

Random Probability Measures on Polish Spaces

Hans Crauel

Department of Mathematics,
Technical University of Ilmenau,
Germany



London and New York

Contents

Preface	vii
1 Notations and Some Technical Results	1
Notations	1
Measurability and Completion	2
2 Random Sets	7
Closed and Open Random Sets	8
The Selection Theorem	10
The Projection Theorem	13
Tightness of Compact Random Sets	14
Comments	16
3 Random Probability Measures and the Narrow Topology	17
Random Probability Measures: Definition and Basic Properties . . .	17
Random Continuous Functions	21
Definition of The Narrow Topology	25
The Portmenteau Theorem	26
The Narrow Topologies on Deterministic and on Random Measures	29

4 Prohorov Theory for Random Probability Measures	33
Tightness for Random Measures	33
Random Lipschitz Functions	36
The Stone–Daniell Characterisation of Random Measures	40
A Criterion for Compactness in the Narrow Topology on Random Measures	43
Metrisability of the Narrow Topology	46
Conditional Expectation for Random Measures	57
The Prohorov Theorem for the Narrow Topology on Random Measures	63
5 Further Topologies on Random Measures	67
Topologies on Random Variables in Metric Spaces	67
Topologies on Random Measures	71
Comparison of Different Topologies	73
6 Invariant Measures and Some Ergodic Theory for Random Dynamical Systems	81
Random Dynamical Systems	82
Invariant Measures	86
Invariant Markov Measures	89
Ergodicity and Extremality of Invariant Measures	92
Oxtoby Theory for Random Dynamical Systems	94
Appendix A The Narrow Topology on Non-Random Measures	99
Appendix B Scattered Results	105
Bibliography	113
Index	117