

M.I. Freidlin A.D. Wentzell

Random Perturbations of Dynamical Systems

Second Edition

Translated by Joseph Szücs

With 33 Illustrations



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