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Random Effects, Fixed Effects and Hausman's Test for the Generalized Mixed Regressive Spatial Autoregressive Panel

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Abstract

This paper suggests random and fixed effects spatial two-stage least squares estimators for the generalized mixed regressive spatial autoregressive panel data model. This extends the generalized spatial panel model of Baltagi, Egger and Pfaffermayr (2013) by the inclusion of a spatial lag dependent variable. The estimation method utilizes the Generalized Moments method suggested by Kapoor, Kelejian, and Prucha (2007) for a spatial autoregressive panel data model. We derive the asymptotic distributions of these estimators and suggest a Hausman test a la Mutl and Pfaffermayr (2011) based on the difference between these estimators. Monte Carlo experiments are performed to investigate the performance of these estimators as well as the corresponding Hausman test.

JEL No. C12, C13, C23

Keywords: Panel Data; Fixed Effects; Random Effects; Spatial Model; Hausman Test

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Random Effects, Fixed Effects and Hausman's Test for the Generalized Mixed Regressive Spatial Autoregressive Panel Data Model*

Badi H. Baltagi[†], Long Liu[‡]

November 1, 2014

Abstract

This paper suggests random and fixed effects spatial two-stage least squares estimators for the the generalized mixed regressive spatial autoregressive panel data model. This extends the generalized spatial panel model of Baltagi, Egger and Pfaffermayr (2013) by the inclusion of a spatial lag dependent variable. The estimation method utilizes the Generalized Moments method suggested by Kapoor, Kelejian, and Prucha (2007) for a spatial autoregressive panel data model. We derive the asymptotic distributions of these estimators and suggest a Hausman test a la Mutl and Pfaffermayr (2011) based on the difference between these estimators. Monte Carlo experiments are performed to investigate the performance of these estimators as well as the corresponding Hausman test.

Key Words: *Panel Data; Fixed Effects; Random Effects; Spatial Model; Hausman Test.*

JEL classification: C12; C13; C23

1 Introduction

Anselin (1988) and Kapoor, Kelejian, and Prucha (2007) considered two different variants of a random effects panel data model with spatially correlated errors. The first paper estimated it with maximum likelihood methods and the second estimated it with a generalized moments (GM) method that is computationally simpler. Baltagi, Egger and Pfaffermayr (2013) generalized this random effects spatial model to encompass both cases and derived LM and LR tests to distinguish between these models. The generalized model allows the individual effects and the remainder errors to have different spatial autoregressive parameters. Using maximum likelihood methods, Baltagi, Egger and Pfaffermayr (2008) examined the consequences of model misspecification in this context using Monte Carlo simulations. These papers assume that the underlying

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spatial panel model is *random effects* (RE). Spatial panel data model with *fixed-effects* (FE) have been considered by Baltagi and Li (2006), Mutl and Pfaffermayr (2011), and Lee and Yu (2010) to mention a few. In fact, Baltagi and Li (2006) obtained the maximum likelihood estimator of a first order spatial autoregressive model with fixed effects and used it to forecast the consumption of liquor across a panel of US states, while Lee and Yu (2010) established the asymptotic properties of a quasi-maximum likelihood (QML) estimator for the spatial panel data model with fixed-effects. However, as pointed out by Kapoor, Kelejian, and Prucha (2007), hereafter denoted by (KKP), the QML estimation of Cliff and Ord (1973, 1981) type models entail substantial computational problems if the number of cross sectional units is large. To circumvent these computational problems for the mixed regressive spatial autoregressive (MRSAR) model, Mutl and Pfaffermayr (2011) suggested a fixed effects two-stage least squares (FE-2SLS) estimator based on a generalized moments (GM) estimator a la Kapoor, Kelejian, and Prucha (2007) extending the latter to include a spatial lag of the dependent variable. Mutl and Pfaffermayr (2011) also propose a Hausman test based on the difference between the fixed and random effects specification of this model. This paper applies the FE-S2SLS estimator of Mutl and Pfaffermayr (2011) to the generalized error component model considered by Baltagi, Egger and Pfaffermayr (2013) by adding a spatial lag term. We also suggest a random effects spatial two-stage least squares (RE-S2SLS) estimator using GM estimation of this generalized MRSAR error component model. Following Mutl and Pfaffermayr (2011), we apply a Hausman test based on the difference between the fixed and random effects specification of this generalized MRSAR model. Small sample properties of these estimators as well as the size of the proposed Hausman test are studied using Monte Carlo experiments. We show that a misspecified GM estimator can cause substantial loss in root mean squared error (RMSE) and wrong size for the corresponding Hausman test.

The rest of the paper is organized as follows. Section 2 introduces the RE-S2SLS and FE-S2SLS estimators for the MRSAR model. Generalized moments (GM) estimators a la Kapoor, Kelejian and Prucha (2007) are proposed for this model and their asymptotic distributions are derived. Following Mutl and Pfaffermayr (2011), a Hausman test is proposed based on the difference between the FE-S2SLS and feasible RE-S2SLS estimators of this spatial panel model. Simulation results are reported in section 3, while section 4 concludes the paper. All proofs are relegated to the Appendix.

2 The MRSAR Model

Let us consider the MRSAR model which is based on a generalized spatial error components model studied in Baltagi, Egger and Pfaffermayr (2013) but extends it by adding a spatial lag of the dependent variable as in Mutl and Pfaffermayr (2011). For each time period $t = 1, ..., T$, the data are generated according to the following model:

$$
\begin{aligned}
y_N(t) &= \lambda M_N y_N(t) + X_N(t)\beta + u_N(t) && (1)\\
u_N(t) &= u_{1N} + u_{2N}(t), && (2)\\
u_{1N} &= \rho_1 W_N u_{1N} + \mu_N, && (3)\\
u_{2N}(t) &= \rho_2 W_N u_{2N}(t) + \nu_N(t), && (4)
\end{aligned}
$$

where $y_N(t)$ denotes the $N \times 1$ vector of observations on the dependent variable in period t. $X_N(t)$ denotes the $N \times K$ matrix of observations on exogenous regressors in period t, which may contain the constant term. β is the corresponding $K \times 1$ vector of regression parameters, and $u_N(t)$ denotes the $N \times 1$ vector of disturbance terms. $u_N(t)$ follows an error component model which involves the sum of two disturbances. The $N \times 1$ vector u_{1N} captures the time-invariant unit-specific effects and therefore has no time subscript. The $N \times 1$ vector of the remainder disturbances $u_{2N}(t)$ varies with time. Both u_{1N} and $u_{2N}(t)$ are spatially correlated with the same spatial weights matrix W_N, but with different spatial autocorrelation parameters ρ_1 and ρ_2, respectively. Both M_N and W_N are $N \times N$ weighting matrices of known constants which do not vary over time. M_N and W_N may or may not be the same. This generalizes the model in Baltagi, Egger and Pfaffermayr (2013) by incorporating a spatial lag term $M_N y_N(t)$.

Stacking the cross-sections over time yields

$$
\begin{aligned}
y_N &= \lambda M_N y_N + X_N \beta + u_N && (5)\\
u_N &= Z_\mu u_{1N} + u_{2N}, && (6)\\
u_{1N} &= \rho_1 W_N u_{1N} + \mu_N, && (7)\\
u_{2N} &= \rho_2 (I_T \quad W_N) u_{2N} + \nu_N, && (8)
\end{aligned}
$$

where $y_N = [y'_N(1), \ldots, y'_N(T)]'$, $X_N = [X'_N(1), \ldots, X'_N(T)]'$, $u_N = [u'_N(1), \ldots, u'_N(T)]'$, $u_{2N} = [u'_{2N}(1), \ldots, u'_{2N}(T)]'$ and $v_N = [v'_N(1), \ldots, v'_N(T)]'$. The unit-specific errors u_{1N} are repeated in all time periods using the $NT \times N$ selector matrix $Z_\mu = \iota_T \quad I_N$, where ι_T is a vector of ones of dimension T and I_N is an identity matrix of dimension N, see Baltagi (2013). Let $\{\mu_{i,N}$ and $\{\nu_{it,N}\}$ denote the elements of the $N \times 1$ vector of individual effects μ_N and the $n \times 1$ vector of remainder disturbances ν_N. Following Kapoor, Kelejian and Prucha (2007), we employ the following assumptions:

Assumption 1 *Let T be a fixed positive integer. (a) For all $1 \leq t \leq T$ and $1 \leq i \leq N$, $N \geq 1$ the error components $\nu_{it,N}$ are identically distributed with zero mean and variance σ^2_ν, $0 < \sigma^2_\nu < b_v < \infty$, and finite* *fourth moments. In addition, for each $N \geq 1$ and $1 \leq t \leq T$, $1 \leq i \leq N$ the error components $\nu_{it,N}$ are independently distributed. (b) For all $1 \leq i \leq N$, $N \geq 1$, the unit specific error components $\mu_{i,N}$ are independently distributed with zero mean and variance σ_μ^2, $0 < \sigma_\mu^2 < b_\mu < \infty$, and finite fourth moments. In addition, for each $N \geq 1$, and $1 \leq i \leq N$ the unit specific error components $\mu_{i,N}$ are independently distributed. (c) The processes $\{\mu_{i,N}$ and $\{\nu_{it,N}\}$ are independent.*

Assumption 2 *(a) All diagonal elements of M_N and W_N are zero. (b) $|\lambda| < 1$, $|\rho_1| < 1$ and $|\rho_2| < 1$. (c) The matrices $I_N - \lambda M_N$, $I_N - \rho_1 W_N$ and $I_N - \rho_2 W_N$ are nonsingular. The row and column sums of M_N, W_N, $(I_N - \lambda M_N)^{-1}$, $(I_N - \rho_1 W_N)^{-1}$ and $(I_N - \rho_2 W_N)^{-1}$ are bounded uniformly in absolute values for all $|\lambda| < 1$, $|\rho_1| < 1$ and $|\rho_2| < 1$.*

As pointed out by Baltagi, Egger and Pfaffermayr (2013), this model nests various spatial panel models in the literature. For example, for the case where there is no spatial lag, i.e., $\lambda = 0$, and when $\rho_1 = \rho_2$, this reduces to the spatial random effects model considered in Kapoor, Kelejian and Prucha (2007). When $\lambda = 0$ and $\rho_1 = 0$, it reduces to the Anselin (1988) spatial random effects model also described in Baltagi, Song and Koh (2003) and Anselin, Le Gallo and Jayet (2008). When $\rho_1 = \rho_2 = 0$, it reduces to the familiar random effects (RE) panel data model with no spatial effects, see Baltagi (2013). In the presence of the spatial lag of the dependent variable, it nests the MRSAR model considered by Mutl and Pfaffermayr (2011) and Debarsy and Ertur (2010), to mention a few.

2.1 The RE-S2SLS Estimator

As shown in Kelejian and Prucha (1998) for the cross-section case, the spatial lag $M_N y_N$ is correlated with the vector of disturbances u_N. Therefore, the Ordinary Least Squares (OLS) estimator will be inconsistent. Define $Z_N = (M_N y_N, X_N)$ and $\delta = (\lambda, \beta')'$. With this notation, the MRSAR model can be rewritten as

$$y_N = Z_N \delta + u_N. \tag{9}$$

For the cross-section spatial autoregressive model, Kelejian and Prucha (1998) suggested instruments like $H_N = (X_N, M_N X_N, M_N^2 X_N)$. Define $\ _u = var(u_N)$. As shown in Baltagi, Egger and Pfaffermayr (2013),

$$\ _u^{-1} = \bar{J}_T \quad \left[T\sigma_\mu^2 (A'A)^{-1} + \sigma_\nu^2 (B'B)^{-1}\right]^{-1} + \sigma_\nu^{-2} [E_T \quad (B'B)], \tag{10}$$

where $A = I_N - \rho_1 W_N$ and $B = I_N - \rho_2 W_N$. $E_T = I_T - \bar{J}_T$, $\bar{J}_T = J_T/T$ and J_T is a matrix of ones of dimension T. Multiplying Equation (9) by $\ _u^{-1/2}$, we get

$$\ _u^{-1/2} y_N = \ _u^{-1/2} Z_N \delta + \ _u^{-1/2} u_N. \tag{11}$$

Define $P = \bar{J}_T \quad I_N$ and $Q = I_{NT} - P$, where I_{NT} is an identity matrix of dimension NT. Following Baltagi and Liu (2011), we apply the instruments ${}_u^{-1/2} H_N^*$ with $H_N^* = (QH_N, PH_N)$ to this transformed panel autoregressive spatial model. The random effects spatial two-stage least squares estimator (RE-S2SLS) of δ is given by

$$\widehat{\delta}_{RE-S2SLS} = \left[Z_N' \; {}_u^{-1} H_N^* \left(H_N^{*\prime} \; {}_u^{-1} H_N^* \right)^{-1} H_N^{*\prime} \; {}_u^{-1} Z_N \right]^{-1} Z_N' \; {}_u^{-1} H_N^* \left(H_N^{*\prime} \; {}_u^{-1} H_N^* \right)^{-1} H_N^{*\prime} \; {}_u^{-1} y_N \qquad (12)$$

with variance $var\left(\widehat{\delta}_{RE-S2SLS}\right) = \left[Z_N' \; {}_u^{-1} H_N^* \left(H_N^{*\prime} \; {}_u^{-1} H_N^* \right)^{-1} H_N^{*\prime} \; {}_u^{-1} Z_N \right]^{-1}$.

Kapoor, Kelejian and Prucha (2007) considered the special case where $\lambda = 0$ and $\rho_1 = \rho_2$ and proposed GM estimators of ρ_2 and σ_ν^2 based on the following three moment conditions:

$$\begin{aligned} \frac{1}{N(T-1)} E\left[\nu_N' Q \nu_N\right] &= \sigma_\nu^2, & (13)\\ \frac{1}{N(T-1)} E\left[\bar{\nu}_N' Q \bar{\nu}_N\right] &= \frac{\sigma_\nu^2 tr\left(W_N' W_N\right)}{N}, & (14)\\ \frac{1}{N(T-1)} E\left[\bar{\nu}_N' Q \nu_N\right] &= 0, & (15) \end{aligned}$$

where $\bar{\nu}_N = (I_T \quad W_N)\nu_N$. Define $\bar{u}_{2N} = (I_T \quad W_N) u_{2N}$ and $\bar{\bar{u}}_{2N} = (I_T \quad W_N) \bar{u}_{2N}$. Substitute $\nu_N = u_{2N} - \rho_2 \bar{u}_{2N}$ and $\bar{\nu}_N = \bar{u}_{2N} - \rho_2 \bar{\bar{u}}_{2N}$ into the system of equations given above, we get

$$\begin{aligned} \frac{1}{N(T-1)} E\left[u_{2N}' Q u_{2N}\right] - \frac{2}{N(T-1)} \rho_2 E\left[\bar{u}_{2N}' Q u_{2N}\right] + \frac{1}{N(T-1)} \rho_2^2 E\left[\bar{u}_{2N}' Q \bar{u}_{2N}\right] &= \sigma_\nu^2, & (16)\\ \frac{1}{N(T-1)} E\left[\bar{u}_{2N}' Q \bar{u}_{2N}\right] - \frac{2}{N(T-1)} \rho_2 E\left[\bar{\bar{u}}_{2N}' Q \bar{u}_{2N}\right] + \frac{1}{N(T-1)} \rho_2^2 E\left[\bar{\bar{u}}_{2N}' Q \bar{\bar{u}}_{2N}\right] &= \frac{\sigma_\nu^2 tr\left(W_N' W_N\right)}{N}, & (17)\\ \frac{1}{N(T-1)} E\left[\bar{u}_{2N}' Q u_{2N}\right] - \frac{1}{N(T-1)} \rho_2 E\left[\bar{\bar{u}}_{2N}' Q u_{2N} + \bar{u}_{2N}' Q \bar{u}_{2N}\right] + \frac{1}{N(T-1)} \rho_2^2 E\left[\bar{\bar{u}}_{2N}' Q \bar{u}_{2N}\right] &= 0. & (18) \end{aligned}$$

Note that $Q(\iota_T \quad u_{1N}) = 0$. Therefore, for the general model in Equations (5)-(8), we have $u_N' Q u_N = u_{2N}' Q u_{2N}, \bar{u}_N' Q \bar{u}_N = \bar{u}_{2N}' Q \bar{u}_{2N}$ and $\bar{u}_N' Q u_N = \bar{u}_{2N}' Q u_{2N}$. This system can be expressed as

$$\Gamma_N^0 \left[\rho_2, \rho_2^2, \sigma_\nu^2\right]' - \gamma_N^0 = 0, \qquad (19)$$

where $\Gamma_N^0 = \begin{pmatrix} \frac{2}{N(T-1)} E\left[\bar{u}_N' Q u_N\right] & -\frac{1}{N(T-1)} E\left[\bar{u}_N' Q \bar{u}_N\right] & 1 \\ \frac{2}{N(T-1)} E\left[\bar{\bar{u}}_N' Q \bar{u}_N\right] & -\frac{1}{N(T-1)} E\left[\bar{\bar{u}}_N' Q \bar{\bar{u}}_N\right] & \frac{1}{N} tr\left(W_N' W_N\right) \\ \frac{1}{N(T-1)} E\left[\bar{\bar{u}}_N' Q u_N + \bar{u}_N' Q \bar{u}_N\right] & -\frac{1}{N(T-1)} E\left[\bar{\bar{u}}_N' Q \bar{u}_N\right] & 0 \end{pmatrix}$

and $\gamma_N^0 = \begin{pmatrix} \frac{1}{N(T-1)} E\left[u_N' Q u_N\right] \\ \frac{1}{N(T-1)} E\left[\bar{u}_N' Q \bar{u}_N\right] \\ \frac{1}{N(T-1)} E\left[\bar{u}_N' Q u_N\right] \end{pmatrix}$. Let $\widehat{u}_N$ denote the 2SLS residuals from (5) ignoring the random effects, i.e., $\widehat{u}_N = y_N - Z_N \widehat{\delta}_{2SLS}$, where $\widehat{\delta}_{2SLS} = \left[Z'_N H_N \left(H'_N H_N\right)^{-1} H'_N Z_N\right]^{-1} Z'_N H_N \left(H'_N H_N\right)^{-1} H'_N y_N$. Let G_N and g^0_N be the sample analogues of Γ^0_N and γ^0_N substituting $\widehat{u}_N$ for u_N.We can get a GM estimator by solving

$$\left(\tilde{\rho}_2, \tilde{\sigma}^2_\nu\right) = \arg\min \left\{ \xi^0_N \left(\underline{\rho_2}, \underline{\sigma^2_\nu}\right)' \xi^0_N \left(\underline{\rho_2}, \underline{\sigma^2_\nu}\right), \underline{\rho_2} \in \left[-a_0, a_0\right], \underline{\sigma^2_\nu} \in \left[0, b_0\right] \right\}, \tag{20}$$

where $\xi^0_N \left(\rho_2, \sigma^2_\nu\right) = G^0_N \left[\rho_2, \rho^2_2, \sigma^2_\nu\right]' - g^1_N$, $a_0 \geq 1$ and $b_0 \geq b_v$.

Assumption 3 *The smallest eigenvalues of* $\Gamma'_N \Gamma_N$ *are bounded away from zero.*

Kapoor, Kelejian and Prucha (2007) showed that, for their model with no spatial lag, and where the disturbances are replaced by OLS residuals, $\tilde{\rho}$ and $\tilde{\sigma}^2_\nu$ are consistent. It is worth pointing out that the condition $Q\left(\iota_T \quad u_1\right) = 0$ holds for the general model given in equation (5), and not only for the special case in Kapoor, Kelejian and Prucha (2007). Therefore, for all values of ρ_1 and σ^2_μ in the parameter space, the GM estimator of ρ_2 and σ^2_ν suggested by Kapoor, Kelejian and Prucha (2007) and applied to our model based on 2SLS residuals, will also be consistent. By Theorem 1 of Kapoor, Kelejian and Prucha (2007), we have $\left(\tilde{\rho}_2, \tilde{\sigma}^2_\nu\right) \xrightarrow{p} \left(\rho_2, \sigma^2_\nu\right)$ under assumptions 1-3 as $N \to \infty$. Similarly, we introduce the following GM estimators of ρ_1 and σ^2_μ:

Define $\bar{\mu} = W_N \mu$. We have the following three moment conditions:

$$\begin{aligned} \frac{1}{N} E\left[\mu'_N \mu_N\right] &= \sigma^2_\mu, & (21)\\ \frac{1}{N} E\left[\bar{\mu}'_N \bar{\mu}_N\right] &= \frac{1}{N} \sigma^2_\mu tr\left(W'_N W_N\right), & (22)\\ \frac{1}{N} E\left[\bar{\mu}'_N \mu_N\right] &= 0. & (23) \end{aligned}$$

Similarly define $\bar{u}_{1N} = W_N u_{1N}$ and $\bar{\bar{u}}_{1N} = W_N \bar{u}_{1N}$. Substitute $\mu_N = u_{1N} - \rho_1 \bar{u}_{1N}$ and $\bar{\mu}_N = \bar{u}_{1N} - \rho_1 \bar{\bar{u}}_{1N}$ into the system of equations given above, we get

$$\begin{aligned} \frac{1}{N} E\left[u'_{1N} u_{1N}\right] - \frac{2}{N} \rho_1 E\left[\bar{u}'_{1N} u_{1N}\right] + \frac{1}{N} \rho^2_1 E\left[\bar{u}'_{1N} \bar{u}_{1N}\right] &= \sigma^2_\mu, & (24)\\ \frac{1}{N} E\left[\bar{u}'_{1N} \bar{u}_{1N}\right] - \frac{2}{N} \rho_1 E\left[\bar{\bar{u}}'_{1N} \bar{u}_{1N}\right] + \frac{1}{N} \rho^2_1 E\left[\bar{\bar{u}}'_{1N} \bar{\bar{u}}_{1N}\right] &= \frac{\sigma^2_\mu}{N} tr\left(W'_N W_N\right), & (25)\\ \frac{1}{N} E\left[\bar{u}'_{1N} u_{1N}\right] - \frac{1}{N} \rho_1 E\left[\bar{\bar{u}}'_{1N} u_{1N} + \bar{u}'_{1N} \bar{u}_{1N}\right] + \frac{1}{N} \rho^2_1 E\left[\bar{\bar{u}}'_{1N} \bar{u}_{1N}\right] &= 0. & (26) \end{aligned}$$

This system can be expressed as

$$\Gamma^1_N \left[\rho_1, \rho^2_1, \sigma^2_\mu\right]' - \gamma^1_N = 0, \tag{27}$$

where $\Gamma_N^1 = \begin{pmatrix} \frac{2}{N}E\left[\bar{u}_{1N}'u_{1N}\right] & -\frac{1}{N}E\left[\bar{u}_{1N}'\bar{u}_{1N}\right] & 1 \\ \frac{2}{N}E\left[\bar{\bar{u}}_{1N}'\bar{u}_{1N}\right] & -\frac{1}{N}E\left[\bar{\bar{u}}_{1N}'\bar{\bar{u}}_{1N}\right] & \frac{1}{N}tr\left(W_N'W_N\right) \\ \frac{1}{N}E\left[\bar{\bar{u}}_{1N}'u_{1N}+\bar{u}_{1N}'\bar{u}_{1N}\right] & -\frac{1}{N}E\left[\bar{\bar{u}}_{1N}'\bar{u}_{1N}\right] & 0 \end{pmatrix}$
and $\gamma_N^1 = \begin{pmatrix} \frac{1}{N}E\left[u_{1N}'u_{1N}\right] \\ \frac{1}{N}E\left[\bar{u}_{1N}'\bar{u}_{1N}\right] \\ \frac{1}{N}E\left[\bar{u}_{1N}'u_{1N}\right] \end{pmatrix}$. Define $S = P - \frac{1}{T-1}Q = S_T \quad I_N$, where $S_T = \bar{J}_T - \frac{1}{T-1}E_T$. Also

$$\begin{aligned} \varphi_{kl,N} &= \frac{1}{NT}u_N'\left(I_T \quad W_N^k\right)'S\left(I_T \quad W_N^l\right)u_N \\ &= \frac{1}{NT}u_N'\left(I_T \quad W_N^k\right)'\left(S_T \quad I_N\right)\left(I_T \quad W_N^l\right)u_N \\ &= \frac{1}{NT}u_N'\left(S_T \quad W_N^{k\prime}W_N^l\right)u_N \end{aligned}$$

for $k,l = 0,1,2$. Hence

$$\begin{aligned} E\left(\varphi_{kl,N}\right) &= \frac{1}{NT}E\left[u_N'\left(S_T \quad W_N^{k\prime}W_N^l\right)u_N\right] \\ &= \frac{1}{NT}E\left[\left(Z_\mu u_{1N}+u_{2N}\right)'\left(S_T \quad W_N^{k\prime}W_N^l\right)\left(Z_\mu u_{1N}+u_{2N}\right)\right] \\ &= \frac{1}{NT}E\left[\left(Z_\mu u_{1N}\right)'\left(S_T \quad W_N^{k\prime}W_N^l\right)\left(Z_\mu u_{1N}\right)\right]+\frac{1}{NT}E\left[u_{2N}'\left(S_T \quad W_N^{k\prime}W_N^l\right)u_{2N}\right] \\ &\quad +\frac{2}{NT}E\left[u_{2N}'\left(S_T \quad W_N^{k\prime}W_N^l\right)Z_\mu u_{1N}\right] \\ &\equiv I + II + III. \end{aligned}$$

Note that

$$I = \frac{1}{NT}E\left[\left(Z_\mu u_{1N}\right)'\left(S_T \quad W_N^{k\prime}W_N^l\right)\left(Z_\mu u_{1N}\right)\right] = \frac{1}{NT}E\left[u_{1N}'\left(\iota_T'S_T\iota_T \quad W_N^{k\prime}W_N^l\right)u_{1N}\right] = \frac{1}{N}E\left(u_{1N}'W_N^{k\prime}W_N^l u_{1N}\right)$$

since $\iota_T'S_T\iota_T = \iota_T'\left(\bar{J}_T - \frac{1}{T-1}E_T\right)\iota_T = \iota_T'\bar{J}_T\iota_T - \frac{1}{T-1}\iota_T'E_T\iota_T = T$ using $\bar{J}_T\iota_T = \iota_T$ and $E_T\iota_T = 0$;

$$\begin{aligned} II &= \frac{1}{NT}E\left[u_{2N}'\left(S_T \quad W_N^{k\prime}W_N^l\right)u_{2N}\right] = \frac{1}{NT}E\left[\nu_N'\left(I_T \quad B^{-1}\right)'\left(S_T \quad W_N^{k\prime}W_N^l\right)\left(I_T \quad B^{-1}\right)\nu_N\right] \\ &= \frac{1}{NT}E\left[\nu_N'\left(S_T \quad B^{-1\prime}W_N^{k\prime}W_N^l B^{-1}\right)\nu_N\right] \\ &= \frac{1}{NT}\sigma_v^2 tr\left(S_T\right)tr\left(B^{-1\prime}W_N^{k\prime}W_N^l B^{-1}\right) \\ &= 0 \end{aligned}$$

since $tr\left(S_T\right) = tr\left(\bar{J}_T - \frac{1}{T-1}E_T\right) = 0$; and

$$III = \frac{2}{NT}E\left[u_{2N}'\left(S_T \quad W_N^{k\prime}W_N^l\right)Z_\mu u_{1N}\right] = 0$$

since u_{1N} and u_{2N} are independent by Assumption 1. Hence, one gets $E\left(\varphi_{kl,N}\right) = \frac{1}{N}E\left(u_{1N}' W_N^{k\prime} W_N^l u_{1N}\right)$; $\Gamma_N^1 = \begin{pmatrix} \frac{2}{NT}E\left[\bar{u}_N' S u_N\right] & -\frac{1}{NT}E\left[\bar{u}_N' S \bar{u}_N\right] & 1 \\ \frac{2}{NT}E\left[\bar{\bar{u}}_N' S \bar{u}_N\right] & -\frac{1}{NT}E\left[\bar{\bar{u}}_N' S \bar{\bar{u}}_N\right] & \frac{1}{N}tr\left(W_N' W_N\right) \\ \frac{1}{NT}E\left[\bar{\bar{u}}_N' S u_N + \bar{u}_N' S \bar{u}_N\right] & -\frac{1}{NT}E\left[\bar{\bar{u}}_N' S \bar{u}_N\right] & 0 \end{pmatrix}$; and $\gamma_N^1 = \begin{pmatrix} \frac{1}{NT}E\left[u_N' S u_N\right] \\ \frac{1}{NT}E\left[\bar{u}_N' S \bar{u}_N\right] \\ \frac{1}{NT}E\left[\bar{u}_N' S u_N\right] \end{pmatrix}$. The sample analogues to Γ_N^1 and γ_N^1 are $G_N^1 = \begin{pmatrix} \frac{2}{NT}\tilde{\bar{u}}_N' S \tilde{u}_N & -\frac{1}{NT}\tilde{\bar{u}}_N' S \tilde{\bar{u}}_N & 1 \\ \frac{2}{NT}\tilde{\bar{\bar{u}}}_N' S \tilde{\bar{u}}_N & -\frac{1}{NT}\tilde{\bar{\bar{u}}}_N' S \tilde{\bar{\bar{u}}}_N & \frac{1}{N}tr\left(W_N' W_N\right) \\ \frac{1}{NT}\left(\tilde{\bar{\bar{u}}}_N' S \tilde{u}_N + \tilde{\bar{u}}_N' S \tilde{\bar{u}}_N\right) & -\frac{1}{NT}\tilde{\bar{\bar{u}}}_N' S \tilde{\bar{u}}_N & 0 \end{pmatrix}$ and $g_N^1 = \begin{pmatrix} \frac{1}{NT}\tilde{u}_N' S \tilde{u}_N \\ \frac{1}{NT}\tilde{\bar{u}}_N' S \tilde{\bar{u}}_N \\ \frac{1}{NT}\tilde{\bar{u}}_N' S \tilde{u}_N \end{pmatrix}$, respectively. Hence, a GM estimator can be obtained from

$$\left(\tilde{\rho}_1, \tilde{\sigma}_\mu^2\right) = \arg\min\left\{\xi_N^1\left(\underline{\rho_1}, \underline{\sigma_\mu^2}\right)' \xi_N^1\left(\underline{\rho_1}, \underline{\sigma_\mu^2}\right), \underline{\rho_1} \in \left[-a_1, a_1\right], \underline{\sigma_\mu^2} \in \left[0, b_1\right]\right\}, \tag{28}$$

where $\xi_N^1\left(\rho_1, \sigma_\mu^2\right) = G_N^1\left[\rho_1, \rho_1^2, \sigma_\mu^2\right]' - g_N^1$, $a_1 \geq 1$ and $b_1 \geq b_\mu$.

Theorem 1 *Under Assumptions 1 -3, we have* $\left(\tilde{\rho}_1, \tilde{\sigma}_\mu^2\right) \xrightarrow{p} \left(\rho_1, \sigma_\mu^2\right)$ *as* $N \to \infty$.

With the GM estimators of $\tilde{\rho}_1$, $\tilde{\rho}_2$, $\tilde{\sigma}_\mu^2$ and $\tilde{\sigma}_\nu^2$, the corresponding random effects feasible spatial two-stage least squares estimator $\widehat{\delta}_{RE-FS2SLS}$ is given by

$$\widehat{\delta}_{RE-FS2SLS} = \left[Z_N' \tilde{}_u^{-1} H_N^*\left(H_N^{*\prime} \tilde{}_u^{-1} H_N^*\right)^{-1} H_N^{*\prime} \tilde{}_u^{-1} Z_N\right]^{-1} Z_N' \tilde{}_u^{-1} H_N^*\left(H_N^{*\prime} \tilde{}_u^{-1} H_N^*\right)^{-1} H_N^{*\prime} \tilde{}_u^{-1} y_N, \tag{29}$$

where

$$\tilde{}_u^{-1} = \bar{J}_T \quad \left[T\tilde{\sigma}_\mu^2\left(\tilde{A}'\tilde{A}\right)^{-1} + \tilde{\sigma}_\nu^2\left(\tilde{B}'\tilde{B}\right)^{-1}\right]^{-1} + \tilde{\sigma}_\nu^{-2}\left[E_T \quad \left(\tilde{B}'\tilde{B}\right)\right]. \tag{30}$$

Assumption 4 X_N *is non-stochastic. The elements of* X_N *are bounded uniformly in absolute value. Furthermore, the limit* $\Sigma_0 = \lim_{N\to\infty} \frac{1}{NT} H_N' \left\{\bar{J}_T \quad \left[T\sigma_\mu^2\left(A'A\right)^{-1} + \sigma_\nu^2\left(B'B\right)^{-1}\right]^{-1}\right\} H_N$, $\Sigma_1 = \lim_{N\to\infty} \frac{1}{NT} H_N'\left[E_T \quad (B'B)\right] H_N$, $\Gamma_0 = \lim_{N\to\infty} \frac{1}{NT} H_N' \left\{\bar{J}_T \quad \left[T\sigma_\mu^2\left(A'A\right)^{-1} + \sigma_\nu^2\left(B'B\right)^{-1}\right]^{-1}\right\} Z_N$ *and* $\Gamma_1 = \lim_{N\to\infty} \frac{1}{NT} H_N'\left[E_T \quad (B'B)\right] Z_N$ *are finite and nonsingular.*

The theorem below establishes consistency and asymptotic normality of the random effects feasible spatial two-stage least squares estimator. The proof of the theorem is given in the appendix.

Theorem 2 *Under Assumptions 1-4, we have* $\sqrt{NT}\left(\widehat{\delta}_{RE-FS2SLS} - \delta\right) \xrightarrow{d} N\left(0, \left(\Gamma_0'\Sigma_0^{-1}\Gamma_0 + \sigma_\nu^{-2}\Gamma_1'\Sigma_1^{-1}\Gamma_1\right)^{-1}\right)$ *as* $N \to \infty$.

Next, we turn to the special cases of this general model. Under the KKP model, but now with a spatial lag, we have $\rho_1 = \rho_2$, and as a result $A = B$, and Equation (10) reduces to

$$ {}_{u}^{-1} = \left(\sigma_1^2 \bar{J}_T + \sigma_\nu^{-2} E_T\right) \quad \left(B'B\right), \tag{31}$$

where $\sigma_1^2 = T\sigma_\mu^2 + \sigma_\nu^2$. Kapoor, Kelejian and Prucha (2007) suggest estimating σ_1^2 by

$$\tilde{\sigma}_1^2 = \frac{1}{N}\left(\tilde{u}_N - \tilde{\rho}_2 W_N \tilde{u}_N\right)' Q \left(\tilde{u}_N - \tilde{\rho}_2 W_N \tilde{u}_N\right).$$

In this case, the asymptotic distribution of the corresponding $\widehat{\delta}_{RE-FS2SLS}$ is the same as in Theorem 2 with Σ_0 and Γ_0 reducing to $\Sigma_0 = \lim_{N\to\infty} \frac{1}{NT} H_N' \left[\sigma_1^2 \bar{J}_T \quad (B'B)\right] H_N$, and $\Gamma_0 = \lim_{N\to\infty} \frac{1}{NT} H_N' \left[\sigma_1^2 \bar{J}_T \quad (B'B)\right] Z_N$, respectively.

Under the Anselin model, but now with a spatial lag, we have $\rho_1 = 0$ and hence $A = I_N$. Equation (10) reduces to

$$ {}_{u}^{-1} = \bar{J}_T \quad \left[T\sigma_\mu^2 I_N + \sigma_\nu^2 \left(B'B\right)^{-1}\right]^{-1} + \sigma_\nu^{-2}\left[E_T \quad (B'B)\right]. \tag{32}$$

We can estimate σ_μ^2 from the first Equation in (21) as

$$\tilde{\sigma}_\mu^2 = \frac{1}{NT}\tilde{u}_N' S \tilde{u}_N = \frac{1}{NT}\tilde{u}_N' P \tilde{u}_N - \frac{1}{NT\left(T-1\right)}\tilde{u}_N' Q \tilde{u}_N.$$

Note that it is the same estimator of σ_μ^2 for the random effect model with $\rho_1 = \rho_2 = 0$. From the proof of Theorem 1, we know that under Assumptions 1-3, $\tilde{\sigma}_\mu^2 \xrightarrow{p} \sigma_\mu^2$ as $N \to \infty$. With these GM estimators of $\tilde{\rho}_2$, $\tilde{\sigma}_\mu^2$ and $\tilde{\sigma}_\nu^2$, we obtain the random effects feasible spatial two-stage least squares estimator $\widehat{\delta}_{RE-FS2SLS}$. Similar to Theorem 2, we can show that $\widehat{\delta}_{RE-FS2SLS}$ has the same asymptotic distribution as in Theorem 2, with Σ_0 and Γ_0 reducing to $\Sigma_0 = \lim_{N\to\infty} \frac{1}{NT} H_N' \left\{\bar{J}_T \quad \left[T\sigma_\mu^2 I_N + \sigma_\nu^2 \left(B'B\right)^{-1}\right]^{-1}\right\} H_N$, and $\Gamma_0 = \lim_{N\to\infty} \frac{1}{NT} H_N' \left\{\bar{J}_T \quad \left[T\sigma_\mu^2 I_N + \sigma_\nu^2 \left(B'B\right)^{-1}\right]^{-1}\right\} Z_N$, respectively.

2.2 The FE-S2SLS Estimator

Let $\{u_{1i,N}\}$ and $\{X_{it,N}\}$ denote the elements of the $N \times 1$ vector of u_{1N} and the $NT \times K$ vector of X_N. A critical assumption for the consistency of the RE estimator is that $E\left(u_{1i,N}|X_{it,N}\right) = 0$. If the unobserved individual invariant effects are correlated with X_{it}, then $E\left(u_{1i}|X_{it}\right) \neq 0$ and RE is inconsistent. As pointed out in Lee and Yu (2010), with the fixed effects specification, the panel models in Baltagi, Egger and Pfaffermayr (2013), Kapoor, Kelejian and Prucha (2007) and Anselin (1988) have the same representation. More specifically, premultiplying equation (5) by the fixed effects (or within) transformation $Q = E_T \quad I_N$, one obtains

$$Qy_N = QZ_N\delta + Qu_{2N}, \tag{33}$$

since $Q\left(\iota_T \quad u_{1N}\right) = 0$, see Baltagi (2013). The fixed effects two-stage least squares estimator (FE-2SLS) estimator

$$\widehat{\delta}_{FE-2SLS} = \left[Z_N' Q H_N \left(H_N' Q H_N\right)^{-1} H_N' Q Z_N\right]^{-1} Z_N' Q H_N \left(H_N' Q H_N\right)^{-1} H_N' Q y_N \tag{34}$$

wipes out the individual effects and does not require the estimation of ρ_1 or σ_μ^2. However, this estimator ignores the spatial autocorrelation in the error. To gain efficiency, one can apply the Cochrane-Orcutt type spatial transformation on the within transformed model in Equation (33) to obtain the FE-S2SLS estimator as suggested in Mutl and Pfaffermayr (2011). More specifically, we premultiply equation (33) by $I_T \quad B$, to get

$$\left(E_T \quad B\right) y_N = \left(E_T \quad B\right) Z_N \delta + Q\nu_N. \tag{35}$$

This uses the fact that $\left(I_T \quad B\right) Q = \left(E_T \quad B\right) = Q\left(I_T \quad B\right)$ and $\left(I_T \quad B\right) Q u_{2N} = Q\left(I_T \quad B\right) u_{2N} = Q\nu_N$. Applying the instruments $\left(E_T \quad B\right) H_N$, we get the fixed effects spatial two-stage least squares estimator (FE-S2SLS) of δ given by

$$\begin{aligned}\widehat{\delta}_{FE-S2SLS} \quad = \quad & \left\{Z_N' \left[E_T \quad (B'B)\right] H_N \left(H_N' \left[E_T \quad (B'B)\right] H_N\right)^{-1} H_N' \left[E_T \quad (B'B)\right] Z_N\right\}^{-1} \\ & Z_N' \left[E_T \quad (B'B)\right] H_N \left(H_N' \left[E_T \quad (B'B)\right] H_N\right)^{-1} H_N' \left[E_T \quad (B'B)\right] y_N\end{aligned} \tag{36}$$

with $var\left(\widehat{\delta}_{FE-S2SLS}\right) = \sigma_\nu^2 \left\{Z_N' \left[E_T \quad (B'B)\right] H_N \left(H_N' \left[E_T \quad (B'B)\right] H_N\right)^{-1} H_N' \left[E_T \quad (B'B)\right] Z_N\right\}^{-1}$. If $\rho_2 = 0$, then $B = I_N$ and the FE-S2SLS estimator in Equation (36) reduces to the FE-2SLS estimator in Equation (34). Using the GM estimators of $\tilde{\rho}_2$ and $\tilde{\sigma}_\nu^2$ from Equation (20), the corresponding fixed-effects feasible spatial two-stage least squares estimator (FE-FS2SLS) $\widehat{\delta}_{FE-FS2SLS}$ is obtained by replacing B by its estimator $\tilde{B} = I_N - \tilde{\rho}_2 W_N$, i.e.,

$$\begin{aligned}\widehat{\delta}_{FE-FS2SLS} \quad = \quad & \left\{Z_N' \left[E_T \quad \left(\tilde{B}'\tilde{B}\right)\right] H_N \left(H_N' \left[E_T \quad \left(\tilde{B}'\tilde{B}\right)\right] H_N\right)^{-1} H_N' \left[E_T \quad \left(\tilde{B}'\tilde{B}\right)\right] Z_N\right\}^{-1} \\ & Z_N' \left[E_T \quad \left(\tilde{B}'\tilde{B}\right)\right] H_N \left(H_N' \left[E_T \quad \left(\tilde{B}'\tilde{B}\right)\right] H_N\right)^{-1} H_N' \left[E_T \quad \left(\tilde{B}'\tilde{B}\right)\right] y_N.\end{aligned} \tag{37}$$

This estimator can be computed conveniently as the fixed effects two-stage least squares estimator after pre-multiplying the model in equation (5) by $I_T \quad \tilde{B}$. The theorem below establishes consistency and asymptotic normality of the FE-FS2SLS estimators. The proof of the theorem is given in the appendix.

Theorem 3 *Under Assumptions 1-4, we have* $\sqrt{NT}\left(\widehat{\delta}_{FE-FS2SLS} - \delta\right) \xrightarrow{d} N\left(0, \sigma_\nu^2 \left(\Gamma_1' \Sigma_1^{-1} \Gamma_1\right)^{-1}\right)$ *as* $N \to \infty$.

One of the advantages of the FE-FS2SLS estimator of δ is that it does not depend on σ_μ^2 and ρ_1. Hence, the FE-FS2SLS estimator is robust to different values of σ_μ^2 and ρ_1. Another advantage of the FE-FS2SLS estimator is that it is still consistent when $E\left(u_{1i}|x_{it}\right) \neq 0$, while the RE-FS2SLS estimator is not.

2.3 Hausman's Test

One can perform Hausman's (1978) specification test for this generalized MRSAR panel data model. The null hypothesis is $H_0 : E\left(u_{1i,N}|X_{it,N}\right) = 0$. Under H_0, $\widehat{\delta}_{RE-S2SLS}$ given in (12) is the efficient estimator, while under the alternative $H_1 : E\left(u_{1i,N}|X_{it,N}\right) \neq 0$, $\widehat{\delta}_{RE-S2SLS}$ is inconsistent. In contrast, $\widehat{\delta}_{FE-S2SLS}$ is consistent under the null and alternative. Let $q = \widehat{\delta}_{FE-S2SLS} - \widehat{\delta}_{RE-S2SLS}$ and note that

$$\begin{aligned}
& cov\left(\widehat{\delta}_{FE-S2SLS}, \widehat{\delta}_{RE-S2SLS}\right) \\
= \; & E\left[\left(\widehat{\delta}_{FE-S2SLS} - \delta\right)\left(\widehat{\delta}_{RE-S2SLS} - \delta\right)'\right] \\
= \; & \left\{Z_N'\left[E_T \quad (B'B)\right] H_N \left(H_N'\left[E_T \quad (B'B)\right] H_N\right)^{-1} H_N'\left[E_T \quad (B'B)\right] Z_N\right\}^{-1} \\
& Z_N'\left[E_T \quad (B'B)\right] H_N \left(H_N'\left[E_T \quad (B'B)\right] H_N\right)^{-1} H_N'\left[E_T \quad (B'B)\right] E\left(u_N u_N'\right) \\
& {}_u^{-1} H_N^* \left(H_N^{*\prime} \; {}_u^{-1} H_N^*\right)^{-1} H_N^{*\prime} \; {}_u^{-1} Z_N \left[Z_N' \; {}_u^{-1} H_N^* \left(H_N^{*\prime} \; {}_u^{-1} H_N^*\right)^{-1} H_N^{*\prime} \; {}_u^{-1} Z_N\right]^{-1} \\
= \; & \left[Z_N' \; {}_u^{-1} H_N^* \left(H_N^{*\prime} \; {}_u^{-1} H_N^*\right)^{-1} H_N^{*\prime} \; {}_u^{-1} Z_N\right]^{-1} \\
= \; & var\left(\widehat{\delta}_{RE-S2SLS}\right).
\end{aligned}$$

Hence

$$\begin{aligned}
var\left(q\right) &= var\left(\widehat{\delta}_{FE-S2SLS} - \widehat{\delta}_{RE-S2SLS}\right) \\
&= var\left(\widehat{\delta}_{FE-S2SLS}\right) + var\left(\widehat{\delta}_{RE-S2SLS}\right) - 2cov\left(\widehat{\delta}_{FE-S2SLS}, \widehat{\delta}_{RE-S2SLS}\right) \\
&= var\left(\widehat{\delta}_{FE-S2SLS}\right) + var\left(\widehat{\delta}_{RE-S2SLS}\right) - 2var\left(\widehat{\delta}_{RE-S2SLS}\right) \\
&= var\left(\widehat{\delta}_{FE-S2SLS}\right) - var\left(\widehat{\delta}_{RE-S2SLS}\right).
\end{aligned}$$

Under H_0, the Hausman test $m = q'\left[var\left(q\right)\right]^{-1} q$ has a limiting χ^2 distribution with degrees of freedom equal to the rank of $var(q)$. In practice, estimates of both ρ_1 and ρ_2 are needed to calculate $var\left(\widehat{\delta}_{RE-S2SLS}\right)$. Under the Kapoor, Kelejian and Prucha (2007) random effects spatial model, $\rho_1 = \rho_2$ and under the Anselin (1988) random effects spatial model, $\rho_1 = 0$. One could perform a Hausman test based on these RE-S2SLS versus FE-S2SLS estimators proposed in this paper. In fact, Mutl and Pfaffermayr (2011) suggested a Hausman test assuming $\rho_1 = \rho_2$ for the MRSAR panel data model. Its sensitivity under model misspecification (say $\rho_1 \neq \rho_2$) is checked in the following section via Monte Carlo experiments.

3 Monte Carlo Simulation

This section performs Monte Carlo experiments to study the finite sample performance of the proposed estimators and the corresponding Spatial Hausman test. Following Baltagi, Egger and Pfaffermayr (2013) but adding a spatial lag term, we consider the following MRSAR panel model

$$y_N = \lambda M_N y_N + \alpha + \beta x_N + u_N, \tag{38}$$

where $\lambda = 0.5$, $\alpha = 5$ and $\beta = 0.5$. x_{it} is generated by $x_{it} = \zeta_i + z_{it}$ with $\zeta_i \overset{iid}{\sim} U[-7.5, 7.5]$ and $z_{it} \overset{iid}{\sim} U[-5, 5]$. The individual specific effects are drawn from a normal distribution so that $\mu_i \overset{iid}{\sim} N(0, 20\theta)$. For the remainder error, we let $\sigma_\mu^2 = 10$ and $\sigma_\nu^2 = 10$. This implies that the proportion of the total variance due to the heterogeneity of the individual-specific effects is $\frac{\sigma_\mu^2}{\sigma_\mu^2 + \sigma_\nu^2} = 0.5$. The spatial weight matrix is created following Kapoor, Kelejian and Prucha (2007). The weighting matrix is referred as "3 ahead and 3 behind". This matrix is defined in a circular world so that the non-zero elements in rows 1 and N are, respectively, in positions $(2, 3, 4, N-2, N-1, N)$ and $(1, 2, 3, N-3, N-2, N-1)$. This matrix is row normalized so that all of its non-zero elements are equal to $1/6$. In the Tables below, we reference this weighting matrix by $J = 6$, where J is the number of nonzero elements in a given row. ρ_1 and ρ_2 vary over the set $\{-0.8, -0.5, -0.2, 0, 0.2, 0.5, 0.8\}$. We consider a panel with $N = 100$ regions and $T = 5$ time periods, and we perform 10,000 replications. For each replication, we estimate the model using (i) FE-2SLS allowing for spatial lag but no spatial error correlation; (ii) RE-2SLS allowing for spatial lag but no spatial error correlation; (iii) FE-S2SLS allowing for both spatial lag and spatial error correlation; (iv) KKP RE-S2SLS allowing for both spatial lag and error correlation; (v) Anselin RE-S2SLS allowing for both spatial lag and error correlation; (vi) General RE-S2SLS allowing for both spatial lag and error correlation; and (vii) True RE-S2SLS allowing for both spatial lag and spatial error correlation.

Table 1 reports the relative root mean squared error (RMSE) of each estimator of β with respect to the true RE-S2SLS. Several conclusions emerge from this table. Not surprisingly, true RE-S2SLS is the most efficient estimator in terms of root mean squared error. When the true model is spatial RE, KKP or Anselin with a spatial lag term, the 'correct' feasible RE-S2SLS estimator performs best and is the closest in RMSE to the true RE-S2SLS. FE-S2SLS estimator performs much better than standard FE-2SLS which ignores the spatial correlation. For example, for $\rho_1 = \rho_2 = -0.8$, the relative RMSE of FE-2SLS and FE-S2SLS with respect to true RE-S2SLS is 1.365 and 1.243, respectively. Note that both FE-S2SLS and FE-2SLS estimators perform much worse than any feasible spatial RE-S2SLS estimator. There is also much gain in performing RE-S2SLS allowing for spatial correlation than ignoring it. For $\rho_1 = \rho_2 = -0.8$, the relative RMSE of RE-2SLS ignoring spatial correlation with respect to true RE-S2SLS is 1.118 compared to 1.027 for the RE-S2SLS based on KKP. The General spatial RE-S2SLS estimator of Baltagi, Egger and Pfaffermayr (2013) is second best with relative RMSE of 1.039. For $\rho_1 = 0$ and $\rho_2 = 0.8$, the relative RMSE of RE-2SLS ignoring spatial correlation is 1.179 compared to 1.047 for the RE-S2SLS based on Anselin. The General spatial RE-S2SLS estimator is again second best with relative RMSE of 1.052. The gain in efficiency from using the correct feasible RE-S2SLS for our experiments when the true model is a generalized MRSAR panel model with $\rho_1 = 0.8$ and $\rho_2 = -0.8$, is as follows: The relative RMSE of the RE-S2SLS based on KKP is 1.356 and the RE-S2SLS based on Anselin is 1.220, while the General spatial RE-S2SLS estimator is 1.062. Table 2 reports the relative root mean squared error (RMSE) of each estimator of λ. Similar to the simulation results for β in Table 1, for $\rho_1 = 0.8$ and $\rho_2 = -0.8$, The relative RMSE of the RE-S2SLS based on KKP is 1.262 and the RE-S2SLS based on Anselin is 1.096, while the General spatial RE-S2SLS estimator is 1.024.

Table 3 reports the empirical size (at the 5% level) of the spatial Hausman test for various values of ρ_1 and ρ_2 based on 10,000 replications. This is based on the contrast of the KKP RE-S2SLS estimator and the FE-S2SLS in the first column, and the contrast of the Anselin RE-S2SLS estimator and the FE-S2SLS in the second column and the contrast of the Generalized RE-S2SLS estimator and the FE-S2SLS in the third column. We can see that for $\rho_1 = 0$ and $\rho_2 = -0.8$, the spatial Hausman test based on KKP is over-sized if the true model is an Anselin random effects MRSAR model. It yields a probability of type I error of 0.070 when it should be 0.05. This oversizing of the test gets worse when $\rho_1 = 0.8$ and $\rho_2 = -0.8$. The Hausman test based on KKP yields a type I error of 0.115. In contrast, for $\rho_1 = \rho_2 = -0.8$, the spatial Hausman test based on the Anselin RE-S2SLS estimator is under-sized if the true model is a KKP random effects MRSAR model. It yields a probability of type I error of 0.031 when it should be 0.05. However, this undersizing does not get worse, and the Hausman test based on the Anselin type MRSAR panel model performs reasonably well when the true model is a generalized MRSAR panel model, with size varying between 0.032 and 0.070. The spatial Hausman test based on the generalized spatial RE-S2SLS estimator performs better with size varying between 0.036 and 0.062.

4 Conclusion

This paper suggests simple RE-S2SLS and FE-S2SLS estimators for the generalized MRSAR panel model. This extends the generalized spatial error model considered by Baltagi, Egger and Pfaffermayr (2013) to include a spatial lag term. More specifically, this generalized MRSAR model encompasses the KKP and Anselin spatial error models and allow for the inclusion of a spatial lag of the dependent variable. Our FE and RE-S2SLS estimators apply the usual fixed and random effects transformations and the GM method of KKP and Mutl and Pfaffermayr (2011), and are easy to compute. We derive the asymptotic distribution of these estimators and investigate their performance using Monte Carlo experiments. Our results show that the FE-S2SLS estimator that accounts for the spatial correlation performs much better than the standard FE-2SLS which ignores the spatial correlation. There is also much gain from performing RE-S2SLS allowing for spatial correlation than the standard RE-2SLS estimator which ignores the spatial correlation. Not surprisingly, the 'correct' feasible RE-S2SLS estimator (Anselin, KKP or Generalized) performs best in terms of RMSE when compared to the true RE-S2SLS. We also investigate the performance of the spatial Hausman test based on the contrast involving the FE-S2SLS estimator and the KKP, Anselin and Generalized variants of the RE-S2SLS estimator. We show that this Hausman test can be misleading under misspecification.

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Appendix

A Proof of Theorem 1

Proof. First, let us show that $\Gamma_N^1 = O(1)$, $\gamma_N^1 = O(1)$ and

$$G_N^{1*} - \Gamma_N^1 \xrightarrow{p} 0 \text{ and } g_N^{1*} - \gamma_N^1 \xrightarrow{p} 0 \text{ as } N \to \infty. \tag{39}$$

Let $\xi_N = \left(\xi_{1,N}, \ldots, \xi_{(T+1)N,N}\right)' = (\mu_N', \ldots, v_N')'$ so that $u_N = Z_\mu u_{1N} + u_{2N} = \left(\iota_T \quad A^{-1}\right)\mu_N + \left(I_T \quad B^{-1}\right)\nu_N = \left[\left(\iota_T \quad A^{-1}\right), \left(I_T \quad B^{-1}\right)\right]\xi_N$ and

$$\begin{aligned}
\varphi_{kl,N} &= \frac{1}{NT} u_N' \left(S_T \quad W_N^{k\prime} W_N^l\right) u_N \\
&= \frac{1}{NT} \xi_N' \left[\left(\iota_T \quad A^{-1}\right), \left(I_T \quad B^{-1}\right)\right]' \left(S_T \quad W_N^{k\prime} W_N^l\right) \left[\left(\iota_T \quad A^{-1}\right), \left(I_T \quad B^{-1}\right)\right] \xi_N \\
&= \frac{1}{NT} \xi_N' C_N \xi_N,
\end{aligned}$$

where $C_N = \begin{pmatrix} T & \iota_T' \\ \iota_T & S_T \end{pmatrix} \quad \begin{pmatrix} A^{-1\prime} W_N^{k\prime} W_N^l A^{-1} & A^{-1\prime} W_N^{k\prime} W_N^l B^{-1} \\ B^{-1\prime} W_N^{k\prime} W_N^l A^{-1} & B^{-1\prime} W_N^{k\prime} W_N^l B^{-1} \end{pmatrix}$ using $\iota_T' S_T \iota_T = T$ and $S_T \iota_T = \iota_T$. Note that the first matrix of the Kronecker product in C_N does not depend on N. The row and column sums of the second matrix of the Kronecker product in C_N are bounded uniformly in absolute value by Remark A2(b) in Kapoor, Kelejian and Prucha (2007). Under Assumptions 2 and 4, by Lemma A1 in Kapoor, Kelejian and Prucha (2007), we have $E\left(\varphi_{kl,N}\right) = O(1)$ and $\varphi_{kl,N} - E\left(\varphi_{kl,N}\right) \xrightarrow{p} 0$. Notice that $\varphi_{kl,N}$ are elements of G_N^{1*} and g_N^{1*}. $E\left(\varphi_{kl,N}\right)$ are elements of Γ_N^1 and γ_N^1, Equation (39) is proved.

Second, let us show that

$$G_N^1 - G_N^{1*} \xrightarrow{p} 0 \text{ and } g_N^1 - g_N^{1*} \xrightarrow{p} 0 \text{ as } N \to \infty, \tag{40}$$

provided $\widehat{\delta}_{2SLS} \xrightarrow{p} \delta$ as $N \to \infty$. Note that the elements of G_N^{1*} and g_N^{1*} are $\varphi_{kl,N} = \frac{1}{NT} u_N' \left(S_T \quad W_N^{k\prime} W_N^l\right) u_N$. Since the row and column sums of the elements of W_N are uniformly bounded in absolute value by Assumption 4, it follows that the row and columns sums of the matrices $S_T \quad W_N^{k\prime} W_N^l$ also have that property. Define $\tilde{\varphi}_{kl,N} = \frac{1}{NT} \tilde{u}_N' \left(S_T \quad W_N^{k\prime} W_N^l\right) \tilde{u}_N$, which are the elements of G_N^1 and g_N^1. By the proof of Lemma A3 in Kapoor, Kelejian and Prucha (2007), we have $\tilde{\varphi}_{kl,N} - \varphi_{kl,N} \xrightarrow{p} 0$ as $N \to \infty$. This completes the proof of Equation (40).

Third, Let $\theta = \left(\rho_1, \sigma_\mu^2\right)$ and $\underline{\theta} = \left(\underline{\rho}_1, \underline{\sigma}_\mu^2\right)$. The objective function of the nonlinear least squares estimator and its corresponding nonstochastic counterpart are given by

$$
\begin{aligned}
R_N^1(\underline{\theta}) &= \left[G_N^1\left[\rho_1,\rho_1^2,\sigma_\mu^2\right]' - g_N^1\right]'\left[G_N^1\left[\rho_1,\rho_1^2,\sigma_\mu^2\right]' - g_N^1\right], \\
\bar{R}_N^1(\underline{\theta}) &= \left[\Gamma_N^1\left[\rho_1,\rho_1^2,\sigma_\mu^2\right]' - \gamma_N^1\right]'\left[\Gamma_N^1\left[\rho_1,\rho_1^2,\sigma_\mu^2\right]' - \gamma_N^1\right],
\end{aligned}
$$

respectively. Using Assumption 3, Equations (39) and (40), and the proof of Theorem 1 in Kapoor, Kelejian and Prucha (2007), we get

$$
\sup_{\underline{\rho_1}\in[-a_1,a_1],\underline{\sigma_\mu^2}\in[0,b_1]}\left|R_N^1(\underline{\theta}) - \bar{R}_N^1(\underline{\theta})\right| \xrightarrow{p} 0
$$

as $N\to\infty$. The consistency of $\tilde{\rho}_1$ and $\tilde{\sigma}_\mu^2$ follows directly from Lemma 3.1 in Pötscher and Prucha (1997).
■

B Proof of Theorem 2

Proof. First, using the central limit theorem and the law of large numbers, we have

$$
\begin{aligned}
&\sqrt{NT}\left(\widehat{\delta}_{RE-2SLS} - \delta\right) \\
= &\left[\frac{Z_N' \quad {}_u^{-1}H_N^*}{NT}\left(\frac{H_N^{*\prime} \quad {}_u^{-1}H_N^*}{NT}\right)^{-1}\frac{H_N^{*\prime} \quad {}_u^{-1}Z_N}{NT}\right]^{-1}\frac{Z_N' \quad {}_u^{-1}H_N^*}{NT}\left(\frac{H_N^{*\prime} \quad {}_u^{-1}H_N^*}{NT}\right)^{-1}\frac{H_N^{*\prime} \quad {}_u^{-1}u_N}{\sqrt{NT}} \\
&\xrightarrow{d} N\left(0,\left(\Gamma_0'\Sigma_0^{-1}\Gamma_0 + \sigma_\nu^{-2}\Gamma_1'\Sigma_1^{-1}\Gamma_1\right)^{-1}\right),
\end{aligned}
$$

as $N\to\infty$ since

$$
\begin{aligned}
\frac{1}{NT}H_N^{*\prime} \quad {}_u^{-1}H_N^* &= \begin{pmatrix} \frac{1}{NT}H_N'\left\{\bar{J}_T \quad \left[T\sigma_\mu^2\left(A'A\right)^{-1} + \sigma_\nu^2\left(B'B\right)^{-1}\right]^{-1}\right\}H_N & 0 \\ 0 & \sigma_\nu^{-2}\frac{1}{NT}H_N'\left[E_T \quad (B'B)\right]H_N \end{pmatrix} \\
&\xrightarrow{p} \begin{pmatrix} \Sigma_0 & 0 \\ 0 & \sigma_\nu^{-2}\Sigma_1 \end{pmatrix}
\end{aligned}
$$

$$
\frac{1}{NT}H_N^{*\prime} \quad {}_u^{-1}Z_N = \begin{pmatrix} \frac{1}{NT}H_N'\left\{\bar{J}_T \quad \left[T\sigma_\mu^2\left(A'A\right)^{-1} + \sigma_\nu^2\left(B'B\right)^{-1}\right]^{-1}\right\}Z_N \\ \sigma_\nu^{-2}\frac{1}{NT}H_N'\left[E_T \quad (B'B)\right]Z_N \end{pmatrix} \xrightarrow{p} \begin{pmatrix} \Gamma_0 \\ \sigma_\nu^{-2}\Gamma_1 \end{pmatrix}
$$

and

$$
\frac{1}{\sqrt{NT}}H_N^{*\prime} \quad {}_u^{-1}u_N \xrightarrow{d} N\left(0,\lim_{N\to\infty}\frac{1}{NT}H_N^{*\prime} \quad {}_u^{-1}H_N^*\right) = N\left(0,\begin{pmatrix} \Sigma_0 & 0 \\ 0 & \sigma_\nu^{-2}\Sigma_1 \end{pmatrix}\right)
$$

using Assumption 4.

Second, from Theorem 1, we know the GM estimators of $\tilde{\rho}_1$, $\tilde{\rho}_2$, $\tilde{\sigma}_\mu^2$ and $\tilde{\sigma}_\nu^2$ are consistent. Similar to Lemma 4 of Baltagi, Egger and Pfaffermayr (2013), one can show that

$$\frac{1}{NT} H_N^{*\prime} \tilde{\ }_u^{-1} H_N^* - \frac{1}{NT} H_N^{*\prime} \ _u^{-1} H_N^* \xrightarrow{p} 0$$
$$\frac{1}{NT} H_N^{*\prime} \tilde{\ }_u^{-1} Z_N - \frac{1}{NT} H_N^{*\prime} \ _u^{-1} Z_N \xrightarrow{p} 0$$

and

$$\frac{1}{\sqrt{NT}} H_N^{*\prime} \tilde{\ }_u^{-1} u_N - \frac{1}{\sqrt{NT}} H_N^{*\prime} \ _u^{-1} u_N \xrightarrow{p} 0.$$

Therefore, we have $\sqrt{NT}\left(\widehat{\delta}_{RE-F2SLS} - \widehat{\delta}_{RE-2SLS}\right) \xrightarrow{p} 0$ as $N \to \infty$. This proves the Theorem. ■

C Proof of Theorem 3

Proof. First, using the central limit theorem and the law of large numbers, we have

$$\begin{aligned}
\sqrt{NT}\left(\widehat{\delta}_{FE-2SLS} - \delta\right) &= \left\{ \frac{Z_N' \left[E_T \quad (B'B)\right] H_N}{NT} \left(\frac{H_N' \left[E_T \quad (B'B)\right] H_N}{NT} \right)^{-1} \frac{H_N' \left[E_T \quad (B'B)\right] Z_N}{NT} \right\}^{-1} \\
&\quad \frac{Z_N' \left[E_T \quad (B'B)\right] H_N}{NT} \left(\frac{H_N' \left[E_T \quad (B'B)\right] H_N}{NT} \right)^{-1} \frac{H_N' \left[E_T \quad B'\right] v_N}{\sqrt{NT}} \\
&\quad \xrightarrow{d} N\left(0, \sigma_\nu^2 \left(\Gamma_1' \Sigma_1^{-1} \Gamma_1\right)^{-1}\right),
\end{aligned}$$

as $N \to \infty$ since

$$\frac{1}{NT} H_N' \left[E_T \quad (B'B)\right] H_N \xrightarrow{p} \Sigma_1$$
$$\frac{1}{NT} H_N' \left[E_T \quad (B'B)\right] Z_N \xrightarrow{p} \Gamma_1$$

and

$$\frac{1}{\sqrt{NT}} H_N' \left[E_T \quad B'\right] v_N \xrightarrow{d} N\left(0, \lim_{N\to\infty} \frac{1}{NT} H_N' \left[E_T \quad (B'B)\right] H_N\right) = N\left(0, \sigma_\nu^2 \Sigma_1\right)$$

using Assumption 4.

Second, similar to the proof of Theorem 2, one can show that

$$\frac{1}{NT} H_N' \left[E_T \quad \left(\tilde{B}'\tilde{B}\right)\right] H_N - \frac{1}{NT} H_N' \left[E_T \quad (B'B)\right] H_N \xrightarrow{p} 0$$
$$\frac{1}{NT} H_N' \left[E_T \quad \left(\tilde{B}'\tilde{B}\right)\right] Z_N - \frac{1}{NT} H_N' \left[E_T \quad (B'B)\right] Z_N \xrightarrow{p} 0$$

and

$$\frac{1}{\sqrt{NT}} H_N' \left[E_T \quad \tilde{B}'\right] v_N - \frac{1}{\sqrt{NT}} H_N' \left[E_T \quad B'\right] v_N \xrightarrow{p} 0.$$

Therefore, we have $\sqrt{NT}\left(\widehat{\delta}_{FE-F2SLS} - \widehat{\delta}_{FE-2SLS}\right) \xrightarrow{p} 0$ as $N \to \infty$. This proves the Theorem. ■

Table 1: Relative Efficiencies of Spatial Panel Data Estimators of β in the MRSAR Model

	ρ_1	ρ_2	FE-2SLS	RE-2SLS	FE-S2SLS	RE-S2SLS KKP	RE-S2SLS Anselin	RE-S2SLS General
RE	0	0	1.203	1.005	1.205	0.994	0.985	1.002
KKP	-0.8	-0.8	1.365	1.118	1.243	1.027	1.082	1.039
	-0.5	-0.5	1.280	1.054	1.223	1.017	1.050	1.028
	-0.2	-0.2	1.224	1.017	1.217	1.005	1.016	1.021
	0.2	0.2	1.187	1.011	1.181	0.956	0.969	0.974
	0.5	0.5	1.157	1.050	1.166	0.963	0.974	0.970
	0.8	0.8	1.155	1.053	1.215	1.054	1.073	1.064
Anselin	0	-0.8	1.306	1.157	1.185	1.105	1.036	1.026
	0	-0.5	1.222	1.067	1.169	1.051	1.031	1.021
	0	-0.2	1.186	1.022	1.180	1.004	1.009	1.009
	0	0.2	1.242	1.029	1.234	0.976	0.968	0.989
	0	0.5	1.307	1.111	1.317	1.047	0.965	0.968
	0	0.8	1.380	1.179	1.465	1.226	1.047	1.052
General	-0.8	-0.5	1.307	1.084	1.248	1.024	1.082	1.032
	-0.8	-0.2	1.312	1.084	1.298	1.058	1.087	1.019
	-0.8	0	1.366	1.115	1.366	1.095	1.084	1.016
	-0.8	0.2	1.419	1.163	1.405	1.117	1.059	1.008
	-0.8	0.5	1.444	1.256	1.445	1.216	1.032	1.005
	-0.8	0.8	1.426	1.294	1.520	1.332	1.075	1.054
	-0.5	-0.8	1.347	1.111	1.224	1.036	1.060	1.036
	-0.5	-0.2	1.265	1.033	1.254	1.027	1.047	1.015
	-0.5	0	1.310	1.061	1.306	1.037	1.036	1.010
	-0.5	0.2	1.363	1.103	1.350	1.059	1.007	1.004
	-0.5	0.5	1.414	1.204	1.422	1.155	1.004	0.995
	-0.5	0.8	1.415	1.248	1.509	1.287	1.063	1.051
	-0.2	-0.8	1.319	1.119	1.197	1.063	1.030	1.031
	-0.2	-0.5	1.246	1.056	1.192	1.028	1.035	1.028
	-0.2	0	1.248	1.025	1.250	1.005	1.000	1.008
	-0.2	0.2	1.293	1.048	1.283	1.005	0.977	0.998
	-0.2	0.5	1.361	1.152	1.366	1.091	0.987	0.983
	-0.2	0.8	1.392	1.201	1.476	1.245	1.045	1.054
	0.2	-0.8	1.292	1.206	1.170	1.152	1.054	1.032
	0.2	-0.5	1.193	1.093	1.140	1.089	1.032	1.006
	0.2	-0.2	1.160	1.036	1.155	1.033	1.020	1.010
	0.2	0	1.171	1.015	1.174	0.995	1.000	0.998
	0.2	0.5	1.252	1.077	1.263	1.015	0.964	0.961
	0.2	0.8	1.348	1.163	1.435	1.185	1.031	1.039
	0.5	-0.8	1.268	1.337	1.149	1.243	1.092	1.028
	0.5	-0.5	1.161	1.183	1.109	1.173	1.068	1.011
	0.5	-0.2	1.115	1.093	1.114	1.098	1.055	1.003
	0.5	0	1.115	1.052	1.118	1.052	1.047	0.990
	0.5	0.2	1.119	1.026	1.110	0.974	1.009	0.959
	0.5	0.8	1.279	1.128	1.360	1.128	1.025	1.041
	0.8	-0.8	1.253	1.815	1.138	1.356	1.220	1.062
	0.8	-0.5	1.145	1.572	1.105	1.287	1.186	1.054
	0.8	-0.2	1.085	1.387	1.097	1.228	1.164	1.035
	0.8	0	1.063	1.268	1.083	1.165	1.142	1.009
	0.8	0.2	1.066	1.176	1.076	1.092	1.110	0.977
	0.8	0.5	1.069	1.070	1.098	0.996	1.046	0.985

Notes: (a) Relative mean square error with respect to the true RE-S2SLS. (b) 10,000 replications.

(c)$N = 100, T = 5, J = 6, \theta = 0.5$ and $\lambda = 0.5$.

Table 2: Relative Efficiencies of Spatial Panel Data Estimators of λ in the MRSAR Model

	ρ_1	ρ_2	FE-2SLS	RE-2SLS	FE-S2SLS	RE-S2SLS KKP	RE-S2SLS Anselin	RE-S2SLS General
RE	0	0	1.261	1.004	1.265	0.998	1.000	1.006
KKP	-0.8	-0.8	1.322	1.068	1.251	1.013	1.033	1.011
	-0.5	-0.5	1.278	1.026	1.249	1.003	1.009	0.999
	-0.2	-0.2	1.261	1.008	1.256	1.001	1.003	1.004
	0.2	0.2	1.267	1.010	1.260	1.000	1.012	1.010
	0.5	0.5	1.288	1.032	1.270	1.004	1.025	1.005
	0.8	0.8	1.264	1.062	1.286	1.021	1.069	1.021
Anselin	0	-0.8	1.368	1.050	1.302	1.034	1.001	1.004
	0	-0.5	1.306	1.023	1.280	1.014	1.000	1.002
	0	-0.2	1.272	1.008	1.268	1.003	1.003	1.004
	0	0.2	1.265	1.009	1.258	1.005	1.003	1.009
	0	0.5	1.301	1.051	1.283	1.017	1.002	1.007
	0	0.8	1.295	1.121	1.319	1.057	1.044	1.023
General	-0.8	-0.5	1.257	1.030	1.229	1.003	1.018	0.998
	-0.8	-0.2	1.228	1.024	1.222	1.014	1.019	0.997
	-0.8	0	1.232	1.028	1.235	1.030	1.023	1.001
	-0.8	0.2	1.239	1.037	1.236	1.036	1.015	0.999
	-0.8	0.5	1.282	1.073	1.257	1.059	1.011	1.007
	-0.8	0.8	1.293	1.186	1.303	1.102	1.051	1.038
	-0.5	-0.8	1.341	1.057	1.271	1.010	1.015	1.012
	-0.5	-0.2	1.244	1.014	1.238	1.006	1.007	1.000
	-0.5	0	1.245	1.020	1.249	1.021	1.016	1.003
	-0.5	0.2	1.253	1.025	1.248	1.019	1.009	1.004
	-0.5	0.5	1.287	1.059	1.263	1.032	0.998	1.008
	-0.5	0.8	1.291	1.160	1.311	1.082	1.041	1.035
	-0.2	-0.8	1.358	1.047	1.292	1.025	1.008	1.011
	-0.2	-0.5	1.295	1.022	1.267	1.006	1.000	1.001
	-0.2	0	1.252	1.006	1.256	0.998	1.002	1.003
	-0.2	0.2	1.268	1.020	1.262	1.013	1.013	1.013
	-0.2	0.5	1.292	1.045	1.269	1.023	0.997	1.007
	-0.2	0.8	1.296	1.140	1.318	1.067	1.043	1.026
	0.2	-0.8	1.380	1.056	1.315	1.049	1.002	1.007
	0.2	-0.5	1.315	1.027	1.289	1.022	0.997	1.002
	0.2	-0.2	1.272	1.008	1.271	1.000	1.000	1.003
	0.2	0	1.268	1.007	1.273	1.001	1.011	1.008
	0.2	0.5	1.294	1.036	1.276	1.006	1.001	1.002
	0.2	0.8	1.289	1.107	1.311	1.044	1.040	1.018
	0.5	-0.8	1.399	1.085	1.332	1.096	1.023	1.012
	0.5	-0.5	1.328	1.059	1.300	1.069	1.024	1.007
	0.5	-0.2	1.279	1.033	1.282	1.035	1.025	1.009
	0.5	0	1.265	1.027	1.269	1.018	1.029	1.003
	0.5	0.2	1.264	1.021	1.264	1.006	1.026	0.998
	0.5	0.8	1.286	1.095	1.306	1.039	1.040	1.020
	0.8	-0.8	1.413	1.185	1.351	1.262	1.096	1.024
	0.8	-0.5	1.340	1.157	1.319	1.211	1.096	1.021
	0.8	-0.2	1.287	1.131	1.294	1.141	1.096	1.020
	0.8	0	1.271	1.113	1.279	1.109	1.098	1.015
	0.8	0.2	1.263	1.096	1.261	1.077	1.097	1.018
	0.8	0.5	1.268	1.070	1.254	1.030	1.078	1.010

Notes: (a) Relative mean square error with respect to the true RE-S2SLS. (b) 10,000 replications.

(c)$N = 100, T = 5, J = 6, \theta = 0.5$ and $\lambda = 0.5$.

Table 3: Size of the Spatial Hausman Test in the MRSAR Model

	ρ_1	ρ_2	RE-S2SLS KKP	RE-S2SLS Anselin	RE-S2SLS General
RE	0	0	0.050	0.049	0.051
KKP	-0.8	-0.8	0.059	0.031	0.061
	-0.5	-0.5	0.061	0.041	0.062
	-0.2	-0.2	0.054	0.048	0.055
	0.2	0.2	0.046	0.046	0.045
	0.5	0.5	0.046	0.040	0.044
	0.8	0.8	0.053	0.044	0.054
Anselin	0	-0.8	0.070	0.049	0.053
	0	-0.5	0.063	0.048	0.053
	0	-0.2	0.055	0.050	0.051
	0	0.2	0.049	0.048	0.052
	0	0.5	0.049	0.049	0.054
	0	0.8	0.053	0.064	0.060
General	-0.8	-0.5	0.053	0.033	0.062
	-0.8	-0.2	0.048	0.038	0.061
	-0.8	0	0.047	0.041	0.060
	-0.8	0.2	0.047	0.045	0.060
	-0.8	0.5	0.055	0.052	0.061
	-0.8	0.8	0.055	0.070	0.062
	-0.5	-0.8	0.071	0.039	0.062
	-0.5	-0.2	0.051	0.043	0.058
	-0.5	0	0.048	0.043	0.057
	-0.5	0.2	0.048	0.045	0.057
	-0.5	0.5	0.051	0.050	0.059
	-0.5	0.8	0.053	0.070	0.064
	-0.2	-0.8	0.072	0.048	0.057
	-0.2	-0.5	0.064	0.048	0.056
	-0.2	0	0.050	0.046	0.053
	-0.2	0.2	0.049	0.048	0.056
	-0.2	0.5	0.049	0.051	0.057
	-0.2	0.8	0.053	0.066	0.059
	0.2	-0.8	0.068	0.046	0.046
	0.2	-0.5	0.061	0.047	0.046
	0.2	-0.2	0.051	0.044	0.045
	0.2	0	0.049	0.044	0.046
	0.2	0.5	0.046	0.047	0.050
	0.2	0.8	0.052	0.061	0.059
	0.5	-0.8	0.075	0.042	0.039
	0.5	-0.5	0.063	0.042	0.038
	0.5	-0.2	0.048	0.040	0.038
	0.5	0	0.044	0.039	0.036
	0.5	0.2	0.040	0.038	0.039
	0.5	0.8	0.053	0.050	0.055
	0.8	-0.8	0.115	0.041	0.040
	0.8	-0.5	0.101	0.040	0.042
	0.8	-0.2	0.076	0.038	0.038
	0.8	0	0.057	0.034	0.038
	0.8	0.2	0.046	0.032	0.040
	0.8	0.5	0.046	0.036	0.049

Notes: (a) 10,000 replications. (b) $N = 100, T = 5, J = 6, \theta = 0.5$ and $\lambda = 0.5$.