RANDOM FIXED POINTS AND APPROXIMATIONS IN RANDOM CONVEX METRIC SPACES[1]

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ABSTRACT

Some random fixed point theorems in random convex metric spaces are obtained. Results regarding random best approximation on random convex metric spaces are also proved.

Key words: Random fixed point, random approximation, metric space.

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1. INTRODUCTION AND PRELIMINARIES

Random fixed point theory has received much attention for the last two decades, since the publication of the paper by Bharucha-Reid [3]. On the other hand, random best approximation has recently received further attention after the papers by Sehgal and Waters [13], Sehgal and Singh [12], Papageorgiou [11], Lin [10] and Beg and Shahzad [1]. The purpose of this paper is to prove some invariant random approximation theorems in random convex metric spaces.

Let $(\Omega, \mathcal{A})$ be a measurable space, (X, d) a metric space, 2^x family of all subsets of X, $CK(X)$ family of all nonempty compact convex subsets of X, $K(X)$ family of all nonempty compact subsets of X and $CB(X)$ family of all nonempty closed bounded subsets of X. A mapping $T: \Omega \to 2^x$ is called *measurable* if for any open subset C of X, $T^{-1}(C) = \{\omega \in \Omega: T(\omega) \cap C \neq \phi\} \in \mathcal{A}$. A mapping $\xi: \Omega \to X$ is said to be a *measurable selector* of a measurable mapping $T: \Omega \to 2^x$ if ξ is measurable and for any $\omega \in \Omega$, $\xi(\omega) \in T(\omega)$. A mapping

$f{:}\,\Omega \times X {\to} X$ is called a *random operator* if for any $x \in X$, $f(\,\cdot\,,x)$ is measurable. A mapping $T{:}\,\Omega \times X {\to} CB(X)$ is a *random multivalued operator* if for any $x \in X$, $T(\,\cdot\,,x)$ is measurable. A measurable mapping $\xi{:}\,\Omega {\to} X$ is called a *random fixed point* of a random multivalued (single valued) operator $T{:}\,\Omega \times X {\to} CB(X)$ $(f{:}\,\Omega \times X {\to} X)$ if for every $\omega \in \Omega$, $\xi(\omega) \in T(\omega,\xi(\omega))(\xi(\omega) = \; f(\omega,\xi(\omega)))$. A mapping $T{:}\,X {\to} CB(X)$ is *upper (lower) semicontinuous* if for any closed (open) subset C of X, $T^{-1}(C)$ is closed (open). A mapping T is called *continuous* if T is both upper and lower semicontinuous. A random operator $T{:}\,\Omega \times X {\to} CB(X)$ is called *Lipschitzian* if $H(T(\omega,x),T(\omega,y)) \leq L(\omega)d(x,y)$ for any $x,y \in X$ and $\omega \in \Omega$, where $L{:}\,\Omega {\to} [0,\infty)$ is a measurable map and H is the Hausdorff metric on $CB(X)$, induced by the metric d. When $L(\omega) < 1$ $(L(\omega) = 1)$ for each $\omega \in \Omega$, then T is called *contraction (nonexpansive)*. Let K be a subset of X. A random operator $T{:}\,\Omega \times K {\to} K$ is said to be a *Banach operator* if there exists a measurable map $\beta{:}\,\Omega {\to} [0,1)$ and for each $x \in K$ and $\omega \in \Omega$, $d(T(\omega,T(\omega,x)),$ $T(\omega,x)) \leq \beta(\omega)\; d(T(\omega,x),x)$. A continuous mapping $V{:}\,X \times X \times [0,1){\to}X$ is said to be *convex structure* on X, if for all x,y in X and $\lambda \in [0,1]$ the following condition is satisfied:

$$d(u,V(x,y,\lambda)) \leq \lambda d(u,x) + (1-\lambda)d(u,y)$$

for all $u \in X$. A metric space X with convex structure is called a *convex metric space*. Banach space and each of its convex subsets are simple examples of convex metric space. There are many convex metric spaces which can not be imbedded in any Banach space. For examples and other details we refer to Takahashi [14]. A subset K of a convex metric space X is said to be convex if $V(x,y,\lambda) \in K$ for all $x,y \in K$ and $\lambda \in [0,1]$. The set K is said to be *starshaped* if there exists some $p \in K$ such that $V(x,p,\lambda) \in K$ for all $x \in K$ and $\lambda \in [0,1]$. The point is called *starcentre* of K. Clearly starshaped subsets of X contain all convex subsets of X as a proper subclass. A convex metric space is said to satisfy property (I), if for all $x,y \in X$ and $\lambda \in [0,1]$,

$$d(V(x,p,\lambda),V(y,p,\lambda)) \leq \lambda d(x,y).$$

Property (I) is always satisfied in any normed space X. For details we refer to Guay, Singh and Whitfield [4]. Let $W{:}\,\Omega \times X \times X \times [0,1]{\to}X$ be a mapping having the following properties:

(*i*)   For each $\omega \in \Omega, W(\omega, \cdot, \cdot, \cdot)$ is a convex structure on X,

(*ii*)   For each $x, y, \in X,$ $\lambda \in [0,1],$ $W(\cdot, x, y, \lambda)$ is measurable.

The mapping W is called a random convex structure on X and X is random convex metric space.

2. RANDOM FIXED POINTS

In this section, random fixed point theorems in random convex metric spaces are proved. For corresponding fixed point theorems, we refer to Guay, Singh and Whitfield [4] and Beg, Shahzad and Iqbal [2].

Theorem 2.1. *Let E be a closed subset of a separable complete metric space X. Let $T: \Omega \times E \to E$ be a continuous Banach operator, then T has a random fixed point.*

Proof: Let $\xi_0: \Omega \to E$ be a measurable mapping. It follows by induction for each $\omega \in \Omega$,

$$d(T^{n+1}(\omega, \xi_0(\omega)), T^n(\omega, \xi_0(\omega))) \leq \beta^n(\omega) d(\xi_0(\omega), T(\omega, \xi_0(\omega))).$$

Put $\xi_1(\omega) = T(\omega, \xi_0(\omega))$. The mapping ξ_1 is measurable by Himmelberg [6] and a sequence of measurable mappings can be defined as follows:

$$\xi_n(\omega) = T(\omega, \xi_{n-1}(\omega)) = T^n(\omega, \xi_0(\omega)) \quad \text{(for each } \omega \in \Omega, \ n = 1, 2, \ldots).$$

Assuming $n \leq m$, we have for any $\omega \in \Omega$,

$$d(\xi_n(\omega), \xi_m(\omega)) = d(T^n(\omega, \xi_0(\omega)), T^m(\omega, \xi_0(\omega)))$$

$$\leq \beta^n(\omega) d(\xi_0(\omega), T(\omega, \xi_{m-n-1}(\omega)))$$

$$\leq \beta^n(\omega)[d(\xi_0(\omega), \xi_1(\omega)) + d(\xi_1(\omega), \xi_2(\omega)) + \ldots + d(\xi_{m-n-1}(\omega), \xi_{m-n}(\omega))]$$

$$< \frac{\beta^n(\omega)}{1 - \beta(\omega)} d(\xi_0(\omega), \xi_1(\omega)).$$

Since $0 < \beta(\omega) < 1$ for each $\omega \in \Omega$, $\{\xi_0(\omega)\}$ is a Cauchy sequence in E. Since E being a closed subset of a complete metric space, is complete, therefore $\{\xi_n(\omega)\}$ converges to some $\xi(\omega) \in E$. Thus $lim_{n \to \infty} \xi_n(\omega) = \xi(\omega)$. By continuity of T, $T(\omega, \xi(\omega)) = \xi(\omega)$ for each $\omega \in \Omega$.

Remark 2.2. Theorem 2.1 remains true if E is a closed subset of a separable metric space X and closure of $T(\omega, E)$ is compact for each $\omega \in \Omega$.

Theorem 2.3. *Let X be a separable random convex metric space satisfying property (I) and E be a closed and p-starshaped subset of X. If $T: \Omega \times E \to E$ is a nonexpansive random operator and closure of $T(\omega, E)$ is compact for each $\omega \in \Omega$, then T has a random fixed point.*

Proof: Define a sequence of random operators $T_n: \Omega \times E \to E$ by $T_n(\omega, x) = W(\omega, T(\omega, x), p, \beta_n(\omega))$, where β_n is a fixed sequence of measurable mappings $\beta_n: \Omega \to (0,1)$ and $\beta_n(\omega)$ converging to 1. Each T_n is a continuous Banach operator:

$$d(T_n(\omega, x), T_n^2(\omega, x)) = d(W(\omega, T(\omega, x), p, \beta_n(\omega)), W(\omega, T(\omega, T_n(\omega, x)), p, \beta_n(\omega)))$$

$$\leq \beta_n(\omega)d(T(\omega, x), T(\omega, T_n(\omega, x)))$$

$$\leq \beta_n(\omega)d(x, T_n(\omega, x)),$$

for each $x \in E$ and $\omega \in \Omega$.

Since closure of $T(\omega, E)$ is compact, closure of $T_n(\omega, E)$ is compact too for each $\omega \in \Omega$, and Remark 2.2, further implies: for each T_n there exists a random fixed point ξ_n such that for any $\omega \in \Omega$,

$$\xi_n(\omega) = T_n(\omega, \xi_n(\omega)) = W(\omega, T(\omega, \xi_n(\omega)), p, \beta_n(\omega)).$$

For each n, define $G_n: \Omega \to K(E)$ by $G_n(\omega) = cl\{\xi_i(\omega): i \geq n\}$. Define $G: \Omega \to K(E)$ by $G(\omega) = \bigcap_{n=1}^{\infty} G_n(\omega)$. Then G is measurable by Himmelberg [6, Theorem 4.1] and by Kuratowski and Ryll-Nardzewski [9] has a measurable selector ξ. As closure of $T_n(\omega, E)$ is compact for each $\omega \in \Omega$, $\{\xi_n(\omega)\}$ has a subsequence $\{\xi_{n_j}(\omega)\}$ converging to $\xi(\omega)$. By continuity of T and W, $T(\omega, \xi_{n_j}(\omega))$ converges to $T(\omega, \xi(\omega))$. Thus $T(\omega, \xi(\omega)) = \xi(\omega)$ for each $\omega \in \Omega$.

Theorem 2.4. *Let X be a compact starshaped subset of a separable random convex metric space satisfying property (I). Let $T: \Omega \times X \to X$ be a nonexpansive random operator. Then T has a random fixed point.*

Proof: Choose a starcentre x_0 of X and a sequence $\{k_n\}$ of measurable mappings $k_n: \Omega \to (0,1)$ and $k_n(\omega) \to 1$ as $n \to \infty$. Define the random operator $T_n: \Omega \times X \to X$ by

$$T_n(\omega, x) = W(\omega, T(\omega, x), x_0, k_n(\omega)).$$

The operator T_n is a contraction. Indeed,

$$d(T_n(\omega, x), T_n(\omega, y)) = d(W(\omega, T(\omega, x), x_0, k_n(\omega)), \ W(\omega, T(\omega, y), x_0, k_n(\omega)))$$

$$\leq k_n(\omega) d(T(\omega, x), T(\omega, y))$$

$$\leq k_n(\omega) d(x, y)$$

for all $x, y \in X$ and $\omega \in \Omega$. By Hans [5], T_n has a unique random fixed point ξ_n. Define a sequence of mappings $G_n : \Omega \to K(X)$ and a mapping $G : \Omega \to K(X)$ by the same way as in the proof of Theorem 2.3. Then G is measurable and has a measurable selector ξ. This ξ is a random fixed point of T.

Theorem 2.5. *Let X be a compact and starshaped subset of a separable random convex metric space satisfying property (I). Let $F : \Omega \times X \to CK(X)$ be a nonexpansive random operator, then F has a random fixed point.*

Proof: Choose a starcentre p of X and a sequence $\{k_n\}$ of real valued measurable mapping such that $0 < k_n(\omega) < 1$ and $k_n(\omega) \to 1$ as $n \to \infty$. For each n, defined contraction random operator, $F_n : \Omega \times X \to CK(X)$ by $F_n(\omega, x) = W(\omega, F(\omega, x), p, k_n(\omega))$, then by Itoh [7] F_n has a random fixed point ξ_n. For each n, define $G_n : \Omega \to K(X)$ by

$$G_n(\omega) = cl\{\xi_i(\omega) : i \geq n\}.$$

Define $G : \Omega \to K(X)$ by $G(\omega) = \bigcap_{n=1}^{\infty} G_n(\omega)$. The mapping G is measurable by Himmelberg [6, Theorem 4.1]. Thus by Kuratowski and Ryll-Nardzewski [9], G has a measurable selector ξ. This selector ξ is the desired random fixed point of F. For each n, there exists $y_n \in F(\omega, \xi_n(\omega))$ such that $\xi_n(\omega) = W(\omega, y_n, p, k_n(\omega))$. It implies that $\{y_n\}$ converges to $\xi(\omega)$ and since F is continuous, it follows that $\xi(\omega) \in F(\omega, \xi(\omega))$ for each $\omega \in \Omega$.

Let (X, d) be a metric space. A random operator $f : \Omega \times X \to X$ is called *asymptotically regular* if for any $x \in X$ and $\omega \in \Omega$, $d(f^n(\omega, x), f^{n+1}(\omega, x)) \to 0$ as $n \to \infty$. A mapping $f : X \to X$ is said to *commute* with a mapping $F : X \to CB(X)$ if for each $x \in X$, $f(F(x)) \subset F(f(x))$. Also, a random operator is said to *commute* with a random operator $F : \Omega \times X \to CB(X)$ if for each $\omega \in \Omega$, $f(\omega, \cdot)$ and $F(\omega, \cdot)$ commute.

Theorem 2.6. *Let X be a compact starshaped subset of a separable random convex metric space satisfying property (I). Let $f:\Omega \times X \to X$ be a nonexpansive and asymptotically regular random operator, $F:\Omega \times X \to CK(X)$ be a nonexpansive random operator. Suppose f commutes with F, then there exists a common random fixed point of f and F.*

Proof: By Theorem 2.5, F has a random fixed point ξ_1. The mapping $\xi_2:\Omega \to X$ defined by $\xi_2(\omega) = f(\omega, \xi_1(\omega))$ is measurable by Himmelberg [6]. Since f and F commute, ξ_2 is a random fixed point of F. By induction, the sequence $\{\xi_n\}$ of mapping $\xi_n:\Omega \to X$ for which $\xi_{n+1}(\omega) = f(\omega, \xi_n(\omega))$ $(\omega \in \Omega,\ n = 1, 2, \ldots)$ are random fixed points of F. Define a sequence of mappings $G_n:\Omega \to K(X)$ and a mapping $G:\Omega \to K(X)$ by the same way as in the proof of Theorem 2.5. Then G is measurable and has a measurable selector ξ. This ξ is common random fixed point of f and F.

Remark 2.7.   With the notion of random convex metric space, our Theorems 2.5 and 2.6 generalize Theorems 3.4 and 3.6 of Itoh [8].

3. RANDOM BEST APPROXIMATIONS

The aim of this section is to prove some results regarding best approximation in random convex metric spaces.

A continuous function S from a closed convex subset C of a convex metric space X, into itself is said to be affine if $S(V(x,y,\lambda)) = V(Sx, Sy, \lambda)$ whenever $\lambda \in [0,1] \cap Q$ and x, y in C, where Q denotes, the set of rational numbers. Let (X,d) be a metric space and C be a nonempty subset of X. Suppose $x \in X$. An element $y \in C$ is called an element of best approximation of x (by the elements of the set C) if we have

$$d(x,y) = \inf_{z \in C} d(x,z).$$

We will denote by $P_c(x)$ the set of best C-approximations to x, that is, $P_c(x) = \{y \in C:\ d(x,y) = \inf_{z \in C} d(x,z)\}$ and boundary of C by ∂C.

Theorem 3.1. *Let X be a separable random convex metric space satisfying condition (I). Let $T, S:\Omega \times X \to X$ be two random operators, C a subset of X such that $T(\omega, \cdot):\partial C \to C$ and $x^* = T(\omega, x^*) = S(\omega, x^*)$ for each $\omega \in \Omega$. Further*

$$d(T(\omega,x),T(\omega,y)) \leq d(S(\omega,x),S(\omega,y)), \ and$$

$$d(S(\omega,x),S(\omega,y))$$

$$\leq \mu(\omega)max\{d(x,y),d(x,S(\omega,x)),d(y,S(\omega,y)),\frac{d(x,S(\omega,y))+d(y,S(\omega,x))}{2}\}$$

[where $\mu:\Omega \to [0,1)$ is a measurable map]

for all $x,y \in P_c(x^*) \cup \{x^*\}$ and $\omega \in \Omega$. Let S be continuous and affine on $P_c(x^*)$ and $S(\omega,T(\omega,x)) = T(\omega,S(\omega,x))$ for all $x \in P_c(x^*)$. If $P_c(x^*)$ is nonempty, compact, and q-starshaped with respect to $q = S(\omega,q)$ and $S(\omega,P_c(x^*)) = P_c(x^*)$, then there exists a measurable map $\xi:\Omega \to P_c(x^*)$ which is a common random fixed point of T and S.

Proof: If $y \in P_c(x^*)$, then for any $\omega \in \Omega$

$$d(T(\omega,y),x^*) = d(T(\omega,y),T(\omega,x^*))$$

$$\leq d(S(\omega,y),S(\omega,x^*))$$

$$= d(S(\omega,y),x^*).$$

Now using $S(\omega,P_c(x^*)) = P_c(x^*)$, we obtain that $T(\omega,y) \in P_c(x^*)$ for each $\omega \in \Omega$.

Let $\{k_n\}$ be a sequence of measurable mappings $k_n:\Omega \to (0,1)$ and $k_n(\omega) \to 1$ as $n \to \infty$. Define a random operator

$$T_n:\Omega \times P_c(x^*) \to P_c(x^*)$$

as $T_n(\omega,x) = W(\omega,T(\omega,x),q,k_n(\omega))$. Since S is affine and commutes with T on $P_c(x^*)$, we have for each $\omega \in \Omega$,

$$T_n(\omega,S(\omega,x)) = W(\omega,T(\omega,S(\omega,x)),S(\omega,q),k_n(\omega))$$

$$= W(\omega,S(\omega,T(\omega,x)),S(\omega,q),k_n(\omega))$$

$$= S(\omega,W(\omega,T(\omega,x),q,k_n(\omega)))$$

$$= S(\omega,T_n(\omega,x)).$$

Thus S commutes with T_n on $P_c(x^*)$ for each n and $T_n(\omega,P_c(x^*)) \subseteq P_c(x^*) = S(\omega,P_c(x^*))$ for each $\omega \in \Omega$. Furthermore for any $\omega \in \Omega$ and $x,y \in P_c(x^*)$,

$$d(T_n(\omega, x), T_n(\omega, y)) \leq k_n(\omega) d(T(\omega, x), T(\omega, y))$$

$$\leq k_n(\omega) d(S(\omega, x), S(\omega, y)).$$

By Beg and Shahzad [1, Theorem 3.2] there exists a measurable map $\xi_n(\omega)$ $= T_n(\omega, \xi_n(\omega)) = S(\omega, \xi_n(\omega))$ for each $\omega \in \Omega$. Define a sequence of mappings $G_n \colon \Omega \to K(P_c(x^*))$ and a mapping $G \colon \Omega \to K(P_c(x^*))$ by the same way as in the proof of Theorem 2.3. Then G is measurable and has a measurable selector ξ. Since $P_c(x^*)$ is compact, $\{\xi_n(\omega)\}$ has a subsequence $\xi_{n_j}(\omega) \to \xi(\omega)$. Now $\xi_{n_j}(\omega) = T_{n_j}(\omega, \xi_{n_j}(\omega))$. Since $k_{n_j}(\omega) \to 1$, therefore $\xi_{n_j}(\omega) \to T(\omega, \xi(\omega))$. Hence $T(\omega, \xi(\omega)) = \xi(\omega)$ for each $\omega \in \Omega$. The continuity of S further implies that

$$S(\omega, \xi(\omega)) = S(\omega, \lim_{n \to \infty} \xi_{n_j}(\omega)) = \lim_{n \to \infty} \xi_{n_j}(\omega) = \xi(\omega)$$

for all $\omega \in \Omega$.

A metric space X is called η-chainable if for every $a, b \in X$, there exists an η-chain that is a finite set of points $a = x_0 < x_1 < x_2 < \ldots < x_n = b$ (n may depend on both a and b) such that $d(x_{i-1}, x_i) < \eta$ ($i = 1, 2, \ldots, n$).

A random operator $T \colon \Omega \times X \to X$ is said to be *locally contractive* if for every $x \in X$ there exists $\epsilon > 0$ and a measurable map $\lambda \colon \Omega \to [0, 1)$ which may depend on x such that $p, q \in S(x, \epsilon) = \{y \in X \colon d(x, y) < \epsilon\}$ implies that $d(T(\omega, p), T(\omega, q)) \leq \lambda(\omega) d(p, q)$ for all $\omega \in \Omega$. A random operator T is called $(\epsilon, \lambda(\omega))$-uniformly locally contractive if it is locally contractive and both ϵ and $\lambda \colon \Omega \to [0, 1)$ do not depend on x.

Theorem 3.2. *Let X be a separable random convex η-chainable metric space satisfying (I) and $T \colon \Omega \times X \to X$ be a random operator. Let C be a $T(\omega, \cdot)$-invariant subset of X and x^* be a $T(\omega, \cdot)$-invariant point for each $\omega \in \Omega$. If $P_c(x^*)$ is nonempty, compact and p-starshaped and T is*

> (i) *continuous on $P_c(x^*)$,*
>
> (ii) *$d(x, y) \leq d(x^*, C)$ implies $d(T(\omega, x), T(\omega, y)) < d(x, y)$ for all $x, y \in P_c(x^*)$ and $\omega \in \Omega$.*

Then there exists a measurable map $\xi \colon \Omega \to P_c(x^*)$ such that for any $\omega \in \Omega$ $T(\omega, \xi(\omega)) = \xi(\omega)$.

Proof: Similar to the proof of Theorem 3.1, only need to notice that the corresponding T_n are uniformly locally contractive random operators and

have random fixed points ξ_n from Beg and Shahzad [2, Theorem 5.1].

We recall that an operator $T: X \rightarrow X$ is *compact*, if for any bounded subset S of X, $Cl(T(S))$ ic compact. If an operator $T: X \rightarrow X$ leaves subset Y of X invariant, then a restriction of T to Y will be denoted by the symbol T/Y.

Theorem 3.3. *Let X be a separable random convex metric space satisfying property (I) and $T: \Omega \times X \rightarrow X$ be a nonexpansive random operator. Let C be a $T(\omega, \cdot)$-invariant subset of X and $T(\omega, \cdot)/C$ be compact and x^* be a $T(\omega, \cdot)$-invariant point for each $\omega \in \Omega$. If $P_c(x^*)$ is nonempty, convex and compact, then there exists a measurable map $\xi: \Omega \rightarrow P_c(x^*)$ such that $T(\omega, \xi(\omega)) = \xi(\omega)$ for each $\omega \in \Omega$.*
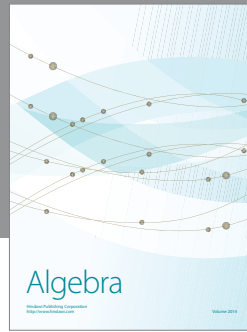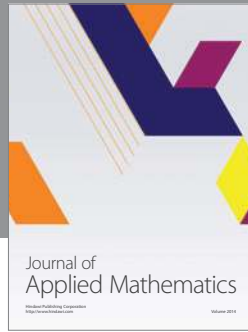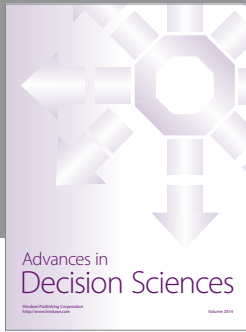
Proof: The set $P_c(x^*)$ is $T(\omega, \cdot)$-invariant, closed and convex. Since $P_c(x^*)$ is bounded subset of c and $T(\omega, \cdot)/C$ is compact, closure of $(T(\omega, P_c(x^*)))$ is compact for each $\omega \in \Omega$. Theorem 2.3 implies that T has a random fixed point ξ in $P_c(x^*)$.

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