www.s2is.org
International Journal on Smart Sensing and Intelligent Systems
ISSN 1178-5608

RANDOM SIGNAL FREQUENCY IDENTIFICATION BASED ON AR MODEL SPECTRAL ESTIMATION

Chunhuan Song
School of Electronic and Information Engineering
Xi'an Technological University
Xi'an, 710021, China
Email: schuan1990@sina.com

Submitted: Jan. 6, 2016 Accepted: Mar. 22, 2016 Published: June 1, 2016

Abstract-The power spectral estimation is an important element in the random signal analysis. The paper will introduce the principles of the classical power spectral estimation and modern power spectral estimation, analyses their characteristics and application in MATLAB simulation. The variance obtained by the classical power spectral estimation is inversely proportional to its resolution, the resolution of the modern spectral estimation are not subject to this restriction, but also the variance achieve greatly improvement, which is a great importance for improving the accuracy of the power spectral estimation. This paper mainly studies AR model of parametric modeling in the modern spectral estimation, and then uses the simulation between the classical power spectral estimation and modern power spectral estimation for comparison, verifies the analysis of the modern power spectral estimation based on ARmodel is more accurate than the classical power spectral estimation.

Index terms: AR model; power spectral estimation; Burg algorithm.

I. INTRODUCTION

Signal is usually divided into two categories: energy signal and power signal, generally, the Fourier transform of the signal energy is convergent, and power signal is usually not convergent. The signal is information carrying tool, information and randomness are closely related, so the majority signals in the daily life are the random signals, its main feature is that the energy of sample is infinite. In other words, most of random signals are power signal instead of energy signal, therefore its Fourier transform does not exist.

Due to the value of the random signal determined at each point in time is not a priori , and each of its samples is different, the random signal is not like the determinate signal what can use mathematical expressions or an exact chart to represent itself, except for using its various statistical average quantities. Among them, the auto-correlation as a function of time is the most complete characterize its specific statistical average value. The power spectral density of a random signal is the Fourier transform of the auto-correlation function [1].

For a random signal, its Fourier Transforms do not exist, and can only use the power spectral density to characterize its statistical average spectral characteristics, so the power spectral density of a random signal is one of the most important forms of representation, if we want to know a random signal in a statistical significance, the power spectral density is very important.

Power signal spectral describes the relationship between frequency and signal power; the peak in the power spectral is the representation of periodic components in the signal and the stationary signal power spectral density is just the Fourier transform of the auto-correlation function. Power spectral can estimate the variation of relationship between the power and frequency of the received signal by means of the signal correlation, due to the practical random signal only is an implementation or a fragment of the sample sequence, the content of power spectral estimation is how to estimate the auto-correlation function or power spectral density of the signal on the basis of the sequence of finite-sample [2]. The classification of the Power spectral estimation is shown in figure 1.

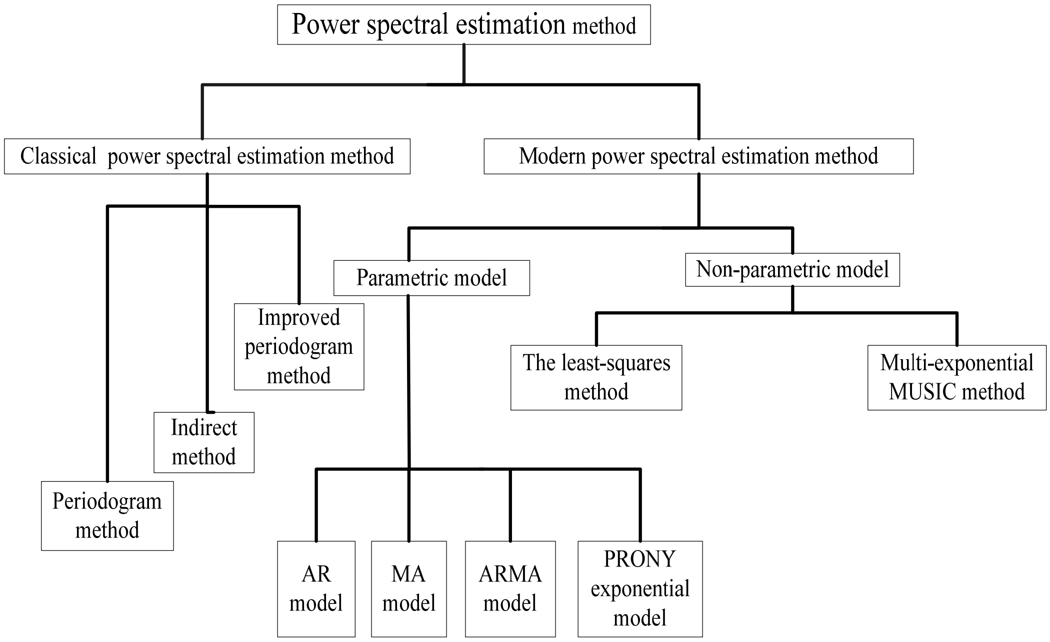

Figure 1 The classification of the power spectral estimation

The variance of the power spectra obtained by classical power spectral estimation is inversely proportional to its resolution, resolution of the modern spectral estimation are not subject to this restriction, but also the variance properties greatly improved, which of great importance for improving the accuracy of power spectral, therefore this paper mainly studies AR model of parametric modeling in the modern spectral estimation[3].

This paper makes a comparison of respective principles, respective characteristics and the MATLAB simulation between classical power spectral estimation and modern power spectral estimation; find the best method of power spectral estimation.

II.PRINCIPLE

The power of the random signal reflect the law of the power changes with time, and the power spectral of random signal reflect the distribution situation of its energy in frequency domain, the mathematics method of studying random signal power spectral is called power spectral estimated technology, power spectral estimation is one of the main content of the digital signal processing, it mainly deal with the signal various features in the frequency domain, and then depend them on extracting the useful signal in the frequency domain from the noise. This method is widely used and has played an important role in radar, sonar, communication, exploration, vibration analysis, and bio-medical, astronomical and so on [4].

Power spectral estimation is based on the theory of Wiener-Khinchine theorem.

$$P_{xx}(\omega) = \sum_{m=-\infty}^{\infty} \phi_{xx}(m) e^{-j\omega m} \tag{1}$$

Among them, $P_{xx}(\omega)$ is the power spectral density, m for the time difference, $\phi_{xx}(m)$ isa auto-correlation sequence.

While the experiment is making use of the fast Fourier transforms (FFT)

$$P_{xx}(k) = \sum_{m=0}^{N-1} \phi_{xx}(m) W_N^{kn} \tag{2}$$

And, N is the sampling number.

There are four parameters as the quality assessment of power spectral estimation:

(1)Bias: If the bias is equal to 0, this estimate is unbiased estimation.

$$Bia[\hat{\phi}] = E[\hat{\phi} - \phi] = E[\hat{\phi}] - \phi \tag{3}$$

(2) Variance: It refers to the fluctuation of the signal and reflects the signal stability.

$$Var[\hat{\phi}] = E\{[\hat{\phi} - E(\hat{\phi})]^2\} \tag{4}$$

(3) Standard deviation: If it is equal to 0, this estimate is consistent estimation.

$$MSE[\hat{\phi}] = E\{[\hat{\phi} - \phi]^2\} = Var[\hat{\phi}] + Bia^2[\hat{\phi}] \tag{5}$$

(4) Resolution: It refers to the sharpness of the peaks in the power spectral and reflects the intensity of ability that the power spectral estimation identify signal.

We usually will regard resolution and variance as main evaluation parameters of the power spectral estimation.

III.CLASSICAL POWER SPECTRAL ESTIMATION

Classical power spectral estimation is divided into periodogram method (direct method), and indirect method and improved periodogram method, their principles and characteristics are given below. Both periodogram method and indirect method contains two important prerequisites:

Prerequisite one: the random signal not only is the stationary random signal, but also is ergodic;

Prerequisite two: must do the necessary pre-processing before power spectral estimation (A/D, DC, filter, etc.).

A. Indirect method of power spectral estimation

The indirect method was proposed in 1958, it is also known as relations diagram method, this is due to the principle of this method is a sampling sequence of the random signal getting its auto-correlation sequence by the auto-correlation operation[5].

$$\hat{\phi}_{xx}(m) = \frac{1}{N}\sum_{n=0}^{N-1} x(n)x(n+m) \quad |m| \le N-1 \tag{6}$$

Then,formula (6) carrying out a Fourier transform ,we can obtain the power spectral.

$$\hat{P}_{xx}(\omega) = \sum_{m=-M}^{M} \hat{\phi}_{xx}(m)e^{-j\omega m} \qquad M \le N-1 \tag{7}$$

In order to understand estimated results of the indirect method, let’s analyze its deviation and variance.

$$Bia[\hat{\phi}_{xx}(m)] = E[\hat{\phi}_{xx}(m)\ ] - \phi_{xx}(m) = -\frac{|m|}{N}\phi_{xx}(m) \tag{8}$$

$$Var[\hat{\phi}_{xx}(m)] = \frac{N-|m|-|i|}{N^2}\sum_{i=-(N-1-|m|)}^{N-1-|m|}[\phi_{xx}^2(i) + \phi_{xx}(i+m)\phi_{xx}(i-m)] \tag{9}$$

$$\lim_{N\to 0} MSE[\hat{\phi}] = \lim_{N\to 0} Var[\hat{\phi}] + \lim_{N\to 0} Bia^2[\hat{\phi}] = 0 \tag{10}$$

Form formula (8), (9) and (10), we can get that the indirect method is asymptotically unbiased estimates, meanwhile it meeting consistent estimation. But this algorithm uses correlation functions, when sampling number N is large, the computation increases [6].

B. Direct method of the power spectral estimation

Direct method also known as the periodogram method, the concept of periodogram was first proposed in 1899, the method of power spectral estimation that adopted the periodogram (including the smoothed periodogram) could perform FFT calculation, which has high-efficiency advantage, so this method commonly used where do not ask for high resolution. Its main disadvantage is the low frequency resolution; this is because the direct method regards the data of except for the finite N data as 0 in the calculation process [7]. This is clearly deviate form reality. Regard the unknown data as 0, this not only equivalent that multiply by a rectangular window function in the time domain, but also is equal to the convolution with Sinc function in the frequency domain. Because the Sinc function has two characteristics, one of which is its main lobe is not infinitely narrow, another is it has the side-lobe, thus the convolution invariably results in distortion.Due to main lobe is not infinitely narrow, if the original power spectral is narrow and making the convolution with the main lobe, which will lead to power spread to near frequency domain, signal become fuzzy, resolution reduced, and main lobe wider resolution worse. There are two consequences result from the main lobe, one of which is the energy in the main lobe will "leak" to the side lobes and the variance will increase, another of which is the power spectral that making convolution with side lobe entirely belong to the interference. If the convolution of strong signal and side lobes may be larger than the weak signal and main lobe, the weak signal will be submerged by the interference of strong signal and then it cannot be detected [8]. These are two main weaknesses of the direct method estimate the power spectral.

Because the direct method regards n observed data of the random signal as a finite energy signal, directly performs the Fourier transform, then takes the square of the amplitude and divided by N, the result is the signal's power spectral estimation. So power spectral estimation is indicated by $P_{xx}(\omega)$, and it shall be calculated in accordance with the following formula:

$$P_{xx}(\omega) = \frac{1}{N}\left|X_N(\omega)\right|^2 = \frac{1}{N}\left|\sum_{n=0}^{N-1} x(n)e^{-j\omega n}\right|^2 \tag{11}$$

Since 1965, after fast Fourier Transforms (FFT) appeared, this method was often used in the power spectral estimation, and it shall be calculated in accordance with the following formula:

$$P_{xx}(k) = \frac{1}{N}|X(k)|^2 \tag{12}$$

In order to understand estimation results of the direct method, let's analyze its deviation and variance.

$$Bia[\hat{P}_{xx}(\omega)] = E[\hat{P}_{xx}(\omega)] = \frac{N-|m|}{N}\sum_{m=-(N-1)}^{N-1}\phi_{xx}(m)e^{-j\omega n} \tag{13}$$

$$Var[\hat{P}_{xx}(\omega)] = \sigma_x^4\{1+[\frac{\sin(\omega N)}{N\sin\omega}]^2\} \tag{14}$$

$$MSE[\hat{\phi}] = Var[\hat{\phi}] + Bia^2[\hat{\phi}] \neq 0 \tag{15}$$

According to the formula (11), it can be known that when n tends to infinite, the calculation is neither has average nor has limit, it can only be seen as a sample that performing a mean value calculation on the real spectral[9]. When determining signal length, if lack of the statistical averaging, to ensure a high resolution, variance of the power spectral estimation will become large, it is not the consistent estimates of the real spectral.

The direct method is unbiased estimation, but does not meet the consistent estimation, if onlywe smooth the periodogram, which will reduce the variance(average is a major method of smoothing) and obtain the consistent spectral estimation. There are two main smooth methods, usually adopt the window function to smooth before the FFT appeared and have been widely used, select the appropriate window function to accelerate the convergence of weighted mean as a weighting function. Another method is dividing equally the periodogram, that it,firstly segment the data, then the average of the periodogram[10]. The latter is also known as Bartlett algorithm, which had been extensively used for thesmoothing method. Welchalgorithm is the improvement of Bartlett algorithm and proposed the specific calculation on FFT.

C. The improved algorithm of the direct method

The spectral resolution of the direct method is high, but variance performance is poor and power spectral fluctuation violent, it is prone to product the false peak, so it is necessary to improve this method. Bartlett algorithm is a modified periodogrammethod, its guiding principle is to firstly put a data segment of length N, and respectively calculate each segment of the power spectral, then average the data, that be able to achieve the desired wish[11].Bartlett algorithm is as follows:

(1) Given or get a random signal sampling sequence, divide it into p segments, each segment is m points.

(2) Then calculate each periodogram:

$$I_P(\omega) = \frac{1}{M}\left|\sum_{n=0}^{M-1} X_P(n)e^{-j\omega n}\right|^2 = \frac{1}{M}\left|X_P(e^{j\omega})\right|^2 \tag{16}$$

(3) Finally, the average of each periodogram

$$B_x(\omega) = \frac{1}{P}\sum_{p=0}^{p-1} I_p(\omega) \tag{17}$$

With the Bartlett algorithm to estimate the variance of power spectral:

$$Var[B_x(\omega)] = \frac{1}{P}\sigma_x^4\{1 + [\frac{\sin(\omega N)}{N\sin\omega}]^2\} \tag{18}$$

The variance of formula (18) and (14) in comparison, we can see that the section p of the spectral variance obtained by Bartlett algorithm greater, its variance smaller. This is as a result of the data segment, each piece of data reduce and spectral leakage effect increase, reduce the spectral resolution[12].

Welch method is the improvement of Bartlett method. Major improvements in two respects: one is that the segmentation is allowing some overlap in each piece of data; the second is that every data window is not necessarily a rectangular window. That can improve the effect of poor resolution caused by rectangular window. Then we can accord the Bartlett method to find out every piece of the power spectral, and the result is normalized, thereby we can get the further modified periodogram. Bartlett algorithm is as follows:

(1) Given or get a random signal sampling sequence, divide it into p segments, each segment is m points;

(2) Each piece of data is processed by Windows $x_p(n)w(n)$;

(3) Calculated for each modified periodogram:

$$I_p(\omega) = \frac{1}{MU}\left|\sum_{n=0}^{M-1} x_p(n) w(n) e^{-j\omega n}\right|^2 \tag{19}$$

$$U = \frac{1}{M}\sum_{n=0}^{M-1} w^2(n) \tag{20}$$

(4) Finally, it should average over each modified periodogram:

$$B_x(\omega) = \frac{1}{P}\sum_{p=0}^{p} I_p(\omega) \tag{21}$$

Formula (19): U is a normalization factor.

Because the Welchmethod allows overlap, thereby increasing the number of p, so it can better improve the variance characteristic. However, the data overlaps cause each paragraph of uncertainty is reduced, reduced the variance will not reach the level of theoretical calculations. In addition, choosing the right window function can reduce spectral leakage and improve resolution [13]. In addition, the differences of window function and window width will result in the change of its main lobe width as well as the side-lobe attenuation rate, thus the spectral resolution is not the same. I will verify these factors through the MATLAB simulation below.

Many scholars tried to select the appropriate window function to improve the spectral resolution of the classical methods, but finally they found that the reduction of the side-lobe width at the expense of the growth of the main lobe width; and vice versa. In any way, these two shortcomings can only be counter balance, but not be improved meanwhile. Therefore, the classical methods can not able to overcome the shortcoming of low resolution [14]. In recent years, many experts and scholars put forward many new methods to improve the resolution of the power spectral estimation.

IV.AR MODEL AND ITS SOLUTION

A. AR model

The variance of the periodogram method is poor, which prompted the people to study others methods. The classical spectral estimation technology requires the measured signal is a stationary random signal, the signal may be truncated and the signal of out of truncated range is treated as zero. But in real life a lot of random signals are very difficult to meet the requirements of stability. Furthermore, the classical power spectral estimation with FT as core to estimate the power spectral, its main defect is a lack of time orientation. Contradictory, the resolution and variance got by the classical power spectral estimation techniques, and one of them is to be improved at the expense of another, so that inevitably lead to the emergence of these shortcomings[15].

On the basis of the observed data, we can choose the proper model and regard $x(n)$ as a white noise generated by this model, so you don't have to think the data outside of N is 0, and then maybe get a better estimation.This method can be divided into the following steps:

(1) Selects a model;

(2) Uses the known data to determine the model parameters;

(3) Calculates the power spectral estimation by parameters.

AR model belongs to the modern power spectral estimation and the reason that the modern power spectral estimation was proposed and gradually developed is due to the performance about variance and resolution of the classical spectral estimation is poor, which can be divided into parameters model spectral estimation and non-parametric model spectral estimation[16]. The non-parametric model spectral estimation mainly includes MTM method, MUSIC method, feature vector method and so on. The parameter model spectral estimation mainly has AR model, MA model, ARMA model, PRONY model and so on, among which the AR model is the most used. The three commonly used models are AR model, MA model and ARMA model, and the following three models are compared:

(1) ARMA model: An Auto regressive moving average model is one of high resolution spectral analysis method in the model parameters method. This is a typical method on stationary random process rational spectral method research and it can be applied in a large class of problems.

Sets up a discrete-time linear system, the input u(n) is a white noise sequence of its average is zero and its variance is σ, and the output isx(n), the relationship between output and input of discrete-time linear systems can be shown with a differential equation as formula (22):

$$x(n)+\sum_{k=1}^{P} a_k x(n-k)=\sum_{r=0}^{M} b_r u(n-r) \tag{22}$$

And then use the z-transform, its system functions is shown as follows:

$$H(Z)=\frac{X(Z)}{U(Z)}=\frac{\sum_{r=0}^{M} b_r Z^{-r}}{1+\sum_{K=1}^{P} a_k Z^{-k}} \tag{23}$$

In the formula (22), X(Z) is the Z-transform of the output signal, U(Z) is Z-transform of the input signal, and this formula is known as ARMA model. Once the parameters of ARMA (P, M) model are determined, the power spectral estimation will be obtained. ARMA model method has more accurate spectral estimation and better spectral resolution than AR model method and MA model method, however, not only its parameter estimation is more tedious but optimal parameter estimation methods has two shortcomings about the computation amount is large and it makes no guarantee the parameter converge, so AR model is more practical than the ARMA model[17].

(2) MA model: moving average model is one of the spectral analysis method of model parameter method, is also the modern used in the modern spectral estimation.

Sets up a discrete-time linear system, the input u(n) is a white noise sequence of its average is zero and its variance is σ, and the output is x (n), the relationship between output and input of discrete-time linear systems can be shown with a differential equation as formula (24):

$$x(n)=\sum_{r=0}^{M} b_r u(n-r) \tag{24}$$

And then use the Z-transform, its system functions is shown as follows:

$$H(Z)=\frac{X(Z)}{U(Z)}=\sum_{r=0}^{M} b_r Z^{-r} \tag{25}$$

In the formula (25), X(Z) is the Z-transform of the output signal, U(Z) is Z-transform of the input signal, $b_r\,(r=0,\ldots,M)$ is a factor, and this formula is known as ARMA model.

In the ARMA model spectral estimation, most of parameters need to estimate the AR parameters, and then estimate the MA parameters on the basis of the AR parameters, so MA model is often used as a process on calculating the ARMA power spectral estimation.

(3) AR model is a linear forecast based on periodogrammethod; it can deduce the data before and after the Nth point by model and known data. so its nature is similar to interpolation, which aim at increasing the effective data. They only difference is AR model deduce data by N points and interpolation is by two points, therefore AR model is better than interpolation [18].

The physical meaning of the AR model is the system response of the white noise acts on a linear time-invariant system. N-order AR model difference equation:

$$y(n) = -\sum_{k=1}^{N} a_k y(n-k) + x(n) \tag{26}$$

And then use the Z-transform, its system functions is shown as follows:

$$H(Z) = \frac{Y(Z)}{X(Z)} = \frac{1}{1-\sum_{k=1}^{N} a_k Z^{-k}} \tag{27}$$

On the basis of $P_{yy}(\omega) = P_{xx}(\omega)\left|H(e^{j\omega})\right|^2$, so the power spectral estimation of $y(n)$ is that:

$$P_y(\omega) = \sigma_w^2 \frac{1}{\left|1-\sum_{k=1}^{N} a_k e^{-j\omega k}\right|^2} \tag{28}$$

Because during the translation of any ARMA model or MA model in the finite variance may be expressed by the infinite-order AR model, if we choose a model which is not matched with the signal in these three models, we can still get a better approximation by using the low order. The estimation of AR model parameters can obtain a linear equation, so the AR model is better than the ARMA model and MA model in the calculation. Meanwhile, the practical physical systems tend to be all-pole system, so in the research that the model of the rational fraction transfer function, mainly research the AR model and its practical application is more extensive.

B. Solving YULE-WALKER equation

If we want to get the power spectral estimation $P_y(\omega)$, it is necessary to know two relevant parameters a_k (k=1，2，3，...N) and ,so power spectral estimated by model method in actually just is solving the model parameters. It is necessary for obtaining the model parameters to solve the Nth order NYULE-WALKER equation and NYULE-WALKER equation is as follows:

$$\begin{bmatrix} \phi_{xx}(0) & \phi_{xx}(1) & \phi_{xx}(2) & .....\phi_{xx}(N) \\ \phi_{xx}(1) & \phi_{xx}(0) & \phi_{xx}(1) & .....\phi_{xx}(N-1) \\ ... & ... & ... & ... \\ \phi_{xx}(N) & \phi_{xx}(N-1) & \phi_{xx}(N-2) & .....\phi_{xx}(0) \end{bmatrix} \begin{bmatrix} 1 \\ a_1 \\ ... \\ a_N \end{bmatrix} = \begin{bmatrix} \sigma_w^2 \\ 0 \\ ... \\ 0 \end{bmatrix} \tag{29}$$

Directly solving the Yule-Walker equation requires the inverse matrix operations, its shown that when N is large, it has large amount of calculation, and when the model order is increased, the matrix dimension also increases, therefore it need to calculate once again[19]. For this problem, the auto-correlation method is providing a efficient solution for Yule-Walker equation.

(1) The steps of solving AR parameters by the auto-correlation method:

First step: estimate the auto-correlation coefficient matrix of the observation sequence;

Second step: use Lenvinson-Durbin recursive method to solve AR model parameters.

Levinson-Durbin recursive method solve the equations by the backward forecasting method, whose aims at obtaining the next order parameter based on the recursive formula of the previous order parameter and the calculate complexity from $o(P^3)$ down to $o(p^2)$. It should be noted that the first subscript of “a” refers to the order of AR model, finally the solution of p order solution is just the practical answer. Algorithm steps are as follows:

① According to $\begin{bmatrix} \phi(0) & \phi(1) \\ \phi(1) & \phi(0) \end{bmatrix} \begin{bmatrix} 1 \\ a_{11} \end{bmatrix} = \begin{bmatrix} \sigma_1^2 \\ 0 \end{bmatrix}$, a_{11} and σ_1^2 can be calculated:

$$a_{11} = -\frac{\phi(1)}{\phi(0)}, \sigma_1^2 = (1-|a_{11}|^2)\sigma_0^2 \tag{30}$$

② a_{22} and σ_2^2 will be calculated by putting a_{11} and σ_1^2 into the following formula:

$$a_{pp} = -\frac{\phi(p) + \sum_{k=1}^{p-1} a_{p-1,k}\phi(p-k)}{\sigma_{p-1}^2} \tag{31}$$

$$a_{pk} = a_{p-1,k} + a_{pp} a_{p-1,p-k} \tag{32}$$

$$\sigma_p^2 = (1-|a_{pp}|^2)\sigma_{p-1}^2 \tag{33}$$

③ The rest can be done in the same manner, we can calculate the parameter a_k (k=1,2,3,...N) and σ_w^2.

Levinson-Durbin recursive algorithm for solving the Yule-Walker equation start from the lower order to p order, it can calculate all of the parameters of each order, which will help choose the practical order of AR models. AR parameter can simplify the calculation, but actually if we want to get the auto-correlation sequence only from the finite data of the time series, which is equal to add a window in the sequence [15]. Window will result in lower frequency resolution of auto-correlation method. When the time sequence is shorter, the estimation error of Levinson-Durbin recursive algorithm is bigger, it will produce severe error to calculation of the AR parameter, so that lead to producing any bad phenomena such as the splitting of spectral lines and the spectral peak deviation.

(2) Burg recursive method is not need to auto-correlation functions, and it is based on linear prediction and it can achieve the prediction error filter by the lattice structure.In the transfer process from left to right, we can get forward and backward prediction errors of different orders at the same time, and then calculate each order reflection coefficient, finally we can calculate AR parameters by the reflection coefficient. In the Burg algorithm, $e(n)$ is the error, the subscript donates the order, $e_m^f(n)$ and $e_m^b(n)$ is respectively the forward error and backward error and the concrete steps in the following:

① According to $e_0^f(m) = e_0^b(m) = x(n)$, k_m can be calculated:

$$k_m = -\frac{2\sum_{n=m}^{N}[e_{m-1}^f(n) + e_{m-1}^b(n-1)]}{\sum_{n=m}^{N-1}\left\{e_{m-1}^f(n)] + [e_{m-1}^b(n-1)]\right\}} \tag{34}$$

② $e_m^f(n)$ and $e_m^b(n)$ will be calculated by the following formula:

$$e_m^f(n) = e_{m-1}^f(n) + k_m e_{m-1}^b(n-1) \tag{35}$$

$$e_m^b(n) = e_{m-1}^b(n) + k_m e_{m-1}^f(n-1) \tag{36}$$

③ $a_m(i)$ will be calculated by the following formula:

$$a_m(i) = a_{m-1}(i) + k_m a_{m-1}(m-i), i = 1,2,..., m-1 \tag{37}$$

④ ρ_m will be calculated by the following formula:

$$\rho_m = \rho_{m-1}(1-k_m^2)\ , \sigma_w^2 = \rho_p \tag{38}$$

⑤ The rest can be done in the same manner, we can calculate the parameter a_k (k=1,2,3,...N) and σ_w^2.In General, the order is not known in advance, we should choose order p=k when the k-th order meet the allowed data.

Comparison between auto-correlation algorithm and Burg algorithm, it can be find that auto-correlation algorithm calculation is simple, but resolution of the power spectral estimation is poor. Burg algorithm is based on the data sequence, it avoid the calculation of auto-correlation function, so Burg algorithm has better frequency resolution than the auto-correlation algorithm. Because Burg algorithm has smaller computational complexity and better spectral estimation quality,is more general of method, so it is wider use than auto-correlation algorithm[11].

From the above two parameters calculation algorithm, It can be shown that the signal modeling nature of AR model is actually using the linear prediction, so in the the power spectral estimation, the data out of the sampling data can extrapolate by way of predicting, effectively avoid the spectral leakage effect of the classical spectral estimation algorithm in data windowing truncation, so it will be good to improve the resolution in the power spectral estimation.

V.MATLAB SIMULATION

Based on the above analysis, this random signal can be selected as the research object:

$$x(n) = 3\sin(2\pi 200n) + \sin(2\pi 230n) + randn(n) \tag{39}$$

The randn(n) is the random array with the normal distribution, next we can develop MATLAB simulation to estimate the power spectral, and then intuitively show the comparison of pros and cons between the classical and modern power spectral estimation method. The sampling frequency Fs=1000 Hz, the sampling points is equal to 512, the rectangle window in the Bartlett algorithm uses the window width is 100 and the overlap is 0, the hamming window in the Welch algorithm uses the window width is 100 and the overlap is 50, the order in the Burg algorithm is 30. The simulation results are shown in figure 2. It is show that the time domain simulation diagram of the random signals in (a); It is show that the simulation diagram of the indirect method of the classical power spectral estimation in (b); It is show that the simulation diagram of the periodogram method of the classical power spectral estimation in (c); It is show that the simulation diagram of the Bartlett algorithm improved by the periodogram method in (d); It is show that the simulation diagram of the Welch algorithm improved by the periodogram method in (e); It is show that the simulation diagram of the burg algorithm of the AR model in (f):

Figure 2 illustrates the comparison of simulation results with different power spectral estimation method, it can be found that the fluctuation amplitude of the curve in (b) is small, but the resolution is poor and the wave crest is not obvious; The resolution in (c) is better (b), but the fluctuation amplitude of the curve in (c) is stronger than (b), the variance is poor and it is prone to produce the false crest; the Bartlett algorithm in (d) and the Welch algorithm in (e) are the modified periodogram method, but them only improve the variance and apparently the improvement at the cost of reduced resolution; Above these algorithms not only cannot distinguish two signals frequency, but the resolution and variance are contradictory. Compared with above several methods, the AR model of the modern power spectral estimation not just improved resolution, which is easy todistinguish two signals frequency is 200Hz and 230Hz and even can

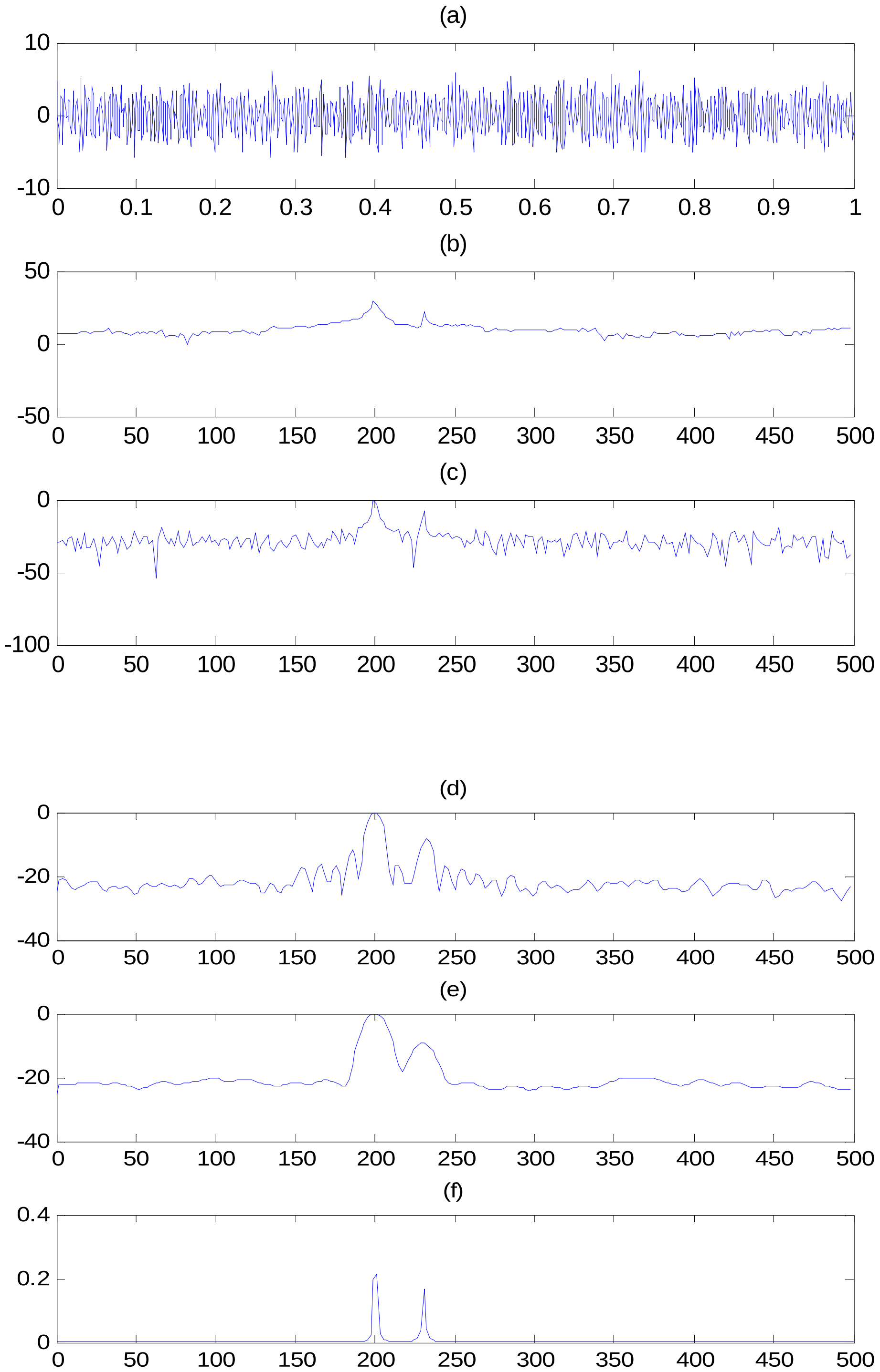

Figure 2 Comparison of simulation results with different power spectral estimation method

get the amplitude relationship of two signal, but alsothe variance is quite small andthe remaining invalid noise or disturbance would be barely noticeable. And the Burg algorithm is not need to calculate the auto-correlation function, thus it has an advantage over classical spectral estimation method. In the treatment of short data or the requirement of variance and the resolution is high, using the AR model parameter method, in particular the Burg algorithm has an advantage over the others[20].

Due to the difference of windows for the Welch algorithm will changes the variance and resolution of the power spectral estimation, and then through the MATLAB simulation for comparison. Just the same random signal is used, whose width is 100 and overlap is 50, and then develops the MATLAB simulation by selecting different window function. The simulation results are shown in figure 3, the analysis and comparison of the windows for the Welch algorithm show: (a) the Hamming window, (b) the Kaiser window, (c) the Blackman window, (d) the Chebyshev window.

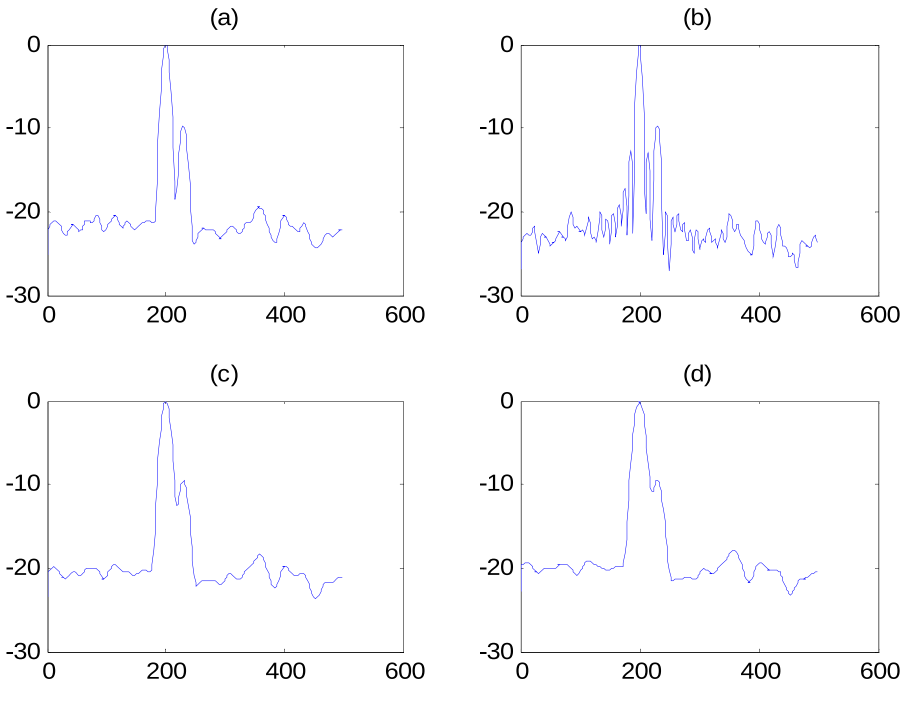

Figure 3 Comparison of the windows for the Welch algorithm

Figure 3 shows the comparison of windows for the Welch algorithm, the fluctuation amplitude of the curve in (c) and (d) are small and the variance well, but the resolution is poor and the wave crest is not obvious; The resolution of (a) and (b) is better (c) and (d), but the fluctuation amplitude of the curve on the contrary, the variance is poor and it is prone to produce the false crest. Therefore the fact can be found that the different windows indeed can affect the performance of the Welch algorithm. If the result of higher resolution is required, the Kaiser window is a better choose; If the result of higher variance is required, the Chebyshev window is a better choose, whereas it is clear that changing the shape of the windows cannot solve the problem of variance and resolution are contradictory.

Due to the difference of window width for the Welch algorithm will changes the variance and resolution of the power spectral estimation, and then through the MATLAB simulation for comparison. Just the same random signal is used, whose the overlap is 50 and hamming window will be selected, and then develops the MATLAB simulation by selecting different window width. The simulation results are shown in figure 4, the analysis and comparison of the window width for the Welch algorithm show: (a) 256, (b) 150, (c) 80.

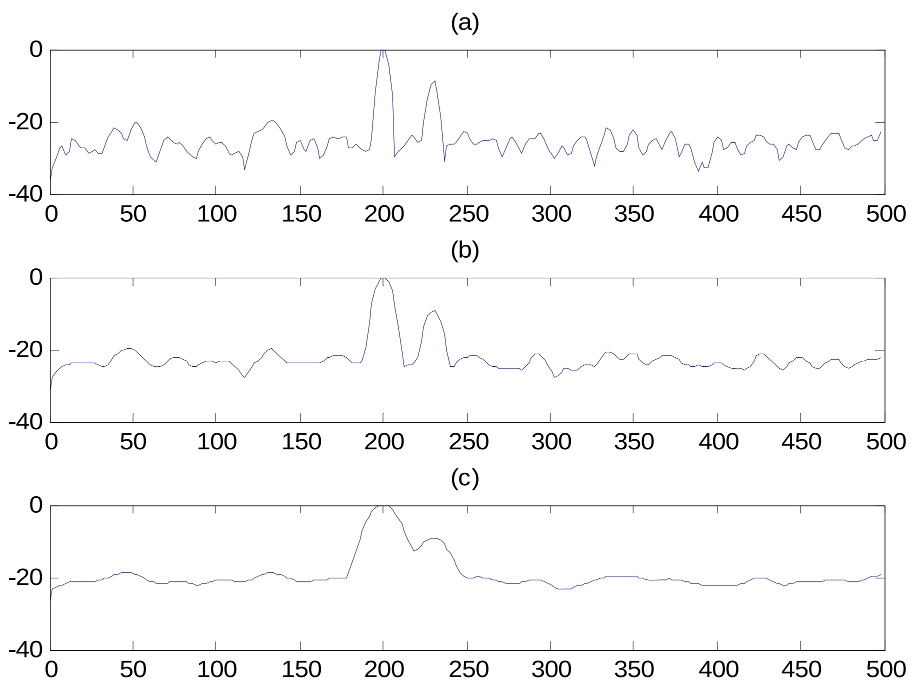

Figure 4 Comparison of the window width for the Welch algorithm

Figure 4 shows the comparison of window width for the Welch algorithm. Along with the window width growing, the resolution is becoming better, but the fluctuation amplitude of the curve on the contrary, the variance is poor and it is prone to produce the false crest. So the fact can be found that the different window width indeed can affect the performance of the Welch algorithm. If the result of higher resolution is required, the wider window is a better choose; If the result of higher variance is required, the narrow window is a better choose, However it is obvious that changing the window width can also not solve the problem of variance and resolution are contradictory.

The difference of the overlap for the Welch algorithm will change the variance and resolution of the power spectral estimation, and then through the MATLAB simulation for comparison. Just the same random signal is used, who's the window width is 50 and hamming window will be selected, and then develops the MATLAB simulation by selecting different overlap. The simulation results are shown in figure 5, the analysis and comparison of the overlap for the Welch algorithm show: (a) 80%, (b) 40%, (c) 0.

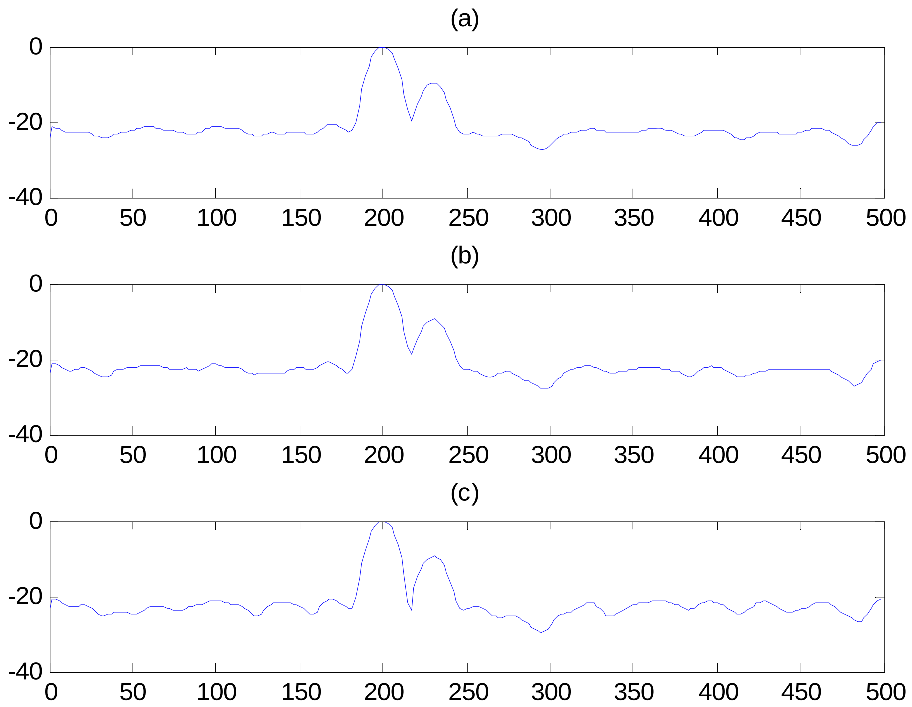

Figure 5 Comparison of the overlap for the Welch algorithm

Figure 5 shows the comparison of the overlap for the Welch algorithm. It can be found that the overlap affect the Welch algorithm a little.

According to the simulation results of figure3, 4 and 5, it can concluded that no matter how to change above factors of the Welchalgorithm, the contradiction between variance and the resolution cannot be resolved, the existence of these two shortcomings can only shift and can not be improved simultaneously. Therefore, the classical method is not as good as the modern method when estimates the power spectral.

The AR model in the modern power spectral estimation not choose the better order, too high will cause a split spectral and produce the false peaks; too low maybe produce smoothing and spectral peak is not prominent. Next this conclusion will be verified by the MATLAB simulation, the simulation results are shown in figure 6. Just the same random signal is used, whose the sampling frequency is 1000Hz, the signal sampling point N is 1024, and the analysis and comparison of the orders for the AR model show: (a) 3, (b) 30, (d) 950.

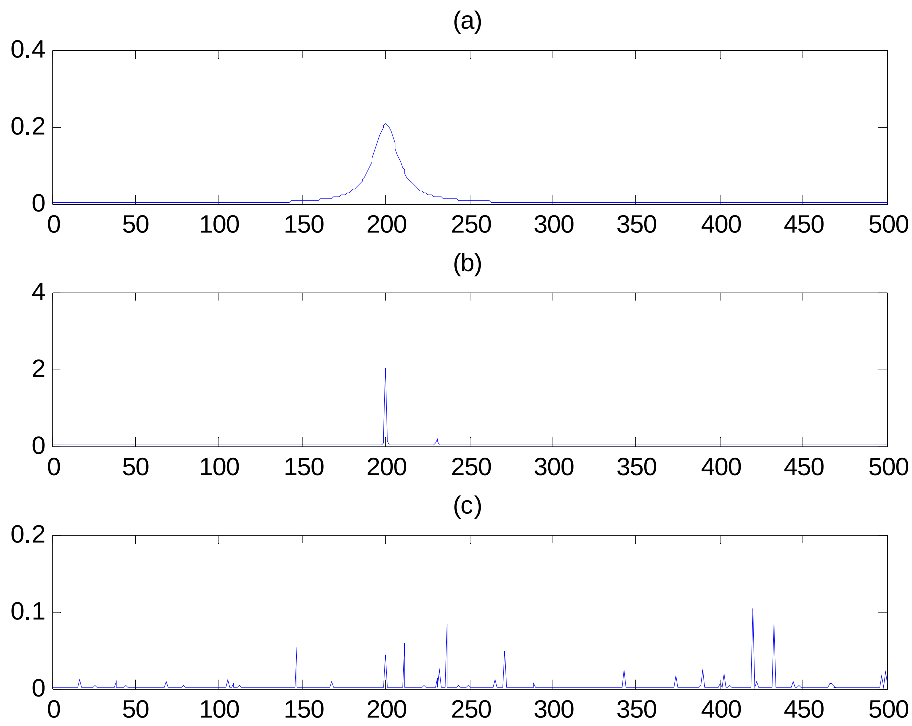

Figure 6 Comparison of the orders for long data in the AR model

Figure6describes the comparison of the orders for long data in the AR model.If the order selected is too low (for example, P=3), which results in the AR model pole-less and the spectral peaks decreasing, and then the spectral estimation will be smoothed, it difficult to distinguish real spectral peak and the resolution poorer. If the order selected is too high (such as P=950), although the resolution of spectral estimation can be improved, at the same time the false peak (false peaks) of the spectral maybe produces. Therefore, when the data of the random signal is long, the best choice of the order in the AR model is at around $\sqrt{N}$ (such as P=30).

Next the simulation results are shown in figure 7. Just the same random signal is used, when the sampling frequency is 512Hz, the signal sampling point N is 512, and the analysis and comparison of the orders for the AR model show: (a) 3, (b) 230, (d)500.

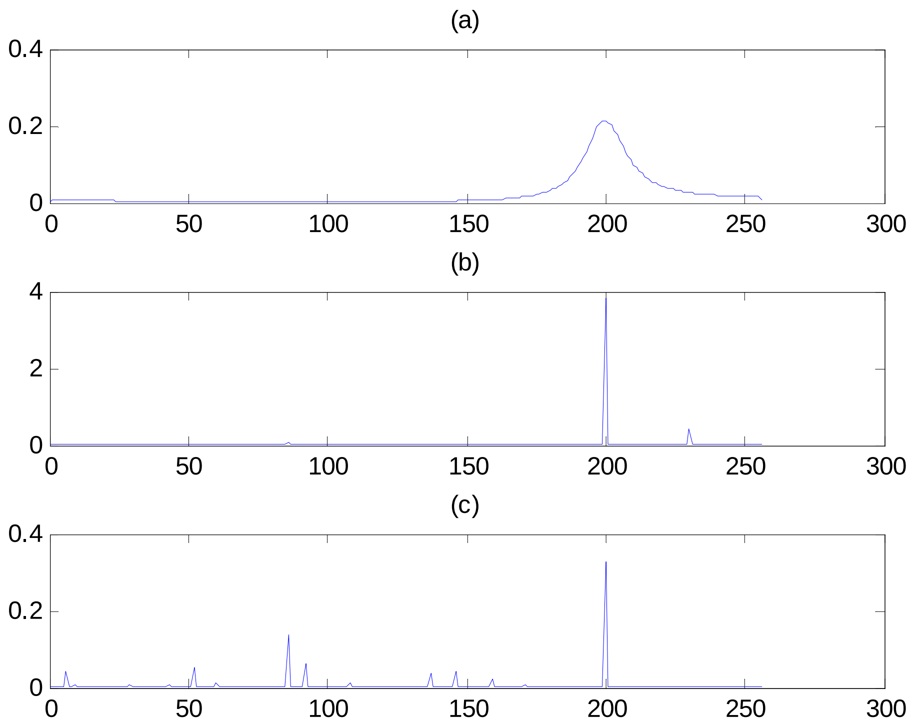

Figure 7 Comparison of the orders for short data in the AR model

Figure7shows the comparison of the orders for short data in the AR model. If the order selected is too low (for example, P=3), which results in the AR model pole-less and the spectral peaks decreasing, and then the spectral estimation will be smoothed, it difficult to distinguish real spectral peak and the resolution poorer. If the order selected is too high (such as P=500), although the resolution of spectral estimation can be improved, at the same time the false peak (false peaks) of the spectral maybe produces. Therefore, when the data of the random signal is short, the best choice of the order in the AR model is at around $\frac{N}{3} < P < \frac{N}{2}$ (such as P=230).

VI.CONCLUSIONS

In summary, the conclusion compared simulation can be drawn that the modern power spectral estimation is better than the classical power spectral estimation in the signals identification (frequency extraction). In particular, it overcomes the difficulty about the variance and resolution in contradiction of the classical spectral estimation. In the modern spectral estimation, which theory and research method has been quite thorough? The modern power spectral estimation only contains the amplitude information and does not contain the phase information; however a lot of practical problems often require the phase information.

VII.REFERENCES

[1]Wang J D,Chen D R,Kong X M, “Improvement of two-dimension AR model and application in simulation on surface profiles of steel plates”,Science in China Series E: Technological Sciences,Vol.42,No.6,1999, pp.637-643.

[2]Cheng J S,YuD J,Yang Y., “A fault diagnosis approach for roller bearings based on EMD method and AR model”,Mechanical Systems and Signal Processing, Vol.20,No.2,2004, pp.350-362.

[3]Masaya Matsuura,“On a recursive method including both CG and Burg’s algorithms”, Applied Mathematics and Computation, Vol.219, No.3,2012, pp.773-780.

[4]S V Narasimhan,E I Plotkin,M N S Swamy, “Power spectrum estimation of complex signals and

its application to Wigner-Ville distribution: A group delay approach",Sadhana, Vol.23,No.1,1998, pp.57-7.

[5]K. Kazlauskas,Ya.Kazlauskas,G. Petreykite, "The comparative analysis of methods for estimation of the power spectrum",Automatic Control and Computer Sciences, Vol.43,No.6,2009, pp.317-327.

[6]Mehmet Tahir Ozden, "Adaptive multichannel sequential lattice prediction filtering method for ARMA spectrum estimation in subbands",EURASIP Journal on Advances in Signal Processing, Vol.2013,No.1,2013, pp.1-36.

[7]Michal Lewandowski,JanuszWalczak, "Current spectrum estimation using Prony's estimator and coherent resampling",COMPEL: The International Journal for Computation and Mathematics in Electrical and Electronic Engineering,Vol.33,No.3,2014, pp.989-997.

[8]Huang T K,Jiang J L,Zha D F, "Multi-segmental Smoothing Cyclic Spectrum Estimation based on Welch-Bartlett Method",Energy Procedia, Vol.13,2011, pp.6079-6082.

[9]AlirezaGhodsi,MahendranShitan, "Some Properties of the Normalized Periodogram of a Fractionally Integrated Separable Spatial ARMA (FISSARMA) Model",Communications in Statistics - Theory and Methods, Vol.42,No.8,2013, pp.1515-1530.

[10]LiC, "Spectral Estimation of Irregularly Sampled Nonstationary Multidimensional Processes by Time-Varying Periodogram",Mathematical Geology, Vol.30,No.1,1998, pp.43-56.

[11]P.M. Robinson,D. Marinucci, "The Averaged Periodogram for Nonstationary Vector Time Series",Statistical Inference for Stochastic Processes, Vol.3,(1),No.1,2000, pp.149-160.

[12]M. Akin,M. Kemal Kiymik,"Application of Periodogram and AR Spectral Analysis to EEG Signals",Journal of Medical Systems, Vol.24,No.4,2000, pp.247-256.

[13]Serrano-Pedraza Ignacio,Sierra-Vázquez Vicente,Derrington Andrew, "Power spectrum model of visual masking: simulations and empirical data",Journal of the Optical Society of America, A, Optics, image science, and vision, Vol.30,No.6,2013, pp.111-935.

[14]Ploquin Marie,Basarab Adrian,Kouamé Denis, "Resolution enhancement in medical ultrasound imaging",Journal of medical imaging (Bellingham, Wash.), Vol.2,No.1,2015,

pp.01-7001.

[15]M. Piechocki,A. Herment,A. Delouche,Ph. Dumée,P. Peronneau, “Smoothed power spectrum estimate applied for analysis of the Doppler signal from blood flow”, Medical & Biological Engineering & Computing, Vol.29,No.2,1991, pp.167-174.

[16]Laguna P,Moody G B,Mark Rs G, “Power spectral density of unevenly sampled data by least-square analysis: performance and application to heart rate signals”,IEEE transactions on bio-medical engineering, Vol.45,No.6,1998, pp.698-715.

[17]Jameson, A R,Kostinski, A B, “Partially Coherent Backscatter in Radar Observations of Precipitation”,Journal of the Atmospheric Sciences,Vol.67,No.6,1998, pp.1928-1936,1938-1946x.

[18]J.B. Florindo,O.M. Bruno, “Closed contour fractal dimension estimation by the Fourier transform”,Chaos, Solitons and Fractals, Vol.44,No.10,2011, pp.851-861.

[19]A.K. Tanc,A.H. Kayran, “Iterative maximum entropy power spectrum estimation for multirate systems”,AEUE - International Journal of Electronics and Communications, Vol.64,No.2,2008, pp.93-98.

[20]Abdelaziz Trabelsi,OtmaneAit Mohamed,Yves Audet, “Robust Parametric Modeling of Speech in Additive White Gaussian Noise”,Journal of Signal and Information Processing,Vol.6,No.2,2015, pp.99-108.