

TECHNICAL NOTE

# Rainfall-induced slope failure considering variability of soil properties

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**KEYWORDS:** slopes; unsaturated soils

## INTRODUCTION

Both hydraulic properties and shear strength properties of unsaturated soils can affect the stability of a soil slope during a rainstorm. The uncertainties of hydraulic property parameters for unsaturated soils and their effects on the reliability of slopes under rainfall condition have been studied recently (Chong *et al.*, 2000; Tung & Chan, 2003). For example, Chong *et al.* (2000) studied the influences of the uncertainty of each parameter in the soil–water characteristic curve (SWCC) and the permeability function on the safety factor of unsaturated soil slopes using Monte Carlo simulations. Tung and Chan (2003) performed probabilistic analyses of rainfall infiltration and slope stability considering uncertainties in SWCCs by the Latin hypercube sampling technique. In both studies the deterministic model used is an uncoupled seepage analysis program Seep/W (Geo-slope Ltd, 2001a) and a companion slope stability analysis program Slope/W (Geo-slope Ltd, 2001b). Seep/W is formulated for conditions of constant net normal stress and non-deformable soil media. However, under real situations, the seepage and flow processes in a deformable soil are influenced by soil deformations. A coupled hydromechanical model is therefore preferred to analyse the behaviour and stability of a deformable soil slope subjected to rainfall.

The objectives of this paper are to develop a coupled hydromechanical model and a finite element based slope stability program to study the behaviour and stability of deformable unsaturated soil slopes, and to illustrate their application in probabilistic study by an example of a hypothetical unsaturated soil slope considering the variability of soil properties.

## SLOPE STABILITY ANALYSIS BASED ON COUPLED HYDROMECHANICAL NUMERICAL MODELLING

Pore water pressure changes due to seepage will lead to changes in stresses and therefore to deformation of a soil. Conversely, stress changes will modify the seepage process because soil hydraulic properties such as porosity, permeability and water storage capacity are affected by the changes in stresses. Hence the seepage and stress-deformation problems are strongly linked. In this study, a coupled hydromechanical finite element modelling program is developed to study the performance of an unsaturated soil slope during a rainstorm. A finite element based slope stability program is also developed to calculate the safety factor of the slope based on the stress distribution in the slope during

raining. Therefore both deformation and the factor of safety can be obtained to assess the stability of the slope.

## Theory of coupled hydromechanical modelling

The seepage through and deformation of an unsaturated soil under isothermal conditions require the coupled solution of the governing equations describing the equilibrium of the soil structure and the mass flow of the water phase. Constitutive relationships for the solid and water are also required. Formulations for coupled analysis of unsaturated soils and numerical solutions of combined seepage and deformation problems have been reported (Lloret *et al.*, 1987; Fredlund & Rahardjo, 1993; Thomas & He, 1995; Alonso *et al.*, 1996; Pereira, 1996).

The constitutive relationships for the soil structure and the water phase (Fredlund & Morgenstern, 1976) are

$$\frac{dV_v}{V_0} = m_1^s d(\sigma_{\text{mean}} - u_a) + m_2^s d(u_a - u_w) \quad (1)$$

$$\frac{dV_w}{V_0} = m_1^w d(\sigma_{\text{mean}} - u_a) + m_2^w d(u_a - u_w) \quad (2)$$

where  $V_0$  is the initial overall volume of the referential soil element,  $V_v$  is the volume of soil solid,  $V_w$  is the volume of water,  $\sigma_{\text{mean}}$  is the mean total normal stress,  $u_a$  is the pore air pressure,  $u_w$  is the pore water pressure,  $\sigma_{\text{mean}} - u_a$  is the mean net normal stress,  $u_a - u_w$  is the matric suction,  $m_1^s$  is the coefficient of volume change of solid with respect to a change in mean net normal stress,  $m_2^s$  is the coefficient of volume change of solid with respect to a change in matric suction,  $m_1^w$  is the coefficient of volume change of pore water with respect to a change in mean net normal stress, and  $m_2^w$  is the coefficient of volume change of pore water with respect to a change in matric suction.

The coefficients of volume change can be calculated from the constitutive surfaces for the void ratio  $e$  and the volumetric water content  $\theta_w$  of the soil:

$$m_1^s = \frac{1}{1 + e_0} \frac{de}{d(\sigma_{\text{mean}} - u_a)} \quad (3)$$

$$m_2^s = \frac{1}{1 + e_0} \frac{de}{d(u_a - u_w)} \quad (4)$$

$$m_1^w = \frac{\partial \theta_w}{\partial (\sigma_{\text{mean}} - u_a)} = \frac{S}{1 + e_0} \frac{de}{d(\sigma_{\text{mean}} - u_a)} + \frac{e}{1 + e_0} \frac{dS}{d(\sigma_{\text{mean}} - u_a)} \quad (5)$$

$$m_2^w = \frac{\partial \theta_w}{\partial (u_a - u_w)} = \frac{S}{1 + e_0} \frac{de}{d(u_a - u_w)} + \frac{e}{1 + e_0} \frac{dS}{d(u_a - u_w)} \quad (6)$$

where  $S$  is the degree of saturation, and  $e_0$  is the initial void ratio.

Manuscript received 5 May 2004; revised manuscript accepted 16 December 2004.

Discussion on this paper closes on 1 September 2005, for further details see p. ii.

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Assuming that the soil behaves as an isotropic and incrementally linear elastic material, and considering the equilibrium of the soil structure and the continuity of water, the coupled non-linear partial differential equations for seepage and deformation for the plane-strain condition can be expressed as follows (Pereira, 1996):

$$\frac{\partial}{\partial x} \left\{ \frac{E}{(1-2\mu)(1+\mu)} \times \left[ (1-\mu) \frac{\partial u}{\partial x} + \mu \frac{\partial v}{\partial y} - \frac{1+\mu}{H} (u_a - u_w) \right] \right\} + \frac{\partial}{\partial y} \left[ G \left( \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right) \right] + b_x = 0 \quad (7)$$

$$\frac{\partial}{\partial x} \left[ G \left( \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right) \right] + \frac{\partial}{\partial y} \left\{ \frac{E}{(1-2\mu)(1+\mu)} \times \left[ \mu \frac{\partial u}{\partial x} + (1-\mu) \frac{\partial v}{\partial y} - \frac{1+\mu}{H} (u_a - u_w) \right] \right\} + b_y = 0 \quad (8)$$

$$\frac{\partial}{\partial x} \left[ k_x \frac{\partial}{\partial x} \left( \frac{u_w}{\gamma_w} + y \right) \right] + \frac{\partial}{\partial y} \left[ k_y \frac{\partial}{\partial y} \left( \frac{u_w}{\gamma_w} + y \right) \right] = \frac{\partial}{\partial t} \left\{ m_1^w \frac{E}{3(1-2\mu)} \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) + \left[ m_2^w - \frac{m_1^w E}{(1-2\mu)H} \right] (u_a - u_w) \right\} \quad (9)$$

where  $u$  is horizontal displacement,  $v$  is vertical displacement,  $\mu$  is Poisson's ratio,  $E$  is the elastic modulus for the soil structure with respect to a change in mean net normal stress [ $E = 3(1-2\mu)/m_1^s$ ],  $H$  is the elastic modulus for soil structure with respect to a change in matric suction [ $H = 3/m_2^s$ ],  $b_x$  and  $b_y$  are body forces in the  $x$  and  $y$  directions respectively,  $k_x$  and  $k_y$  are coefficients of permeability of water in the  $x$  and  $y$  directions respectively,  $\gamma_w$  is the unit weight of water,  $y$  is the elevation, and  $t$  is time.

The solution of the complicated coupled governing equations requires the use of numerical procedures. In this study, a computer program FlexPDE (PDE Solutions Inc., 2004), which is a scripted finite element model builder and numerical solver for the solution of systems of partial differential equations, is implemented to solve the coupled governing equations. A time incremental procedure is adopted to perform the coupled numerical modelling. The whole seepage and deformation process is divided into several major time intervals. In each major time interval, the seepage analysis is first performed. In each seepage analysis, the time interval is automatically divided into many time steps based on a specified convergence criterion. Then the stress–deformation analysis is performed by applying the changes in the pore water pressure during this time interval as the loading. In both the seepage analysis and stress analysis in each major time interval, the coefficients of deformation and hydraulic properties are updated according to the stresses and pore water pressures from the results of the previous time interval. The void ratio and the unit weight of the soil are also updated based on the deformation of the soil structure and the changes in the water content. Considering the deformation of the soil structure, the saturated permeability is defined as a function of the porosity based on the Kozeny–Carman estimation for hydraulic conductivity (Ahuja *et al.*, 1989).

#### Finite element based slope stability analysis

The stability of an unsaturated soil slope is evaluated by a finite element based slope stability analysis computer program FESSA, given the distributions of stresses, pore water pressures and void ratio in the slope obtained from the coupled numerical modelling. The slip surfaces of slope failure are assumed to be circular. Each trial slip surface is divided into many sections (say, 250 sections) based on a rectangular grid mesh. The safety factor FS for a given slip surface can be calculated by

$$FS = \frac{\sum \tau_f \Delta L}{\sum \tau \Delta L} \quad (10)$$

where  $\tau$  is the shear stress,  $\tau_f$  is the shear strength, and  $\Delta L$  is the length of a section of the slip surface. The shear stress and shear strength are evaluated based on the net normal stress vector and the pore water pressure at the middle point of the section of the slip surface. The global safety factor is the minimum safety factor obtained from all the trial slip surfaces. Applying the same stability analysis approach at different times during a rainstorm, the stability of the slope during the whole rainfall process can be evaluated.

#### VARIABILITY OF SOIL PROPERTIES

In this study, the stability of an unsaturated soil slope with completely decomposed granite (CDG), which is a coarse-grained sand with an appreciable amount of silt and clay, will be investigated by numerical modelling. The properties of the soil and the variability of the soil properties are described in the following sections.

#### Soil properties affecting slope stability under rainfall condition

The coefficients of volume change for solid,  $m_1^s$  and  $m_2^s$ , can be derived from the void ratio state surface of an unsaturated soil (equations (3) and (4)). Based on experimental data from 1-D virgin wetting tests for CDG in Cha Kwo Ling, Hong Kong (Kam, 1999), a void ratio state surface (Fig. 1) can be fitted using a model proposed by Lloret & Alonso (1985):

$$e = a + b \ln(\sigma_{\text{mean}} - u_a) + c(u_a - u_w) + d \ln(\sigma_{\text{mean}} - u_a)(u_a - u_w) \quad (11)$$

where  $a$ ,  $b$ ,  $c$ ,  $d$  are fitting coefficients. For Cha Kwo Ling

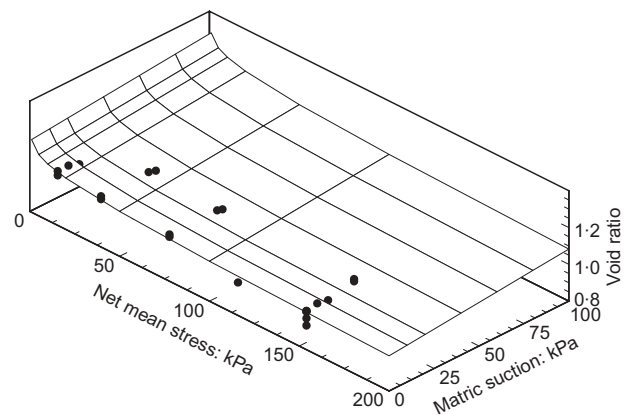


Fig. 1. Void ratio state surface of CDG at Cha Kwo Ling, Hong Kong

CDG, their values are  $a = 1.2187$ ,  $b = -0.04412$ ,  $c = -0.00239$  and  $d = 0.000747$ . The use of 1-D wetting tests is an approximation because no triaxial wetting test results are available.

The coefficients of volume change for the pore water,  $m_1^w$  and  $m_2^w$ , can be estimated from a model for the volumetric water content  $\theta_w$  (i.e. SWCC) using equations (5) and (6). The SWCCs used in this study are obtained from standard SWCC tests using pressure extractors (ASTM D6836-02). The effect of confining pressure on SWCCs is not considered. The Fredlund & Xing (1994) model is used as a mathematical model for SWCC in this study.

When CDG soil is fully saturated, it exhibits a strain-softening behaviour under undrained conditions (Ng *et al.*, 2004). In the  $p'-q$  plane, in which  $p'$  is the mean effective stress and  $q$  is the deviator stress, the stress path of a strain-softening soil under undrained shearing will reach a peak state first and then the critical state. The collapse surface is defined by a straight line that joins the peak stress points (Sladen *et al.*, 1985). The critical state line is defined by a line that joins the critical stress state points. The slopes of the critical state line and the collapse surface in the  $p'-q$  plane are  $M$  and  $M_{col}$  respectively. In the  $v-\ln p'$  plane, where  $v = 1 + e$  is the specific volume, the intercept and the slope of the critical state line are  $\Gamma$  and  $\lambda$  respectively.

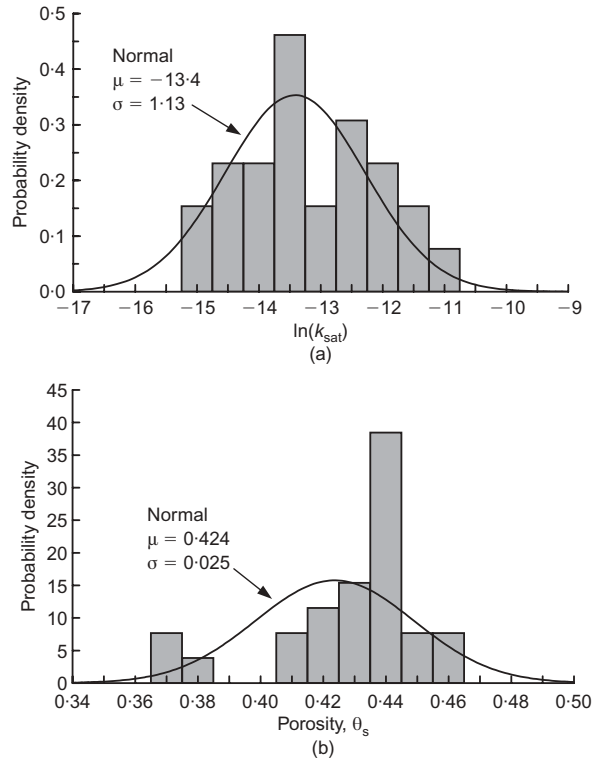
The extended Mohr–Coulomb shear strength model (Fredlund *et al.*, 1978) is used for the unsaturated CDG soil, with the friction angle assumed to be the critical state friction angle. When the CDG soil is saturated and the stress state is on the collapse surface, the shear strength of the soil is determined by the collapse surface.

*Uncertainties of soil properties*

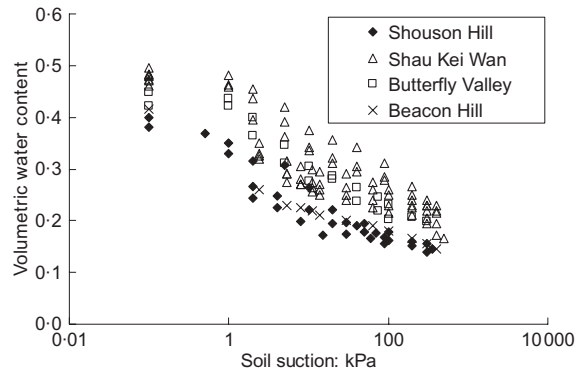
In this study, porosity  $\theta_s$ , saturated permeability  $k_{sat}$ , parameters  $a_1$  and  $n_1$  in the Fredlund & Xing (1994) SWCC model, and shear strength parameters  $M$ ,  $M_{col}$ ,  $\Gamma$  and  $\lambda$  are selected as random variables, because these are considered to be important parameters that may influence the pore pressure distributions in a slope and hence the stability of the slope during a rainstorm. The basic statistics of these random variables are listed in Table 1. The uncertainties of the void ratio state surface are omitted owing to lack of data. The uncertainties of initial conditions and boundary conditions in the slope are not considered.

Figure 2 shows the frequency diagrams and probability density functions of  $\theta_s$  and  $\ln(k_{sat})$  obtained from 26 soil samples taken from bore holes at Sau Mau Ping, Hong Kong (Knill *et al.*, 1999). The probability density functions for  $\theta_s$  and  $\ln(k_{sat})$  are fitted using normal distributions. At this site the variability of  $\ln(k_{sat})$  is significant. The standard deviation of the measured  $\ln(k_{sat})$  is 1.13. In comparison, the standard deviation of the measured porosity is only 0.025.

Figure 3 shows 13 measured SWCCs for CDG soils from four sites based on experimental results reported by Gan & Fredlund (1997) and Fung (2001). The range of applied



**Fig. 2. Frequency diagrams and probability density functions: (a)  $\ln(k_{sat})$ ; (b) porosity  $\theta_s$  (soil samples from Sau Mau Ping slope)**



**Fig. 3. Experimental soil–water characteristic curves for CDG soils at four sites in Hong Kong (data from Gan & Fredlund, 1997, and Fung, 2001)**

matric suction is from 0.1 kPa to 500 kPa. Normal distributions can be substantiated for both  $\ln(a_1)$  and  $\ln(n_1)$  with a 5% significance level based on Kolmogorov–Smirnov tests. The uncertainties of  $\ln(a_1)$  and  $\ln(n_1)$  are significant, with standard deviations of 1.44 and 1.04 respectively.

**Table 1. Mean  $\mu$ , standard deviation  $\sigma$  and distribution of random variables**

	$\mu$	$\sigma$	Distribution	Sources of data
$\ln(k_{sat})$	-13.41	1.13	Normal	Knill <i>et al.</i> (1999)
$\theta_s$	0.42	0.03	Normal	
$\ln(a_1)$	0.08	1.44	Normal	Gan & Fredlund (1997), Fung (2001)
$\ln(n_1)$	0.78	1.04	Normal	
$M$	1.50	0.23	Normal	Sun (1998), Fung (2001)
$M_{col}$	0.98	0.15	Normal	
$\Gamma$	2.22	0.22	Normal	
$\ln(\lambda)$	-2.09	0.20	Normal	

Statistics of the shear strength parameters  $M$ ,  $M_{col}$ ,  $\Gamma$  and  $\lambda$  for CDG in Table 1 are evaluated based on laboratory triaxial test results from nine sites in Hong Kong (Sun, 1998; Fung, 2001). The distributions of  $M$ ,  $M_{col}$ ,  $\Gamma$  and  $\ln(\lambda)$  are all assumed to be normal.

Table 2 presents the correlation matrix for  $\ln(a_1)$ ,  $\ln(n_1)$ ,  $\ln(k_{sat})$  and  $\theta_s$ . The coefficient of correlation between  $\ln(k_{sat})$  and  $\theta_s$  is 0.744, which is reasonable, because the larger the void space in a soil, the larger the saturated permeability of the soil. The coefficients of correlation between  $\ln(k_{sat})$  and  $\ln(a)$  and between  $\ln(k_{sat})$  and  $\ln(n)$  are  $-0.188$  and  $0.299$  respectively.  $\theta_s$  is negatively correlated with  $\ln(a_1)$  but positively correlated with  $\ln(n_1)$ . This implies that the greater the porosity of a soil, the smaller the air-entry value of the soil and the greater the desaturation rate. In this study, the correlations between the soil hydraulic properties and the soil shear strength parameters, and those among the shear strength parameters themselves, are not considered because of the lack of experimental data.

#### RELIABILITY OF A DEFORMABLE UNSATURATED SOIL SLOPE

A 30 m soil slope with a slope angle of  $35^\circ$  (Fig. 4) is studied in this paper. The slope is composed of a CDG soil with the underlying natural ground. The groundwater table is beneath the boundary between the CDG layer and the natural ground.

The initial unit weight of the CDG soil is assumed to be  $15 \text{ kN/m}^3$ , and the initial pore pressure distribution is hydrostatic with a maximum suction of 50 kPa. The initial stress distribution in the slope is determined by switching on the self-weight of the soil. A rainstorm with a uniform flux rate of  $2.0 \times 10^{-5} \text{ m/s}$  is applied along the slope surface.

#### Verification of deterministic models

A deterministic analysis using mean values of the random variables is conducted using the coupled hydromechanical finite element model together with FESSA. The results are compared with those obtained from uncoupled analyses using Seep/W and Slope/W.

At the initial state, the shear strength model used in this study for CDG soil is the same as the extended Mohr–

Coulomb shear strength model available in Slope/W. Therefore it is decided to compare the safety factors calculated by FESSA and by Slope/W at the initial condition. Fig. 4 illustrates the two critical slip surfaces determined by FESSA and Slope/W. The safety factors are 1.570 and 1.572 respectively. Therefore the developed finite element based slope stability program FESSA yields comparable results with respect to the traditional limit equilibrium slope stability program Slope/W.

The pore water pressure profiles at four time steps along section X-X are shown in Fig. 5. The depths of the wetting front calculated by the coupled model are slightly less than those calculated by Seep/W at the same time of rainfall. The reason for this is that the CDG soils experience contractive deformation due to wetting and become less permeable to the infiltrated rainwater. Therefore the movement of the wetting front predicted by a coupled hydromechanical model should be slower in the contractive soil slope than in a non-deformable soil slope.

#### Uncertainty analysis of safety factor and displacement during rainfall

The Latin hypercube sampling technique is adopted to generate samples of random variables. Sixteen sets of random samples of the soil parameters were generated based on the mean values, standard deviations, distribution types and coefficients of correlation of the random variables shown in Tables 1 and 2. For each set of soil parameters the SWCC of the soil is first determined using the generated values of porosity and parameters  $a_1$  and  $n_1$  of the Fredlund & Xing (1994) model. Then the corresponding permeability function is estimated from the SWCC and  $k_{sat}$  using the Fredlund *et al.* (1994) prediction method. Coupled hydromechanical analyses and slope stability analyses were conducted for each of the 16 sets of soil properties. The uncertainties of the safety factor and the displacements can therefore be evaluated.

Figure 6(a) shows the variation of the mean and coefficient of variation (COV) of the global safety factor with time. At the beginning of the rainstorm, the COV of the safety factor is 14% because only the uncertainties of the shear strength parameters propagate to the uncertainty of the safety factor. After rain for 36 h the COV increases to 22%, because both the variations of the shear strength properties and the hydraulic properties influence the pore water pressure distributions and consequently the variation of the safety factor.

Figure 6(b) shows the changes with time of the mean value and COV of the magnitude of the resultant displacement.

Table 2. Correlation matrix for random variables

	$\theta_s$	$\ln(a_1)$	$\ln(n_1)$	$\ln(k_{sat})$
$\theta_s$	1			
$\ln(a_1)$	-0.148	1		
$\ln(n_1)$	0.090	-0.120	1	
$\ln(k_{sat})$	0.744	-0.188	0.299	1

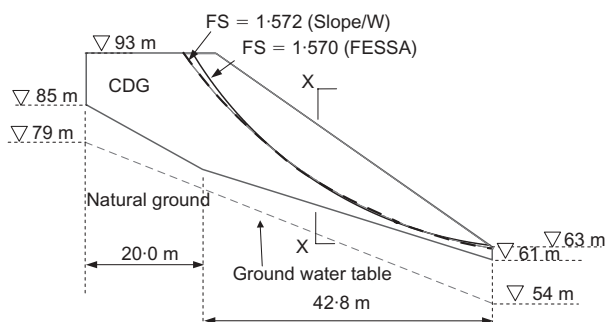


Fig. 4. Cross-section of a soil slope in this study

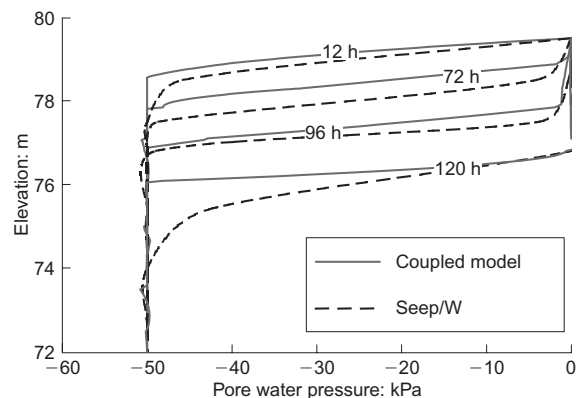


Fig. 5. Comparison of pore water pressure profiles by the coupled model and Seep/W

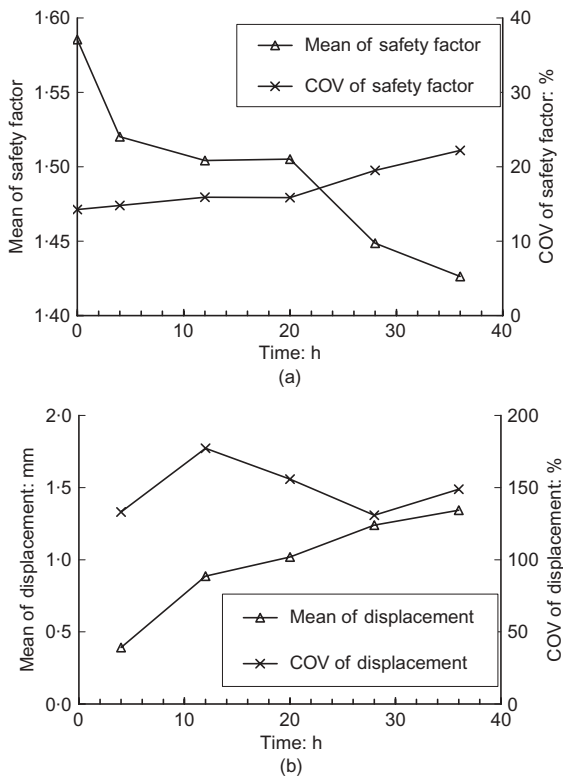


Fig. 6. Variation of mean and COV with time: (a) safety factor; (b) resultant displacement vector at crest of slope

ment vector at the crest of the slope. As the storm progresses, the mean of the displacement increases from zero to 1.3 mm. The COV of the displacement also increases, from 133% to 149%. The variation of the displacement is significant because, for some combinations of soil parameters, the wetting front may be limited to a very small depth of the slope during the 36 h of rainstorm, and, as a result, there is almost no deformation in the slope, whereas for some other combinations of soil parameters the wetting front can reach a relatively larger depth, and hence the displacement in the slope can be large. Consequently, the difference among the displacements obtained can vary by orders of magnitude.

## CONCLUSIONS

In this study, uncertainties in soil hydraulic properties and shear strength properties affecting the stability and deformation of slopes are analysed. The uncertainties of measured  $\ln(k_{\text{sat}})$ ,  $\ln(a_1)$  and  $\ln(n_1)$  for CDG soils are found to be significant. The standard deviations of the three random variables are 1.13, 1.44 and 1.04 respectively. In comparison, the standard deviation of the measured porosity is only 0.025.

A coupled hydromechanical finite element modelling program and a finite element based slope stability analysis program FESSA are developed to study the performance of an unsaturated soil slope during a rainstorm and to include the uncertainties of soil properties in the analyses. Solutions obtained are found to be reasonable, when compared with the results from Slope/W and Seep/W.

A probabilistic study for a hypothetical unsaturated CDG soil slope during a rainstorm is presented, considering the variability of soil properties using the Latin hypercube sampling technique. Before the storm starts, only the uncertainties in soil strength parameters influence the variation of the safety factor. After rain, the soil hydraulic properties begin to influence pore water pressures and hence the per-

formance of the slope. Consequently, the uncertainties of the safety factor and the slope displacement increase as the storm progresses. The coefficient of variation of the safety factor increases from 14% to 22%. The coefficient of variation of the displacement increases more significantly, from 133% to 149%.

## ACKNOWLEDGEMENTS

The work in this paper was substantially supported by grants from the Research Grants Council of the Hong Kong Special Administrative Region, China (Project No. CA99/00.EG01 and Project No. HKUST 6229/01E).

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