Raman-noise-induced noise-figure limit for $\chi^{(3)}$ parametric amplifiers

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Received August 22, 2003

The nonzero response time of the Kerr [$\chi^{(3)}$] nonlinearity determines the quantum-limited noise figure of $\chi^{(3)}$ parametric amplifiers. This nonzero response time of the nonlinearity requires coupling of the parametric amplification process to a molecular-vibration phonon bath, causing the addition of excess noise through Raman gain or loss at temperatures above 0 K. The effect of this excess noise on the noise figure can be surprisingly significant. We derive analytical expressions for this quantum-limited noise figure for phase-insensitive operation of a $\chi^{(3)}$ amplifier and show good agreement with published noise-figure measurements. © 2004 Optical Society of America

OCIS codes: 060.2320, 270.5290.

Fiber-optical parametric amplifiers (FOPAs) are currently the subject of much research for use in wavelength conversion[1] and efficient broadband amplification.[2] They are also candidates for performing all-optical network functions.[3,4] Advances in pumping techniques have permitted improvements of the noise figure[1,5,6] (NF), and the manufacture of high-nonlinearity and microstructure fibers has improved the gain slope[7,8] of fiber parametric amplifiers. To date, the lowest published NF measurements in phase-insensitive operation of a $\chi^{(3)}$ amplifier have been 3.7,[9] 3.8,[1] and 4.2 dB.[5] In Ref. 1 it is stated that the NF of a well-designed FOPA should be slightly above 3 dB because of the presence of small amounts of linear loss in the fiber. In our previous experiment,[9] which measured the NF with only parametric fluorescence and thus was not subject to pump-noise-induced signal-gain modulation,[5] the cause of measured excess noise was not understood. Underlying the premise that the high-gain NF of a lossless parametric phase-insensitive amplifier (PIA) is 3 dB is the assumption that the $\chi^{(3)}$ nonlinearity is instantaneous or that the effect of a nonzero $\chi^{(3)}$ response time on the NF is negligible. To the best of our knowledge, the nonzero response time of the nonlinearity has not yet been used to derive a correct NF limit for a $\chi^{(3)}$ parametric amplifier. In this Letter we do so and find good agreement with our previously published parametric-amplifier NF measurements.

The frequency response of the $\chi^{(3)}$ nonlinearity can be written as $F(\Omega) = \int \mathrm{d}t f(t) \exp(i\Omega t)$, where $f(t)$ is the response function of the Kerr interaction. We write the response function in the frequency domain as $F(\Omega) = F_e + F_r r(\Omega)$, which is composed of an electronic response ($\ll$1 fs) that is similar to a time domain delta function and is constant over the bandwidths of interest and a time-delayed Raman response ($\approx$50 fs) that varies over frequencies of interest and is caused by back action of nonlinear nuclear vibrations on electronic vibrations. Published measurements of the real part of the Kerr nonlinearity in common optical fibers, although widely varying, yield $F(0)$ when nonlinear interaction times in the measurements are of much longer duration than the Raman response time but are of shorter duration than the electrostriction time constant (typically of nanosecond duration). Along with measurement of the Raman gain profile, one may, by means of the Kramers–Kronig transformation, obtain $F(\Omega)$ at the frequencies of interest.[10] Here we have assumed symmetry in the Raman-gain profile, i.e., $F(\Omega) = F(-\Omega)^*$. The asymmetric case will be treated in a subsequent longer paper. We also note here the relation between the published spectra of the Raman-gain coefficient and the coefficients used in this Letter. Typical measurements of the counterpropagating pump-and-signal Raman-gain spectrum yield the polarization-averaged power-gain coefficient $g_r(-\Omega) = [g_\parallel(-\Omega) + g_\perp(-\Omega)]/2$. At the Raman-gain peak, $g_\perp \simeq 0$. We define a nonlinear coefficient $H(\Omega) = 2\pi F(\Omega)/(\lambda A_{\mathrm{eff}})$, where λ is the pump wavelength and A_{eff} is the fiber effective area. For copropagating, copolarized optical waves $\mathrm{Im}\{H(-\Omega)\} = g_\parallel(-\Omega)/2$. We estimate the spectrum of $g_\parallel$, normalized to its maximum value, from Ref. 11 and take its magnitude from Ref. 12 for both dispersion-shifted fiber (DSF) and standard single-mode fiber (SMF). For $F(0)$ we use measurements from Ref. 13.

A self-consistent quantum theory of light propagation in a nonzero $\chi^{(3)}$ response-time medium has been developed,[14] and the associated Raman-noise limit on the generation of squeezing in such a medium through fully frequency-degenerate four-wave mixing has been found.[15] This theory is consistent with the classical mean-field solutions and preserves the continuous-time field commutator. Although the theory in Ref. 14 provides integral-form expressions for propagation of a multimode total field, dispersion was not explicitly included. In the following we present a theory for parametric amplification in the undepleted-pump approximation that yields analytical expressions for the NF while preserving the commutators for the signal and idler fields.

Consider the field operator $\hat{A} = \hat{A}_p + \hat{A}_s \exp(i\Omega t) + \hat{A}_a \exp(-i\Omega t)$ for the total field propagating through a FOPA with a frequency- and polarization-degenerate pump. We call the lower frequency field the Stokes field, $\hat{A}_s$, and the higher frequency field the anti-Stokes field, $\hat{A}_a$. The fields propagate in a lossless, polarization-preserving, single-transverse-mode fiber under the slowly varying envelope approximation. Here the frequency deviation from the pump frequency is $\Omega = \omega_a - \omega_p = \omega_p - \omega_s$. The quantum equation of motion for the total field can be written as[14,15]

$$\frac{\partial \hat{A}(t)}{\partial z} = i\left[\int \mathrm{d}\tau h(t-\tau)\hat{A}^\dagger(\tau)\hat{A}(\tau)\right]\hat{A}(t) + \hat{m}(z,t)\,, \quad (1)$$

where the operator $\hat{m}(z,t)$ is a phase-noise operator that is required to preserve the continuous-time commutators $[\hat{A}(t), \hat{A}^\dagger(t')] = \delta(t-t')$ and $[\hat{A}(t), \hat{A}(t')] = 0$. Taking the Fourier transform and separating it into frequency-shifted components, we obtain the following differential equations after making the undepleted-pump approximation by neglecting terms with fewer than two pump operators contributing. Under the undepleted-pump approximation it is also acceptable to neglect the fluctuation operators at all frequencies except the Stokes and anti-Stokes frequencies because only the pump mean field will interact with the modes of interest to a nonnegligible degree, as can be shown by linearization of the quantum fluctuations. Unlike in Ref. 16, we obtain $\mathrm{d}\overline{A}_p/\mathrm{d}z = iH(0)|\overline{A}_p|^2\overline{A}_p$,

$$\frac{\mathrm{d}\hat{A}_a}{\mathrm{d}z} = i[H(0) + H(\Omega)]|\overline{A}_p|^2\hat{A}_a + iH(\Omega)\overline{A}_p^2\hat{A}_s^\dagger \exp(-i\Delta k z) + \widehat{M}(z,\Omega)\overline{A}_p\,, \quad (2)$$

$$\frac{\mathrm{d}\hat{A}_s}{\mathrm{d}z} = i[H(0) + H(-\Omega)]|\overline{A}_p|^2\hat{A}_s + iH(-\Omega)\overline{A}_p^2\hat{A}_a^\dagger \exp(-i\Delta k z) + \widehat{M}(z,-\Omega)\overline{A}_p\,, \quad (3)$$

where $\Delta k = \beta_2\Omega^2$ and β_2 is the group-velocity dispersion coefficient at ω_p and mean fields are written as $\langle\hat{A}_j\rangle = \overline{A}_j$ for $j \in \{p, a, s\}$. In Eqs. (2) and (3) all interactions are photon number preserving, and all but the Raman loss and gain terms conserve energy in the multimode optical field. Thus only the Raman terms require the addition of commutator-preserving quantum-noise operators that couple the field to the molecular-vibration modes in the $\chi^{(3)}$ medium. The solution for the mean fields can be written as $\overline{A}_p(z) = \overline{A}_p(0)\exp[iH(0)|\overline{A}_p(0)|^2 z]$, $\overline{A}_a(z) = \mu_a(z)\overline{A}_a(0) + \nu_a(z)\overline{A}_s^*(0)$, and $\overline{A}_s(z) = \mu_s(z)\overline{A}_s(0) + \nu_s(z)\overline{A}_a^*(0)$, where

$$\mu_a(z) = \exp(i\phi_c z)\left[\cosh(gz) + \frac{i\kappa}{2g}\sinh(gz)\right], \quad (4)$$

$$\mu_s(z) = \exp(i\phi_c z)\left[\cosh(g^* z) + \frac{i\kappa^*}{2g^*}\sinh(g^* z)\right], \quad (5)$$

$$\nu_a(z) = \exp(i\phi_c z)\,\frac{iH(\Omega)|\overline{A}_p|^2}{g}\sinh(gz)\,, \quad (6)$$

$$\nu_s(z) = \exp(i\phi_c z)\,\frac{iH(-\Omega)|\overline{A}_p|^2}{g^*}\sinh(g^* z)\,, \quad (7)$$

with $g = \{-(\kappa/2)^2 + [H(\Omega)|\overline{A}_p|^2]^2\}^{1/2}$, $\kappa = \Delta k + 2H(\Omega)|\overline{A}_p|^2$, and $\phi_c = -\Delta k/2 + H(0)|\overline{A}_p|^2$. The units of $|A_j|^2$ are in $\mathrm{W/m^2}$.

Figure 1 shows the power gain versus the signal-pump detuning for a FOPA made with DSF when the nonlinearity is assumed to be instantaneous (solid curve) and when the complex nonlinear response at 1.3 THz is included as explained above (dotted curve). We note that the power gain of the mean field is modified only slightly by the nonzero time response of the nonlinearity in the DSF.

Each differential element of the fiber couples in noise from an independent reservoir of phonon oscillators to the Stokes and anti-Stokes modes with a coupling strength that preserves the mode commutators. Each phonon mode is assumed to be in a thermal state with a mean phonon occupation number of $n_{\mathrm{th}} = 1/[\exp(\hbar|\Omega|/kT) - 1]$ and commutator $[\widehat{M}(z, \pm\Omega), \widehat{M}(z', \pm\Omega')] = \pm 2\mathrm{Im}\{H(\Omega)\}\delta(z - z') \times \delta(\pm\Omega - \pm\Omega')$. Here $\hbar$ is Planck's constant over 2π, k is Boltzmann's constant, and T is the temperature. Under these conditions the operator Eqs. (2) and (3) yield PIA-valid expressions

$$\hat{A}_a(z) = \mu_a(z)\hat{A}_a(0) + \nu_a(z)\hat{A}_s^\dagger(0) + c_{a1}\hat{t}_1 + c_{a2}\hat{t}_2\,, \quad (8)$$

$$\hat{A}_s(z) = \mu_s(z)\hat{A}_s(0) + \nu_s(z)\hat{A}_a^\dagger(0) + c_s\hat{t}_1^\dagger\,, \quad (9)$$

where $c_s = (|\mu_s|^2 - |\nu_s|^2 - 1)^{1/2}$, $c_{a1} = Kc_a$, and $c_{a2} = (1 - |K|^2)^{1/2}c_a$, with $K = (\mu_s\nu_a - \mu_a\nu_s)/(c_s c_a)$ and $c_a = (-|\mu_a|^2 + |\nu_a|^2 + 1)^{1/2}$, and where $\hat{t}_1$ and $\hat{t}_2$ are the thermal-field operators representing the sum of the contributions by each differential element operator $\widehat{M}(z)$ propagated through the remaining length of fiber.

The NF is defined as $\mathrm{SNR}_{\mathrm{in},j}/\mathrm{SNR}_{\mathrm{out},j}$, where $\mathrm{SNR}_j(z) = \overline{n}_j(z)^2/\langle\Delta\hat{n}_j(z)^2\rangle$, with $\hat{n}_j(z) = \hat{A}_j^\dagger(z)\hat{A}_j(z)$,

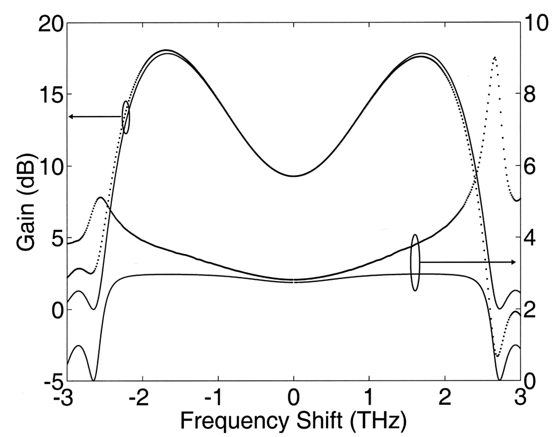

Fig. 1. Gain and NF spectra for 1-km-long FOPA pumped at 1537.6 nm with 1.5 W of power. The fiber's dispersion zero is at 1537 nm, the dispersion slope is 0.064 ps/(nm^2 km), and the nonlinear coefficient is $H(0) = 1.8\ \mathrm{W^{-1}\,km^{-1}}$. $\mathrm{Im}\{H(\Omega)\}$ calculated from Raman measurements[11–13] (dotted curve) and $\mathrm{Im}\{H(\Omega)\} = 0$ (solid curve).

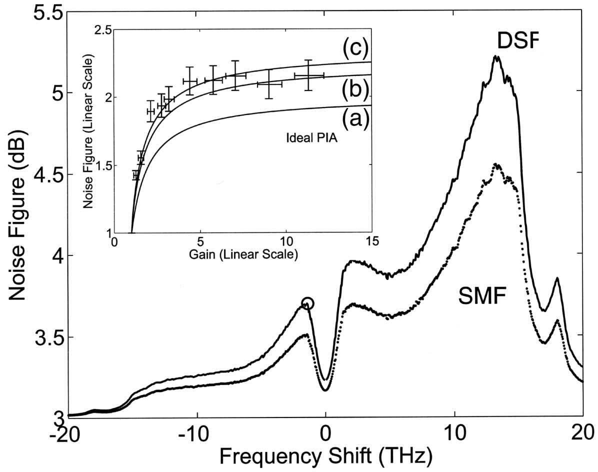

Fig. 2. High-gain NF versus pump-signal detuning for a FOPA phase matched at each detuning. Solid curve, DSF; dotted curve, SMF; circle, experimental data point from Voss *et al.*[9] Inset, NF versus gain for a (a) phase-matched ideal PIA, (b) PIA made with SMF-28, and (c) PIA made with DSF at 1.38-THz pump-signal detuning and 300 K. Experimental data points for DSF are from Voss *et al.*[9] No fitting parameters are used, and $H(\Omega)$ is calculated from measured nonlinear coefficients.[11–13]

$\overline{n}_j = \langle \hat{n}_j \rangle$, and $\Delta\hat{n}_j = \hat{n}_j - \overline{n}_j$ for $j \in \{a, s\}$. For a PIA with a coherent-state input signal of photon number much greater than the amplifier gain, Eqs. (8) and (9) lead to the following expression for the NF:

$$\mathrm{NF}_{j,\,\mathrm{PIA}} = 1 + \frac{|\nu_j|^2 + (1 + 2n_{\mathrm{th}})\,|-1 + |\mu_j|^2 - |\nu_j|^2|}{|\mu_j|^2}, \tag{10}$$

where $j = s(a)$ for signal frequency on the Stokes (anti-Stokes) side. Results for wavelength conversion will be presented elsewhere.

In Fig. 1 we also plot NF versus signal-pump detuning for the experimental setup described in the caption. In the inset of Fig. 2 we plot NF versus gain for DSF (c) and SMF (b). We note that plot (c) matches well with experimental data obtained for DSF in Voss *et al.*[9] We stress that no fitting parameters have been used and the nonlinear coefficients have been calculated directly from reported measurements of the fiber nonlinearity.

In Fig. 2 we show the NF versus the signal-pump detuning where the gain in the DSF and the SMF has been phase matched at each signal frequency ($\mathrm{Re}\{\kappa\} = 0$). Thus the response is not that of a real fiber but shows what the quantum limit would be at a particular pump-signal detuning if phase matched at that detuning. The differences between the quantum-limited NF for the DSF and the SMF arise from the differing ratio $\mathrm{Im}\{H(\Omega)\}/\mathrm{Re}\{H(\Omega)\}$ in the two fibers. The ratio varies depending on the dopants introduced into the core of these fibers. Thus, in designing ultrawideband FOPAs, there is a NF advantage in choosing fibers with dopant that minimize the Raman-gain coefficient for a given magnitude of nonlinear coefficient $H(\Omega)$.

Even though the effects of $\mathrm{Im}\{H(\Omega)\}$ on the mean field are small, the contribution of the Raman gain to the NF is surprisingly large. This is due to the large excess-noise factor at low frequencies and due to the larger relative contribution of the Raman noise in the earliest stages of the amplifier. The Raman gain scales linearly $[\propto \mathrm{Im}\{H(\Omega)\}|\overline{A}_p|^2 L]$, whereas the parametric gain scales quadratically $\{\propto[\mathrm{Re}\{H(\Omega)\}|\overline{A}_p|^2 L]^2\}$ in the early parts of the amplifier. When Ω is near the Raman loss peak, the large noise figure is explained by competition between the Raman loss and the parametric gain.

In conclusion, we have derived analytical expressions for the quantum-limited noise figure of $\chi^{(3)}$ parametric amplifiers that take into account the nonzero response time of the nonlinearity, explaining to a large extent why no group has produced parametric amplifiers with a NF below 3.7 dB. As microstructure fibers permit newfound flexibility in amplifier design, it will be important to properly model the nonlinear interaction to predict the gain and noise performance of $\chi^{(3)}$ amplifiers.

This work was supported by the U.S. Office of Naval Research under grant N00014-03-1-0179 and by the U.S. Army Research Office under a Multidisciplinary University Research Initiative grant DAAD19-00-1-0177. The authors thank Sang-Kyung Choi, Kahraman G. Köprülü, and Ranjith Nair for useful discussions. P. Voss's e-mail address is voss@ece.northwestern.edu.

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