

Rainfall interception and drop size—development and calibration of the two-layer stochastic interception model

IAN R. CALDER

Institute of Hydrology, Crowmarsh Gifford, Wallingford, Oxfordshire OX10 8BB, U.K.

Received March 2, 1995

Summary This paper reviews the development of the stochastic interception model from the original, single-layer, drop-size-dependent model to the two-layer model that recognizes that vegetation canopies are wetted through both the primary impact of raindrops to the top layer of the canopy and secondary impacts from drops falling from the vegetation to lower layers of the canopy.

It is shown that drop volumes of primary raindrops can be calculated from the Marshall-Palmer distribution and drop volumes of secondary drops can be estimated from disdrometer measurements of the characteristic volume appropriate to the particular vegetation species. It is recognized that, in addition to the volume-dependent stochastic wetting effect, there is also another drop-size-dependent wetting effect that is related to the kinetic energy of the raindrops, which reduces the maximum storage that can be achieved on the canopy.

The predicted wetting functions for canopies of different density are described and compared with observations made with the use of a rainfall simulator. It is also shown that the species-dependent model parameters can be determined from measurements made with the rainfall simulator. The improved performance of the model compared with conventional interception models is demonstrated for a tropical forest in Sri Lanka. Application of the two-layer model may explain why interception losses from coniferous, fine-leaved forests in the temperate, low-intensity rainfall climate of the uplands of the U.K. are among the highest in the world, whereas interception losses from tropical broad leaved forest in high-intensity rainfall climates of the tropics are among the lowest.

Keywords: canopy wetting, drop volume, kinetic energy, Marshall-Palmer distribution, rainfall simulator.

Introduction

Arguably, the processes controlling the retention and subsequent evaporation of water droplets on vegetation, commonly known as interception, are among the most extensively studied, but most poorly understood, in hydrology. An understanding of the processes controlling interception is of considerable interest to hydrologists because, in wet climates, this

component of the total evaporation can be the dominant term and can explain why evaporative losses from forests can greatly exceed those from grassland (Calder 1990). An understanding of the processes that determine interception is also relevant to studies of splash-induced erosion (Morgan 1985).

The single-layer stochastic interception model

The importance of drop size in determining interception losses from canopies was first established through experimental studies of the rainfall interception process in the tropical climates of Indonesia and India (Calder et al. 1986, Hall et al. 1992) and by theoretical development of the stochastic interception model (Calder 1986). This model used Poisson probability statistics to describe the manner in which elemental areas of vegetation are wetted by raindrop strikes of different drop size. The model predicts that for storms with the same total rainfall, interception losses will be greater for those with smaller drop sizes (Calder 1986, Calder 1990, Hall et al. 1992). A brief description of this model is given below.

It was proposed that the manner in which vegetation surfaces are wetted by raindrops is a random process governed by Poisson statistics. If the surface of vegetation is considered to be composed of elemental areas, each of which has the same probability of being struck by raindrops it can be shown that the mean number of drops retained per elemental area of the canopy, n , can be calculated from the mean number of drop strikes per element, m , and a parameter q , defined as the maximum number of drops that could be retained per elemental area of the canopy.

The basic equation of the model gives the mean number of drops retained per element as:

$$n = q(1 - e^{-m}) + e^{-m} \sum_{i=1}^r (i - q) \frac{m^i}{i!}, \quad (1)$$

where r is the truncated value of q .

The depth of canopy storage per unit ground area, C (mm), and the depth of rainfall striking the canopy per unit ground area (excluding the free throughfall component), P (mm), are

related to the model parameters (Table 1) through the relationships:

$$C = nvL, \quad (2)$$

and

$$P = mvL, \quad (3)$$

where v (mm^3) is the volume of the raindrops and L is the number of elemental surfaces per unit ground area (mm^{-2}). The maximum storage capacity can then be defined as:

$$C_{\max} = qvL. \quad (4)$$

If C_{\max} is invariant and independent of v , which is not a prerequisite of the stochastic hypothesis, then the drop retention number, q , has a simple inverse relationship with v , i.e.:

$$q = C_{\max}/vL. \quad (5)$$

Thus, for small drops the drop retention number, q , becomes large and, in the limit, as drop size decreases, q approaches infinity; for this special case the wetting function predicted by

the stochastic model is identical with that implicit in the Rutter interception model (Rutter et al. 1971).

The normalized wetting functions, expressed in terms of the normalized canopy storage, C/C_{\max} , and normalized rainfall, P/C_{\max} , for different values of drop retention number q , predicted by the stochastic model, are shown in Figure 1.

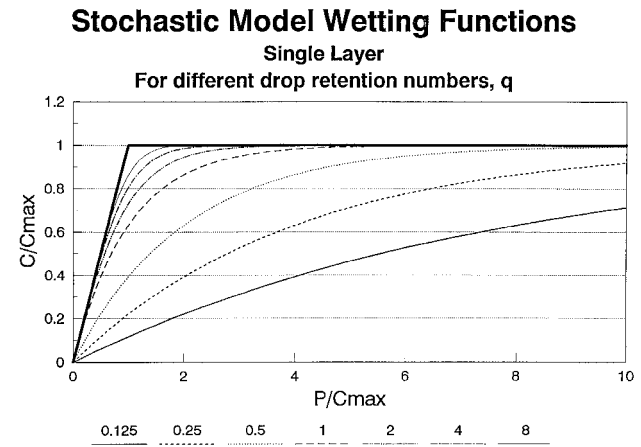


Figure 1. Normalized wetting functions showing C/C_{\max} against normalized rainfall P/C_{\max} , for different values of drop retention number q , predicted by the original stochastic model.

Table 1. Summary of stochastic model parameters, input data and state variables. For the two-layer model the same names apply but a subscript 1 or 2 is used to denote the top or bottom layers for the parameters and state variables C_{\max} , C_m , C , q , L , L_p^* , m , n , v and P .

VEGETATION PARAMETERS

Primary parameters

C_{\max}	Maximum canopy storage for zero kinetic energy drops, $= qv_0L$, (mm)
v_{e0}	Elemental volume for zero kinetic energy drops, (mm^3)
v_c	Characteristic volume of drops falling from vegetation, (mm^3)
L_p^*	Projected leaf area index (dimensionless), $= rL^*$, where r is related to leaf angle and L^* = leaf area index
k_a, k_b, k_c	Parameters governing the kinetic energy dependence of C_m/C_{\max} as a function of v

Derived parameters

$L_{p,1}^*$	Projected leaf area index (dimensionless) of top layer, $L_{p,1}^* = 1 - \exp(-L_p^*)$
$L_{p,2}^*$	Projected leaf area index (dimensionless) of bottom layer, $L_{p,2}^* = L_p^* - L_{p,1}^*$
C_{\max}	Maximum canopy storage achievable for the two-layer formulation with zero kinetic energy drops incident on the top layer, $C_{\max} = C_{\max}(L_{p,1}^*/L_p^*) + C_{\max}(L_{p,2}^*/L_p^*)(k_a + k_b \exp(-k_c v_c))$
C_m	Maximum canopy storage achievable for the two-layer formulation with non-zero kinetic energy drops incident on the top layer, $C_m = C_{\max}(L_{p,1}^*/L_p^*)(k_a + k_b \exp(-k_c v)) + C_{\max}(L_{p,2}^*/L_p^*)(k_a + k_b \exp(k_c v_c))$
v_e	Elemental volume for non-zero kinetic energy drops, $v_e = (C_m/C_{\max})v_{e0}$, (mm^3)
q	Mean maximum drop retention number per element for non-zero kinetic energy drops, $q = v_e/v$
L	Number of elements per unit ground area, $L = C_{\max}/v_{e0}$ for the two-layer formulation, $L_1 = (L_{p,1}^*/L_p^*)(C_{\max}/v_{e0})$ and $L_2 = (L_{p,2}^*/L_p^*)(C_{\max}/v_{e0})$
f_1	Fraction of rain striking the top layer $= L_{p,1}^*$
f_2	Fraction of the shed rain from the top layer striking the bottom layer of the canopy $= 1 - \exp(-L_{p,2}^*)$

INPUT DATA

P	Depth of precipitation, (mm); R , Precipitation intensity, (mm h^{-1})
E_w^{PM}	Evaporation rate estimated from the Penman-Monteith equation for a wet surface (mm)

STATE VARIABLES

v	Median volume of drops, (mm^3)
m	Mean number of raindrop strikes per element
n	Mean number of drops retained per element
C	Canopy storage $= nvL$, (mm)
S	Shed rainfall (mm)

Evaporation can be incorporated explicitly into the model by redefining m as the effective mean number of raindrop strikes per element. Thus, m has the same effect on canopy storage as the combined effect of the actual number, due to rainfall, and evaporation. The evaporation increment is normally calculated from the Penman-Monteith equation with the surface resistance term equated to zero to represent wet canopy surface conditions.

Development of the two-layer stochastic interception model

The two-layer stochastic model (Calder 1995) considers the canopy to be composed of two layers, the top layer receiving primary raindrops and the bottom layer receiving secondary drops falling from the top layer. A brief description of this model is given below.

Dependence of q on drop volume and kinetic energy

Previous applications of the single-layer stochastic model considered q to be fixed for a particular vegetation type and rainfall regime. The two-layer model (Calder 1995) takes into account the dependence of q on three factors: (1) the variation of raindrop volume with rainfall intensity; (2) the dependence of the drop retention number q not only on drop volume but also on drop kinetic energy; and (3) the volumes of drops falling off higher levels of the canopy.

To incorporate the dependence of q on both drop volume and drop kinetic energy within the stochastic interception model, a new model parameter was introduced, specific to the particular vegetation type, termed the maximum elemental volume, v_e . For drops impacting the surface with zero kinetic energy, the maximum elemental volume was defined as:

$$v_{e0} = qv_0, \tag{6}$$

where subscript 0 denotes zero kinetic energy.

The term C_{max} can then be redefined as the maximum storage capacity that is obtained when the canopy is wetted with drops of zero kinetic energy, i.e.:

$$C_{max} = v_{e0}L = qv_0L. \tag{7}$$

Further, a new model parameter, C_m , can be defined as the maximum storage capacity obtained with non-zero kinetic energy drops:

$$C_m = v_eL. \tag{8}$$

For drops of non-zero kinetic energy, the maximum elemental volume is given by:

$$v_e = (C_m/C_{max})v_{e0}. \tag{9}$$

The maximum drop retention number for drops of non-zero kinetic energy, q , can be calculated from

$$q = v_e/v, \tag{10}$$

that is

$$q = (C_m/C_{max})v_{e0}/v. \tag{11}$$

To operate the model for a particular vegetation type requires values for two vegetation specific parameters C_{max} (which defines L), and v_{e0} . Also required is the functional relationship between C_m/C_{max} and v . Preliminary measurements with a rainfall simulator (Calder et al. 1995), suggest the empirical exponential relationship:

$$\begin{aligned} C_m/C_{max} &= 1 && \text{for } v < 0.065 \\ C_m/C_{max} &= 0.5 + 0.73 \exp(-5.5v) && \text{for } v \geq 0.065. \end{aligned} \tag{12}$$

Estimation of drop volume

Primary drops Marshall-Palmer (1948) established an empirical relationship between the volume of raindrops and rainfall intensity. Following on from this work, Torres and colleagues (Torres et al. 1994) have provided a general formula for raindrop size distribution. With this general formula, and from field observations, they have shown that the median-volume diameter, D_0 (cm), of raindrops is related to rainfall intensity, R (mm h^{-1}), by the equation:

$$D_0 = 0.087R^{0.0228}. \tag{13}$$

Secondary drops Hall and Calder (1993), from measurements made with an optical disdrometer, showed that the size spectrum of drops falling from tree canopies is characteristic of the particular tree species (Figure 2). Secondary drops, falling from the top layer, are therefore considered to be of the characteristic volume v_e .

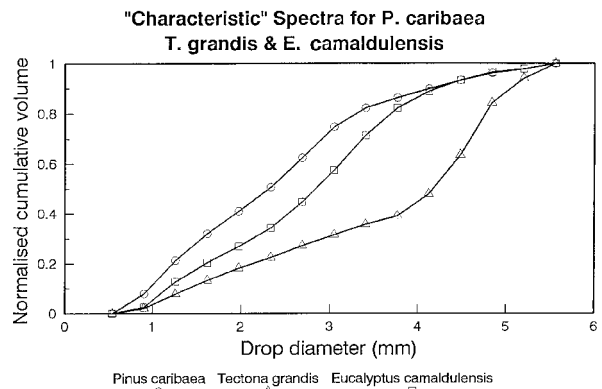


Figure 2. Cumulative frequency distribution of throughfall drop spectra for three tree species subject to spray with median volume drop diameter of 3.2 mm. Median volume drop diameter (the characteristic volume) was 2.3 mm for *Pinus caribaea* Morelet., 2.8 mm for *Eucalyptus camaldulensis* Dehnh. and 4.2 mm for *Tectona grandis* L. f.

Layer partition

Partitioning of the number of elemental areas between layers can be achieved from a knowledge of the projected leaf area index of the canopy, L_p^* . Assuming that leaves are distributed randomly within the canopy space, the projected leaf area index of the top layer of the canopy, $L_{p,1}^*$, which receives incident, vertically falling, primary raindrops is given by Beer's Law as:

$$L_{p,1}^* = 1 - \exp(-L_p^*), \quad (14)$$

The projected leaf area index of the bottom layer of the canopy, $L_{p,2}^*$, which receives secondary drops, is given by:

$$L_{p,2}^* = L_p^* - L_{p,1}^*. \quad (15)$$

Note that, with the above definitions, as the projected leaf area of the canopy increases, the projected leaf area index of the top layer approaches unity, implying that no primary drops fall through the canopy without a strike; the projected leaf area of the bottom layer then approaches that of the total projected leaf area index of the canopy less one. Partitioning of the number of elemental areas per unit ground is assumed to be proportional to these projected leaf area index values.

Rain and shed-rain strike fractions Partitioning of rainfall between the top layer of the canopy and the ground, and partitioning of shed-rain from the top layer, between the bottom layer and the ground, can also be achieved from a knowledge of the projected leaf area index of the two canopy layers, i.e., rainfall striking the top layer of the canopy, P_1 , is expressed as a proportion of total rainfall, P :

$$P_1 = f_1 P = L_{p,1}^* P. \quad (16)$$

The input to the bottom layer of the canopy, P_2 , resulting from a depth of water, S , shed from the top layer is given by:

$$P_2 = f_2 S = (1 - \exp(-L_{p,2}^*)) S. \quad (17)$$

Operation of the two-layer stochastic model

The steps in the operation of the model are essentially the same as those described for the original stochastic model (Calder 1986) except that raindrop volume, v , is now calculated from rainfall intensity, and q for the top layer is calculated from the elemental volume, v_e , divided by v . For the bottom layer, q is calculated from v_e divided by v_c . In the two-layer model, evaporation is considered to occur only from the top layer while it retains water. When the top layer is completely dry, evaporation is considered to deplete the bottom layer. Further details of the operation of the model are given in Calder 1996.

Predicted wetting functions

The manner in which the incorporation of the two-layer formulation alters the predicted wetting functions can best be understood by comparing wetting functions obtained with varying

values of projected leaf area index, L_p^* , while holding the other canopy parameters constant.

The wetting functions obtained with the two layer stochastic model with a projected leaf area index, $L_p^* = 0.45$, an elemental volume, v_e , of 4 mm^3 , a characteristic volume, v_c , of 11.5 mm^3 and incorporating the q dependence on kinetic energy specified by $k_a = 0.5$, $k_b = 0.73$, $k_c = 5.5$, are shown in Figure 3 for different values of rainfall intensity, (based on the Torres relationship between drop size and rainfall intensity).

The wetting functions obtained with similar canopy parameter values but with drop volumes appropriate to those used with a rainfall simulator on studies of *Eucalyptus camaldulensis* are shown in Figure 4. The matching of predicted wetting func-

Layer Stochastic Model Wetting Functions

$V_e = 4, V_c = 11.5, L_p^* = 0.45$
for different rainfall intensities

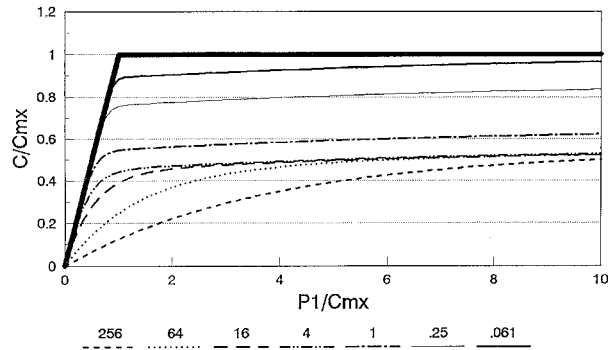


Figure 3. Normalized wetting functions showing C/C_{mx} against normalized rainfall P_1/C_{mx} , predicted by the two-layer stochastic model for different rainfall intensities with $L_p^* = 0.45$, depicting a sparse canopy, $v_c = 11.5 \text{ mm}^3$, $v_e = 4 \text{ mm}^3$, $k_a = 0.5$, $k_b = 0.73$, and $k_c = 5.5$.

Wetting Functions for *E. camaldulensis*

$V_e = 4, V_c = 11.5, L_p^* = 0.598$

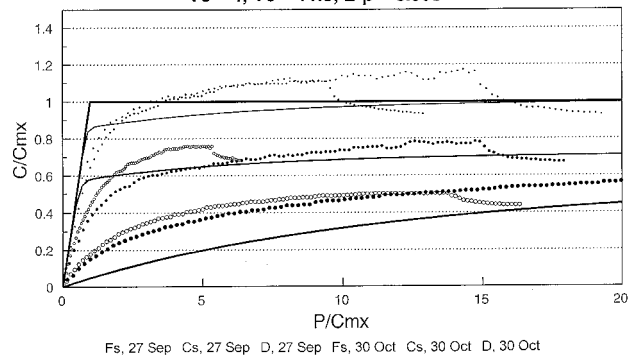


Figure 4. Normalized wetting functions, determined on September 27 and October 30, 1993, for *Eucalyptus camaldulensis*, showing C/C_{mx} against normalized depth of water applied P/C_{mx} , for different sizes of drops obtained with a fine spray (Fs), coarse spray (Cs), and the drifter system (D) with median volumes of 0.065, 0.27 and 73 mm^3 , respectively. Also shown, as solid lines, are the wetting functions predicted by the stochastic model for these drop sizes with elemental volume, $v_e = 4 \text{ mm}^3$, $v_c = 11.5 \text{ mm}^3$, and $L_p^* = 0.598$.

tions with those observed with the rainfall simulator provides a means of determining the elemental volume appropriate to a particular species.

For forests with even greater L^*_p , the dependence of the wetting function on the size of raindrops will be further reduced. The wetting response will be determined principally by the volume and kinetic energy of the “characteristic” drops. Figure 5 shows the type of wetting response that might be expected for a forest with a dense canopy, such as a humid tropical rainforest with an $L^*_p = 2.7$. For dense canopies, the wetting response converges to that expected with a single-layer formulation with a drop retention number equal to the ratio of the elemental to the characteristic volume, i.e., $q = v_e/v_c$. It is interesting to note that the q value obtained for tropical rainforest in Indonesia (Calder et al. 1986) by means of the single-layer formulation of the stochastic model indicated a v_e/v_c ratio that was less than unity.

Implementation and assessment of the two-layer model

The two-layer stochastic model has been implemented and run with climate and interception data collected in Sri Lanka for tropical forest (Hall and Calder 1995). Comparison of the performance of the two-layer stochastic model with that of the Rutter model is demonstrated by Figure 6. Although both models relied on parameter optimization, the Rutter model is clearly unable to reproduce the onset of net rainfall at the beginning of the storm.

Discussion and conclusions

The two-layer stochastic model allows predictions, based on Poisson probability statistics, of the rate at which vegetation canopies are wetted in relation to drop size. Studies carried out with the rainfall simulator have verified the predictions made

Layer Stochastic Model Wetting Functions

$v_e = 4, v_c = 11.5, L^*_p = 2.7$
for different rainfall intensities

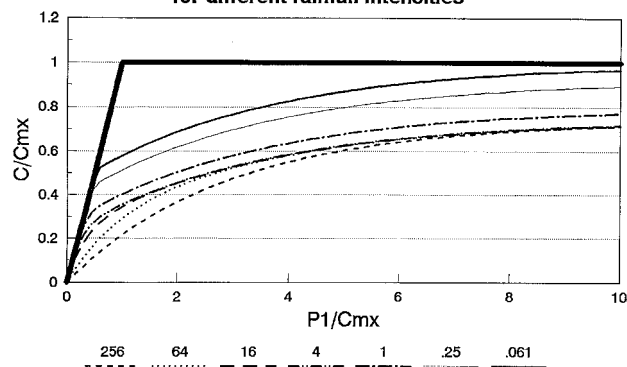


Figure 5. Normalized wetting functions showing C/C_{mx} against normalized rainfall P_1/C_{mx} , predicted by the two-layer stochastic model for different rainfall intensities with $L^*_p = 2.7$, depicting a dense canopy, $v_c = 11.5 \text{ mm}^3$, $v_e = 4 \text{ mm}^3$, $k_a = 0.5$, $k_b = 0.73$, and $k_c = 5.5$.

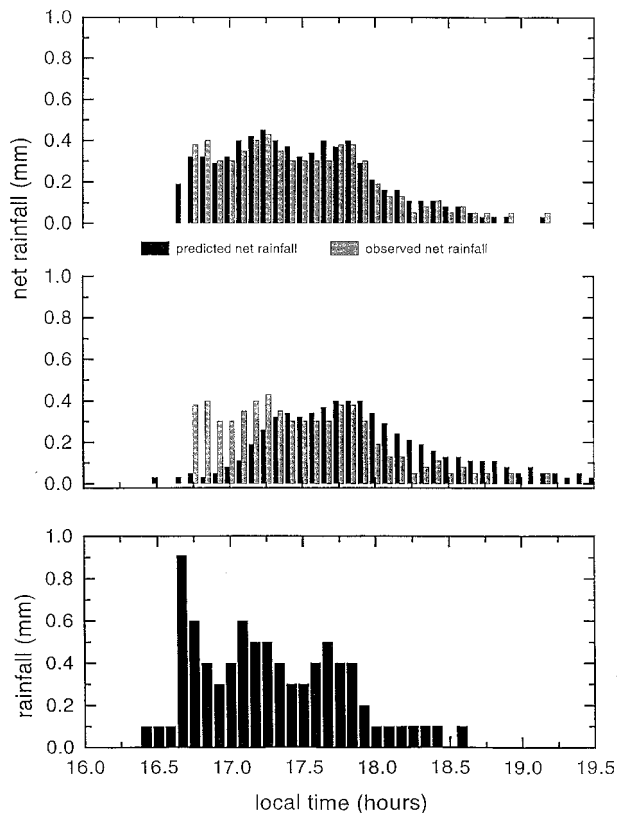


Figure 6. A comparison of the gross (bottom) and net rainfall observed and predicted by the Rutter model (middle) and two-layer stochastic model (top) for a storm recorded on May 28, 1993 at Dodongolla, Sri Lanka.

by the stochastic wetting hypothesis with respect to the rate at which canopies wet in relation to drop size. These studies have also established that the maximum storage value is not a constant, as has been assumed in previous interception studies, but that it is strongly dependent on drop size through an assumed kinetic energy dependence.

Application of the model to a real situation, using interception data collected from a site in Sri Lanka, has shown that the model is better able to describe the initial wetting-up phase of the storm than conventional interception models that do not incorporate a drop size dependence. The two-layer stochastic model may provide insights into some of the reasons why forest interception varies so much worldwide, and may help to explain why interception losses from coniferous temperate forests are so much higher than those from tropical forests.

The model shows that canopy wetting will be achieved most rapidly and maximum canopy storage will be highest, leading to high interception losses overall, when both raindrop volume is small and the characteristic volume of drops draining from the canopy is also small. These conditions apply for coniferous forests in the low-intensity, small rain drop size climate of the uplands of the U.K. By contrast, when both rain drop volumes and characteristic volumes are large, canopy wetting will be achieved much more slowly and interception losses are likely

to be much reduced. This situation is typified by tropical rainforest experiencing high intensity convective storms of large drop size.

References

- Calder, I.R. 1986. A stochastic model of rainfall interception. *J. Hydrol.* 89:65–71.
- Calder, I.R. 1990. *Evaporation in the uplands*. John Wiley Publishers, New York, 148 p.
- Calder, I.R. 1996. Dependence of rainfall interception on drop size: 1. Development of the Two-Layer Stochastic Model. *J. Hydrol.* In press.
- Calder, I.R., I.R. Wright and D. Murdiyarso. 1986. A study of evaporation from tropical rainforest—West Java, *J. Hydrol.* 89:13–33.
- Calder, I.R., R.L. Hall, P.T.W. Rosier, H.G. Bastable and K.T. Prasanna. 1996. Dependence of rainfall interception on drop size 2. Experimental determination of the wetting functions and stochastic model parameters for five tropical tree species. *J. Hydrol.* In press.
- Hall, R.L., I.R. Calder, P.T.W. Rosier, M.H. Swaminath and J. Mumtaz. 1992. Measurements and modelling of interception loss in eucalypt plantations in southern India. *In Growth and Water Use of Forest Plantations*, Bangalore. Eds. I.R. Calder, R.L. Hall and P.G. Adlard. John Wiley Publishers, Chichester, England, pp 167–179.
- Hall, R.L. and I.R. Calder. 1993. Drop size modification by forest canopies—measurements using a disdrometer, *J. Geophys. Res.* 90:465–470.
- Hall, R.L. and I.R. Calder. 1995. Modelling interception from Kandyan forest garden; dependence on rainfall intensity and drop size. *In Proc. Phase I Review Seminar Workshop of the ODA Forestry Programme*. Eds. J. Burley, H.P.M. Gunasena and J. Vincent. UP-OFI Link Project, University of Peradeniya, Sri Lanka. In press.
- Marshall, J.S. and W.M. Palmer. 1948. The distribution of raindrops with size. *J. Meteorol.* 5:165–166.
- Morgan, R.P.C. 1985. Establishment of plant cover parameters for modelling splash detachment. *In Soil Erosion and Conservation*. Eds. S.A. El-Swaify, W.C. Moldenhauer and A. Lo. Soil Conservation Society of America, pp 377–383.
- Rutter, A.J., K.A. Kershaw, P.C. Robins and A.J. Morton. 1971. A predictive model of rainfall interception in forests. I. Derivation of the model from observations in a plantation of Corsican pine. *Agric. Meteorol.* 9:367–384.
- Torres, D.S., J.M. Porra and J.-D. Creutin. 1994. A general formulation for rain drop size distribution. *J. Appl. Meteorol.* 33:1491–1502.