Random Vibration of Systems with Viscoelastic Memory

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Abstract: The equation of motion of linear dynamic systems with viscoelastic memory is usually expressed in a integrodifferential form, and its numerical solution is computationally heavy. In two recent papers, the writers suggested that the system memory be accounted for through the introduction of a number of additional internal variables. Following this approach, the motion of the system is governed by a set of first-order, linear differential equations, whose solution is quite easy. In this paper, the approach is extended to single-degree-of-freedom systems subjected to random, nonstationary excitation. The equations governing the time variation of the second-order statistics are derived, and an effective step-by-step solution procedure is proposed. Numerical example shows the accuracy of the procedure for white and nonwhite excitations.

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Introduction

When the stress-strain relationship of a material at a given time instant depends on the previous strain history, the material is said to have memory. For systems including materials with memory, knowledge of strain and strain rate at a given time is therefore insufficient to fully characterize the state, as some additional information is required to predict their evolution. Among materials with memory are materials with linear viscoelastic damping, for which the stress-strain relationship is linear, and can be expressed through a convolution integral.

In the last decades materials with memory have been used in civil engineering, as they proved effective in mitigating structural vibrations due to natural loads, such as earthquakes, winds, and waves. These actions are usually described through stochastic models, and stochastic procedures have to be used to assess the effects of the viscoelastic devices on the structural response.

The theory of linear viscoelasticity is well established and a number of textbooks are available for references, e.g., Lockett (1972); Drozdov (1998); and Lakes (1998). In more specific books, such as Nashif et al. (1985); Sun and Lu (1995); and Soong and Dargush (1997), the use of viscoelastic material in

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vibration damping is covered. In Tseitlin and Kusainov (1990) the problem of computing the dynamic response of viscoelastic structures is addressed, with particular attention to the case of frequency independent damping. Furthermore, a large number of papers exist dealing with the dynamic response of linear viscoelastic systems under deterministic excitation. Starting from the papers of Bagley and Torvik (1983, 1985), a great effort has been devoted to the development of fractional-derivative models for viscoelastic devices. Contributions in structural engineering were given by Koh and Kelly (1990), who applied fractional calculus to analyze the viscoelastic behavior of elastomeric bearings used for seismic isolation. Makris and co-workers (Makris and Constantinou 1991; Makris et al. 1993) proposed a fractionalderivative model for dampers used for the isolation of industrial equipment. Another form of memory behavior which received great attention in the last decade is the linear hysteretic (frequency-independent) damping. In particular, Inaudi and Kelly (1995) and Inaudi and Makris (1996) proposed two different approaches for the time-domain analyses of structures with such form of damping; later, Makris (1997) proposed a causal version of this model. More recently, Makris and Zhang (2000) used the Biot model (Biot 1958) to describe the nearly frequencyindependent cyclic behavior of earth structures, and suggested the use of Prony series to compute the time-domain response with commercial software. As an alternative, Spanos and Tsavachidis (2001) proposed a recursive procedure and an autoregressive moving average (ARMA) approximation. Finally, Palmeri et al. (2003) proposed a new method, termed Laguerre polynomial approximation (LPA), to evaluate both time- and frequency-domain responses of viscoelastic systems with given relaxation function. Independently, Yuan and Agrawal (2002) proposed a similar approach for the dynamic analysis of single-degree-of-freedom (SDoF) oscillators in which the damping force is proportional to a fractional derivative of the strain. In Palmeri et al. (2004) the LPA model is applied to assess the effects of the viscoelastic memory of dampers incorporated in tall buildings subjected to gust buffeting. With respect to similar approaches (e.g., Lee 1997), the LPA method allows reducing the computational effort through the definition of approximated modal relaxation functions, with a lower number of additional internal variables (AIVs).

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