowly over the bandwidth B_{ch} of the channel in question. When preemphasis is not used, the CTC is given by Eq. (45) with the term in $G_{Do}(f_{ch})$ omitted. For use in Eq. (45), 10 log $\overline{G}_N$ is given for various types of weighting by Table C-1, and 10 log $G_{Do}(f_{ch})$ can be read from Fig. C-2. It remains only to calculate the value of the term involving $W_n(f)$ for the modulation method and type of unwanted signal of interest.

White Noise Unwanted Signal. When the unwanted rf signal has the characteristics of white noise, $W_n(f)$ is given by column 4 of Table 1, and hence

$$10 \log \frac{W_n(f_{ch})}{\overline{W}_n} = \begin{cases} 0 & \text{AM, DSB, SSB, PM} \\ 4.8 + 20 \log \dfrac{f_{ch}}{f_m} & \text{FM} \end{cases} \tag{46}$$

where, in writing the expression for FM, terms in f_ℓ/f_m were neglected. Substituting Eq. (46) into Eq. (45) yields the CTC for the channel at frequency f_{ch} in an n-channel FDM telephone baseband when the unwanted signal is white noise. With psophometric weighting, the result for PM and for the amplitude modulation methods is the same for all channels:

$$\text{CTC} = \begin{cases} 19.1 & n > 240 \\ 5.1 + 6 \log n & 12 < n < 240 \end{cases} \quad \begin{array}{l} \text{FDM telephony} \\ \text{white noise un-} \\ \text{wanted signal,} \\ \text{AM, DSB,} \\ \text{SSB, PM} \end{array} \tag{47}$$

It is instructive to note the way in which the nominal 19-dB difference between the channel signal-to-noise ratio and the baseband signal-to-noise ratio arises when n is large. To begin with, the weighted channel noise is about 3.5 dB less than would be the case if the total baseband noise were simply divided by the number of channels. Of this, 2.5 dB represents the effect of noise weighting and 1 dB the difference between the channel bandwidth and the average channel spacing. On the other hand, the channel test-tone power is about 16 dB higher than it would be if the total busy hour baseband signal power were divided equally among the channels. Of this, about 10 dB represents the ratio of test-tone power to the average talker power and the remaining 6 dB accounts for the fact that on the average only a quarter of the channels are active--i.e., carrying speech power.

For FM, the result of combining Eqs. (45) and (46) depends on the location of the channel within the baseband and on whether or not preemphasis is used. The case of greatest interest is the worst or top channel $f_{ch} \cong f_m$, for which Eq. (46) gives $10 \log [W_n(f_{ch})/\overline{W}_n] = 4.8$, and Eq. (C-6) gives $10 \log G_{Do}(f_{ch}) = -4.0$. Hence, in this case,

$$\mathrm{CTC} = \begin{cases} \left.\begin{array}{ll} 14.3 & n > 240 \\ 0.3 + 6 \log n & 12 < n < 240 \end{array}\right\} & \text{FM with no preemphasis} \\ \\ \left.\begin{array}{ll} 18.3 & n > 240 \\ 4.3 + 6 \log n & 12 < n < 240 \end{array}\right\} & \text{FM with preemphasis} \end{cases} \tag{48}$$

The foregoing expressions for the CTC with white noise are plotted in Fig. 7. The discontinuity in slope at n = 240 is a consequence of a similar discontinuity in the baseband average power curve shown in Fig. A-2.

Expressed in dB, the overall receiver transfer characteristic is given by

$$\mathrm{RTC} = \mathrm{DTC} + \mathrm{CTC} \tag{49}$$

where DTC is the demodulator transfer characteristic. For a white noise unwanted signal, the DTC is given in column 1 of Table 1. These expressions may be combined with those of Eqs. (47) and (48) to obtain the RTC. For example, in the case of FM with preemphasis, the RTC for the top channel, psophometrically weighted, is

$$\mathrm{RTC} = \begin{cases} 23.1 + 10 \log \frac{W}{B} + 20 \log D_r & n > 240 \\ \\ 9.1 + 6 \log n + 10 \log \frac{W}{B} & \\ \qquad + 20 \log D_r & 12 < n < 240 \end{cases} \quad \begin{array}{l} \text{FDM telephony,} \\ \text{white noise unwanted signal,} \\ \text{FM with preemphasis} \end{array} \tag{50}$$

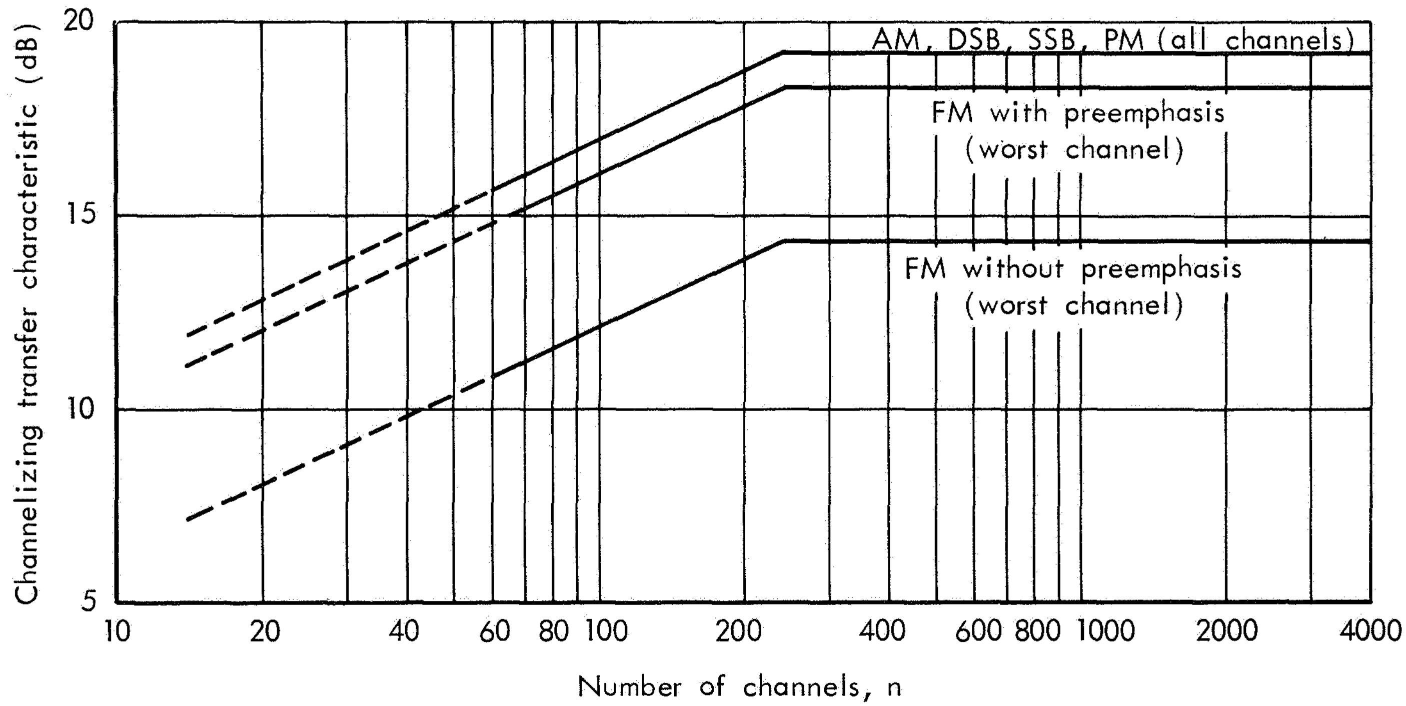


Fig.7—Channelizing transfer characteristic for white noise interference, CCIR busy hour load, and psophometric weighting

An alternative general equation for this case, utilizing different parameters, may be obtained by first expressing the ratio S_{ch}/S in terms of rms frequency deviations:

$$10 \log S_{ch}/S = 20 \log(f_{rch}/f_r) \tag{51}$$

When this is substituted with Eq. (43) into Eq. (40) and the resultant expression for the CTC is added to the DTC for FM given in Table 1, the end product for a psophometrically weighted voice channel at baseband frequency f_{ch} is

$$\begin{aligned} RTC = 2.5 - 10 \log G_{Do}(f_{ch}) &+ 20 \log \frac{f_{rch}}{f_{ch}} \\ &+ 10 \log \frac{W}{B_{ch}} \end{aligned} \qquad \text{FDM telephony, white noise unwanted signal, FM with preemphasis} \tag{52}$$

The CCIR recommended values of f_{rch}, the rms frequency deviation due to a channel test tone, are given in Table A-1 for terrestrial radio relay systems.

For most purposes, however, the RTC for white noise interference can be obtained with sufficient accuracy simply by adding the values of DTC and CTC read from the curves of Figs. 3 and 7 respectively.

Arbitrary Unwanted Signal. When the unwanted signal is not white noise, the CTC is still given by Eq. (45) except that $W_n(f)$ is now given in terms of the wanted and unwanted signal spectra by expressions such as Eq. (34) for the amplitude modulation methods and Eq. (36) or Eq. (39) for phase modulation. Having computed the CTC, the RTC may then be found by adding to it the DTC in accordance with Eq. (49). But the DTC now also depends on the shape of the output noise spectrum $W_n(f)$, and it is just as simple to compute the RTC directly.

In the case of the amplitude modulation methods, it is easy to show that the RTC with psophometric weighting is given for SSB modulation by

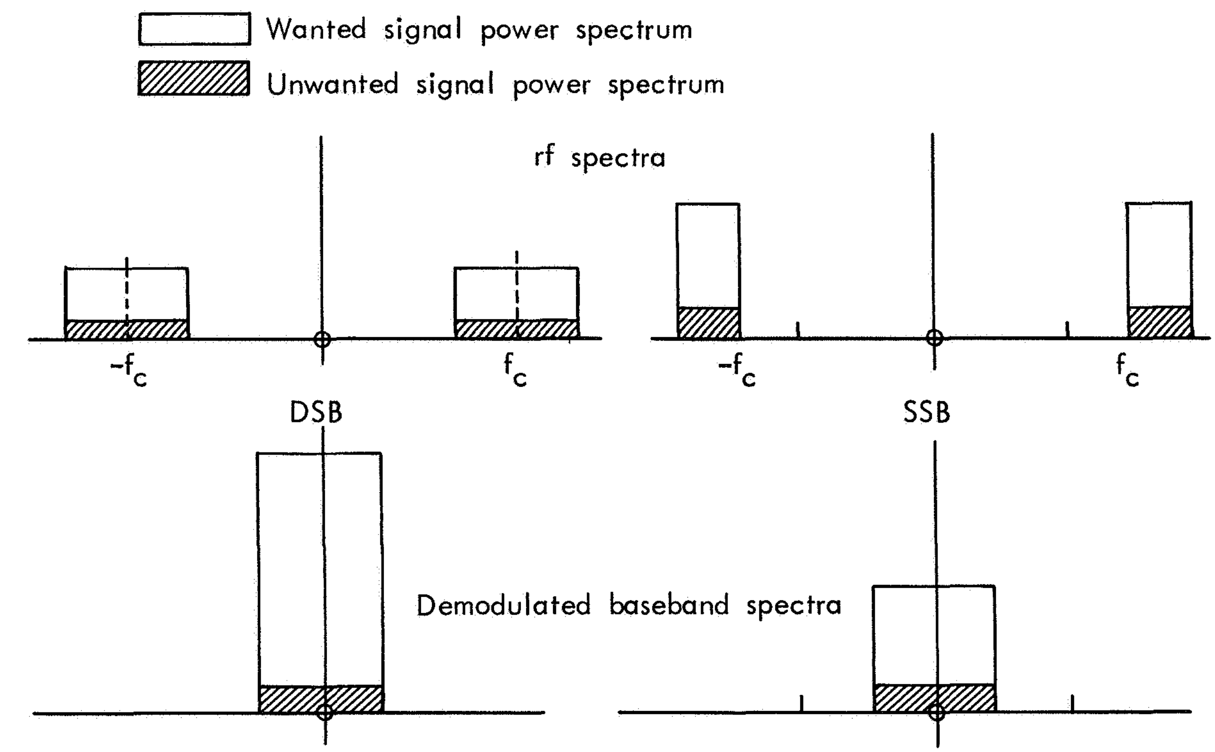
$$
\mathrm{RTC} = \begin{cases} 19.1 - 10 \log \dfrac{W_n(f_{ch})}{\overline{W}_n} & n > 240 \\[2ex] 5.1 + 6 \log n & \\ \qquad - 10 \log \dfrac{W_n(f_{ch})}{\overline{W}_n} & 12 < n < 240 \end{cases} \qquad \begin{array}{l}\text{FDM telephony,} \\ \text{arbitrary} \\ \text{interference,} \\ \text{SSB}\end{array} \tag{53}
$$

where $W_n(f)$ is the output noise spectrum given in Eq. (34) and $\overline{W}_n$ is its average value over the baseband as defined in Eq. (42). The RTC for DSB modulation may be obtained by adding 3 dB to the right side of the equation. Comparing this result with the values of RTC implied by Eq. (47) and Table 1 for white interference, it is seen that with non-white interference contained entirely within the wanted signal pass-band, the RTC is reduced by 10 log $W_n(f_{ch})/\overline{W}_n$ compared with its white noise value.

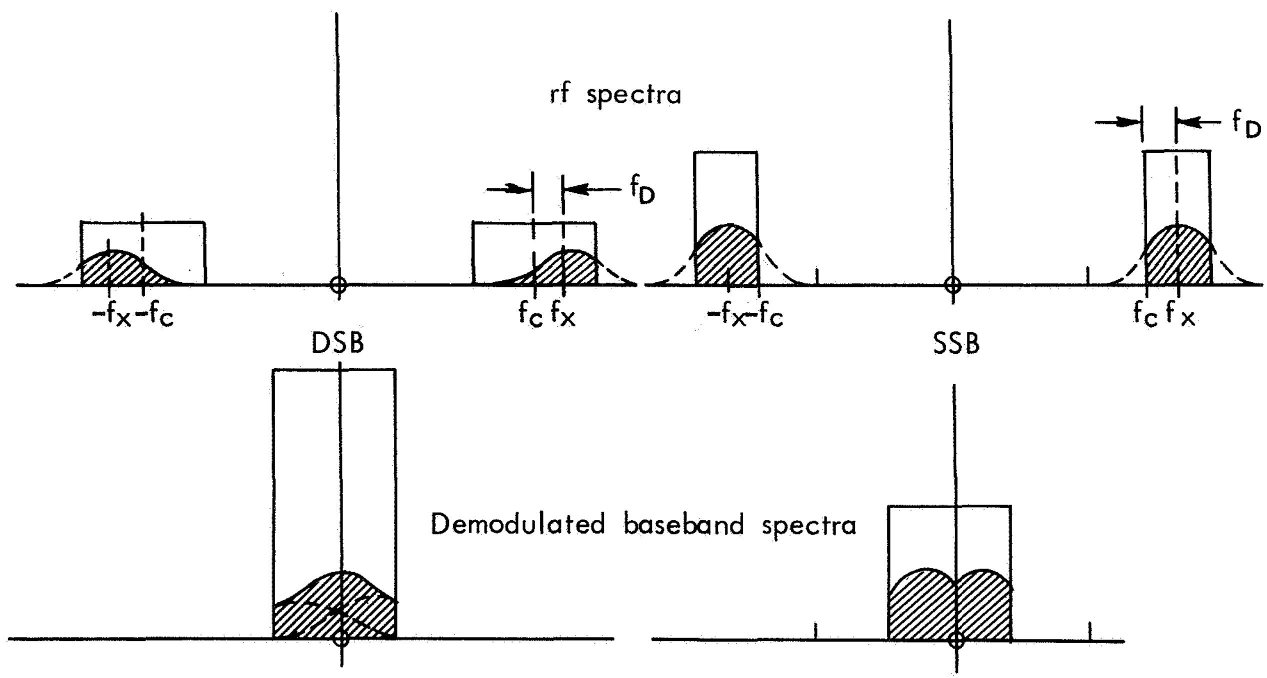
Some of the pertinent spectral relations are sketched in Fig. 8 to illustrate the effects of white and colored interference on DSB and SSB systems. Note in particular that when the unwanted signal spectrum extends beyond the passband of the wanted signal, DSB and SSB can yield significantly different output noise spectra and hence different RTCs for the same unwanted signal. Analytically this result stems from the fact that the effective power spectrum of the unwanted signal (namely, that portion of the unwanted signal spectrum included in the wanted signal passband) can be quite different for the two types of modulation. In general, however, the greatest interference, and hence the lowest values of RTC, correspond to the case of strictly cochannel operation, $f_D = 0$.

When the wanted signal is modulated in angle rather than amplitude, the RTC may be found from Eq. (49) by substituting N_{ch} from either Eq. (C-9) or Eq. (C-10) and setting

$$
S_{ch} = \begin{cases} \varphi_{rch}^2 & \text{PM} \\[2ex] f_{rch}^2 & \text{FM} \end{cases} \tag{54}
$$



(a) White noise unwanted signal



(b) Arbitrary unwanted signal

Fig.8—Spectral relations for interference to DSB and SSB systems

The result for PM is

$$\text{RTC} = 20 \log \varphi_{rch} - 10 \log \left[\frac{C}{X} 2W_n(f_{ch})B_{ch}\right] - 10 \log \overline{G}_N \tag{55}$$

and for FM with preemphasis,

$$\text{RTC} = 20 \log \frac{f_{rch}}{f_{ch}} - 10 \log G_{Do}(f_{ch})$$

$$-10 \log \left[\frac{C}{X} 2W_n(f_{ch})B_{ch}\right] - 10 \log \overline{G}_N \tag{56}$$

where in both equations, $W_n(f)$ is the PM noise spectrum as given either in Eq. (36) or Eq. (39).

A comparison of Eq. (56) with Eq. (52) shows that for the channel at frequency f_{ch}, the RTC with arbitrary interference differs from that with white noise interference by an amount

$$\Delta\text{RTC} = \text{RTC} - (\text{RTC})_{ref} = -10 \log \left[W \frac{C}{X} 2W_n(f_{ch})\right] \tag{57}$$

where W is the rf bandwidth of the wanted signal. When the expression for $W_n(f)$ given in Eq. (36) is substituted in Eq. (57) and it is noted that $C = C_o$ for FM, the result is an expression for ΔRTC directly in terms of the normalized low-pass versions of the one-sided spectra of the total wanted and unwanted signals

$$\Delta\text{RTC} = -10 \log \frac{W}{2} \left[W(f_{ch} - f_D) + W(f_{ch} + f_D)\right] \tag{58}$$

where W(f) is given by Eq. (37)

$$W(f) = w_x(f) * w_c(-f) \tag{59}$$

Plots of ΔRTC versus carrier frequency difference f_D for the worst channel in the case of interference between wideband FM systems are shown in Fig. 9. The output interference noise spectra on which these results depend were displayed earlier in Fig. 6.

Referring to Fig. 9, it should be noted that when the wanted and unwanted signals are strictly cochannel ($f_D = 0$), ΔRTC is negative. That is, the RTC for an arbitrary unwanted signal is always smaller (worse interference) than for the reference white noise case. However, as f_D is increased beyond about one-quarter of the rf bandwidth, ΔRTC becomes positive in most cases and continues to increase rapidly with increasing frequency separation. For example, with $f_D = W/2$, the RTC is from 6 to 7 dB larger than with white noise.

The exceptions to this rule occur when the bandwidth of the unwanted signal is much wider than that of the wanted signal. In this case, well illustrated by the curve for $D_c = 3$, $B_x D_x / B_c = 30$, the unwanted signal spectrum is nearly constant over the wanted signal passband and, as should be expected, the RTC is nearly equal to that for white noise for all values of f_D.

Television Basebands

The CTC and RTC for television signals may be computed in much the same manner as those for telephone messages. The principal difference is, that as a result of the large bandwidth of the video signal, the baseband usually includes only one TV channel. Thus, in contrast to a telephone channel, neither the preemphasis function nor the output noise spectrum is constant over the channel bandwidth.

The output channel noise is

$$N_{ch} = \begin{cases} 2 \int_0^B G_N(f) W_n(f) df & \text{no preemphasis} \\ \\ 2 \int_0^B G_N(f) G_{Do}(f) W_n(f) df & \text{with preemphasis} \end{cases} \tag{60}$$

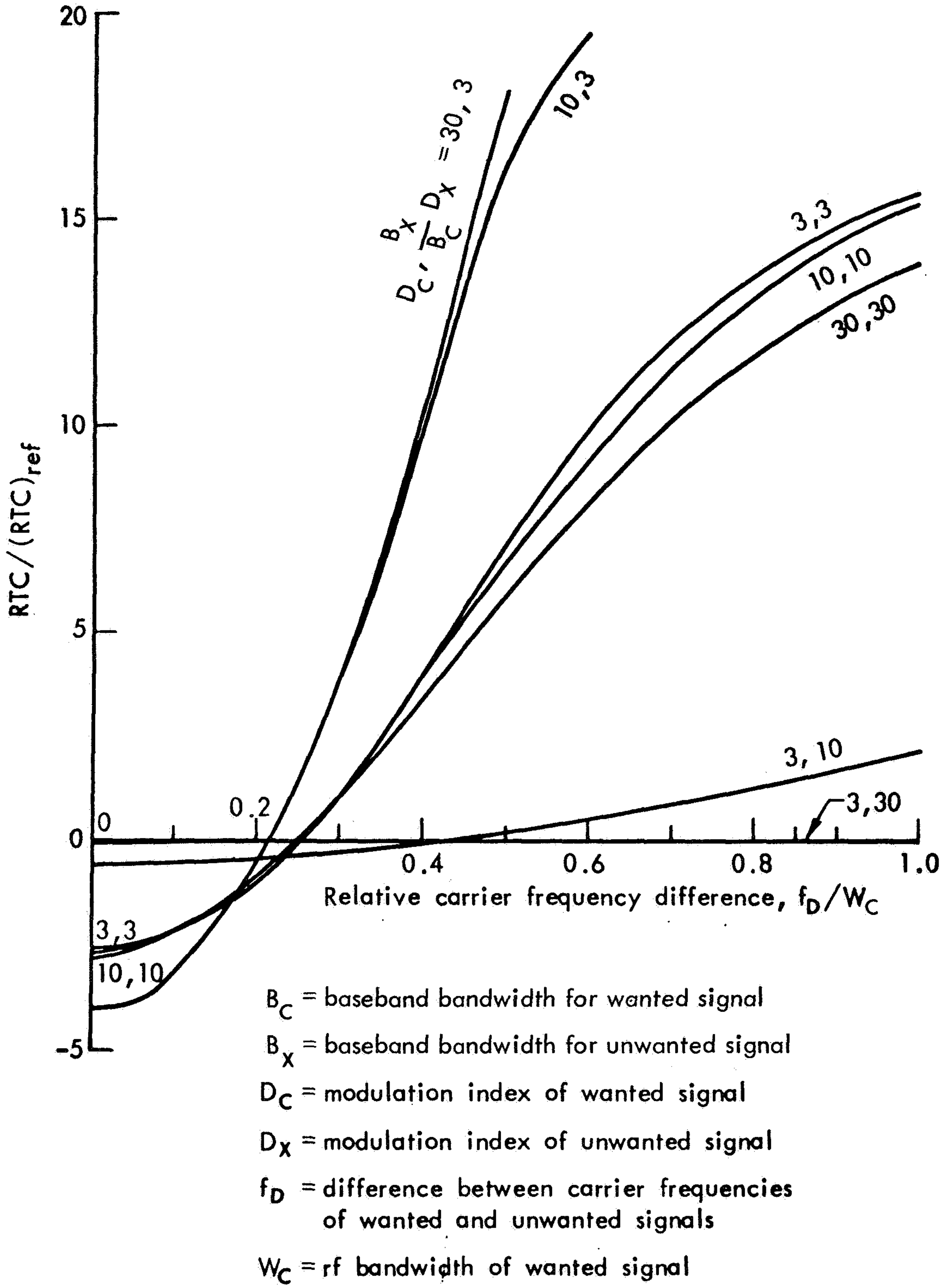


Fig. 9—Receiver transfer characteristic for interference between wideband FM signals relative to that for white noise interference

and the total noise at the demodulator output is

$$N = 2 \int_0^B W_n(f)\, df \tag{61}$$

Substituting Eqs. (C-17), (60), and (61) into Eq. (40) yields the general expressions for the CTC for TV

$$\mathrm{CTC} = \begin{cases} 6 + 10 \log \Lambda - 10 \log \dfrac{\int_0^B G_N(f)\, W_n(f)\, df}{\int_0^B W_n(f)\, df} & \text{TV: no preemphasis} \\ 6 + 10 \log \Lambda - 10 \log \dfrac{\int_0^B G_N(f)\, G_{Do}(f)\, W_n(f)\, df}{\int_0^B W_n(f)\, df} & \text{TV: with preemphasis} \end{cases} \tag{62}$$

Here, $G_{Do}(f)$ and $G_N(f)$ are given by Eqs. (C-12) and (C-15) respectively, and as before, the output noise spectrum $W_n(f)$ depends on the spectrum of the unwanted signal and on the type of modulation.

<u>White Noise Unwanted Signal</u>. With white noise interference and either FM or one of the amplitude modulation methods, $W_n(f)$ is constant, no preemphasis is used, and the third term in Eq. (62) reduces to

$$10 \log \frac{1}{B} \int_0^B G_N(f)\, df = -6.2 \text{ dB}$$

With FM and preemphasis, the noise spectrum is parabolic, and the value of the third term is -12.8 dB. Thus the CTC for white noise interference to TV transmission is given by

$$\mathrm{CTC} = \begin{cases} 12.2 + 10 \log \Lambda & \text{TV: white noise; AM, DSB, SSB} \\ 18.8 + 10 \log \Lambda & \text{TV: white noise; FM with preemphasis} \end{cases} \tag{63}$$

To obtain the RTC for white noise, it is only necessary to add the DTC given by Table 1 or Fig. 3 for the modulation method and modulation index in question. In the case of FM, it is useful to note from Table 1 that

$$\mathrm{DTC} = 7.8 + 10 \log D^2(D+1) - 10 \log \Lambda \tag{64}$$

so that, upon adding the DTC to the CTC given in Eq. (63), the terms in Λ cancel one another, leading to the result

$$\mathrm{RTC} = 26.6 + 10 \log D^2(D+1) \qquad \text{TV: white noise; FM with preemphasis} \tag{65}$$

Also note from Eq. (63) that curves such as those in Fig. 5 which tell directly the input wanted-to-unwanted signal ratio needed to yield a specified value of peak baseband signal-to-noise ratio can be relabeled in terms of the corresponding channel output signal-to-noise ratio by merely adding 18.8 dB to the labeled values. In using these curves, note that the FM modulation index D and Carson's rule bandwidth W may be expressed directly in terms of the deviation f_p corresponding to the peak video signal voltage s_p and the maximum baseband frequency f_m.

$$D = f_p/f_m \,, \qquad W = 2(f_p + f_m) \tag{66}$$

Arbitrary Unwanted Signal. When the unwanted signal is not white noise, the appropriate expression for $W_n(f)$ can be used in Eq. (62) to give the CTC but it is no more trouble in this case to compute the RTC directly. For the amplitude modulation methods, however, the DTC is unaffected by the change in the unwanted signal spectrum, and the resultant RTC is

$$\mathrm{RTC} = \begin{cases} 3 + \mathrm{CTC} & \mathrm{AM,\ DSB} \\ \\ \mathrm{CTC} & \mathrm{SSB} \end{cases} \tag{67}$$

where the CTC is given by Eq. (62) in which $W_n(f)$ is specified in terms of the unwanted signal spectrum by Eq. (34).

For preemphasized FM, the result is

$$\mathrm{RTC} = 6 + 20 \log f_p - 10 \log 2 \int_0^{f_m} f^2 \frac{C}{X} W_n(f) G_N(f) G_{Do}(f) df \tag{68}$$

where $2f_p$ is the peak-to-peak frequency deviation and $W_n(f)$ is the phase noise spectrum given by Eq. (36) or (39). The result for PM is the same except for the replacement of f_p by the corresponding peak phase deviation and the omission of the f^2 factor in the integrand.

Values of the RTC corresponding to the types of rf interference likely to be encountered in TV transmission by space and terrestrial relay systems may now be calculated in much the same fashion as for telephone transmission. To make such computations, however, it is necessary to know something about the amplitude distribution for the television baseband. Both the peak-to-average power ratio needed for the amplitude modulation methods and the power spectrum of the modulated carrier needed for FM are determined by this amplitude distribution. The ubiquitous gaussian distribution is sometimes assumed for this purpose, but little data on the actual distribution appear to be available.

As a result, or perhaps because theoretical predictions using the data that were available proved unreliable, the effect of nonwhite rf interference on television picture quality seems to have been determined in practice directly from special experiments with viewer panels.

III. DIGITAL MODULATED SYSTEMS

Radio relay systems employing digital modulation methods such as amplitude-shift keying (ASK), phase-shift keying (PSK), and frequency-shift keying (FSK)--the digital equivalents of AM, PM, and FM--are designed to carry not only messages which are inherently digital (computer data, teletype, and certain types of facsimile) but also analog messages (voice and television) which have been converted to digital form using encoding techniques such as pulse code modulation (PCM) and delta modulation (ΔM).

GENERAL RELATION BETWEEN SIGNAL ENVIRONMENT AND OUTPUT MESSAGE QUALITY

The problem of relating output message quality to the wanted and unwanted signal environment may be treated in much the same fashion as for analog modulated systems. Referring to Fig. 1 and working backward from the output of a selected channel, one first relates the appropriate measure of output message quality to the quality of the digital baseband as it is recovered by the demodulator in the terminal receiver.

For analog messages, this means expressing the output signal-to-noise ratio S_{ch}/N_{ch} in terms of the baseband error probability p_n at the terminal receiver output. In functional notation, this expression will be written

$$N_{ch}/S_{ch} = \gamma(p_n) \tag{69}$$

For digital messages, the output message quality is measured by the channel error probability p_{ch}, which in all cases of interest is simply equal to p_n. In either case, this first step in approaching the digital transmission problem is equivalent to determining the channelizing transfer characteristic in an analog modulated system. Accordingly, the function $\gamma(p)$ will be called the channelizing function.

The next step is equivalent to finding the demodulator transfer characteristic in an analog system. And in doing so, it is necessary, just as in an analog system, to distinguish between two types of repeaters: those in which the digital baseband is recovered and used to remodulate a fresh carrier (baseband repeater), and those in which the modulated carrier is merely shifted in frequency, amplified, and retransmitted (rf or i-f repeaters).

When the system employs rf or i-f repeaters, the modulated carrier is corrupted by noise and interference at each repeater, but the errors which these unwanted signals cause in the baseband digits are not perceived until the terminal receiver where the baseband is recovered for the first time. The resultant error probability p_n at the output of the terminal demodulator is determined by the effective unwanted-to-wanted signal ratio X_n'/C_n' at the terminal receiver input. The corresponding functional relationship

$$p_n = \delta_n(X_n'/C_n') \tag{70}$$

depends not only on which digital modulation method is used but also on the type of demodulator (coherent or incoherent) used by the terminal receiver and on the statistical properties of the unwanted signal. The function $\delta_n(r)$ will be called the demodulator function for the terminal receiver.

The effective unwanted-to-wanted signal ratio at the terminal receiver input is in turn determined by the apparent unwanted-to-wanted signal ratios at the repeater inputs in exactly the same fashion as for an analog modulated system employing i-f or rf repeaters--viz., by Eq. (7), which may be written

$$X_n'/C_n' = \sum_{i=1}^{n} (X_i/C_i) \qquad \text{rf or i-f repeaters} \tag{71}$$

By combining Eqs. (69), (70), and (71), the quality of an analog message at the output of a single channel in a digitally modulated radio relay system employing rf or i-f repeaters can be expressed directly in terms of the signal and noise environment at the repeater inputs:

$$N_{ch}/S_{ch} = \gamma \left\{ \delta_n \left[\sum_{i=1}^{n} (X_i/C_i) \right] \right\} \qquad \text{rf or i-f repeaters} \qquad (72)$$

This is to be compared with the corresponding relationship for an analog modulated system given in Eq. (8).

When the system employs baseband repeaters, as most digital systems do because of the opportunity such repeaters afford for reshaping and retiming pulses, the baseband is recovered with errors at each repeater. But it modulates a fresh carrier at each repeater so that the additional baseband errors observed at the demodulator output in the next repeater result only from the noise at that repeater input, and there is no distinction between apparent and effective unwanted and wanted signals at the repeater inputs. As a result, the probability p_i of new errors contributed by the i-th repeater is related to the input unwanted-to-wanted signal ratio X_i/C_i by an equation similar to Eq. (70),

$$p_i = \delta_i(X_i/C_i) \qquad \text{Baseband repeaters} \qquad (73)$$

where $\delta_i(r)$ is the demodulator function for the i-th repeater.

When the error probability p_i is very small, as will be assumed throughout this study, the total baseband error probability at the terminal receiver is simply the sum of the incremental error probabilities $p_i^{(11)}$

$$p_n' = \sum_{i=1}^{n} p_i \qquad \text{Baseband repeaters} \qquad (74)$$

Combining Eqs. (69), (73), and (74) gives the output analog signal quality in a single channel of a digital modulated radio relay system employing baseband repeaters:

$$N_{ch}/S_{ch} = \gamma \left\{ \sum_{i=1}^{n} \delta_i(X_i/C_i) \right\} \qquad \text{Baseband repeaters} \qquad (75)$$

This relation should of course be compared with its counterpart in an analog modulated system, which is given by Eq. (15). More important, it should be compared with Eq. (72) for a digital system using rf or i-f repeaters. The latter comparison becomes more vivid when the actual function $\delta(X_i/C_i)$ for a particular digital modulation method is substituted. For example, in the case of an idealized PSK modulated carrier noncoherently detected in the presence of a white-noise-like unwanted signal, the demodulator function is[11]

$$\delta_i(u) = \frac{1}{2} \exp\,(-1/u) \qquad i = 1,\cdots,n \qquad (76)$$

In this case, the baseband error rate p_n' which forms the argument of the function γ in Eqs. (72) and (75) is given by

$$p_n' = \begin{cases} \frac{1}{2} \exp\left\{-\left[\sum_{i=1}^{n} (X_i/C_i)\right]^{-1}\right\} & \text{rf or i-f repeaters} \\ & \qquad\text{Noncoherent PSK} \\ \frac{1}{2} \sum_{i=1}^{n} \exp\,(-C_i/X_i) & \text{Baseband repeaters} \end{cases} \qquad (77)$$

In the special case where the wanted-to-unwanted signal ratio is the same at all repeaters, e.g., $C_i/X_i = C/X$, these expressions simplify to

$$p_n' = \begin{cases} \frac{1}{2} \exp\,(-\frac{1}{n}\frac{C}{X}) & \text{rf or i-f repeaters} \\ & \qquad\text{Noncoherent PSK} \\ \frac{1}{2}\, n \exp\,(-\frac{C}{X}) & \text{Baseband repeaters} \end{cases} \qquad (78)$$

The marked advantage of the baseband repeater in systems involving more than two or three links is apparent. For example, with n = 10, a baseband error rate $p_n = 5 \times 10^{-4}$ would require wanted-to-unwanted signal ratios of 18 dB at the inputs to rf or i-f repeaters, but only 9.5 dB for baseband repeaters.

DEMODULATOR FUNCTIONS

As defined in Eqs. (70) and (73), a demodulator function expresses the error rate at the output of a digital demodulator in terms of the effective wanted-to-unwanted signal ratio at the receiver input, thus performing a function equivalent to the DTC for an analog demodulator. Like the DTC, the demodulator function depends on the nature of both the wanted and unwanted signal. But where for given signal characteristics the DTC was a simple proportionality factor, the demodulator functions are highly nonlinear. In addition, the demodulator function depends significantly on whether the demodulator is coherent or not, and on the number m of discrete values that the modulation parameter (amplitude, phase, or frequency) can assume.

White Noise Unwanted Signal

Formulas for the demodulator functions when the unwanted signal is white gaussian noise are presented in Table 2 for the binary (m=2) forms of ASK, FSK, and PSK modulation and, except for ASK, for both coherent and noncoherent demodulation.[11,12] The distinction between these types of demodulator is indicated by prefixing the abbreviation for the modulation method by the letters C or NC, respectively. The parameter A which appears in these expressions is a measure of the carrier-to-noise ratio C/X. Specifically, $A = \sqrt{E/N_o}$, where E is the average energy per bit, and, as before, N_o is the (one-sided) rf noise power density at the receiver input. The relation between A and C/X depends on the i-f bandwidth W of the demodulator, which in turn depends on the shape of the pulses used to represent the bits. In terms of the parameter k, which will be used shortly to relate bit rate to i-f bandwidth,

$$A^2 = E/N_o = kC/X \qquad (79)$$

Table 2

DEMODULATOR FUNCTIONS FOR BINARY DIGITAL SYSTEMS
($A^2 = E/N_o = kC'/X'$)

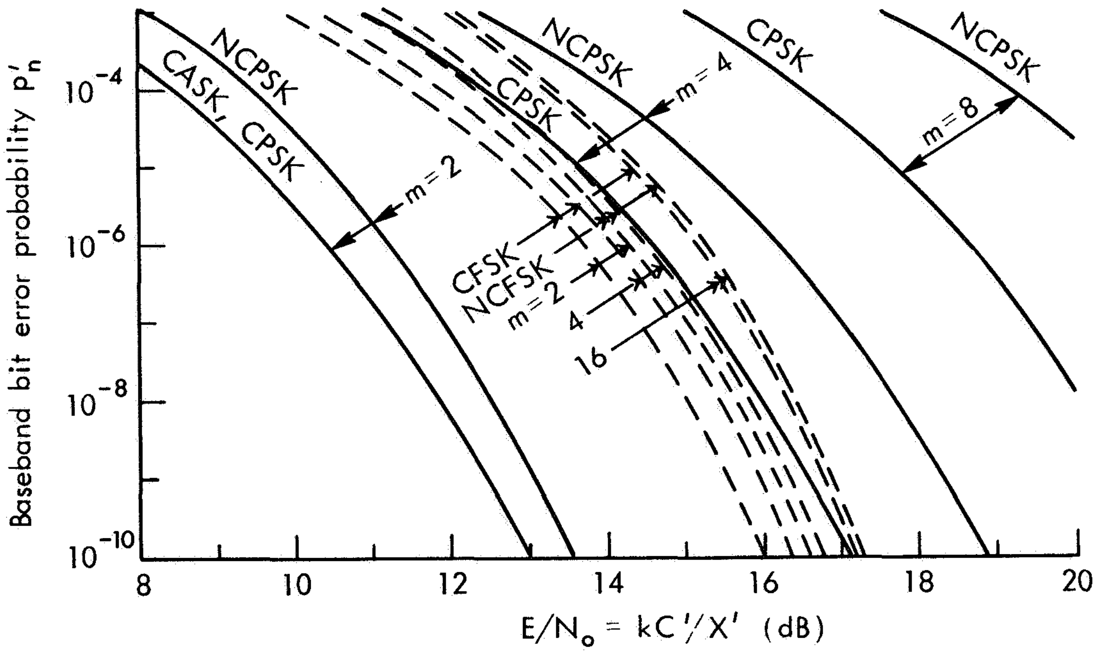
Modulation	Coherent	Noncoherent
Amplitude	$\frac{1}{\sqrt{2\pi}} \int_{A\sqrt{2}}^{\infty} \exp(-x^2/2)\, dx$	
Frequency	$\frac{1}{\sqrt{2\pi}} \int_{A}^{\infty} \exp(-x^2/2)\, dx$	$\frac{1}{2} \exp(-A^2/2)$
Phase	$\frac{1}{\sqrt{2\pi}} \int_{A\sqrt{2}}^{\infty} \exp(-x^2/2)\, dx$	$\frac{1}{2} \exp(-A^2)$

The demodulator functions for binary digital modulation (m = 2) are plotted in Fig. 10a along with those for multilevel or m-ary modulation with m = 4,8,... .[13] In all cases, the error probability shown refers to bit errors; in the case of the m-ary methods it is assumed that a Gray code has been used to insure than an error of one level leads to only one bit error. It also should be noted that in the case of the m-ary methods, the energy E in the ratio E/N_o represents the average energy per baud--i.e., per transmitted level--rather than per bit. Note that in general, coherent detection permits a given error probability to be achieved with a few-dB smaller value of C/X, and that in all cases, a few-dB change in C/X can lead to order-of-magnitude changes in error rate. The value of C/X required for a specified error probability also increases as the number of levels m is increased, but, as will be seen in a moment, there is often a compensatory savings in bandwidth.

Arbitrary Unwanted Signal

When the unwanted signal does not have the characteristics of white noise, there are two important distinguishable cases to consider. First, the unwanted signal may arise from a sum of other signals too few in number to constitute a signal having gaussian characteristics. In this case, which represents a few interfering carriers for example, the amplitude distribution function allows no probability for large peaks as does the gaussian. But the number of lesser peaks is greater. Hence for the same total average unwanted signal power, a few sinusoids behave--so far as peaks are concerned--much better than gaussian noise. Hence, if the same wanted-to-unwanted signal power ratio is used, the observed error probability will be smaller. But the sensitivity to peaks is more pronounced for a few sinusoids, so that once a few peaks appear, only the slightest change in average power brings about a major change in error probability. Therefore, with this type of interference, the demodulator functions for white noise will be used.

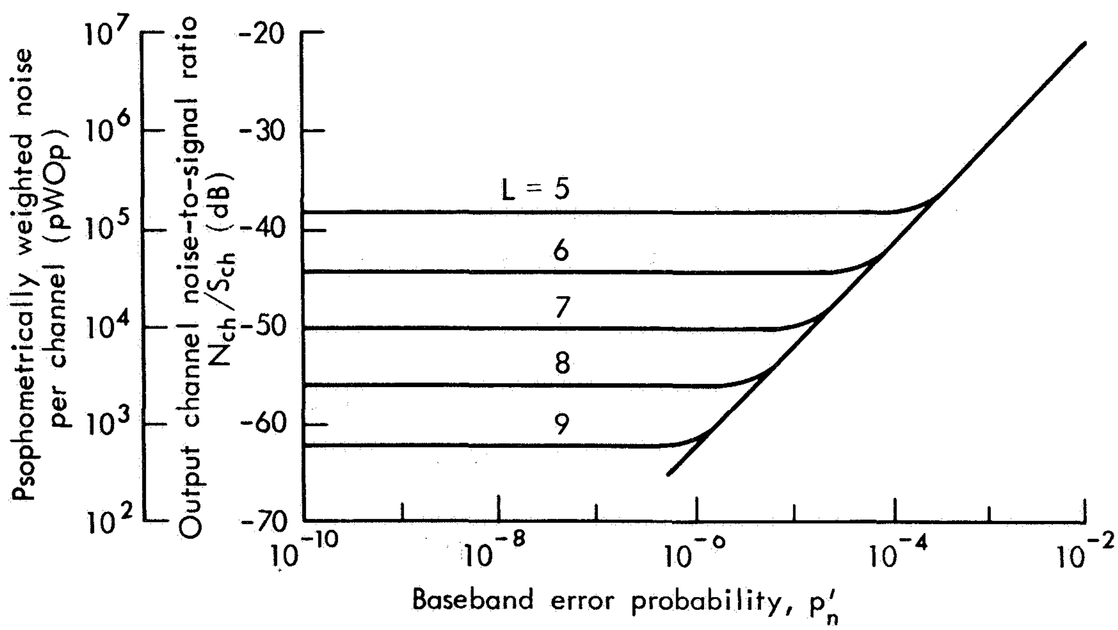
The second case of interest is when the interference is due to a single signal with extremely high peaks (radar, ignition noise, dial pulses in telephone circuits, etc.). In this situation, the error probability is determined by the peak rather than the average level. Peaks



E = wanted signal energy per baud
N_o = one-sided noise spectral density
m = number of modulation levels

k = pulse shape factor
C'/X' = effective wanted-to-unwanted-signal ratio

(a) Demodulator functions for m-ary digital modulation methods



(b) Channelizing functions for PCM-encoded telephone channels

Fig. 10 — Demodulator and channelizing functions for white noise unwanted signal

above a certain level will cause errors every time they occur and not merely when they happen to randomly add in phase. It will be assumed that for the systems of interest in this Memorandum, unwanted signals of this type can be reduced to negligible levels.

CHANNELIZING FUNCTIONS

The form of the channelizing function, which, as defined in Eq. (69), relates output channel message quality to the baseband error probability, depends on the type of message carried by the channel. For time-division multiplexed (TDM) digital messages, output channel quality is measured by the error probability in the channel. Since this is generally equal to the baseband error rate, the channelizing function is unity.

For digitally encoded analog messages, the channelizing function depends on the method used to convert the analog message to digital form. The two most commonly used methods are pulse code modulation (PCM) and delta modulation (ΔM).

Pulse Code Modulation

When PCM is used, an analog message of bandwidth B_{ch} is first sampled at the rate of $2B_{ch}$ samples per second, the minimum rate that will permit perfect message recovery from the samples. The sample values are then "quantized," i.e., represented by the closest approximation among a set of 2^L discrete levels, each of which is in turn represented by an L-bit code word. The sequence of bits representing these words then comprises the PCM representation of the sequence of values obtained in the sampling process.

Since each sample requires the transmission of L bits and there are $2B_{ch}$ samples per second, the bit rate per channel is $2LB_{ch}$. Hence the bit rate for a TDM baseband of n such channels is

$$f_{bit} = 2nLB_{ch} = 2LB \tag{80}$$

where $B = nB_{ch}$ is the total information bandwidth of the messages comprising the baseband.

The accuracy with which a PCM-encoded baseband is recovered at the terminal receiver is less than perfect for two reasons. First, the levels corresponding to some message samples will be incorrectly identified because of bit errors. Second, the sample values, even if received without error, are only discrete approximations to the actual analog amplitudes. Each of these sources of impairment to the quality of the output message can be characterized by a signal-to-noise ratio.[12] Thus, when the bit error probability p is low, the ratio of mean signal to mean error noise power is to good approximation

$$\left(\frac{S}{N}\right)_e = \frac{1}{4p} \qquad \text{PCM} \tag{81}$$

And when L, the number of bits per word, exceeds three, the ratio of mean signal to mean quantizing noise power is given by

$$\left(\frac{S}{N}\right)_q = 4^L \qquad \text{PCM} \tag{82}$$

provided only that, before sampling, the amplitude distribution of the message is modified (by means of instantaneous companding) to make all levels equally likely. The average noise-to-signal ratio for the companded signal at the channel output is the sum of the reciprocals of these two component signal-to-noise ratios.

To convert this ratio into a measure of quality comparable with the test-tone-to-weighted-noise ratio (S_{ch}/N_{ch}) used for FDM telephone channels, it is necessary to allow for the following factors:

- o The peak-to-average ratio of an ideal signal with a uniform amplitude distribution (5 dB)
- o The average-to-peak ratio of the companded voice signal (-10.5 dB)
- o The ratio of the peak companded voice signal to the peak of an ideal signal (0 dB assumed)
- o The ratio of test-tone power to average speech power (11 dB)
- o The ratio of unweighted-to-weighted noise power (2.5 dB)

Assuming the values indicated in parentheses, the net conversion factor is 8 dB; hence, the channelizing function for a PCM-encoded telephone channel is given by

$$N_{ch}/S_{ch} = \left(4p + 4^{-L}\right)/6.3 \qquad \text{PCM} \tag{83}$$

These channelizing functions are plotted in Fig. 10b for values of L ranging from 5 to 10. Note that the output noise-to-signal ratio is dominated by the error noise contribution for high error probabilities and by the constant quantizing noise contribution for low error probabilities. Thus, as the input wanted-to-unwanted signal ratio is increased to reduce the baseband error probability in accordance with the applicable demodulator function, a threshold value is reached such that a further decrease in C/X has no effect on the channel output quality.

For any given value of L, this threshold corresponds to the "knee" of the curve where the quantizing and error noise contributions are approximately equal. For example, with L = 7, Fig. 10b shows that the threshold occurs at an error probability of about 2×10^{-5}. The threshold wanted-to-unwanted signal ratio for a particular digital modulation method may then be obtained from Fig. 10a as the value of C/X corresponding to this error rate. Thus with binary CPSK and k = 1.5, the threshold is about 8 dB.

Delta Modulation

Delta modulation (ΔM) is an analog-to-digital encoding method wherein the resultant digital signal can be converted back to analog form merely by passing it through a filter.(12) This technique has seen only limited use so far, but it holds considerable promise for future applications. In ΔM, the digital signal is generated by means of a controlled pulse generator, a filter, and a comparator. The comparator controls the generation of digits in such a way that the filtered output at the receiver is the best approximation to the input analog signal. If simple linear filtering is used at the receiver, the effect of errors may be judged readily once the filter characteristic is known. The best ΔM schemes, however, employ both nonlinear and time-varying filters, and an analysis of the effect of transmission errors is not easy in such a case.

For the simplest delta encoder using an RC integrator to convert an 800-Hz sinusoid to a 50-kbit binary signal, the dependence of the ratio of output signal power to error-induced noise power on bit error probability is given by[12]

$$\left(\frac{S}{N}\right)_e = \frac{1185}{p} \qquad \Delta M \tag{84}$$

With other linear filtering systems a similar dependence can be expected to hold, but with different constants of proportionality. Although general expressions for nonlinear time-varying filters and for actual message inputs are not available, it appears safe to conclude that, as implied by a comparison of Eq. (84) with Eq. (81), ΔM is considerably less susceptible than PCM to error-induced noise.

Error resistance is not the only consideration, however. Just as with PCM, when the error probability is low, the output signal-to-noise ratio with ΔM is determined by quantizing noise rather than by error noise. Here again, a generally satisfactory analysis is not available, but experimental data suggest that for the same bit rate, ΔM is superior to PCM, at least for voice transmission. Unfortunately, the data needed to describe channelizing functions for ΔM do not appear to be available.

BANDWIDTH CONSIDERATIONS

As mentioned earlier, digital modulation methods differ not only in the carrier-to-noise ratios required to achieve a specified error probability, as described by the demodulator function, but also in the amount of rf bandwidth W required for transmission at a given information rate.

For digital messages, the information rate is measured by the bit rate f_{bit}. The ratio W/f_{bit} depends on the type of modulation (ASK, PSK, FSK), the number of digital levels m, the type of demodulator (coherent or incoherent), and on the pulse shape that describes the transitions from one digital level to another. Describing the effect of pulse shape by a factor k, it can be shown that[13]

$$\frac{W}{f_{bit}} = \begin{cases} k/\log_2 m & \text{ASK, CASK, PSK, CPSK} \\ (m - 1 + k)/\log_2 m & \text{NCFSK} \\ \frac{1}{2}\,(m - 1 + 2k)/\log_2 m & \text{CFSK} \end{cases} \tag{85}$$

where $k = 1$ represents a theoretical minimum, and $1.5 \leq k \leq 2$ represents current engineering practice.

For analog messages, the appropriate measure of information rate is the information bandwidth B. For a baseband comprising n channels, each with bandwidth B_{ch}, the total information bandwidth is

$$B = n\, B_{ch} \tag{86}$$

The bit rate required for transmission of such a baseband in digital form depends on the technique used for analog to digital conversion. For PCM, the bit rate is given by Eq. (80), and substitution into Eq. (85) leads to the following expressions for the ratio of rf bandwidth to information bandwidth

$$\frac{W}{B} = \left.\begin{cases} 2kL/\log_2 m & \text{ASK, PSK} \\ 2(m - 1 + k)L/\log_2 m & \text{NCFSK} \\ (m - 1 + 2k)L/\log_2 m & \text{CFSK} \end{cases}\right\} \text{PCM} \tag{87}$$

OVERALL RECEIVER FUNCTIONS

Just as it is possible to combine the demodulator transfer characteristic and channelizing transfer characteristic of an analog receiver to obtain an overall receiver transfer characteristic, so also may the demodulator and channelizing functions of a digital receiver be combined to yield a function which describes the overall relation between input wanted-to-unwanted signal ratio and output message quality. For example, in a digital system transmitting PCM-encoded telephone messages, the difference in dB between the output channel signal-to-noise ratio and the threshold input wanted-to-unwanted signal ratio may be compared directly with the RTC for an analog system carrying FDM telephone traffic.

Such a comparison is given in Fig. 11 for binary and four-level CPSK systems with $k = 1.5$ and an FM system using preemphasis. In the case of the digital systems, the bandwidth relations given in Eq. (87) were used to show their cost in rf bandwidth. It will be noted that for PCM with 7 or more bits per sample, CPSK appears to offer both power and rf bandwidth savings compared with an FM system delivering the same output message quality.

In making these comparisons, however, it should be remembered that CPSK is the most efficient of the digital techniques in the use both of power and bandwidth. Most other digital techniques are likely to be less efficient than FM. Moreover, if an FM and a PCM system which offer the same output quality when operated at their respective thresholds are compared, any allowance for fading margin will yield higher output message quality in the FM system but not in the PCM system.

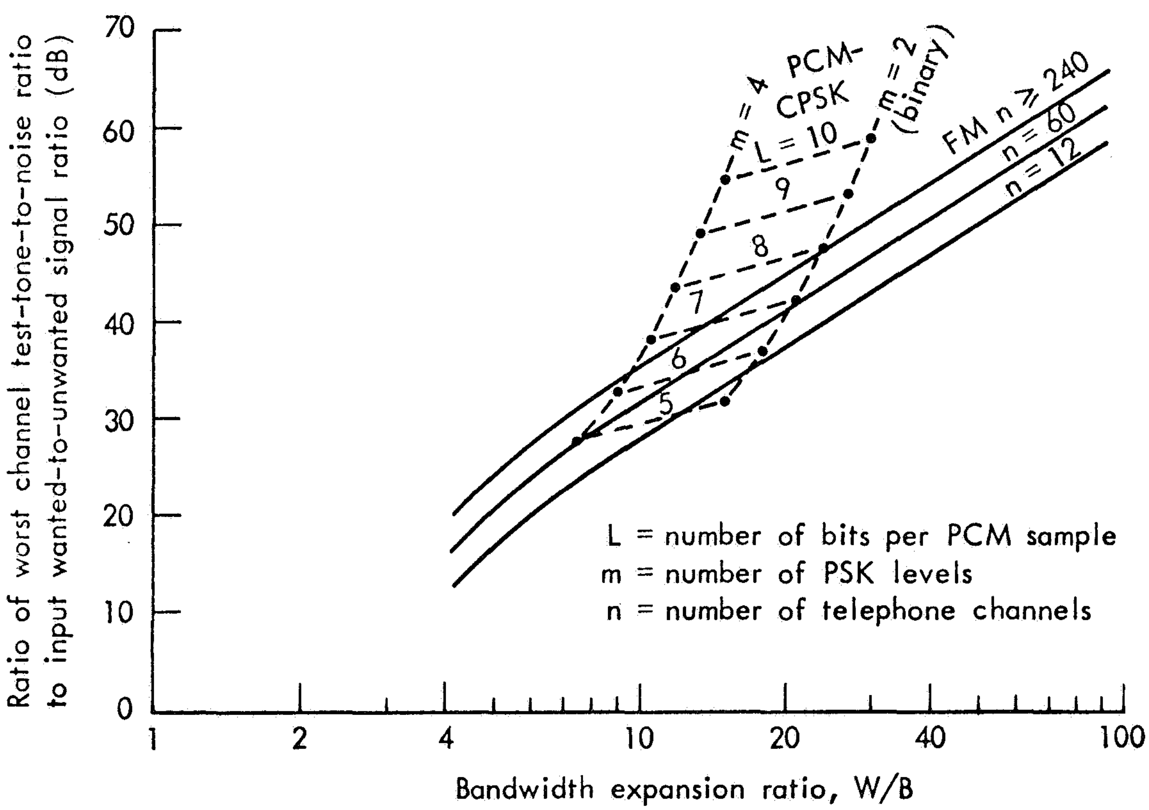


Fig. 11—Comparison of interference reduction capability of FM with preemphasis to PCM-CPSK at threshold when used for multichannel telephony

IV. MESSAGE OBJECTIVES AND PROTECTION RATIOS

The preceding sections have described the general relationship between message quality at the output of a selected channel in a multichannel radio relay system and the wanted and unwanted signal environment at the repeater input terminals. When written for the worst channel in the system, such a relationship can be applied to determine how the wanted-to-unwanted signal ratios must be constrained in order to achieve specified message-quality objectives. Such constraints are expressed in terms of "protection ratios"--the values of the apparent wanted-to-unwanted signal ratios that must be exceeded at the repeater inputs in order to maintain the specified objectives.

MESSAGE OBJECTIVES

The specification of minimum acceptable message quality or transmission fidelity must take into account a number of factors. To begin with, the evaluation of message quality is inherently a subjective process. A TV picture whose quality is acceptable to one viewer might be objectionable to another. Thus, experiments with a large number of representative users are normally conducted to determine that quality which a specified large fraction of the users will find satisfactory.

Next, the quality specification must recognize that impairments to quality can arise from a variety of different kinds of unwanted signal such as thermal noise, undesired rf transmissions, circuit mismatches, and circuit nonlinearities. Moreover, the annoying effects of equal amounts of the different types of unwanted signal may be quite different, so that additional subjective user experiments are required to determine the appropriate weighting functions.

Finally, at each repeater the wanted signal and the different components of the unwanted signal can vary with time, causing the output message quality to fluctuate in a complex way. Thus the message objective should also indicate the fraction of the time that various degrees of quality degradation can be tolerated.

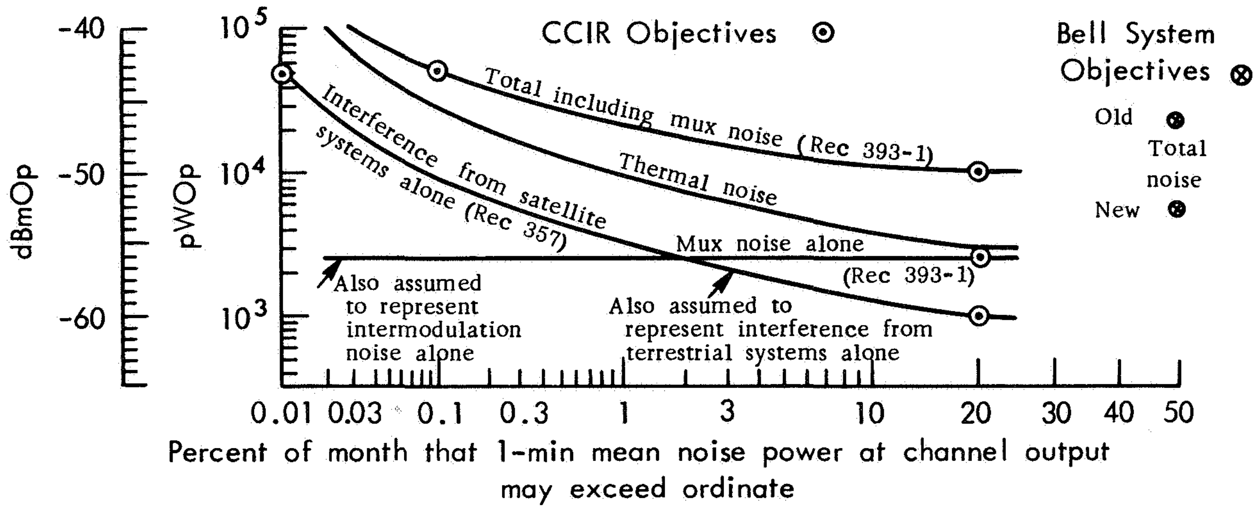
So far as the user is concerned, if the channel noise from each source is properly weighted to reflect its relative annoyance potential, only the weighted total noise at the channel output (relative to the signal there) need be specified. From the standpoint of the system designer, however, such a total noise objective is a very loose constraint, since it leaves open a wide variety of tradeoffs among equipment parameters, technique choices, and repeater locations. To restrict these tradeoffs somewhat, it is common practice to divide the total noise objective between its major components. For example, both the Bell System and the CCIR recommend that output noise due to unwanted rf signals be only about one tenth of the total noise from all causes.

But even a detailed resolution of total output noise-to-signal ratio into components by sources is not sufficiently restrictive to determine the protection ratios. For this purpose, it is necessary to partition the output noise from each type of source among the different repeaters. When all links are essentially the same, it is common practice to divide the noise or interference objective equally between them. When the links are different, e.g., the up link and down link of a communication satellite system, considerable engineering judgment may be required to find the optimum partition of the output noise objective.

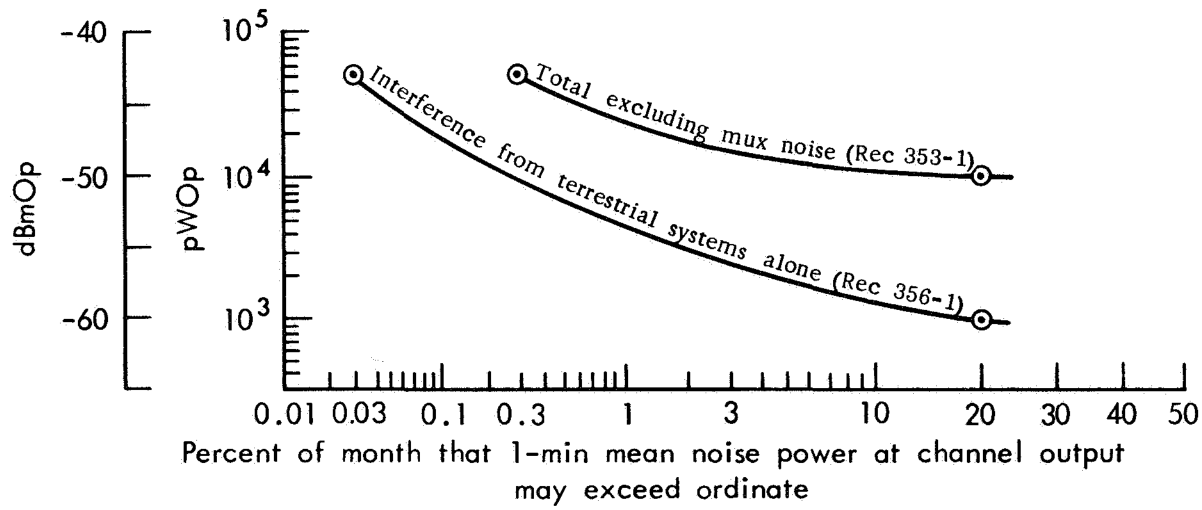
Analog Messages

Many of the foregoing remarks are illustrated by the objectives for telephone and TV messages shown in Fig. 12. These are based primarily on recommendations of the International Radio Consultative Committee (CCIR) for so-called hypothetical reference circuits.[14] Also shown for comparison at 50-percent probability are the old and new Bell System telephone objectives for total noise in the absence of fading.[15,16]

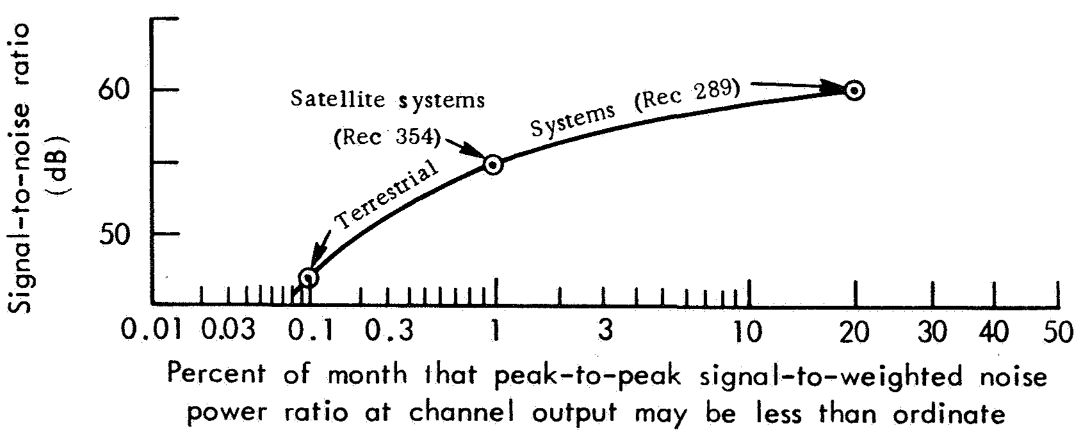
Note that in the case of FDM telephone channels the CCIR has specified only the total noise and the components of noise originating from multiplexing equipment and from interference due to sources outside the system, as indicated by the circles at 0.01 percent, 0.1 percent, and 20 percent in Fig. 12a for terrestrial relay systems and at 0.03 percent, 0.3 percent, and 20 percent in Fig. 12b for satellite relay systems. All of these objectives represent one-minute averages. Thus, the



(a) Objectives for FDM telephone channels of terrestrial systems



(b) CCIR objectives for FDM telephone channels of satellite systems



(c) CCIR objectives for 525-line TV channels

Fig. 12—Noise and interference objectives

percentages on the abscissa scale may be thought of in terms of the number of minutes per month to which they correspond. For example, 0.01 percent is 4.3 min, 0.1 percent is 43 min, etc.

The telephone objectives shown in Figs. 12a and 12b for other noise components and for other percentages of time represent reasonable subdivisions and extrapolations of the recommended values. For this purpose it has been assumed that in the absence of fading, the thermal noise and full-load intermodulation noise are equal to each other, and also that the interference from terrestrial systems is the same as the interference from satellite systems. Finally, the distribution of fading for both components of interference has been assumed to be the same as that for thermal noise.

In specifying telephone objectives, it is customary to indicate the desired signal-to-noise ratio S_{ch}/N_{ch} by giving the allowable value of N_{ch} at a so-called point of 0 relative level where, by definition, S_{ch} = 1 mW. As explained in Appendix C, the CCIR normally gives N_{ch} in units of pW0p (picowatts at a point of 0 relative level, psophometrically weighted), whereas the Bell System uses dBrnc0 (dB above reference noise of 1 picowatt, c message weighted at a point of 0 relative level), or dBa0 (dB above reference noise adjusted at a point of 0 relative level). In Figs. 12a and 12b, all of these units have been converted to dBm0p (dB above a milliwatt at a point of 0 relative level, psophometrically weighted) using the identities

$$N_{dBm0p} = 10 \log N_{pW0p} - 90 = N_{dBa0} - 84.5 = N_{dBrnc0} - 90.5 \qquad (88)$$

It is also useful to note that N_{dBm0p} is numerically equal to the weighted-noise-to-test-tone ratio $(N_{ch}/S_{ch})_{dB}$ since S_{ch} = 0 dBm0.

The CCIR objectives for television channels shown by the circles in Fig. 12c apply to the ratio of peak-to-peak video signal (excluding synchronizing pulses) to the 1-sec average power of continuous random noise, using the CCIR-recommended weighting network. If a fading distribution similar to that assumed for telephone systems is adopted, the television objectives for satellite and terrestrial systems are seen to lie on the same curve.

Digital Messages

Formal standards of system performance for digital systems have not yet been agreed upon. Digital communication is expanding rapidly, with new applications arising almost daily, and it would be premature to set rigid standards. However, it is instructive to interpret different possible error probability objectives in terms of the average time between errors implied for various digital message data rates as shown in Table 3. From this it can be inferred that for most applications, error probability objectives will lie in the range from 10^{-4} to 10^{-8}, with some special-purpose systems likely to require a corrected error probability of 10^{-10}. Values in the latter category are most easily achieved with error-correction equipment applied to a poorer channel.

Table 3

MEAN TIME BETWEEN ERRORS AS A FUNCTION OF ERROR PROBABILITY AND DATA RATE

Channel Output Error Probability	Teletype (100 wpm)[a]	Digital Message Data Rate: Low-Speed Data Transmission (2400 bits/sec)	Digital Message Data Rate: High-Speed Data Transmission (50,000 bits/sec)
10^{-4}	166 sec	4.2 sec	0.2 sec
10^{-6}	4½ hr	7 min	20 sec
10^{-8}	19 days	11.5 hr	33 min
10^{-10}	5.3 yr	48 days	2.3 days

[a]Equivalent to 60 bits/sec.

PROTECTION RATIOS

The principles involved in using the message objective of Fig. 12 to derive protection ratios are the same for satellite systems and terrestrial radio relay systems, but basic differences in the two types of systems affect the relative importance of the factors which must be taken into account. Thus, a typical long-haul microwave relay system has a large number of links (e.g., n = 140 for a transcontinental system), each having quite similar equipment and path parameters, and each subject to severe signal fading during certain times of the day. A typical satellite link, however, has only two links (n = 2), which are likely to have very different equipment parameters but exhibit practically no diurnal fading.

Terrestrial FM Systems

Consider an n-link microwave relay system carrying at least 240 FDM telephone channels using i-f-type repeaters and low-index FM without feedback. Such a system is typical of the TD-2 and TH systems[17,18] operated by the Bell System for transcontinental telephone and TV transmission. Protection ratios at the repeater inputs are needed for both the thermal noise and the rf interference (RFI) components of the unwanted signal. For this purpose, it will be assumed that in the absence of fading, all repeaters have the same apparent carrier power input and that the fading on individual links, while uncorrelated between links, is governed by the same distribution relative to the free-space or unfaded carrier level.

Thermal Noise. The protection ratio for thermal noise may be found by using experimentally determined fading distributions (such as those shown in Fig. 2) in conjunction with Eq. (8) and the transfer characteristics for white gaussian noise to predict the fading distribution of 1-min mean noise power at the output of the worst channel. The protection ratio is then that unfaded value of the single-link carrier-to-noise ratio which makes the predicted output distribution lie as close as possible to the objective thermal noise distribution without exceeding it at any point.

If the predicted distribution matches the objective at all points, the thermal noise protection ratio may be determined entirely from the message objectives that apply in the absence of a fade. Thus, using the subscript o to indicate unfaded values, and setting $C_i/X_i = (C/X)_o$, Eq. (8) gives for the thermal noise components of X_i and N_{ch}:

$$\left(\frac{C}{X}\right)_o = n\left(\frac{C_n'}{X_n'}\right)_o = \frac{n}{R}\left(\frac{S_{ch}}{N_{ch}}\right)_o \tag{89}$$

where $(C_n'/X_n')_o$ is the effective carrier-to-noise ratio at the terminal receiver and $R = R_{ch}R_n$ is the RTC for a white noise unwanted signal, as determined from Figs. 3 and 7, or from Eq. (52).

For example, with a modulation index D = 1.5, R = 23.5 dB. Hence, setting n = 140 and using the 20-percent total thermal noise objective of 3000 pW0p, or $10 \log(S_{ch}/N_{ch})_o = 55$ dB, from Fig. 12a, Eq. (89) gives, for the thermal noise protection ratio,

$$10 \log(C/X)_o = 21.5 - 23.5 + 55 = 53 \text{ dB} \tag{90}$$

This is to be compared with the unfaded single-link carrier-to-noise ratio of 50 to 53 dB obtained in practice with the TD-2 system, whose modulation index with a 600-channel baseband is about 1.5.

The effect of changing the FM modulation index or bandwidth expansion ratio from that assumed in the example is easily seen with the aid of Figs. 3 and 7. Thus, if the index is reduced from 1.5 to 0.5, the RTC will decrease from 23.5 to 12 dB, and the protection ratio must be increased from 53 to 64.5 dB. This is close to the 63-dB carrier-to-noise ratios typical of the TH system, whose modulation index with 1860 channels is about 0.5. Conversely, doubling the modulation index will increase the RTC by about 8 dB and permit the protection ratio to be relaxed to 45 dB.

In this connection, however, it is important to note from Eq. (8) that when the carrier-to-noise ratio on a single link fades by a factor ρ, the effective carrier-to-noise ratio at the terminal receiver input is reduced to

$$\left(\frac{C_n'}{X_n'}\right)_F = \frac{1}{\rho + n - 1}\left(\frac{C}{X}\right)_o \tag{91}$$

If the fade is deep, ρ will be much larger than n - 1, and it follows that $(C_n'/X_n')_F$ will be approximately equal to the apparent carrier-to-noise ratio $(C/X)_o/\rho$ on the faded link. Therefore, in choosing the modulation index, the protection ratio must not be reduced below the sum of the FM threshold and the maximum allowable single-link fade measured in dB; otherwise the terminal input carrier-to-noise ratio would drop below threshold during the fade. For example, if the maximum fade is $F = 10 \log \rho = 35$ dB, the modulation index should not exceed the value for which the protection ratio is equal to $F + 10 = 45$ dB.

The preceding calculations of thermal noise protection ratios using the unfaded noise objective assumed that the predicted distribution of 1-min output noise power matched the thermal noise objectives at all points. This method of calculation is still valid when the predicted fading is less severe than that implied by the objective, but when it is more severe, the noise objective corresponding to small percentages of the month is the dominating consideration and leads to somewhat higher values of protection ratio. For example, if actual fading is comparable to that depicted in Fig. 2 for a part of the TD-2 system, it can be shown that the protection ratios must be increased by about 8 dB over those calculated from the unfaded objectives.

Interference. In computing protection ratios for RFI in terrestrial systems, it should be recognized that the carrier-to-interference ratio at the repeater inputs will usually exhibit fading with a distribution quite similar to that for the carrier-to-thermal-noise ratio. Thus, the calculation procedure will be the same as that used for thermal noise except that (1) the RTCs will differ from white noise values to the extent that the power spectrum of the interfering signals departs from that of white noise, and (2) the unfaded carrier-to-interference ratios may not be the same at all repeaters.

To take these differences into account, the relationship between the unwanted signal on a particular link and the corresponding output channel noise component given in Eq. (20) is extended to the components of this unwanted signal. Thus for a component with power X_{ik},

$$\frac{S_{ch}}{N_{ik}} = R_{ik} \frac{C_i}{X_{ik}} \tag{92}$$

where N_{ik} is the component of the total output channel noise N_{ch} caused by the unwanted signal of type k entering the system at repeater i, and R_{ik} is the RTC for unwanted signals of this type.

In order to use this equation to calculate interference protection ratios, it is of course necessary to subdivide the total interference objectives among the repeaters and the different interference sources. Such a subdivision, consistent both with the 20-percent CCIR objectives shown in Fig. 12a and with commercial practice for intrasystem interference,[20] is shown in Table 4. The corresponding interference protection ratios, calculated from these objectives on the assumption that fading is no worse than that implied by the objectives for the smaller percentages of the month, are also presented in Table 4 for the same FM modulation indices for which thermal noise protection ratios were calculated--i.e., D = 0.5, 1.5, 3. It will be noted that the TV objectives lead to protection ratios that are about 6 dB less stringent than those imposed by the telephone objectives.

For simplicity, white noise reference case values of the RTCs were used in computing all of the protection ratios. This is a fairly good approximation for interference from terrestrial stations, and it also holds quite well for interference from satellite systems if their carrier frequencies are separated by no less than one-quarter the rf bandwidth. The protection ratios for interference from satellites are quite dependent on how the total 1000 pW0p objective for this kind of interference is partitioned--i.e., on how the 1000 pW0p is divided between earth stations and satellite repeater sources and how many entries of each kind are assumed. The result in the table illustrates the simple case where this objective is divided equally among all 140 repeaters.

Effect of Changing the Message Objectives. It is interesting to speculate on some of the design changes that would be possible if the message objectives were modified. For example, if in Table 4, the total objectives for thermal noise and for interference from satellite systems were to be interchanged, the protection ratios for thermal noise would all be increased by 5 dB while those for interference could be reduced by the same amount. The new protection ratio for thermal noise

Table 4

EXAMPLES OF MESSAGE OBJECTIVES AND PROTECTION RATIOS FOR TERRESTRIAL FM RADIO RELAY SYSTEMS

A. FDM Telephone

Source of Unwanted Signal	Assumed Number of Entries	20% Message Objective (pWOp)		Protection Ratio (dB) FM Modulation Index D		
		Per Entry	Total	0.5 (TH)	1.5 (TD-2)	3
Mux equipment	-	-	2,500[a]	-	-	-
Intermodulation (IM)	-	-	2,500	-	-	-
Thermal noise	140	21.4	3,000	65	53	45
RFI from terrestrial systems						
Trunk repeaters	280	2.1	1,000	75	63	55
Spur repeaters	68	6		70.5	58.5	50.5
RFI from satellite systems	140	7.1	1,000[a]	70	58	50
Total from all sources			10,000[a]	38.5[b]	26.5[b]	18.5[b]

B. Television

Source of Unwanted Signal	Assumed Number of Entries	20% Message Objective (dB)	Protection Ratio (dB) FM Modulation Index D		
			0.5 (TH)	1.5 (TD-2)	3
Thermal noise	140	60[a]	58.5	46.5	38.5
RFI from satellites	140	65	63.5	51.5	43.5
Total from all sources		55	32[b]	20[b]	12[b]

[a]CCIR recommended value.

[b]Minimum effective wanted-to-unwanted-signal ratio at terminal receiver input.

could then be achieved, for example, by increasing the transmitter power at all of the repeaters by 5 dB. This change in repeater power would not affect the observed carrier-to-interference ratios for intrasystem interference, and so the protection ratio for this type of interference would continue to be met. But the carrier-to-interference ratios observed for interference from satellites would be enhanced by 5 dB. Since the protection ratio for this type of interference is now 5 dB lower than before, satellite EIRP could be raised by nearly 10 dB.

Satellite FM Systems

As previously noted, a satellite relay system has only two links to consider, and fading on these links is normally less severe than that indicated in the CCIR noise objectives. On the other hand, the output noise objectives are not normally partitioned equally between the up link and the down link.

For any specified partition of the output noise, however, the corresponding protection ratios may be calculated from Eq. (92). Thus, designating the up link by the subscript i = 1 and the down link by i = 2, the protection ratios for the thermal noise component of the unwanted signals are

$$\left(\frac{C_1}{X_{iN}}\right)_o = \frac{1}{R_{iN}}\left(\frac{S_{ch}}{N_{iN}}\right)_o \qquad i = 1,2$$

where R_{iN} and $\left(S_{ch}/N_{iN}\right)_o$ are respectively the RTC and the unfaded message objective for thermal noise entering the system at the receiver for link i. Similarly, the protection ratios for interference are given by

$$\left(\frac{C_1}{X_{iI}}\right)_o = \frac{1}{R_{iI}}\left(\frac{S_{ch}}{N_{iI}}\right)_o \qquad i = 1,2$$

where R_{iI} and $\left(S_{ch}/N_{iI}\right)_o$ are respectively the RTC and message objectives for interference entering the system by link i.

Application of these equations will be illustrated for telephone and television channels using the FM modulation parameters proposed for the INTELSAT III system and for the domestic pilot program of the Communications Satellite Corporation.[21,22] The unfaded message objectives assumed for these systems are displayed in Table 5A for telephone channels and in Table 5B for television. The telephone objectives are consistent with a noise budget used in planning the INTELSAT III system.[21] In accordance with Fig. 12c, the television objective for random noise is the same as that assumed in Table 4B for terrestrial systems. In partitioning the thermal noise objectives between up link and down link, a 3:1 ratio has been assumed, whereas the RFI objective has been divided equally.

As with the examples used to illustrate the calculation of protection ratios for terrestrial systems, white noise RTCs are used. This is believed to be a conservative assumption for a model environment in which carrier frequency interleaving makes the total interference spectra approximately uniform across the passband of any given receiver.

In INTELSAT III, three capacities of rf channels are to be provided for telephone messages. These will employ wideband FM to carry FDM basebands of 24, 60, and 132 channels in nominal rf bandwidths of 5, 10, and 20 MHz, respectively. The proposed FM modulation indices are such that the white noise RTC is about 44.5 dB for all three capacities. The resultant up link and down link protection ratios are shown in Table 5A. It should be noted that these protection ratios apply to the total unwanted noise and the total unwanted interference. This means, for example, that if two interfering sources contribute equally, the protection ratio against each would be 3 dB higher than the value shown in the table.

The trunk message channels of the domestic satellite system will carry 1800-channel telephone basebands using narrowband FM with a modulation index of about 1.25. The RTC for such a circuit and white noise interference is about 21 dB, leading to the protection ratios cited in Table 5A.

Table 5

EXAMPLES OF MESSAGE OBJECTIVES AND PROTECTION RATIOS FOR SATELLITE FM SYSTEMS

A. FDM Telephone

Unwanted Signal Component	20-percent Message Objective (pWOp)			Protection Ratio (dB)			
				INTELSAT III		Pilot Program Trunk Messages (D = 1.25)	
	Total	Up Link	Down Link	Up Link	Down Link	Up Link	Down Link
IM and other noise	3,340	2,840	500	..	..	..	..
Thermal noise	5,660	1,410	4,250	14	9	37.5	32.5
RFI	1,000[a]	500	500	18.5	18.5	42	42
Total	10,000[a]	4,500	5,500	5.5[b]		29[b]	

B. Television

Unwanted Signal Component	20-percent Message Objective (dB)			Protection Ratio (dB)			
				INTELSAT III (D = 3.75)		Pilot Program (D = 3)	
	Total	Up Link	Down Link	Up Link	Down Link	Up Link	Down Link
Thermal noise	60[a]	66	61.5	20	15.5	23	18
Interference	67.5	70.5	70.5	24.5	24.5	27.5	27.5
Total	57.5	63.5	59	11.5[b]		14.5[b]	

[a] CCIR recommended value.

[b] Minimum effective wanted-to-unwanted signal ratio at terminal receiver input.

For TV transmission with INTELSAT III, three quality levels are contemplated. The highest quality level uses an FM modulation index of 3.75 and requires a nominal 40-MHz rf channel. The white noise value of the RTC for such a channel is 45 dB, which when combined with the message objectives used for terrestrial TV transmission yield the protection ratios shown in Table 5B. Finally, this table shows the protection ratios for TV transmission using the pilot system, with an FM modulation index of 3, corresponding to a white noise RTC of about 43 dB.

Digital Systems

Frequency modulation is used almost exclusively in existing terrestrial and satellite radio relay systems, but both types of systems can be expected to make increasing use of digital modulation methods in the future. As previously noted, no international standards for message quality have been adopted for digital systems, but it is easy to give numerical illustrations of the protection ratios required for service equivalent to that provided by the FM systems.

For example, consider a 140-link terrestrial system using 4-phase NCPSK to transmit 8-bit PCM-encoded TDM telephone channels via baseband repeaters. As indicated in Fig. 10b, the quantizing noise will be about 2500 pW0p, which is roughly equal to the unfaded thermal noise allowance shown in Fig. 12a for an FDM telephone channel. In such a system, the total error probability p_n' at the terminal receiver is simply the sum of the link error probabilities as shown by Eq. (74). From Fig. 10b, it is seen that at the PCM threshold $p_n' = 4 \times 10^{-6}$. Hence, if there were no fading and all links were identical, the required link error probability would be

$$p_i = p_n'/n = 2.9 \times 10^{-8}$$

From Fig. 10a, it is seen that for 4-phase NCPSK and a pulse shape factor k = 1.5, this error rate requires an input wanted-to-unwanted signal ratio of 19.3 dB.

In practice, of course, there will be fading, and even a 1-dB fade will lead to more than an order-of-magnitude increase in error rate. This means that a small fade on only a single link will cause the error probability for that link to dominate the contributions to p_n'. Hence, no link can be permitted to drop significantly below the unfaded wanted-to-unwanted signal ratio of 19.3 dB if output message quality is to be maintained at the threshold level determined by the quantizing noise. Even if the output channel signal-to-noise ratio were allowed to drop 15 dB during severe single link fades, as in the case of the FDM objectives of Fig. 12a, the minimum permissible wanted-to-unwanted signal would be reduced by only 1 dB.

The conclusion is that for the system in question the protection ratio for total noise and interference is about

$$19 + F \text{ dB}$$

where F is the maximum single link fade through which the system is expected to operate.

As a second illustration, consider a satellite relay system which uses the same type of signal processing and modulation to carry telephone channels through an rf-type repeater. In this case, the total error probability at the earth station terminal receiver depends on the apparent up link and down link wanted-to-unwanted signal ratios C_1/X_1 and C_2/X_2, respectively, in the manner indicated by Eqs. (70) and (71) with $n = 2$. The threshold error probability is again $p_n' = 4 \times 10^{-6}$ and, referring to Fig. 10a, the corresponding effective wanted-to-unwanted signal ratio for $k = 1.5$ is $C_2'/X_2' = 56$ (17.5 dB).

If there were no fading, C_1/X_1 and C_2/X_2 would thus have to meet the condition

$$X_1/C_1 + X_2/C_2 = X_2'/C_2' \; 1/56$$

in accordance with Eq. (71). Within this constraint, the values of the individual ratios will depend on the desired division of noise and interference between the up link and the down link. With an equal division, for example, the minimum wanted-to-unwanted signal ratios would be 17.5 + 3 = 20.5 dB, and the protection ratios for total noise and interference would be

$$20.5 + F \text{ dB}$$

where F is the fading margin appropriate to the up link and down link.

Appendix A

ANALOG MESSAGE AND BASEBAND SIGNAL CHARACTERISTICS

As background for deriving the transfer characteristics of various types of analog demodulators and terminal equipment, it will be useful to describe briefly the properties of the single channel messages and basebands which these circuits are designed to handle. For most signals, the description will necessarily be statistical in nature and will use the amplitude distribution and power spectrum to characterize the signal in time and frequency respectively. In the case of telephone messages, numerical values of signal and noise power will usually be given at a "point of zero relative level" or "0-dB transmission level point." This is simply an agreed reference point in the message circuit such as the toll-transmitting switchboard in a telephone system. Power levels at other points can then be calculated, provided the transmission gain or loss relative to the point of zero relative level is known.

SINGLE CHANNEL SPEECH

One of the most irregular message waveforms, and certainly the most commonly transmitted, is that of human speech. An important property of speech, observed in one of the two one-way telephone channels that comprise a voice circuit, is its discontinuous nature. A typical telephone channel is "active" (actually carrying speech power) only a fraction of the time that it is "busy" (being used by a customer to complete a call or carry on a conversation). This is due not only to the fact that each party listens to the other about half the time, but that ordinary speech contains many silent pauses. When nonbusy periods are also counted, a typical one-way telephone channel in a multichannel trunk is active only about one-quarter of the time even during the busiest traffic hour of the day.

When a channel is active, the instantaneous amplitude of the speech signal varies over a wide dynamic range. Although the amplitude remains below its rms value nearly 90 percent of the time, it rises to peaks 10 dB above the rms level at least 2 percent of the time and to

20 dB above rms almost 0.1 percent of the time. In addition, since some people talk louder than others, the rms level in an active channel also varies over a considerable range from one talker to another.

The loudness of speech is measured in terms of a quantity called "volume" with the aid of a special instrument called a vu-meter or volume indicator. Since, to a good approximation, volume is proportional to the logarithm of average speech power, the fact that speech volume is normally distributed implies that the distribution of average speech power is log-normal. The mean of the speech power distribution defines an "average talker," and the volume of the average talker is about 4 dB higher than the mean of the volume distribution, the standard deviation of which is about 6 dB. At a point of zero relative level, the average speech power of the average talker is about 0.1 mW or -10 dBm, although the exact value depends somewhat on the overall transmission quality of the telephone circuits being measured.

The power spectrum of a speech signal shows that more than 99 percent of speech energy lies at frequencies below 3000 Hz, with maximum energy located at about 400 Hz. The passband of a typical telephone channel extends from 300 Hz to 3400 Hz.

SINGLE CHANNEL TEST TONE

Due to the complexity and variability of the speech waveform, voice channels are normally tested and channel quality is specified in terms of a sinusoidal "test tone" at 1000 Hz (800 Hz in Europe) with amplitude adjusted to provide an average power of 1 mW (0 dBm) at a point of zero relative level.

FREQUENCY DIVISION MULTIPLEXED SPEECH BASEBANDS

When more than one speech channel is to be transmitted over the same route by an analog modulated relay system, advantage can be taken of the sporadic and dynamic nature of the individual speech channels. This may be done, for example, by combining them into a single composite signal or baseband by means of frequency division multiplexing (FDM). In this technique, each telephone channel is connected to the input of one of a collection of SSB modulators whose carrier frequencies are separated by exactly 4 kHz. The modulator outputs are then simply added to obtain a baseband consisting of SSB versions of each speech input stacked in frequency at 4 kHz intervals. The combination of channels is usually carried out in a series of steps until the desired total circuit capacity is obtained. Twelve channels are first multiplexed to form a "group." Five groups are then multiplexed to form a "supergroup" of 60 channels. Ten supergroups can then be multiplexed to form a "mastergroup" of 600 channels. If basebands of still larger numbers of voice channels are desired, mastergroups and supergroups can be further multiplexed.[23]

None of the successively larger basic basebands (group, supergroup, etc.) extends down to zero frequency, and when supergroups or mastergroups are combined, they are separated by spectral guard bands. As a result, the highest frequency in an n channel FDM baseband is not 4n kHz, but is given approximately by

$$f_m = 4.5n \text{ kHz} \tag{A-1}$$

The actual limits of the baseband spectrum and of the ratio f_m/n are tabulated for various numbers of voice channels in Table A-1*.

The statistical characteristics of an FDM voice baseband at any particular time depend not so much on the total number of channels it comprises as on how heavily the channels are "loaded"--i.e., on how many channels are simultaneously active and on the speech volumes in these channels. Since the number of active channels in an n-channel baseband can range from zero to n, and the channel volumes are normally distributed, the short-term amplitude distributions and power spectra of the baseband signal will be different from one interval to the next.

As the number of simultaneously active channels increases, the amplitude distribution of the FDM baseband signal changes gradually from that of a single voice channel to that characteristic of a gaussian random process, the change being virtually complete by the time the number of active channels reaches 64. The baseband power spectrum is

*See Ref. 4, Recommendations 380-1, 398-1, and 401-1.

Table A-1

CCIR-RECOMMENDED FDM TELEPHONE BASEBAND CHARACTERISTICS

Number of Telephone Channels n	Lowest Modulating Frequency, f_ℓ (kHz)	Maximum Modulating Frequency, f_m (kHz)	$\frac{f_m}{n}$	rms Frequency Deviation Due to Single Channel Test Tone, f_{rch} (kHz)
12	12	60	5.0	35
24	12	108	4.52	35
60	12 60	252 300	4.2 5.0	50, 100, 200
120	12 60	552 552	4.6 4.6	50, 100, 200
300	60 64	1300 1296	4.33 4.32	200
600	60 64	2540 2660	4.23 4.43	200
960	60 316	4028 4188	4.20 4.36	200
1260	60 60 316	5636 5564 5564	4.47 4.42 4.42	140, 200
1800	312 316 312	8204 8204 8120	4.56 4.56 4.51	140
2700	312 316 312	12,388 12,388 12,336	4.59 4.59 4.57	Not specified

even simpler to describe, since the SSB modulators involved in the multiplexing merely shift the single channel spectra in frequency without changing their shapes or relative magnitudes. Thus, the baseband power spectrum is just the superposition of the spectra of the active channels each shifted to its own 4 kHz frequency slot. Slots in the baseband spectrum corresponding to inactive channels are of course empty.

In order to design circuits to handle such a variable signal without objectionable distortion, it is important to know just how high its rms and peak values rise during periods of heavy telephone usage. It is the peak value of the baseband signal during such periods of maximum load that determines the amplitude range over which the circuits of an amplitude modulated system must be linear. Similarly, in a phase or frequency modulated system, the peak baseband load determines the bandwidth occupied by the modulated carrier and hence the frequency range over which the system must be sensibly free of distortion.

The "peak" baseband load is by no means the highest level that the baseband signal ever reaches, but it must be high enough so that the period of distortion that occurs when it <u>is</u> surpassed is too brief to be objectionable to system users. Quantitatively, the peak load is defined in terms of the variations of the instantaneous baseband power during the busiest hour of the day. The busy hour is first divided into short intervals such that, in each interval, the number of active channels and the volume in each channel remains constant. Then, noting for each interval the level exceeded with 0.1 percent probability,* the peak load is defined as the 99th percentile of the set of 0.1 percent levels.

For an n-channel FDM voice baseband, the value of the peak load thus defined will depend on τ, the "activity factor" or probability of finding a given channel active during the busiest hour, and on the power represented by an average talker. The upper curve in Fig. A-1 shows how the ratio of peak baseband load to average talker power varies with n for $\tau = 1/4$. The lower curve shows the corresponding average

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*For some circuits, a probability of 0.01 percent is chosen.

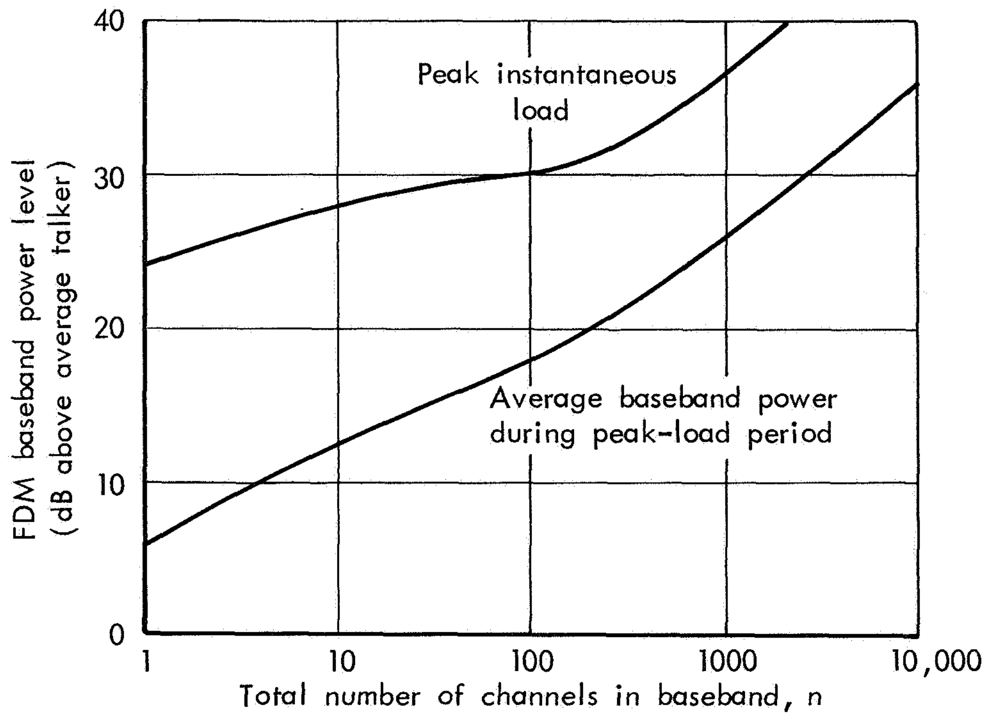

Fig. A-1 — Peak and average power levels exceeded only 1% of busy hour by an n-channel FDM voice baseband with a 25% activity factor

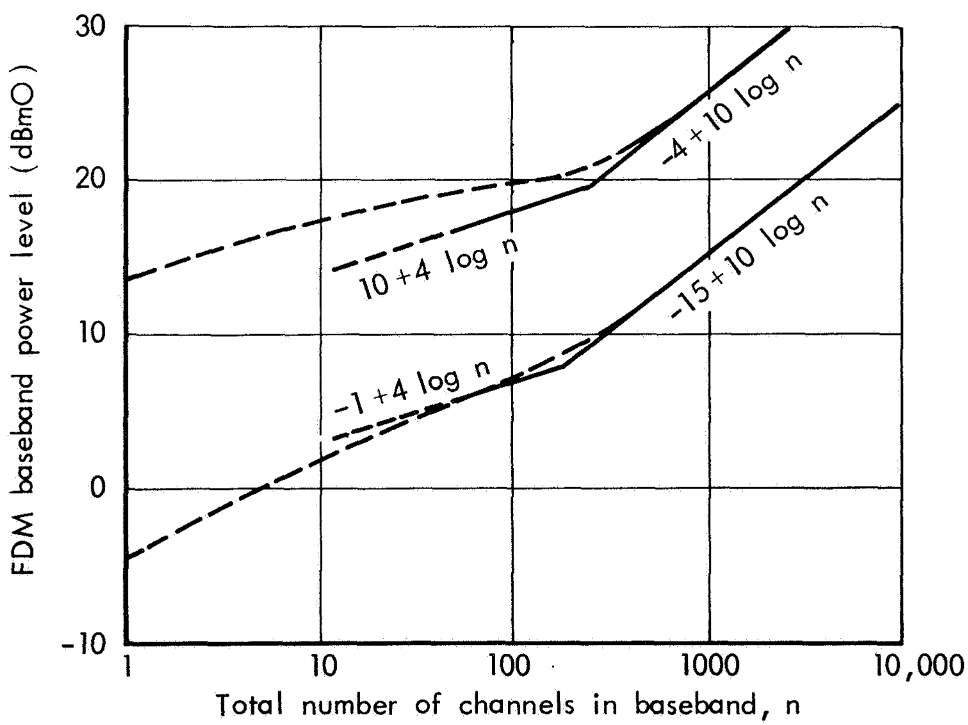

Fig. A-2 — White noise equivalent baseband signal values of the baseband power. For any value of n, the difference in dB between the two curves is of course the peak-to-average power ratio for the peak busy hour baseband. Note that, for large values of n, the average baseband power becomes proportional to n, and the peak-to-average ratio approaches 11 dB, the value characteristic of the 0.1 percent peak level of a gaussian distributed signal.

Probably the most important thing to observe about the maximum busy hour load due to any n-voice channel FDM baseband is that it is much less than n times the maximum load for a single channel. For example, Fig. A-1 shows that the load for 100 channels is only 12 dB (16 times) greater than the single channel load. This is even more true of the peak load. The peak baseband load for 100 channels is only 6 dB (4 times) greater than that of a single channel. These advantageous properties of the baseband stem directly from the discontinuous and "peaky" nature of the speech waveforms in the component channels.

NOISE-EQUIVALENT TELEPHONE BASEBAND

The fact that the baseband amplitude distribution becomes gaussian for large n provides the basis for using a band of white gaussian noise as a test signal to represent the peak-load busy hour FDM baseband. For a given number of channels, the frequency limits of the noise-equivalent baseband are the same as the actual basebands shown in Table A-1, and by international recommendation,* the average power in dBm at a point of zero relative level is taken to be

$$10 \log S = \begin{cases} -15 + 10 \log n, & n \geq 240 \\ -1 + 4 \log n, & 12 < n < 240 \end{cases} \tag{A-2}$$

where S is in mW.

This formula, which is only provisional for $n < 60$, is plotted as the lower solid curve in Fig. A-2. The upper solid curve is the

*See, for example, Ref. 4, Recommendation 353-1, Note 4.

corresponding 0.1-percent peak baseband load. For comparison, the more realistic curves of Fig. A-1 are shown with dashed lines. In order to make this comparison, it is necessary to assign a value in dBm0 to the average talker power because the power levels in Fig. A-1 are expressed in dB relative to average talker power whereas those in Fig. A-2 are expressed in dBm0. It should be noted that the value of average talker power required to achieve the agreement for large n shown in Fig. A-2 is about -11 dBm0, and not -15 dBm0, as some authors erroneously infer from Eq. (A-2). Also, note that the empirical fit which Eq. (A-2) presumably represents becomes increasingly bad for low values of n where the actual baseband signal has a much higher peak-to-average power ratio and in any case, is poorly represented by a gaussian distributed signal. Finally, it should be recognized that the uniform power spectrum of the noise-equivalent busy hour baseband is quite different from that of the signal it represents. Instead of log-normally distributed levels of speech power in roughly one quarter of the channels, it puts smaller but equal amounts of noise power in all of the channels.

MONOCHROME TELEVISION

Next to human speech, the most frequently transmitted analog message waveform is one carrying the information needed to display a television picture. There is no unique way of encoding such information in electrical form, and different countries have developed somewhat different signals for the purpose. Fundamentally, however, they are all alike in that they may be regarded as composite basebands obtained by multiplexing a set of independent waveforms representing the components of a moving, talking picture whose pictorial content has been sampled by a periodic line-scanning process.

For a black-and-white (monochrome) television picture, there are three such component waveforms:

- a "picture" signal which describes the brightness variations along the scan lines
- a "synch" signal which provides synchronizing information for reassembling the scans into a picture
- a "sound" signal

For color television, two additional waveforms are required in conjunction with the brightness signal to specify for each picture element the three characteristics of its color: brightness, hue, and saturation. In this case, it is the combination of these three waveforms that is called the "picture" signal.

In distributing television signals from studio to broadcast stations, the sound waveform is usually sent separately over an independent channel, but the picture and synchronizing signals are always sent together as parts of a single baseband called the "video" signal.* The differences in the basebands of different national systems lie in such details as the number of lines per picture (and hence the resolution) and the number of pictures transmitted per second (which, taken together with the number of lines, determine the baseband bandwidth). A brief discussion of the baseband for the U.S. system will illustrate the general characteristics common to most of the national systems.[24]

Consider first the baseband for black-and-white (monochromatic) television. Here the picture signal describes only the brightness variations in the image being transmitted. It is obtained using a scanner that moves a spot across the picture in a series of vertically separated, horizontal lines and yields an instantaneous voltage proportional to the image brightness at the spot location. Scanning is from left to right beginning at the top, and successive lines are separated by about twice the spot diameter which, in turn, is about 1/500 the height of the picture. At the end of each line, during the time required to sweep the scanning spot back for the start of the next line, a blanking pulse is sent. This simply means that the picture signal drops to a so-called "blanking level" which lies just below the level corresponding to black (zero picture brightness), and insures that the return trace on the picture tube will be invisible. When the bottom of the picture is reached, a much longer blanking pulse is sent and the spot is returned to the top of the picture for the next series of

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*An interesting variation on this practice is found in the Soviet "Molniya" satellite TV relay system in which the sound signal is transmitted in sample form during the blanking interval of the picture signal.

line scans. The lines of this series do not retrace those just scanned but rather lie halfway between them, a technique called "interlacing." Each top-to-bottom series of line scans is called a "field," and it is seen that a complete scan of the picture, a so-called "frame," consists of two successive interlaced fields.

A new frame is scanned every 1/30 of a second and the scanning rate is such that this is exactly 525 times the interval between successive line scans. For this reason the U.S. system is sometimes said to produce a 525-line picture. In actuality, the blanking pulse at the end of each field has a duration of from 13 to 21 times the line-scanning period, which leaves only 483 to 499 lines to form a complete frame. Moreover, although the interval between successive lines is $(30 \times 525)^{-1}$ sec = 63.5 μsec, the line blanking pulse takes about 11 μsec, leaving only 52.5 μsec to write each line of picture information.

The synchronizing waveform consists of a short "horizontal synch pulse" at the end of each line for triggering the next horizontal line sweep at the receiver, and a series of six longer "vertical synch pulses" at the end of each field for triggering the next vertical sweep. The latter are placed between two sets of 6 short pulses called equalizing pulses, and the whole 18-pulse sequence is then followed by from 4 to 12 line synch pulses. The synchronizing pulses occur during the corresponding blanking pulses of the picture signal. As previously noted, the combination of the picture and synch waveforms is called the video signal and may be regarded as a time division multiplex of these two waveforms.

The synchronizing signal is distinguished from the picture signal not only in time but also in amplitude. Unlike voice signals and voice basebands, the video signal has well defined maximum and minimum values. In particular, when the video signal polarity is "positive" (signal level increases in a transition from black to white), the maximum or peak amplitude is called the "peak white level" and represents the amplitude corresponding to maximum picture brightness. The minimum amplitude is the "synch-peak level" which corresponds to the tips of the synch pulses. The difference between these two levels is the "peak-to-peak amplitude" of the video signal. It is the amplitude normally used in describing the overall magnitude of the video signal or the power it represents.

The previously mentioned blanking level is customarily taken as the reference level for signal measurement. It is so positioned in the U.S. system that the nominal amplitude of the picture signal (measured from the blanking level to the peak white level) is about 70 percent of the peak-to-peak video amplitude, while the synch signal amplitude (measured from the blanking level to the synchronizing level) is about 30 percent of the peak-to-peak amplitude. A fourth reference level, the "black level," corresponding to zero picture brightness, lies just above the blanking level and as close to it as the state of the art permits.

When used with vestigial sideband amplitude modulation (VSB) for TV broadcast in the United States, the video baseband is reversed in polarity (negative modulation). The peak synch level then becomes the positive peak of the modulation envelope and all levels are measured with reference to the zero carrier level. Specifically, if the peak-synch level is labeled 100 percent, the blanking level is 75 percent, and the peak white level should lie between 0 and 15 percent, with the latter value preferred.

The third component of a complete monochromatic TV signal, the sound waveform, could be multiplexed with the video signal at baseband, i.e., prior to modulation of the rf carrier, but usually is not. As already noted, in distributing TV program material to broadcast stations over terrestrial relay facilities, the sound signal is normally sent over an independent channel. Moreover, in broadcasting to home receivers in the U.S. system, the video and sound signals are fed to separate transmitters which use different modulation methods: VSB for the video signal and FM with a peak deviation of 25 kHz for the sound signal.*

On the other hand, an exact difference of 4.5 MHz is maintained between the video and the sound carrier frequencies, the ratio of sound

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*AM is used for the sound signal in certain other systems.

carrier power to the peak power of the modulated video carrier is kept at a fixed value--usually 50 percent--and the two rf signals are combined before reaching the transmitting antenna. Thus, so far as the rf signal which leaves the transmitting antenna is concerned, the result is much the same as if a single rf carrier had been VSB modulated by an FDM baseband consisting of the sum of the video signal and a 4.5 MHz subcarrier frequency modulated by the sound signal. Indeed, the output of the first detector in a home receiver is just such a baseband.

The spectrum of the monochrome video baseband signal is extremely complicated. To begin with, it should be noted that it is basically a line, rather than a continuous spectrum. This is to be expected from the periodically repetitive nature of the video waveform. The repetitiveness is obvious in the case of the synchronizing signal, and especially for the horizontal or line synch pulses whose period of 63.5 μsec corresponds to a fundamental frequency of exactly 15.75 kHz. The brightness information in the line scans is also quite repetitive. Not only is one line similar to its immediate neighbors, but successive frames are almost identical with each other since motion within the picture is usually slow compared with the 30-Hz frame rate. Finally, the vertical and horizontal blanking pulses impose obvious periodicities of 15.75 kHz and 60 Hz on the picture signal respectively.

Another striking feature of the video spectrum is that most of the space between the dominant spectral lines is free of energy. This is true not only of the harmonics of 15.75 kHz, but also for the lines spaced at 60-Hz intervals about these harmonics.

Despite its apparent complexity, the basic structure of the video baseband spectrum may be understood with the aid of elementary Fourier analysis. The complete spectrum is simply a superposition of the spectra due to the waveforms which comprise the video signal (picture and synch signals) with proper attention to their relative phases. Thus the spacing of the spectral lines for each component waveform is equal to the reciprocal of the waveform's period, while the relative amplitude of the line is determined by such things as the maximum rate of change of the waveform--e.g., by pulse rise and fall times--and by its "duty cycle," i.e., the ratio of pulse duration to pulse spacing.

For example, the spectral lines corresponding to the synchronizing waveform are spaced at 15.75 kHz, and can have amplitudes exceeding 1/1000 that of the fundamental as far out as the 320 harmonic which lies at about 5 MHz. The spectrum of the picture signal for the simplest possible picture (a uniform gray field) has a similar 15.75-kHz line structure, albeit with different line amplitudes, and occupies a similar bandwidth. However, the vertical blanking part of the picture signal imposes a 60-Hz square-wave modulation on the brightness information and hence on each of the 15.75-kHz harmonics which comprise the spectrum. As a result, each such harmonic is accompanied by symmetrical sidebands consisting of lines spaced 60 Hz apart. The location of the first zero in the envelope of these sidebands is determined by the duty cycle of the vertical blanking pulses and occurs at the 14th 60-Hz harmonic, or at about 840 Hz on either side of the 15.75-kHz line. This accounts for the previously noted fact that the bulk of the space between the 15.75-kHz harmonics is empty. The structure of the sidebands is further complicated by lines of smaller amplitude at 30-Hz spacing due to field interlace.

Except for the relative amplitude of the lines, the picture signal spectrum for more complicated pictures will be similar to that for a flat grey field so long as there are no diagonal patterns and no image motion. When there are diagonal lines, however, such as those in a TV test pattern, successive line scans will not be the same, and the sidebands accompanying the 15.75-kHz line-scan harmonics will be more diffuse. Moreover, the width of these sidebands will increase with the order of the harmonic. Thus the space between line harmonics becomes less and less empty as frequency within the baseband is increased. Nonetheless, with no image motion the spectra are still line spectra; the sidebands are wider but still consist of lines at 30-Hz spacing about each 15.75-kHz harmonic.

When there is image motion, not even successive frames are repeated exactly. The result is that each of the 30-Hz-spaced lines in the sidebands is broadened to an extent which depends on the nature of the motion. If the motion itself is repetitive and commensurate with the field rate, the broadening of each 30-Hz line will consist of a subsidiary line spectrum of "motion harmonics" at a spacing given by the period of the motion.

In summary, then, the spectrum of a monochrome video signal consists of major concentrations of energy at 15.75-kHz intervals. Each concentration in turn consists of a cluster of lines separated by 30 Hz with each line broadened to a degree which depends on the type and amount of motion in the image. The relative amplitude and spectral width both of the clusters and of the lines which comprise them are determined by the nature of the picture being scanned. With the scanning and frame rates of the U.S. system, baseband spectral components with frequencies higher than 4.2 MHz can be ignored without seriously impairing subjective picture quality.

Although the bandwidth of the video baseband is 4.2 MHz, the rf bandwidth allocated for TV broadcast in the United States is 6 MHz. This amount is required to accommodate a VSB modulated picture carrier whose frequency lies 1.25 MHz above the bottom of the rf passband, and an FM modulated sound carrier located 4.5 MHz above the picture carrier, or 0.25 MHz below the top of the rf passband.

COLOR TELEVISION

The reproduction of television pictures in color is based on the elementary fact that light of any color can be simulated by combining appropriate amounts of light in the three primary colors. A color picture can thus be reproduced point by point if three sources of primary light which can be independently controlled are provided for each point in the picture. The picture tube of a conventional color TV set provides just such sources in the form of an array of closely spaced phosphor dots arranged in groups of three on the screen of the tube. Three different phosphors are used so that, when excited, each dot in a group emits one of the three primary colors, red, green, and blue. The tube also contains three independent electron guns so arranged that the beam from a given gun can excite only the dots corresponding to one of the primary colors. The three electron beams are then scanned in unison but independently modulated in intensity so that, for each picture element, the proper amount of each primary color is excited from the corresponding group of phosphor dots.

There are a variety of ways in which the information on the color content of the picture elements can be encoded for transmission. The way actually used in the NTSC (National Television System Committee) system adopted by the United States was chosen to meet several practical objectives, among which two of the most stringent were:

1. The color TV signal was to be compatible with existing monochrome receivers--i.e., produce an acceptable black-and-white picture on such sets.
2. The color signal was to be transmitted in the same rf bandwidth used for monochrome signals.

To help meet the first objective, the three waveforms which tell how the required amount of each primary color varies as the picture is scanned are <u>not</u> transmitted directly. Instead, three new waveforms are derived from them in such a way that one, called the luminance signal, contains information only on brightness variations, while the other two, called color difference or chrominance signals, can be combined to give the variations in hue and saturation. When the picture to be transmitted contains no color, the chrominance signals vanish.

The luminance signal occupies exactly the same position in the color video baseband as does the brightness signal in the monochromatic television baseband. The synch signal component of the color TV baseband is also virtually identical to its monochromatic counterpart. Thus, if the color data were omitted from the video baseband, or if the scene being scanned were entirely in shades of grey, a monochrome TV receiver would display the same picture as if a black-and-white TV camera had generated the baseband. But the second objective requires that the two chrominance signals be transmitted in the same baseband as the luminance signal. In order to do this without creating "visual crosstalk" among these three components of the picture signal, the chrominance signals are constructed to take advantage of both the characteristics of color vision and the spectral properties of the picture signal components.

The pertinent feature about color perception is that the human eye appears to have three-color vision only for objects that subtend angles of a half degree or more, two-color vision for objects down to about one-sixth of a degree, but cannot distinguish colors for objects smaller than this. This fact is used to reduce significantly the bandwidth required for the chrominance signals. In particular, while the luminance signal requires a bandwidth of 4.2 MHz, it is found that a chrominance signal describing the departure of a color from white in the direction of orange or its complement cyan (blue-green) need occupy only a 1.5-MHz bandwidth, and a bandwidth of 0.5 MHz is sufficient for a chrominance signal describing departures from white in the green-magenta direction.

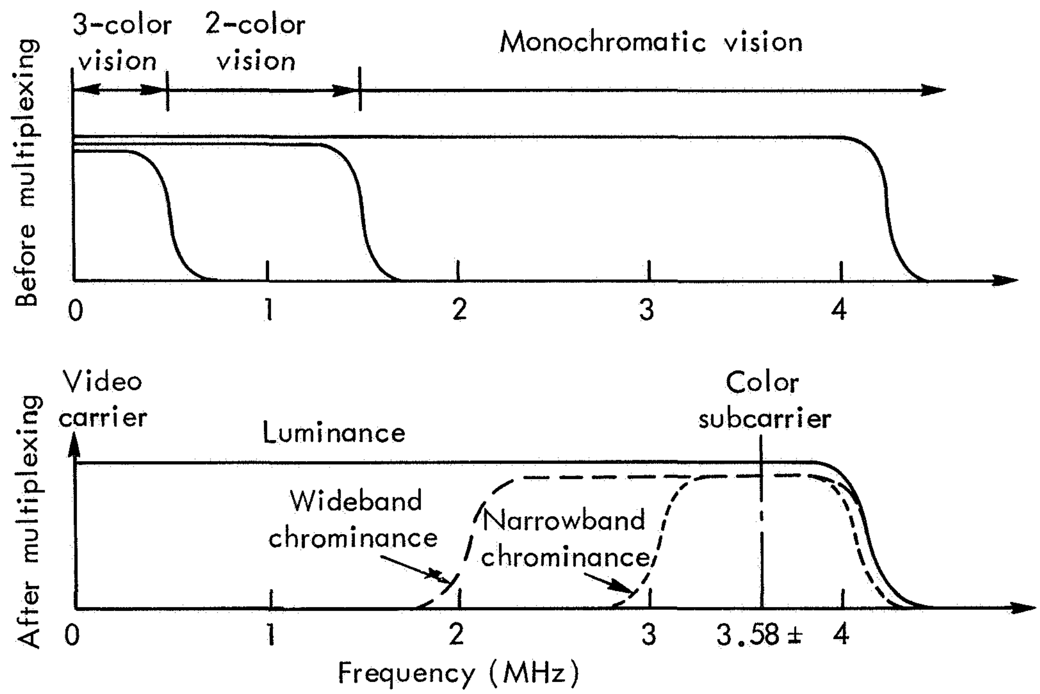
To avoid interference between the chrominance signals after frequency division multiplexing with the luminance signal, they are modulated onto subcarriers which have the same frequency but differ in phase by 90^o. Amplitude modulation is used in order to keep the bandwidth of the modulated subcarriers low, and the subcarriers are suppressed to insure that the chrominance components of the baseband vanish when there is no color. Specifically, DSB is used for the 0.5-MHz chrominance signal and VSB for the 1.4-MHz chrominance signal. The two signals are separated at the receiver with synchronous detectors whose locally generated carriers are kept at the proper frequency and phase with the aid of a "pilot" or reference subcarrier transmitted as part of the video baseband in a "burst" of 8 to 11 cycles' duration during each horizontal blanking interval. This color burst is located on the "back porch" of each horizontal synchronizing pulse.

Since amplitude modulation of quadrature carriers is equivalent to simultaneous phase and amplitude modulation of a single carrier, the color video baseband may also be viewed as a result of adding to the combined luminance-synch signal a subcarrier whose phase difference from the reference carrier is a measure of hue and whose amplitude is a measure of saturation.

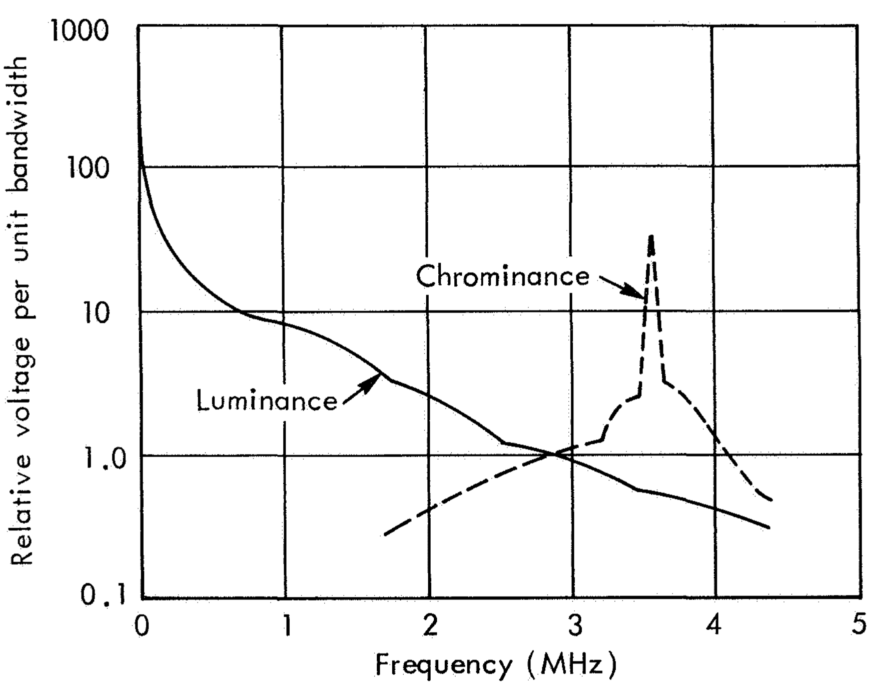
The spectrum of the amplitude modulated color subcarriers is qualitatively similar to that of the luminance signal in that each of its sidebands consists of clusters of lines spaced at 30 Hz about frequencies whose differences from the subcarrier frequency are harmonics of the line scan frequency. By choosing the common subcarrier frequency to be an odd multiple of one-half the line scan frequency, the clusters of the chrominance modulated carrier spectrum are made to fit neatly into the nearly empty spaces between the clusters of the luminance-synch signal. Even where clusters overlap, the individual lines are still separated by 15 Hz.

Under these circumstances, it might be thought that a high-resolution comb filter would be required to separate the chrominance and luminance signals. This is not in fact necessary because the display process effectively discriminates against the spectral components of the chrominance modulated subcarriers. These components do appear on the picture tube, but their frequencies are just those which produce alternating checkerboard patterns of extremely low visibility. Moreover, by choosing the frequency of the chrominance subcarrier near the high end of the baseband, the checkerboard pattern is extremely fine-grained.

To insure that interference due to possible beats between the received chrominance subcarrier and the sound carrier will produce only low-visibility patterns on the picture tube, the difference between these two carrier frequencies is also made an odd multiple of one-half the line scan frequency. However, in order to satisfy both this and the previous condition on the frequency of the chrominance subcarrier simultaneously, it is necessary to change either the line scan frequency or the sound carrier frequency from the values used in monochrome television. In practice the frequency of the sound carrier is left unchanged at exactly 4.5 MHz above the video carrier, with the end result that the line scan frequency for color television is not 15,750 Hz, but 15,734.264 Hz. Of the line scan frequencies permitted by the conditions on the chrominance subcarrier frequency, this frequency is the one closest to the monochrome value, and it makes the frame rate for color TV not 30 Hz, but 15,734.264/525 = 29.97 Hz. The color subcarrier frequency itself is then chosen as the 455th harmonic of half the line scan frequency, or 3,579,545 Hz. A schematic representation of the components of the color video baseband and its spectrum is shown in Fig. A-3.[24] The spectrum of the corresponding monochrome video baseband consists of only that portion associated with the luminance signal.



(a) Schematic pass bands of luminance and chrominance signals



(b) Typical spectral envelopes of luminance and chrominance signals

Fig. A-3—Spectral components of color-picture signal

Appendix B

CHARACTERISTICS OF ANALOG MODULATED CARRIERS

As demonstrated in Section II, the magnitude of the transfer characteristics for a specified channel in a radio relay system employing an analog modulation method depends in general on the statistical characteristics of both the wanted and the unwanted signals. These in turn depend on the type and degree of modulation used with these signals as well as on the type of messages they carry.

The various analog modulation methods of interest to satellite and terrestrial radio relay systems are defined in Table B-1. This table shows how the instantaneous amplitude a(t) and the phase deviation $\varphi(t)$ of the modulated carrier waveform

$$c(t) = a(t)\cos[\omega_c t + \varphi(t)]$$

depend on the waveform b(t) of the modulating signal or baseband and on its Hilbert transform* $\hat{b}(t)$. To suppress constants of proportionality, c(t) has been normalized to the amplitude of the unmodulated carrier, and b(t) to the peak baseband amplitude. With this understanding, the parameters M, Φ, and Ω in the table represent the peak deviations from their unmodulated values of amplitude ratio, phase, and instantaneous angular frequency, respectively.

Also shown in Table B-1 is the dependence on the baseband b(t) of the modulation function m(t), a complex valued function of time defined in terms of the amplitude modulation a(t) and the phase modulation $\varphi(t)$ by[25]

$$m(t) = a(t)e^{j\varphi(t)}$$

---

*Mathematically, the Hilbert transform of b(t) may be described as the convolution of b(t) with 1/t. It may also be described as the waveform obtained from b(t) by shifting the phase of all its Fourier spectrum components by 90°.

Table B-1

DEFINITION OF ANALOG MODULATION METHODS

Modulation Method	Amplitude Ratio $a(t)$	Phase Deviation $\varphi(t)$	Instantaneous Angular Frequency Deviation $\omega(t) = \dot{\varphi}(t)$	Modulation Function $m(t)$	Modulation Index: Peak	Modulation Index: rms
AM	$1 + M\,b(t)$	0	0	$1 + M\,b(t)$	$M = [a(t)]_{pk} - 1$	
DSB	$M\,b(t)$	0	0	$M\,b(t)$	$M = [a(t)]_{pk}$	$M_r = \frac{M}{\sqrt{\Lambda}}$
SSB	$M\sqrt{b^2(t) + \hat{b}^2(t)}$	$\pm\tan^{-1}\frac{\hat{b}(t)}{b(t)}$		$M\,[b(t) + j\hat{b}(t)]$		
PM	1	$\Phi\,b(t)$	$\Phi\,\dot{b}(t)$	$e^{j\Phi b(t)}$	Φ	$\Phi_r = \frac{\Phi}{\sqrt{\Lambda}}$
FM	1	$\Omega\int b(t)dt$	$\Omega\,b(t)$	$e^{j\Omega\int b(t)dt}$	$D = \frac{\Omega}{2\pi f_m}$	$D_r = \frac{D}{\sqrt{\Lambda}}$

Note: $c(t) = a(t)\cos[\omega_c t + \varphi(t)]$ = normalized modulated carrier

$b(t)$ = normalized baseband or modulating signal waveform

$\hat{b}(t) = \text{P.V.}\ \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{b(t)}{t - \tau}\,dt$ = Hilbert transform of $b(t)$

$\Lambda = 1/\overline{b^2(t)}$ = baseband peak-to-average power ratio

f_m = highest frequency in baseband spectrum

In terms of m(t), the normalized modulated carrier may be written

$$c(t) = \mathrm{Re}\left[m(t)e^{j\omega_c t}\right]$$

and it is obvious that the tabulated relations between m(t) and b(t) provide a compact alternative definition of the modulation methods. Moreover, the modulation function also finds great utility in describing not only the spectra of modulated carriers but also the output waveforms and spectra obtained by demodulating these carriers in various ways.[3]

The principal statistical characteristics of the modulated carriers are described by their amplitude distributions and power spectra which determine the peak power and the bandwidth of the rf signal respectively. The rf power spectra are also very important in determining the output interference that results when two or more carriers have overlapping spectra. In general, rf signal statistics will depend on the statistics of the baseband and, particularly for the angle modulation methods, on the modulation indices defined in columns 5 and 6 of Table B-1. In this connection, note that both peak and rms modulation indices are defined for PM and FM.

AMPLITUDE DISTRIBUTIONS

The amplitude distribution for the modulated carrier is in most cases determined by the probability density function $p_a(x)$ of the amplitude ratio a(t). In particular, the average and peak values of modulated carrier power are, respectively,

$$C = \overline{a^2(t)}\, C_o, \qquad C_{pk} = \left[a^2(t)\right]_{pk} C_o$$

where C_o is the average power of the unmodulated carrier. These relations are applied in Table B-2 where, for each analog modulation method, the ratios C/C_o and C_{pk}/C_o, together with the rf peak-to-average power ratio C_{pk}/C, are expressed in terms of the modulation index and the baseband peak-to-average power ratio Λ. It will be noted that C_{pk}/C is

Table B-2

AVERAGE AND PEAK POWER OF ANALOG MODULATED CARRIERS

Modulation Method	Average rf Power $C/C_o \quad \overline{2c^2(t)}$	Peak rf Power $c_{pk}/C_o = 2[c^2(t)]_{pk}$	rf Peak-to-Average Power Ratio C_{pk}/C
AM	$1 + M^2/\Lambda$	$2(1 + M)^2$	$\frac{(1 + M)^2}{M^2 + \Lambda} 2\Lambda \simeq 2(1 + M^2)$
DSB	M^2/Λ	$2M^2$	2Λ
SSB	M^2/Λ	$M^2[b^2(t) + \hat{b}^2(t)]_{pk}$	$[b^2(t) + \hat{b}^2(t)]_{pk}\Lambda$
PM	1	2	2
FM	1	2	2

constant for the angle modulation methods, but proportional to the corresponding baseband quantity for the amplitude modulation methods. Moreover, for SSB modulation, the constant of proportionality depends on the amplitude distribution of the Hilbert transformed baseband as well as that of the baseband itself. It is noted in passing that if the baseband is gaussian distributed, so also will be its Hilbert transform, and the combination $\sqrt{b^2(t) + \hat{b}^2(t)}$ will be Rayleigh distributed.

POWER SPECTRA

The power spectrum of a waveform is defined as the (complex exponential) Fourier transform of its autocorrelation function. For example, the power spectrum of the modulation function m(t) is

$$W_m(f) = \int_{-\infty}^{\infty} R_m(\tau) e^{-2\pi f \tau} \, d\tau$$

where the autocorrelation is defined in terms of m(t) by

$$R_m(\tau) = \overline{m^*(t) \, m(t + \tau)}$$

In this definition $m^*(t)$ is the complex conjugate of m(t) and the bar indicates the time average.

Using these definitions, it is easy to show that under quite general conditions which are met by most basebands, the power spectrum of the normalized modulated carrier c(t) is given in terms of the spectrum of its modulation function by[3]

$$W_c(f) = \tfrac{1}{4}\left[W_m(f - f_c) + W_m(-f - f_c)\right] \qquad f_c \neq 0 \qquad \text{(B-1)}$$

From this equation, it is obvious that the power spectrum of c(t) is an even function of frequency (symmetric about f = 0) as are the power spectra of all real waveforms. As a result, the fraction of the total average power of c(t) contained in the frequency interval $f_1 < f < f_2$

may be obtained not only by integrating $W_c(f)$ over both positive and negative frequencies, $f_1 < |f| < f_2$, but also by integrating over positive frequencies only the one-sided spectrum defined by

$$W_{c1}(f) = 2u(f)\, W_c(f) \tag{B-2}$$

where u(f) is the unit step function

$$u(f) = \begin{cases} 0 & f < 0 \\ 1 & f > 0 \end{cases}$$

It follows directly from Eqs. (B-1) and (B-2) that when c(t) is narrow band, as will be the case for all radio frequency carriers of practical interest, its modulation function may be written

$$W_m(f) = \frac{C}{C_o}\, w_c(f) \tag{B-3}$$

where $w_c(f)$ is the low-pass version of the normalized one-sided power spectrum of the unnormalized carrier $A_c c(t)$ defined by

$$w_c(f) = 2C_o W_{c1}(f+f_c)/C$$

In analyzing the effects of rf interference, the power spectrum of the noise at the demodulator output can be written in terms of the power spectra of the modulation functions of the wanted and unwanted carriers. Thus the interpretation of $W_m(f)$ given in Eq. (B-3) permits the output noise spectrum to be expressed directly in terms of the spectra of the interfering carriers.

In connection with interference analysis, it is also useful to note that, providing c(t) is a wide-sense stationary random process, Eq. (B-1) holds even when the carrier frequency f_c is small compared with the bandwidth of the modulation function.

When $f_c = 0$, however, Eq. (B-1) holds only if $W_m(f)$ is one-sided (as it is for SSB, for example) or if the corresponding positive and negative frequency components in the Fourier spectrum of m(t) are incoherent. In the special case of DSB modulation, m(t) is real (see Table B-1), $W_m(f)$ is symmetrical, and when $f_c = 0$, the sideband components add coherently. Thus, the modulated carrier power spectrum for DSB modulation and any value of carrier frequency is given by

$$W_c(f) = \frac{\varepsilon}{4}\left[W_m(f - f_c) + W_m(f + f_c)\right] \qquad \text{DSB} \qquad \text{(B-4)}$$

where

$$\varepsilon = \begin{cases} 1 & f_c \neq 0 \\ 2 & f_c = 0 \end{cases}$$

With the aid of Eqs. (B-1) and (B-4), the power spectrum of the modulated carrier may be described for any modulation method simply by giving the dependence of the modulation function spectrum on the baseband spectrum and the modulation index. These relations are in turn determined by the expressions for m(t) in terms of the baseband b(t) given in Table B-1. In the case of the amplitude modulation methods, the defining spectral relations are

$$W_m(f) = \begin{cases} \delta(f) + M^2 W_b(f) & \text{AM} \\ M^2 W_b(f) & \text{DSB} \\ u(f) 4M^2 W_b(f) & \text{SSB} \end{cases} \qquad \text{(B-5)}$$

where $\delta(f)$ is the unit impulse or Dirac delta function, and u(f) is again the unit step function.

The situation is not so simple in the case of the angle modulation methods, PM and FM. Here the shape of the rf power spectrum depends not only on the baseband power spectrum but also on the modulation index, and general results can be given in closed form only for the limiting cases of high and low indices. FM is said to be high index or wideband when the rms modulation index D_r is much greater than unity. In this case, the rf power spectrum depends not on the spectrum of the baseband but on its probability density, which is the derivative of its amplitude distribution function. More specifically, if $p_f(x)$ is the probability density of the instantaneous frequency deviation defined by

$$f(t) \equiv \dot{\varphi}(t)/2\pi = Df_m b(t) \qquad \text{FM} \tag{B-6}$$

then the power spectrum of the modulation function is

$$W_m(f) \cong p_f(f) \qquad \text{FM with } D_r = D/\sqrt{\Lambda} \gg 1 \tag{B-7}$$

This result can be applied to high-index phase modulation, $\Phi_r \gg 1$, by interpreting $p_f(x)$ as the probability density of the frequency deviation

$$f(t) = \Phi\dot{b}(t)/2\pi \qquad \text{PM} \tag{B-8}$$

that results from the imposed phase deviation, $\varphi(t) = \Phi b(t)$.

Phase modulation is considered low index when the rms phase deviation Φ_r is much less than unity. In this case, the rf power spectrum is independent of the baseband amplitude distribution and closely resembles the power spectrum for conventional amplitude modulation. Thus, if $W_\varphi(f)$ is the power spectrum of the phase deviation produced by the baseband, the power spectrum of the modulation function is

$$W_m(f) \cong \left(1 - \Phi_r^2\right) \delta(f) + W_\varphi(f) \qquad \text{PM with } \Phi_r \ll 1 \tag{B-9}$$

This result can be applied to low index FM, $D_r \ll 1$, merely by interpreting $W_\varphi(f)$ as the power spectrum of the phase deviation $\varphi(t)$ that results from the imposed frequency modulation. In this connection, note that, if both $\varphi(t)$ and $f(t)$ are stationary,

$$W_\varphi(f) = W_f(f)/f^2 \tag{B-10}$$

The foregoing approximations are of particular interest in the commonly occurring special case where the baseband has a gaussian amplitude distribution and a rectangular low-pass power spectrum. This is just the noise-equivalent FDM baseband described in Appendix A. Formulas for the power spectra when this baseband is used for high and low index PM and FM are displayed in Table B-3.[26,27] It will be noted that when the baseband spectrum does not extend down to zero frequency, these formulas depend to some extent on the magnitude of the lowest modulating frequency f_ℓ.

A general series expansion for the power spectrum of the modulation function for a phase modulated carrier with arbitrary rms modulation index Φ_r is given by[28]

$$\begin{aligned} W_m(f) = e^{-\Phi_r^2} \Big[&\delta(f) + W_\varphi(f) + \frac{1}{2!} W_\varphi(f) * W_\varphi(f) \\ &+ \frac{1}{3!} W_\varphi(f) * W_\varphi(f) * W_\varphi(f) + \dots \Big] \end{aligned} \tag{B-11}$$

Again, if Φ_r is finite, the result for FM may be obtained by substituting W_f/f^2 for W_φ.

Given the power spectrum $W_m(f)$ of the modulation function, it is a simple matter to determine the bandwidth of the modulated carrier according to any one of a number of possible bandwidth definitions. For many purposes, however, the most useful measure of bandwidth is the frequency interval W that includes all but a specified small fraction of the rf signal power (excluding any carrier frequency component).

Table B-3

APPROXIMATE MODULATION FUNCTION POWER SPECTRA FOR

PM AND FM BY WHITE GAUSSIAN NOISE

	Low Index	High Index
PM	$\Phi_r \ll \sqrt{3}$ $e^{-\Phi_r^2}\left[\delta(f) + \frac{\Phi_r^2}{2f_m}\right] \qquad \lvert f\rvert < f_m$ $0 \qquad \lvert f\rvert > f_m$	$\Phi_r \gg \sqrt{3}$ $\frac{1}{\sqrt{2\pi}\, f_r} e^{-\frac{1}{2}(f/f_r)^2} \qquad f_r = \Phi_r f_m/\sqrt{3}$
FM	$D_r = f_r/f_m \ll 0.2$ $\left.\begin{array}{ll} \left[\pi^2 f_r^2/(2f_m) + \frac{f^2}{f_r^2/(2f_m)}\right]^{-1} & f_r^2 \gg f_\ell f_m \\ e^{-f_r^2/(f_\ell f_m)}\delta(f) + \frac{f_r^2}{2(f_m - f_\ell)}\frac{1}{f^2} & f_r^2 \ll f_\ell f_m \end{array}\right\} \lvert f\rvert < f_m$ $0 \qquad \lvert f\rvert > f_m$	$D_r = f_r/f_m > 1.5$ $\frac{1}{\sqrt{2\pi}\, f_r} e^{-\frac{1}{2}(f/f_r)^2}$

With the amplitude modulation methods, the rf bandwidth W is a simple function of the baseband bandwidth B, regardless of how bandwidth is defined. Thus, W = 2B for AM and DSB, and W = B for SSB. For the angle modulation methods, however, the rf bandwidth depends not only on B but on the modulation index as well. No exact expression can be given for the bandwidth W which contains the bulk of the power of an angle modulated carrier, but certain rules of thumb are widely used. In particular, "Carson's rule" gives, for the bandwidth of a frequency modulated carrier,

$$W = 2f_m + 2f_p \cong 2B(1 + D) \qquad \text{FM} \qquad \text{(B-12)}$$

where f_m and B are the highest frequency component and the bandwidth of the baseband respectively, $f_p = \Omega/2\pi$ is the peak frequency deviation imposed by the modulation, and $D = f_p/f_m$ is the peak deviation ratio or FM modulation index. If the baseband amplitude distribution is gaussian, and its power spectrum is flat, the corresponding approximation for PM is

$$W \cong 2B(1 + \Phi/\sqrt{3}) \qquad \text{PM} \qquad \text{(B-13)}$$

where Φ is the peak phase deviation or PM modulation index.

Note from Eqs. (B-12) and (B-13) that when the modulation index D or Φ is small compared with unity (low index, or narrowband, FM or PM) the rf bandwidth is just twice the baseband bandwidth. On the other hand, when D or Φ is much larger than one (high index, or wideband, FM or PM), the rf bandwidth for FM is just twice the peak frequency deviation, independent of baseband bandwidth B and for PM, it is proportional to the product of B and the peak phase deviation.

Appendix C

PREEMPHASIS AND NOISE WEIGHTING

In order to derive expressions for CTCs and RTCs, it is necessary to understand the characteristics of the preemphasis and noise weighting networks used on message channels carrying the two most common analog basebands: telephone and television.

FDM TELEPHONE BASEBANDS

Preemphasis

The characteristics of individual speech waveforms and of composite basebands formed by frequency division multiplexing a number of speech channels were discussed in Appendix A. About the only signal processing technique used at all extensively with such basebands is preemphasis. It is used on FM systems to compensate for the fact that the output noise power spectrum due to thermal noise or to noise-like unwanted signals is parabolic--i.e., proportional to f^2 (see Table 1). As a result, the signal-to-noise ratios at the single channel outputs of an FM system without preemphasis decrease as the inverse square of the frequency at which the channel is located within the baseband.

In a preemphasized FM system, however, the baseband is passed through a filter network before entering the modulator at the transmitting terminal. This preemphasis network has an amplitude response roughly proportional to frequency, so that the message power in a channel at baseband frequency f is increased by the factor f^2 compared with its unprocessed value. If, upon demodulation at the receiving terminal, the preemphasized baseband were demultiplexed without further processing, the single channel signal-to-noise ratios would all be the same, since both the baseband and the noise power spectrum would then be parabolic. But this parabolic dependence would also mean that speech in channels at the higher baseband frequencies would be correspondingly louder.

To maintain uniform volume as well as equal signal-to-noise ratios, the combination of preemphasized baseband and noise is fed from the demodulator to a deemphasis network with a power transfer function* which is proportional to the reciprocal of the power transfer function of the preemphasis net at the transmitting terminal. This restores the baseband power spectrum to its unprocessed state and converts the parabolic thermal noise spectrum to an approximately uniform one. The result is essentially the same as transmitting the unprocessed baseband on a PM system.

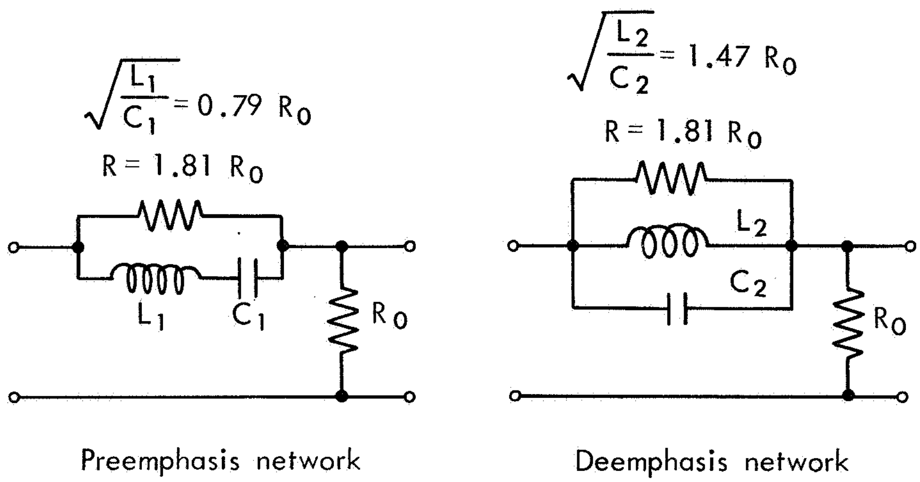
A more important consideration from a practical standpoint is that if the dominant unwanted signal is thermal-noise-like, the equalization of channel signal-to-noise ratios through preemphasis permits a savings in transmitter power and/or rf bandwidth when compared with an FM system without preemphasis delivering the same minimum signal-to-noise ratio. The nature and magnitude of the savings are controlled by the FM modulation index used with the preemphasized system. For example, if, as recommended by the CCIR, the deviation sensitivity of the modulator is chosen to yield the same rms modulation index with preemphasis as without, the rf bandwidth will remain about the same, but as will be seen in a moment, the transmitter power can be reduced by about 4 dB.

When the interference noise spectrum at the demodulator output is not parabolic, as will probably be the case with FM whenever the power spectrum of unwanted signal is not uniform, the deemphasis net will not reduce it to a uniform spectrum, and the channel output signal-to-noise ratios will no longer be even approximately the same for all channels. In order to determine the output signal-to-noise ratio for a selected channel when preemphasis is used, it is in any case necessary to multiply the output noise spectrum $W_n(f)$ by the power transfer function $G_D(f)$ the deemphasis network. Although a variety of deemphasis networks are used in practice, the one recommended by the CCIR and shown in Fig. C-1a is representative.** Its power transfer function is

$$G_D(f) = \frac{1 + 1.52\, g^2(f)}{7.90 + 1.52\, g^2(f)} \qquad \text{(C-1)}$$

*The square of the magnitude of the voltage transfer function.

**Reference 14, Recommendations 275-1.



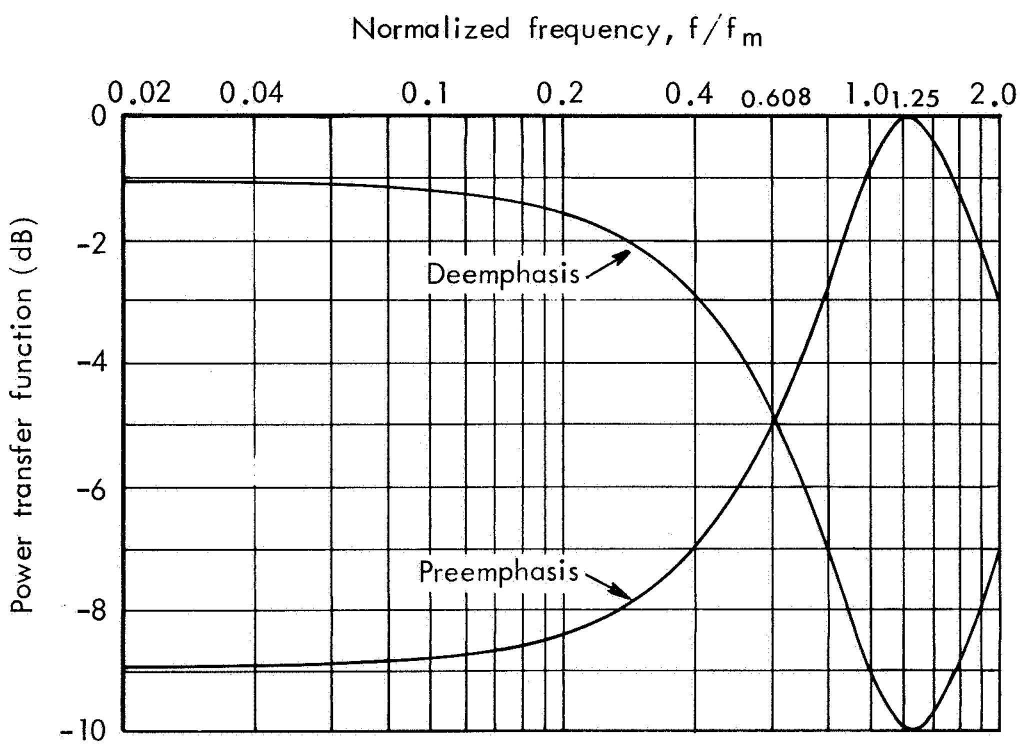


Fig. C-1 — CCIR-recommended preemphasis and deemphasis networks and transfer functions for FDM telephony where

$$g(f) = \frac{f}{f_R} - \frac{f_R}{f} \tag{C-2}$$

and the reactive components of the network are chosen so that the resonant frequency f_R is 25 percent higher than the maximum frequency f_m of the baseband to be processed--i.e.,

$$f_R = 1.25 \ f_m \tag{C-3}$$

The power transfer function of the corresponding CCIR-recommended preemphasis network is proportional to the reciprocal of the deemphasis function

$$G_P(f) = \frac{1}{7.90 \ G_D(f)} \tag{C-4}$$

Plots of $G_D(f)$ and $G_P(f)$ are shown in Fig. C-1b where it will be noted that $G_P(f)$ varies as f^2 (6 dB per octave) only for channels in the upper 40 percent of the baseband, while at the bottom of the baseband, all channels are attenuated about equally. This arrangement permits the lower frequency channels to tolerate the somewhat higher levels of nonthermal noise experienced at the low-frequency end of the baseband, while still maintaining nearly full carrier power savings compared with a nonpreemphasized system.

In practice, the CCIR-recommended condition of keeping the same rms frequency deviation with preemphasis as without is achieved not by using a different modulator, but rather by amplifying the preemphasis filter output by an amount corresponding to the change in deviation sensitivity that otherwise would be required. Thus, in describing the recommended preemphasis characteristic for telephony, the CCIR multiplies the power transfer function $G_P(f)$ by 3.16 (5 dB) and identifies the result, when expressed in dB, as the relative frequency deviation. In symbols, if $(f_{rch})_P$ and f_{rch} are respectively the rms frequency deviations produced by a channel test tone at baseband frequency f, with and without preemphasis, the overall preemphasis characteristic is

$$10 \log G_{Po}(f) = 20 \log \frac{(f_{rch})_P}{f_{rch}} = 10 \log [3.16\ G_P(f)]$$

$$= 5 + 10 \log G_P(f) \qquad \text{(C-5)}$$

where $G_P(f)$ can be expressed in terms of f and f_m with the aid of Eqs. (C-1) through (C-4).

The corresponding overall deemphasis characteristic is the reciprocal of $G_{Po}(f)$

$$G_{Do}(f) = \frac{1}{G_{Po}(f)} = 2.5\ G_D(f) = \frac{2.5 + 3.8\ g^2}{7.9 + 1.5\ g^2} \qquad \text{(C-6)}$$

where g is the function of frequency given in Eq. (C-2). In terms of $G_{Do}(f)$, the interference noise power spectrum at the input to the demultiplexer is

$$W_{nD}(f) = G_{Do}(f)\ W_n(f) \qquad \text{(C-7)}$$

Plots of G_{Do}, and of $W_n(f)$ and $W_{nD}(f)$ for a white noise unwanted signal, are shown in Fig. C-2, where the power spectra have been normalized to $N_o f_m^2/C$ for scaling purposes. From this figure it is evident that, given the same values of N_o, f_m, and C, a CCIR preemphasized system will have 4 dB less noise in its top channel ($f = f_m$) than a system without preemphasis. Thus, to deliver the same output signal-to-noise ratio in the top channel, the wanted signal power for the preemphasized system can be reduced 4 dB. When the unwanted rf signal is not white noise, the effect of deemphasis on the channel output signal-to-interference noise ratio may be numerically different, but the nature of the calculation using Eq. (C-7) is the same.

As previously mentioned, the channel output signal-to-noise ratio S_{ch}/N_{ch} used in defining the channelizing and receiver transfer

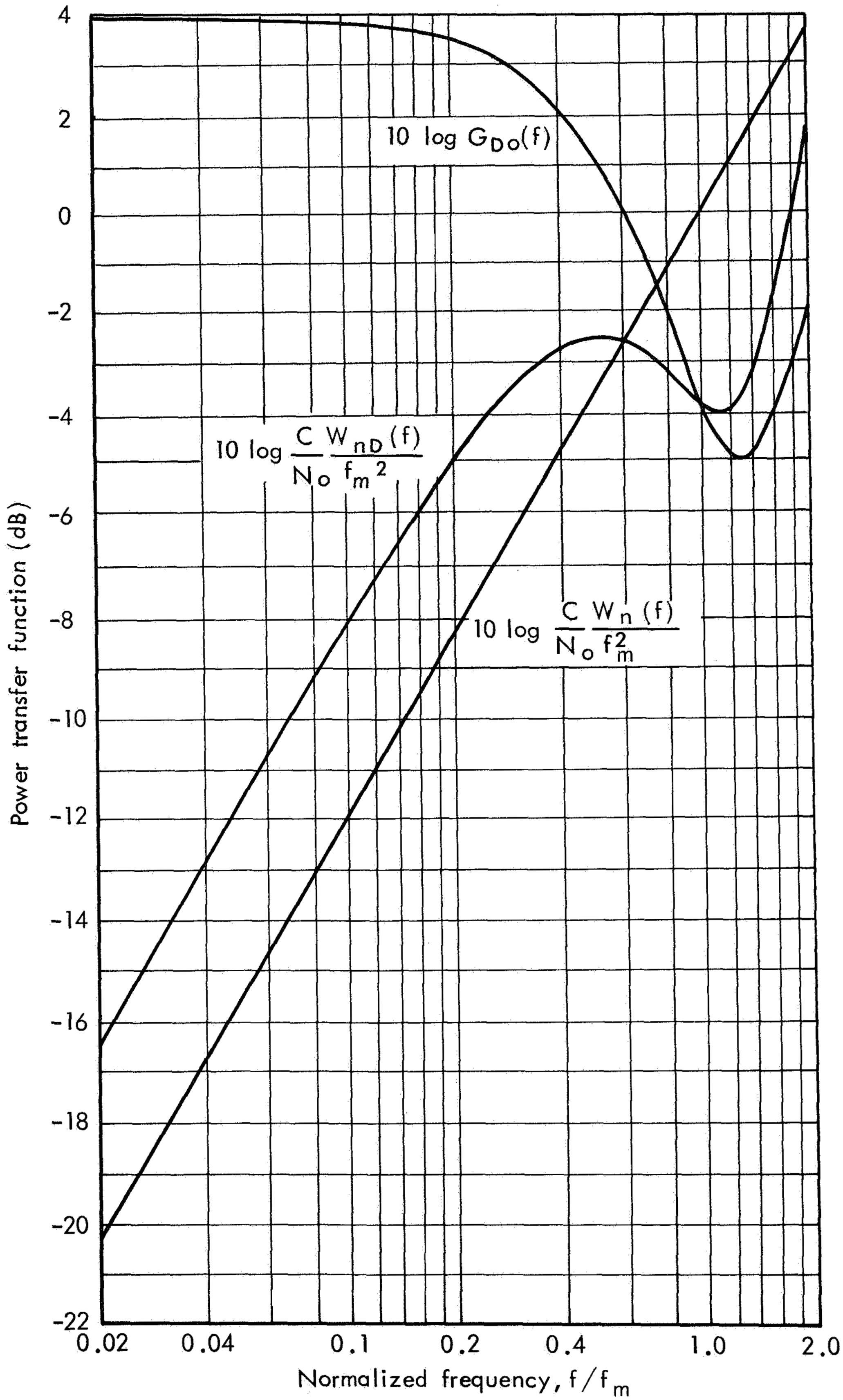


Fig. C-2 — Deemphasis transfer characteristic for telephone basebands and corresponding output noise power spectra when unwanted signal or noise is white and gaussian characteristics is actually the ratio of a standard reference or test signal to the channel noise after frequency weighting to reflect its subjective annoying effect. The reference signal in the case of a speech channel is a sinusoidal test tone with an average power of 1 mW at a point of zero relative level. Appending a zero to the unit to indicate the point of measurement, this convention may be written in a number of equivalent ways:

$$\left.\begin{aligned} S_{ch} &= 1 \text{ mW0} = 10^9 \text{ pW0} \\ 10 \log S_{ch} &= 0 \text{ dBm0} = 90 \text{ dBp0} \end{aligned}\right\} \qquad \text{(C-8)}$$

This is more than ten times the power produced at the same point by the average talker defined in Appendix A.

Noise Weighting

The relative annoyance caused by different spectral components of the noise at the output of a voice channel is described by a weighting function $G_N(f)$. This function takes into account the frequency responses both of the telephone instrument which converts the output signal from electrical to acoustical form, and of the human ear which responds to the sound. In terms of the weighting function and the noise power spectrum at the demultiplexer output, the weighted channel noise is

$$N_{ch} = \begin{cases} 2\int_0^{B_{ch}} G_N(f)\, W_n(f_{ch} + f)\, df & \text{no preemphasis} \\ 2\int_0^{B_{ch}} G_N(f)\, W_{nD}(f_{ch} + f)\, df & \text{with preemphasis} \end{cases} \qquad \text{(C-9)}$$

where f_{ch} is the lower frequency limit of the channel in question, and B_{ch} is its bandwidth. Thus weighted, N_{ch} reflects the annoying effect of the channel noise to an average user and permits direct comparison of channel quality irrespective of the shape of the noise spectra they carry or of the telephone instruments connected to them.

In practice $G_N(f)$ is determined by averaging the results of two types of experiments, each repeated with a large number of human subjects using the telephone handset in question. The first experiment measures the annoying effect of noise components in the absence of speech by adjusting the level of a tone at audio frequency f until it is just as annoying as a reference test tone. The other experiment is similar except that it is conducted in the presence of ordinary speech at the average talker level and measures the effect of the interfering audio tone on speech intelligibility.

The noise weighting curve obtained in this way for the type 500 handset in current use by the Bell System is shown in Fig. C-3.[15] Older Bell System instruments are characterized by curves with similar but less flat frequency responses. For international use, the CCIR and CCITT recommend another frequency weighting curve called "psophometric," also shown in Fig. C-3.[29]

From Eq. (C-9) it is apparent that in general, the effect of frequency weighting will depend not only on the shape of the weighting function $G_N(f)$ but also on the shape of the noise spectrum $W_n(f)$ or $W_{nD}(f)$ within the channel. A given amount of noise power concentrated near the center of the channel obviously leads to a higher value of N_{ch} than the same amount spread uniformly across the channel. In the case of FDM telephone basebands comprising a dozen or more channels, however, the interference noise spectrum, even for FM without preemphasis, is virtually constant across any one channel, provided only that $W_n(f)$ contains no discrete components. With this condition Eq. (C-9) becomes

$$N_{ch} = \begin{cases} 2W_n(f_{ch})\, B_{ch}\, \overline{G}_N & \text{no preemphasis} \\ 2G_{Do}(f_{ch})\, W_n(f_{ch})\, B_{ch}\, \overline{G}_N & \text{with preemphasis} \end{cases} \tag{C-10}$$

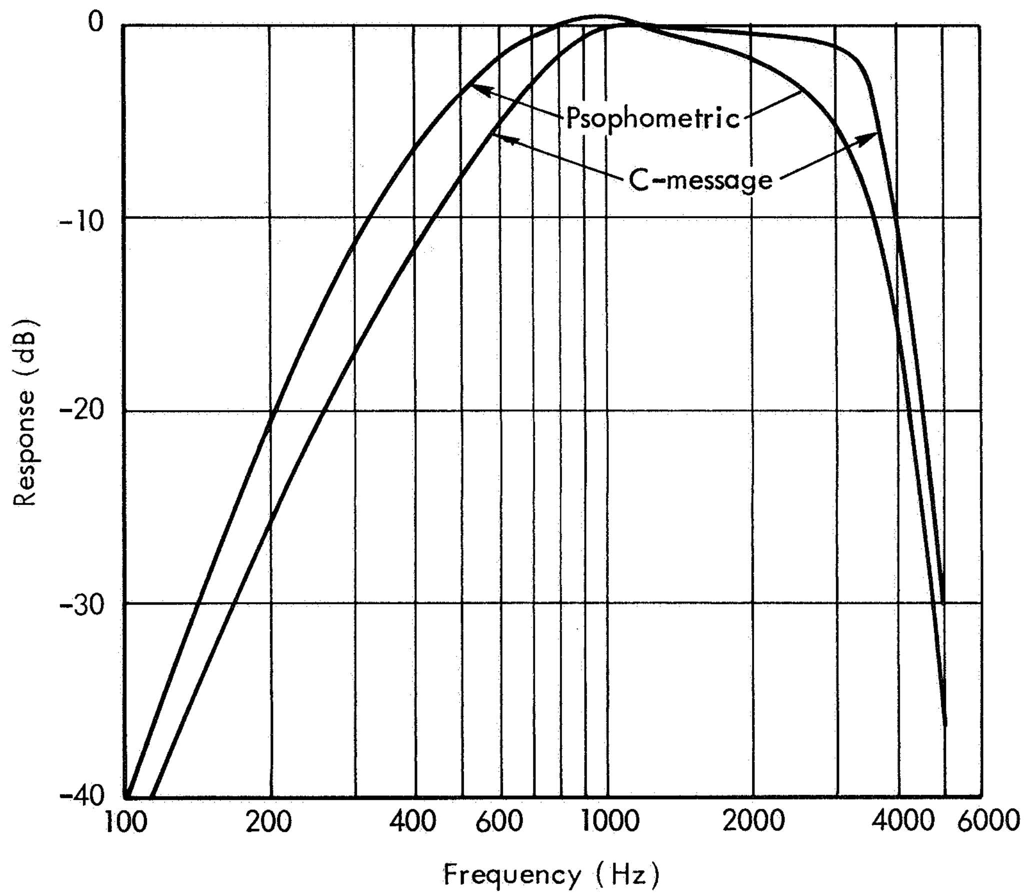


Fig. C-3 — Noise weighting functions for voice channels where $\overline{G}_N$ is the average value of the noise weighting function

$$\overline{G}_N = \frac{1}{B_{ch}} \int_0^{B_{ch}} G_N(f)\, df \qquad \text{(C-11)}$$

From Eq. (C-10) it is apparent that $\overline{G}_N$ is also the factor by which the weighted channel noise power differs from the unweighted noise.

Although the weighted noise N_{ch} characterizes the subjectively evaluated acoustical consequences of the channel noise rather than the power of any actual electrical waveform, it is useful to have a test instrument capable of directly indicating the value of N_{ch} corresponding to an arbitrary noise input. Such a "noise measuring set" incorporates the weighting function $G_N(f)$ as an electrical network and performs the operation indicated by Eq. (C-9), thus simulating both the telephone handset and the ear of the average subscriber.

To express the reading of such a set, or to identify a noise specification as "weighted," special noise power "units" are defined for each of the standard weighting curves. All of the noise measuring sets used by the Bell System are calibrated in dB relative to the reading produced by a 1000-Hz test tone having a specified average power.(15) For the noise set corresponding to telephones of the 1920's, the reference test tone power was 1 pW (-90 dBm) and readings were given in dBrn (decibels above reference noise). Thus a standard 0-dBm test tone would cause a reading of 90 dBrn, whereas 0 dBm of white noise power in a 3.1-kHz band would measure 82 dBrn. With the 2B noise measuring set developed for the telephones used from the 1930's into the 1950's, the reference test-tone power was about 3 pW (-85 dBm) and the corresponding "units" were called dBa (decibels above reference noise adjusted). The change in reference was made so that 0 dBm of white noise would produce a reading numerically the same as that obtained with the older noise-measuring set--viz., 82 dBa. The associated weighting network was called "FIA weighting." The type 3A noise set currently used by the Bell System employs so-called "C message weighting" which corresponds to the type 500 telephone. The 0-dB reference test tone power is 1 pW, as with the earliest set, and the units are again called dBrn (or sometimes dBrn - Cm), but due to the flatness of the weighting, 0 dBm of white noise now gives a reading of 88 dBrn. Thus for any given level of white noise, a type 3A set gives a reading in dBrn about 6 dB higher than the reading in dBa of a type 2B noise set.

In the case of the CCIR-recommended psophometric weighting, unweighted noise powers are usually specified in terms of pW and weighted noise powers in pWp (picowatts psophometric). If the unweighted noise is white, the weighted reading is smaller by a factor of 1/1.8 or -2.5 dB; thus 0 dBm of white noise corresponds to 87.5 dBpWp.

The preceding conventions regarding units are summarized in Table C-1, which gives the value of 10 log $\bar{G}_N$ required to convert unweighted white noise expressed in dBpW into weighted noise expressed in units appropriate to the type of weighting.

Table C-1

AVERAGE NOISE WEIGHTING FUNCTION FOR TELEPHONY

Type Weighting	Weighted Noise Unit	10 log $\bar{G}_N$ (dB)
Early Bell System	dBrn	-8.0
FIA	dBa	-8.0
C	dBrn	-2.0
Psophometric	dBpWp	-2.5

TELEVISION

The television baseband was described in considerable detail in Appendix A. As with an FDM voice baseband, it is normally transmitted without signal processing for all analog modulation methods except FM, where it is passed through a preemphasis circuit before modulation. By suitably attenuating the large low-frequency components of the TV spectrum, preemphasis permits the same FM modulators and demodulators to be used interchangeably for TV and large capacity FDM telephony transmission. Also, as with telephony, a preemphasized TV-FM system affords power and/or bandwidth savings compared to an FM system without preemphasis. Finally, preemphasis can reduce differential gain and phase distortion when there are several repeaters.

Since the baseband spectra are quite different, the preemphasis characteristic for television is somewhat different from that used for telephony. The preemphasis and deemphasis networks recommended* for 525-line TV by the CCIR are shown in Fig. C-4 along with the corresponding overall transfer characteristics, $G_{Po}(f)$ and $G_{Do}(f)$. The latter is represented to a close approximation by**

$$G_{Do}(f) = 9.26 \frac{1 + 1.1\ f^2}{1 + 23.5\ f^2} \tag{C-12}$$

where f is measured in MHz. Using this deemphasis function, the noise power spectrum at the input to the picture tube is given by an equation identical to Eq. (C-7) for telephony

$$W_{nD}(f) = G_{Do}(f)\ W_n(f) \tag{C-13}$$

where $W_n(f)$ is the noise power spectrum at the video demodulator output.

To calculate the subjective effect of this noise spectrum on the viewer, an experimentally determined noise weighting function, again denoted $G_N(f)$, is used to compute the weighted output noise power

$$N_{ch} = 2 \int_0^{B_{ch}} G_N(f)\ W_{nD}(f)\ df \tag{C-14}$$

The noise weighting function for the U.S. 525-line monochrome system is given by(30)

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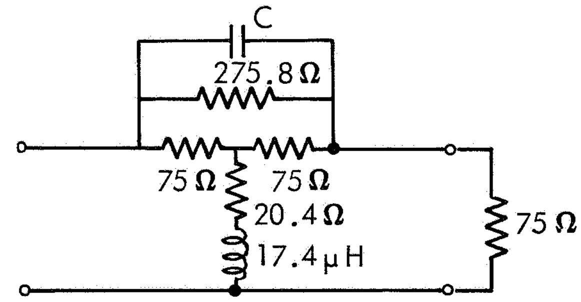
*Reference 14, Recommendation 405.

**J. J. Bisaga, "Effect of Adjacent Satellite Downlink Interference on Minimum Spacing," Communications Systems, Inc., Draft of Technical Memorandum for National Aeronautics and Space Administration, 18 October 1968.



Preemphasis network

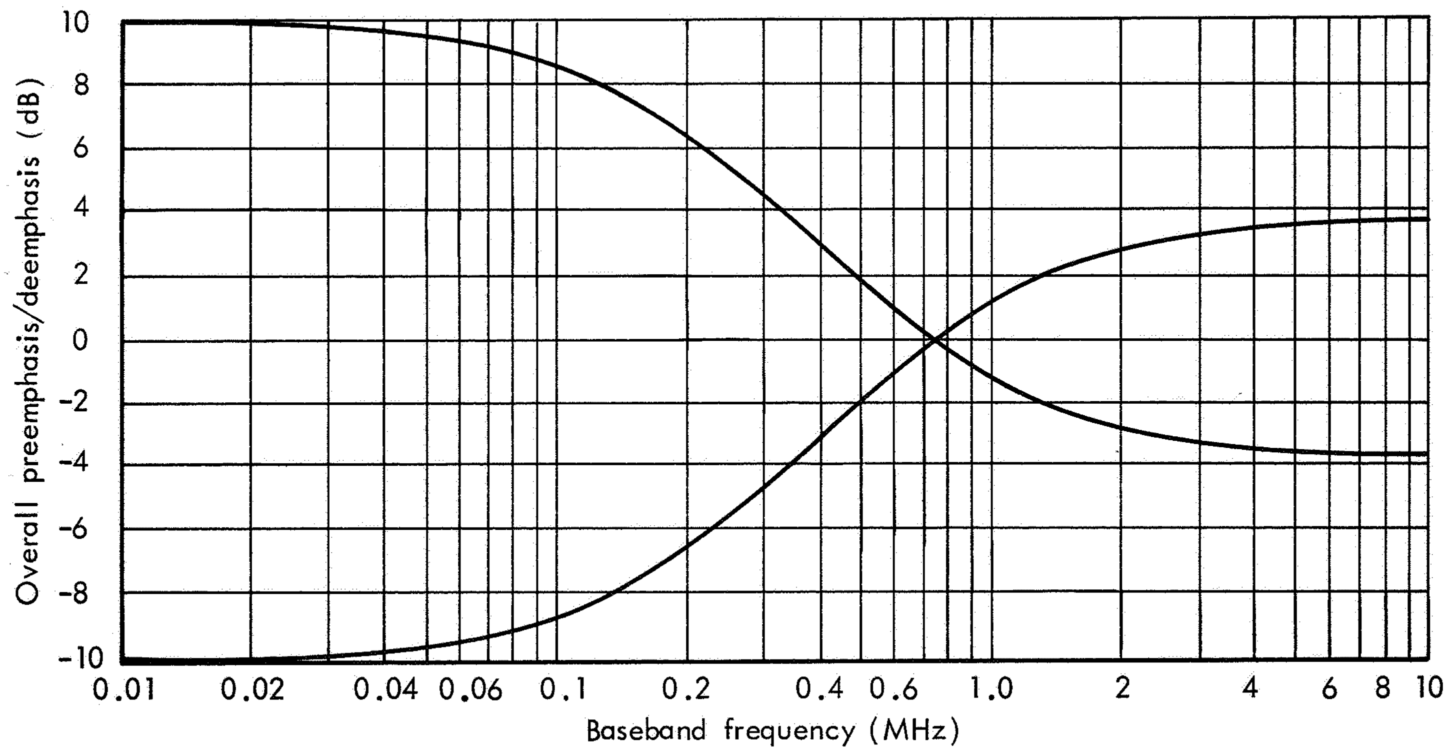


Fig.C-4—CCIR-recommended preemphasis and deemphasis characteristics for 525-line TV

$$G_N(f) = \frac{1 + \left(\frac{f}{0.39}\right)^2}{\left[1 + \left(\frac{f}{0.27}\right)^2\right]\left[1 + \left(\frac{f}{1.37}\right)^2\right]} \qquad \text{(C-15)}$$

where f is measured in MHz. This function is plotted in Fig. C-5, which also includes the weighting function for the Japanese 525-line color system for comparison.

It is apparent from both curves that noise at low baseband frequencies is generally much more annoying to the viewer than the same amount of noise at high baseband frequencies. That this is true for the chrominance as well as the luminance portion of the picture is indicated by the peak in the color TV weighting function at the chrominance subcarrier frequency. Since chrominance spectrum components in this region are demodulated to low video frequencies, the noise that corrupts them also appears at low frequencies.

Although no special test signal is involved in describing the channel output signal S_{ch}, this quantity is neither the average signal power nor is it the power corresponding to peak amplitude as usually defined. It is instead the square of the peak-to-peak voltage of the video signal. The use of the peak-to-peak voltage is justified by the fact that it is well defined and easily measured. However, there is not universal agreement on whether or not to include the synch pulses in specifying this quantity. The CCIR excludes the synch pulses and defines S_{ch} as the peak-to-peak voltage of the picture signal--i.e., the difference s_p between the blanking level and the peak white level.[14] On the other hand, the Bell System includes the synch signal and defines S_{ch} as the peak-to-peak voltage of the video signal--i.e., the difference s_v between the synch level and the peak white level.[15] This difference in convention should be remembered when comparing the channel signal-to-noise objectives of the two organizations. Since the two signal voltages are related by

$$s_p = 0.7s_v \qquad \text{(C-16)}$$

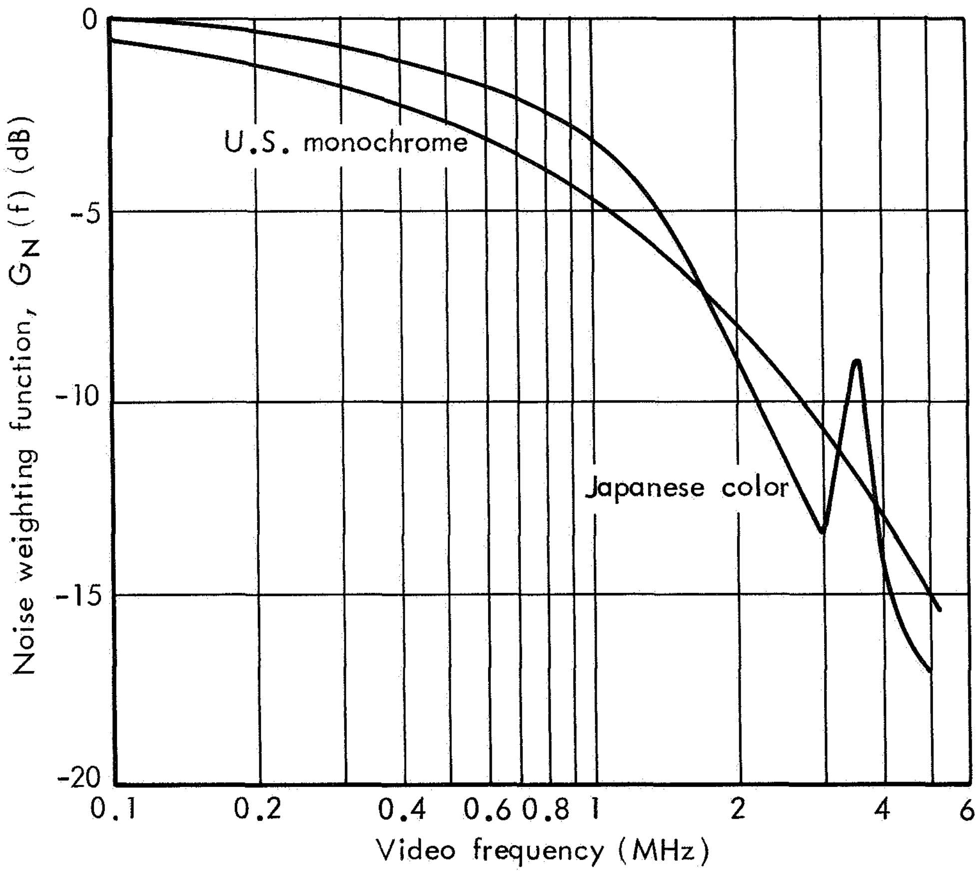


Fig. C-5 — Noise weighting functions for TV the Bell System signal-to-noise ratio objective for a given picture quality should be about 3 dB higher than the CCIR objective.

Whichever definition of peak-to-peak voltage is used in specifying the channel output power, the ratio of S_{ch} to the average baseband power S may be written

$$\frac{S_{ch}}{S} = \frac{4s_{peak}^2}{s_{rms}^2} = 4\Lambda \tag{C-17}$$

where s_{rms} is the standard deviation of the baseband signal at the demodulator output

$$s_{rms}^2 = \overline{[s(t) - \overline{s}]^2} \tag{C-18}$$

and $2s_{peak}$ is the peak-to-peak voltage, s_p or s_v, chosen to represent the baseband at the channel output. In this way, s_{peak}^2 represents the peak value of the a-c component of baseband power, and Λ is the peak-to-average power ratio for the a-c component. Note that if preemphasis is used, s_{rms} is measured on the processed baseband whereas s_{peak} applies to the unprocessed baseband--i.e., after deemphasis.

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