Random walks and non-linear paths in macroeconomic timeseries: some evidence and implications

3. Random walks and non-linear paths in macroeconomic time series: some evidence and implications

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1. INTRODUCTION

The aim of this chapter is to identify the nature of the dynamics of macroeconomic time series. When time series are characterized by zero autocorrelation for all possible leads and lags, the issue of distinguishing between deterministic and stochastic components becomes an impossible task when linear methods are used (Hommes 1998).

This impasse arises because linear methods are appropriate to detect regularities in time series like autocorrelations and dominant frequencies (Conover 1971, Oppenheim and Schafer 1989), while fluctuations in real economic time series are generally characterized by zero autocorrelation and no dominant frequency. Economic fluctuations seem really similar to background noise, which does not possess dominant frequencies and each noise impulse is not serially correlated. The spectral analysis of economic fluctuations, seemingly as complex as noise, has led many economists to consider fluctuations as identically independently distributed (i.i.d.) events.

As a matter of fact the i.i.d. hypothesis is an obvious necessity for all linear models to describe, at least approximately, the irregularities in the observed data. In the past two kinds of linear economic models based on the i.i.d. hypothesis in the residuals have been presented. In the first model, known as the *deterministic trend* model, variables evolve as a function in time along a linear trend. In the second model (the *stochastic trend* model), variables evolve as a function of their forgoing values and a shock shifts the value of the variable from the lagged value (Rappoport and Reichlin 1989). In this second case any shock evidently affects the value of the variable at all leads and, therefore, it has a persistent effect. Moreover the time series is entirely determined by the occurrence of all past shocks (Fuller 1999, Maddala and Kim 1998).

Following the seminal article by Nelson and Plosser (1982), the empirical evidence in the last 20 years has contradicted the linear trend models.

The stochastic trend model put forward by Nelson and Plosser seemed, instead, not to be contradicted by empirical results.

In this chapter the Nelson and Plosser model will be called in question because it is based on the hypothesis that fluctuations are i.i.d. while they are not. The i.i.d. hypothesis, in our opinion, obscures existent non-linearities that may be endogenized in non-linear models.

This chapter is organized as follows. In Section 2 the main stylized facts offered by the recent linear econometric analysis are presented. In Section 3 it is shown how neoclassical economic theory can be fully consistent with recent econometric results. In Section 4 we put forward the hypothesis that non-linearities of the system may be a deterministic cause of the irregularities in economic time series and we introduce a procedure, based on recent non-linear signal processing techniques, that allows us to identify the existence of non-linearities in the system and, it is hoped, to filter out non-linearities (signals) from truly i.i.d. components (noise). In Section 5 we present results obtained using artificial non-linear and autoregressive models; in particular we use the arsenal of tools from non-linear dynamics to identify the hidden deterministic structure that underlies the time series. In Section 6 we present results obtained using non-linear metric techniques applied to monthly seasonally adjusted time series of some real macroeconomic time series of the USA (industrial production, employment, consumer price index, hourly wages, and so on). The common result that stands out from this analysis is that all the time series we have analyzed are also characterized by non-random structures in the residuals and therefore the i.i.d. hypothesis is simply inconsistent with facts. The choice of assuming the residual components as random neglects the existence of a complex phenomenon. Instead, it is even theoretically possible to reduce any stochastic component that perturbs the system unpredictably and thus highlight the non-linear deterministic component. In Section 7 we conclude by showing some theoretical implications that we can infer from our empirical results about real business cycle theory grounded on stochastic components with persistent effects.

2. EMPIRICAL EVIDENCE

In the last 20 years we have witnessed huge progress in the statistical and econometric analysis of time series which has given economists a far more profound knowledge about the relations between economic variables. The discovery and the realization that time series do not show any tendency to evolve along a deterministic log-linear growth trend and the cyclical reversible components, assumed in classical econometrics, do not exist at all has deeply marked the direction of the empirical research in the last two decades.

Recent econometric works have provided a solid empirical basis that is in contrast to the theoretical results of the early neoclassical growth models *à la* Solow (1956) and the business cycle models *à la* Lucas (1972, 1977 and 1980) based on monetary disturbances with transitory effects. Nelson and Plosser (1982) have provided empirical evidence for the theoretical alternative to the real business cycle, despite the conventional wisdom of classical econometrics that assumed *ex ante* stationarity for all the economic variables. Nelson and Plosser have shown that many macroeconomic time series[1] are not stationary at all, and the stationary stochastic models developed in the 1970s do not actually have any empirical foundation.[2]

On the contrary, Nelson and Plosser have shown that the irregularity present in macroeconomic time series could simply be explained by the introduction of random shocks with persistent effects as happens in unit root processes.[3]

These results were in sharp contrast with the classic econometric works, which affirmed that the irregularity in economic time series was due to transitory shocks, and has been crucial in moving the direction of research towards the theory of the real business cycle.

The acknowledged contribution of the Nelson and Plosser work was the discovery of the non-stationarity in the time series and the absence of any deterministic trend. More importantly, the introduction of random external shocks as the unique generator of the irregularity in the behavior of economic systems did not contradict the results put forward by a modern version of neoclassical theory: real business cycle theory. Indeed, without the injection of external shocks, time series would move exactly in the direction that the neoclassical theory predicts. However, in the presence of external shocks, economic systems move irregularly in the way that is described by real business cycle models (Prescott 1998).

In this chapter we try to move a step forward, starting from this empirical evidence. Our aim is to identify the process that generates the non-stationarity in time series without stating *ex ante*, contrary to Nelson and Plosser, that the non-stationarity is the direct consequence of a stochastic process. Actually there may be many possible non-linear deterministic alternatives to the stochastic explanation of the non-stationarity in time series.

Treating economic fluctuations as an endogenous non-linear process, and therefore an object of analysis, may contribute to a better understanding of the temporal evolution of time series. Our purpose is to understand the dynamics of fluctuations as the evolution of the system may depend entirely on them. We believe that assuming fluctuations as i.i.d. variables equivalent to noise is basically wrong since, as we shall see in Section 6, residuals are characterized by a structure that is very different from noise and even from any other kind of random variable. These results will lead us to conclude that it is feasible to discover deterministic laws that shape the underlying non-linear structures.

2.1 Recent Results from the Unit Root Literature

Many recent related works have been published after the Nelson and Plosser paper and their results differ mainly in respect of the test function that has been used in the verification of the non-stationarity hypothesis.

Some papers simply confirm that the non-stationarity of economic time series is a recurrent characteristic in many countries. Similarly to Nelson and Plosser, Lee and Siklos (1991) found that macroeconomic time series for Canada are not stationary. Mills (1992) obtained basically the same results for the UK, McDougall (1995) for New Zealand, Rahman and Mustafa (1997) for the Asian countries, Sosa for Argentina (1997), Gallegati (1996) and de Haan and Zelhorst (1994) for Italy.

The macroeconomic variables that are more frequently analyzed are GDP, GNP, GDP per capita and GNP per capita, industrial production, employment, unemployment rate and the consumer price index. Occasionally other variables such as savings (Coakley et al. 1995), investments (Coorey 1991, Coakley et al. 1995), wages (Coorey 1991), exchange rates (Durlauf 1993, Parikh 1994, Wu and Crato 1995, Serletis and Zimonopoulos 1997, Weliwita 1998), money and velocity of money (Al Bazai 1998, Serletis 1994) have been analyzed.

All these studies pointed out that almost every time series in any country is characterized by the presence of a unit root, or equivalently by a stochastic process like a random walk.[4] The one exception to the existence of a unit root in macroeconomic time series is the unemployment rate. This non-conformity was first noticed by Nelson and Plosser and has been confirmed by the majority of unit roots researchers afterwards.[5]

In Table 3.1 we list the main works that have ascertained the existence of a unit root in macroeconomic time series. For each author we mark with a plus sign (+) the variable that was found to follow a random walk, and with an equals sign (=) the variable for which the results were mixed.

2.2 The Broken Trend Hypothesis

Rappoport and Reichlin (1986, 1988, 1989) put forward the hypothesis that there could exist a broken deterministic trend that cannot be identified by the Dickey–Fuller test. Rappoport and Reichlin showed that in the case of

Table 3.1 *Main works ascertaining existence of a unit root in macroeconomic time series*

Authors	Y	Y/N	Y_i	Y_a	GNP	GNP/N	Exp	S	I	E	U	r	e	p	m	v	Country
Nelson and Plosser (1982)																	USA
Lee and Siklos (1991)																	Canada
McDougall (1995)					+	+				+	=			+			New Zealand
Franses and Kleibergen (1996)																	USA
Wells (1997)																	USA
Nunes et al. (1997)																	USA
Coorey (1991)									+								USA
Mills (1992)																	UK
Durlauf (1993)																	USA
de Haan and Zelhorst (1994)	+																Italy
Sosa (1997)																	Argentina
Bannerjee et al. (1992)	+				+	+				+	+			+			7 OECD, Japan
Fung and Lo (1992)													+				USA
Parikh (1994)																	Japan, UK, Germany
Mocan (1994)					+												USA
Gamber and Sorensen (1994)											+						USA
Haslag et al. (1994)																	USA
Serletis (1994)																+	USA
Bresson and Celimene (1995)																	Caribbean
Dolado and Lopez (1996)											+						Spain
Leybourne and McCabe (1999)																	USA

Wu and Crato (1995)											New Zealand
Serletis and Zimonopoulos (1997)								+			17 OECD
Weliwita (1998)											Asia
Gallegati (1996)	+		+	+							Italy
Rahman and Mustafa (1997)	+				+						Asia
Bohl (1998)		+									G7
Al Bazai (1998)										+	Saudi Arabia
Choi and Yu (1997)									+		OECD
Coakley et al. (1995)						+	+				OECD
Osborn et al. (1999)			+								USA

Notes:
Y = GDP, Y/N = GDP per capita, Y_i = industrial production, Y_a = agriculture, Exp = exports, S = savings, I = investments, E = employment, U = unemployment rate, r = interest rate, e = exchange rate, p = consumer price index, m = money, v = velocity of money

a broken deterministic trend the Dickey–Fuller test produces spurious results, since it is incapable of rejecting a false null hypothesis (the unit root hypothesis). Rappoport and Reichlin have moreover revealed empirical evidence concerning the existence of a broken trend in many macroeconomic time series. They indeed rejected the hypothesis of a random walk for many real variables (such as industrial production, real GNP, real per capita GNP and money supply), though not for all of them.[6]

Perron (1989) as well as Rappoport and Reichlin showed that, when fluctuations are stationary along a broken trend, the Dickey–Fuller test is not able to reject the unit root hypothesis. Perron developed a test that allows to reject the unit root null hypothesis if the series is characterized by a broken trend. He applied his test to the same time series of the USA that was used by Nelson and Plosser, after he had arbitrarily assigned the date at which the structural break occurred. Perron concluded that the null unit root hypothesis could be rejected also at a high confidence level for almost all the time series.

Similar results were obtained by Raj (1992) for the macroeconomic time series of Canada, France and Denmark, by Rudebusch (1992) for England, by Linden (1992) for Finland, by Wu and Chen (1995) for Taiwan and by Soejima (1995) for Japan.

Other authors also looked for a broken trend in specific time series. Diebold and Rudebush (1989), Duck (1992), Zelhorst and de Haan (1993), Ben-David and Papell (1994), Alba and Papell (1995) and McCoskey and Selden (1998) have found a broken trend for the GDP in many countries. Alba and Papell (1995) for GDP per capita and Li (1995), Gil and Robinson (1997) found similar results for industrial production, Simkins (1994) for wages in eight OECD countries and McCoskey and Selden (1998) for the G7 countries, Raj and Slottje (1994) for the US income distribution, Culver and Papell (1995), Leslie et al. (1995), and MacDonald (1996) for exchange rates. Given these results, we could check whether the broken trend hypothesis also explains the dynamics of unemployment rate better than the unit root hypothesis. However, Nelson and Plosser had already found that the US unemployment rate tended to be stationary, and the works by Hansen (1991), Li (1995), Leslie et al. (1995), Song and Wu (1997, 1998), Gil and Robinson (1997) and Hylleberg and Engle (1996) simply confirm the empirical evidence presented by Nelson and Plosser.

In Table 3.2 we present the main works that support the hypothesis of a broken trend in macroeconomic time series. For each author we mark with a minus (−) sign the variable that was found stationary along a broken trend.

Criticisms of both the broken trend and the unit root hypothesis have been put forward by several authors. Zivot and Andrews (1990, 1992)

Table 3.2 *Main works supporting hypothesis of a broken trend in macroeconomic time series*

Authors	Y	Y/N	Y_i	YD	Gnp	Gnp/N	E	U	w	r	e	p	Country
Perron (1989)													USA
Rudebusch (1990)													Australia
Raj (1992)													USA, Canada, France, Denmark
Rudebusch (1992)													Germany
Linden (1992)					–	–	–	–				–	Finland
Wu and Chen (1995)													Taiwan
Soejima (1995)													Japan
Lee (1996)													USA
Lumsdaine and Papell (1997)													USA
Diebold and Rudebusch (1989)													USA
Duck (1992)	–												9 countries
Ben-David and Papell (1994)													16 countries
Gokey (1990)													8 OECD
Capitelli and Scjlegel (1991)													6 countries
Leslie et al. (1995)										–			UK
Wu and Zhang (1996)													OECD
Moosa and Bhatti (1996)													Asia
Hansen (1991)													UK
Song and Wu (1997)								–					48 US states
Song and Wu (1998)													OECD
Hylleberg and Engle (1996)													OECD

Table 3.2 (Continued)

Authors	Y	Y/N	Y_i	YD	GNP	GNP/N	E	U	w	r	e	p	Country
Simkins (1994)							–		–				USA
Raj and Slottje (1994)				–									USA
Caselli and Marinelli (1994)					–								Italy
Culver and Papell (1995)											–		OECD
Wu (1996)													USA
Li (1995)			–										USA
Gil and Robinson (1997)													USA
Alba and Papell (1995)	–	–											Newly industrializing countries
Fleissing and Strauss (1997)									–				G7
McCoskey and Selden (1998)	–												OECD
Cheung and Chinn (1996)					–								USA
Dolmas et al. (1999)		–											USA

Notes:
Y = GDP, Y/N = GDP per capita, Y_i = industrial production, YD = income distribution, E = employment, U = unemployment rate, w = wages, r = interest rate, e = exchange rate, p = consumer price index

estimate the position in time of the structural break and find that the existence of the broken trend is not that clear in many of the time series that were analyzed by Perron. Cushing and McGarvey (1996) found that the fluctuations in the macroeconomic time series are more persistent compared to what stationary models indicate, but they are also less persistent than unit root models suggest. Mixed results were also obtained by Leybourne et al. (1996) for many US macroeconomic time series, Krol (1992) for the production of many US sectors, and Crosby (1998) for Australian GDP.

It seems therefore that not every time series is characterized by a unit root. What does this suggest? Are time series generated by a deterministic process or by chance? This issue has not been well formulated either in the unit root or in the broken trend literature. The problem is that the idea according to which a non-stationary process is a random walk process was implied in most of these studies. As we will see in Section 4, not all the non-stationary processes follow a random walk. Indeed, there may exist many deterministic non-linear processes that are not stationary and become stationary after differentiating with respect to time.

Since the results obtained by the broken trend literature are still open to discussion in the sense that the studies hitherto published do not lead to a general rejection of the random walk hypothesis, we question whether the broken trend hypothesis provides the ultimate explanation of the nature of economic time series. Moreover, as will be shown in the next section, the random walk hypothesis has the great advantage that it may be theoretically fully consistent with the neoclassical framework once it is assumed that real changes occur randomly.

3. THE LINK BETWEEN NEOCLASSICAL GROWTH THEORY AND THE UNIT ROOT LITERATURE

King et al. (1988b) showed that growth theory, which assumes steady growth, may be consistent with the highly irregular behavior of economic time series.

They considered a one-commodity Solow (1956) and Swan (1956) model. The production function, the capital accumulation equation and the resource constraint are:

$$
\begin{gathered}
Y_t = A_t K_t^{1-\alpha} (NX_t)^{\alpha} \qquad 0<\alpha<1 \\
K_{t+1} = I + (1-\delta)K_t = sA_t K_t^{1-\alpha} (NX_t)^{\alpha} + (1-\delta)K_t \\
L_t + N = 1 \\
C_t + I_t = Y_t,
\end{gathered}
$$

where Y_t is the output at time t, K_t is the capital stock available at time t, s the saving rate, N is the labor input that is assumed constant at all time t, A_t is a multiplier factor and its change corresponds to temporary changes of total factor productivity, X_tN is the *effective* labor units and changes of X_t modify permanently the performance of the system, and C_t is the consumption at time t.[7]

Assume constant returns to scale in the production function, and *constant* labor-augmenting technical change rate $\Delta(x)/(x)$. The dynamic equation for the capital stock may be rewritten as:

$$\Delta K_t = sA_tK_t^{1-\alpha}(NX_t)^{\alpha} - \delta K_t \rightarrow \frac{\Delta K_t}{N} = \frac{sA_tK_t^{1-\alpha}(NX_t)^{\alpha} - \delta K_t}{N}$$

$$\Delta k_t = sA_tk_t^{1-\alpha}N^{1-\alpha}N^{\alpha-1}X_t^{\alpha} - \delta k_t, \text{ where } k_t = \frac{K_t}{N}.$$

$$\frac{\Delta k_t}{k_t} = \frac{sA_tk_t^{1-\alpha}(X_t)^{\alpha} - \delta k_t}{k_t} = \frac{sA_tk_t^{1-\alpha}(X_t)^{\alpha}}{k_t} - \delta = \gamma,$$

where γ is the growth rate of the capital per capita.

If

$$\frac{sA_tk_t^{1-\alpha}(X_t)^{\alpha}}{k_t} > \delta, \frac{\Delta k_t}{k_t} > 0,$$

capital per capita grows.

Conversely if

$$\frac{sA_tk_t^{1-\alpha}(X_t)^{\alpha}}{k_t} < \delta, \frac{\Delta k_t}{k_t} < 0,$$

capital per capita decreases.

In steady state

$$\frac{\Delta k_t}{k_t} = 0 \text{ and } \frac{A_tk_t^{1-\alpha}(X_t)^{\alpha}}{k_t} = A_tk_t^{-\alpha}(X_t)^{\alpha} = \frac{\delta}{s} \text{ is constant.}$$

In order that $A_tk_t^{-\alpha}(X_t)^{\alpha}$ is constant over time, k_t and X_t must grow at the same rate γ. The output per capita is $y_t = A_tk_t^{1-\alpha}(X_t)^{\alpha} = kA_tk_t^{-\alpha}(X_t)^{\alpha}$; in steady state, being $A_tk_t^{-\alpha}(X_t)^{\alpha} = \delta/s$, also y_t grows at the same rate of k, γ. Consumption per capita is $c = (1-s)y$ and grows at the same rate γ over time. In this sense, macroeconomic variables follow a (linear) deterministic trend.

This view was in sharp contrast with the empirical evidence from Nelson and Plosser (1982), who showed that the existence of a stochastic trend should not be neglected. However, it is very easy to make stochastic the basic version of the deterministic neoclassical model.

To do that, we consider that the labor-augmenting technical change occurs *stochastically* as a random walk.

We have:

$$X_\tau = X_0 \gamma^\tau e^{\sum_{t=0}^{\tau} \varepsilon_{t-i}} \rightarrow \ln X_\tau = \ln X_0 + \tau \ln \gamma + \sum_{t=0}^{\tau} \varepsilon_{t-i},$$

where $\sum_{t=0}^{\tau} \varepsilon_{t-i}$ represent permanent shifts of $\ln X_\tau$ which are not reabsorbed by the internal dynamics of the system.

Given the dynamic equation for capital accumulation, in steady state,

$$\frac{\Delta k_\tau}{k_\tau} = 0 \text{ and } \frac{A_\tau k_\tau^{1-\alpha}(X_\tau)^\alpha}{k_\tau} = A_\tau k_\tau^{-\alpha} (X_\tau)^\alpha = \frac{\delta}{s} \text{ is constant.}$$

In order that $A_\tau k_\tau^{-\alpha} (X_\tau)^\alpha$ is constant over time, k_τ and X_τ must grow at the same stochastically by

$$\gamma^\tau e^{\sum_{t=0}^{\tau} \varepsilon_{t-i}}:$$

$$\ln k_\tau = \ln k_0 + \tau \ln\gamma + \sum_{t=0}^{\tau} \varepsilon_{t-i}$$

The output per capita is $y_\tau = A_\tau k_\tau^{1-\alpha} (X_\tau)^\alpha = k A_\tau k_\tau^{-\alpha} (X_\tau)^\alpha$ in steady state, being $A_\tau k_\tau{}^{-\alpha} (X_\tau)^\alpha = \delta/s$, y_τ grows also by

$$\gamma^\tau e^{\sum_{t=0}^{\tau} \varepsilon_{t-i}}:$$

$$\ln y_\tau = \ln y_0 + \tau \ln\gamma + \sum_{t=0}^{\tau} \varepsilon_{t-i}$$

Consumption per capita is $c_\tau = (1-s) y_\tau$ and grows by

$$\gamma^\tau e^{\sum_{t=0}^{\tau} \varepsilon_{t-i}}:$$

$$\ln c_\tau = \ln c_0 + \tau \ln\gamma + \sum_{t=0}^{\tau} \varepsilon_{t-i}$$

In this sense, macroeconomic variables follow a *stochastic* trend where all the dynamics is driven by additive random innovations. Most of the empirical studies confirm that: (1) macroeconomic variables follow a stochastic trend, that is, a random walk; (2) macroeconomic variables co-evolve together; that is, they are co-integrated. This is exactly what occurs in the stochastic formulation of the neoclassical model. In fact, the above equations may be equivalently rewritten in terms of an AR(1) process:

$$\ln X_t = \ln X_{t-1} + \ln \gamma + \varepsilon_t$$
$$\ln k_t = \ln k_{t-1} + \ln\gamma + \varepsilon_t$$

$$\ln y_t = \ln y_{t-1} + \ln\gamma + \varepsilon_t$$
$$\ln c_t = \ln c_{t-1} + \ln\gamma + \varepsilon_t,$$

where all the economic variables depend on their previous value, on the average growth rate plus a non-transitory stochastic error term.

What is implicit in the stochastic version of the neoclassical model is that the economic system is essentially stable. In fact, if time series follow a random walk and we remove random innovations, we have a stationary stable system. In absence of technical change the system would never change, except for the occurrence of other exogenous shocks, for instance a change in preferences.

If the term $\Sigma_{i=0}^{T} \varepsilon_{t-i}$ were not random, what would be the consequences for economic theory? The first consequence would be that, understanding the deterministic non-linear dynamics, we could make a better prediction than simple AR-like models, since the best predictor for the residual in the AR models cannot be but its mean value. The second consequence would be that economic systems might be intrinsically unstable, that is, also without the injection of exogenous random inputs the system could not be motionless. Moreover, just because real economic time series prove to be complex, seemingly random but containing some deterministic structure, they could be better forecasted and better controlled.

In the next section we raise the hypothesis that residuals might appear random while they are indeed generated by a deterministic system. Later on, in Section 6, we will test whether or not the residual component of an autoregresssive model is truly random, and we will find, to our surprise, that the hypothesis that $\Sigma_{i=0}^{T} \varepsilon_{t-i}$ is not truly random is indeed found in our inference.

4. THE NON-LINEAR HYPOTHESIS[8]

Twenty years after the publication of the Nelson and Plosser article, we now have two literature streams that debate the nature of the time series: one that underlines the existence of a random walk and one that asserts the complete linear (though with a break) determinism in the economic time series. We will show in Section 6 that the empirical evidence about the nature of economic time series can be clearer than that provided by either the unit root literature or the broken trend literature.

The procedure that will be used in Sections 5 and 6 to detect non-linearities consists of the following steps:

1. Select time series with a minimal number of observations. Brock et al. (1991) have proved that at least 400 observations would be a good start-

ing point, if not a necessary condition, for obtaining trustworthy results from the BDS test. It is therefore necessary to rely on seasonally adjusted monthly data for a sufficiently long period.[9] The time series we used are those of the USA and data were provided by the Bureau of Labor and Statistics and the Federal Reserve.[10]

2. Take the natural logs of the original time series if the time series tend to diverge exponentially.
3. Differentiate the time series once with respect to time, eventually remove linear autocorrelation in the residuals and check for stationarity via the augmented Dickey–Fuller test.
4. Calculate the level of spatio-temporal entropy[11] to measure the degree of disorder of the system. If the time series of the residual was generated by a random process, the level of entropy should be close to the maximal value. However, non-linear processes may also present a high degree of disorder and reach values of entropy close to that of white noise.[12] On the other hand we should expect a low level of entropy for processes that are deterministic and autocorrelated.[13] However, we should not overestimate the importance of the measure of entropy; in fact it does not allow us to distinguish a random process from a complex deterministic one and even between periodic cycles and linear trend. Nevertheless the measure of entropy may help us to better understand the complexity of a time series.
5. Calculate the values of the maximal Liapunov exponents that characterize the time series, to measure how fast nearby trajectories diverge over time. If the maximal Liapunov exponents turn out to be negative, this means that trajectories tend to converge to a stable fixed point. If it were zero, we would have found a limit cycle. If it were positive, the time series is either characterized by chaos or a random walk. We anticipate that the residuals of the linear models that explain economic time series are generally characterized by a positive maximal Liapunov exponent and a high level of entropy, and this indicates how difficult it might be to forecast economic time series in the long run.
6. Generate *Ruelle plots* (recurrence plots) to uncover, from the qualitative point of view, hidden structures in the time series.
7. Perform the BDS test to detect quantitatively and in a reliable way the existence of non-linearity in data.
8. Check results randomly by shuffling the time series and verify whether the results obtained by the BDS test applied on a randomly shuffled time series are indeed different from the results obtained by the BDS test on the original time series.[14] This verification is extremely important since, if the two results turn out to be different, it means that the

time order of the original time series is significant and there exists causality in the data.

5. RESULTS FROM ARTIFICIAL TIME SERIES

Before applying the described procedure to real time series, we present some results obtained from artificial time series, whose deterministic data generating process is known. We present some cases of deterministic systems whose dynamics is very similar to a random walk and we check whether the non-linear dynamics tools allow us to gain more information about the nature and the evolution of the time series. We will see that the information gain ensued from the numerical tools of non-linear time series analysis may be relevant and may lead us to consider the issues of dynamics from a very different perspective.

5.1 Trends

We consider first the most simple case: growth along a linear trend. We first check the results obtained with the Dickey–Fuller test when a linear time series grows deterministically with time. Thereafter we apply non-linear metric tools to see which other information may be obtained. The application of non-linear techniques to a linear system may not seem to be necessary, but this step will allow us to compare the information that can be obtained using linear statistics and non-linear dynamics tools.

In the trend stationary case, residuals have no persistent effects and the time series is stationary along a linear trend. If we consider the variable x_t as a linear function of time t: $x_t = x_0 + \phi t + \varepsilon_t$ where x_0 is the initial value (in our case it is equal to zero), ϕ is a parameter and ε_t is an i.i.d. variable. Running the Dickey–Fuller test we should reject correctly the null hypothesis of a unit root and the Durbin–Watson statistics, DW, should be around 2 (when $DW \simeq 2$, residuals have no serial correlation).

Suppose that we are interested in studying the dynamics of a variable that could be the GDP, y_t. We assume that GDP grows at the yearly rate $g = 2$ per cent:

$$y_t = y_0(1+g)^t \rightarrow \ln y_t = \ln y_0(1+g)^t \rightarrow \ln y_t = \ln y_0 + t\ln(1+g)$$

Suppose that $\ln y_t$ is perturbed by a i.i.d. exogenous shock ε:[15]

$$\ln y_t = \ln y_0 + t\ln(1+g) + \varepsilon_t.$$

Table 3.3 Rejection of the null hypothesis of unit root

	Deterministic trend	Random walk	'Tent map' walk	'Rossler' walk
ADF statistic	−21.3**	−1.98	−1.79	−67.53**
D-W statistic	2.00	1.99	2.00	0.09**
Entropy of residuals	90%	90%	78%	15%
BDS statistic	−1.28	−1.55	99.2**	355.0**

Note: * and ** denote significance at the 5% and 1% levels.

Set $\ln y_t = x_t$ and $\ln(1+g) = \phi$ we obtain:

$$x_t = x_0 + \phi t + \varepsilon_t \text{ where } g = 0.02 \text{ and } \phi = 0.02.$$

Applying the Dickey–Fuller test we decidedly reject the null hypothesis of unit root (Table 3.3). The Dickey–Fuller test turned out to be −21.3 while the critical value at 5 percent significance level is −3.41. For values less than 3.41, the null hypothesis is rejected, as it is in this case. The Durbin–Watson statistics turned out to be close to 2, and this confirms that the residuals are not serially correlated. In this case, the Dickey–Fuller test was able to correctly reject the null hypothesis of a stochastic trend and to accept correctly the alternative hypothesis of a linear trend.

Let us now turn our attention to some qualitative and quantitative measurements obtained with non-linear dynamics tools. The value of *entropy* that characterizes the level of GDP is 0 percent, and this indicates that the time series is characterized by an almost null degree of disorder. In fact residuals are all concentrated around a linear trend, which represents a long-term equilibrium path. If we analyze the residuals, which were assumed to be i.i.d., the level of entropy turns out to be 90 percent, a value relatively close to the ideal limit of 100 percent of a purely random process (a value that is very difficult to reach in series generated by the simple algorithms of a random number generator). This indicates that the degree of disorder of a system characterized only by an i.i.d. variable is very high.

We have calculated the value of the maximal Liapunov exponent for the residuals, in order to measure the rate of sensitive dependence on initial conditions, that is, the rate of divergence of nearby initial states. It turned out to be positive (Table 3.5, row i.i.d. process) and so high that residuals follow a unpredictable dynamics. As we will see in Section 5.3, high values of the maximal Liapunov exponent and entropy are also typical of many non-linear systems.

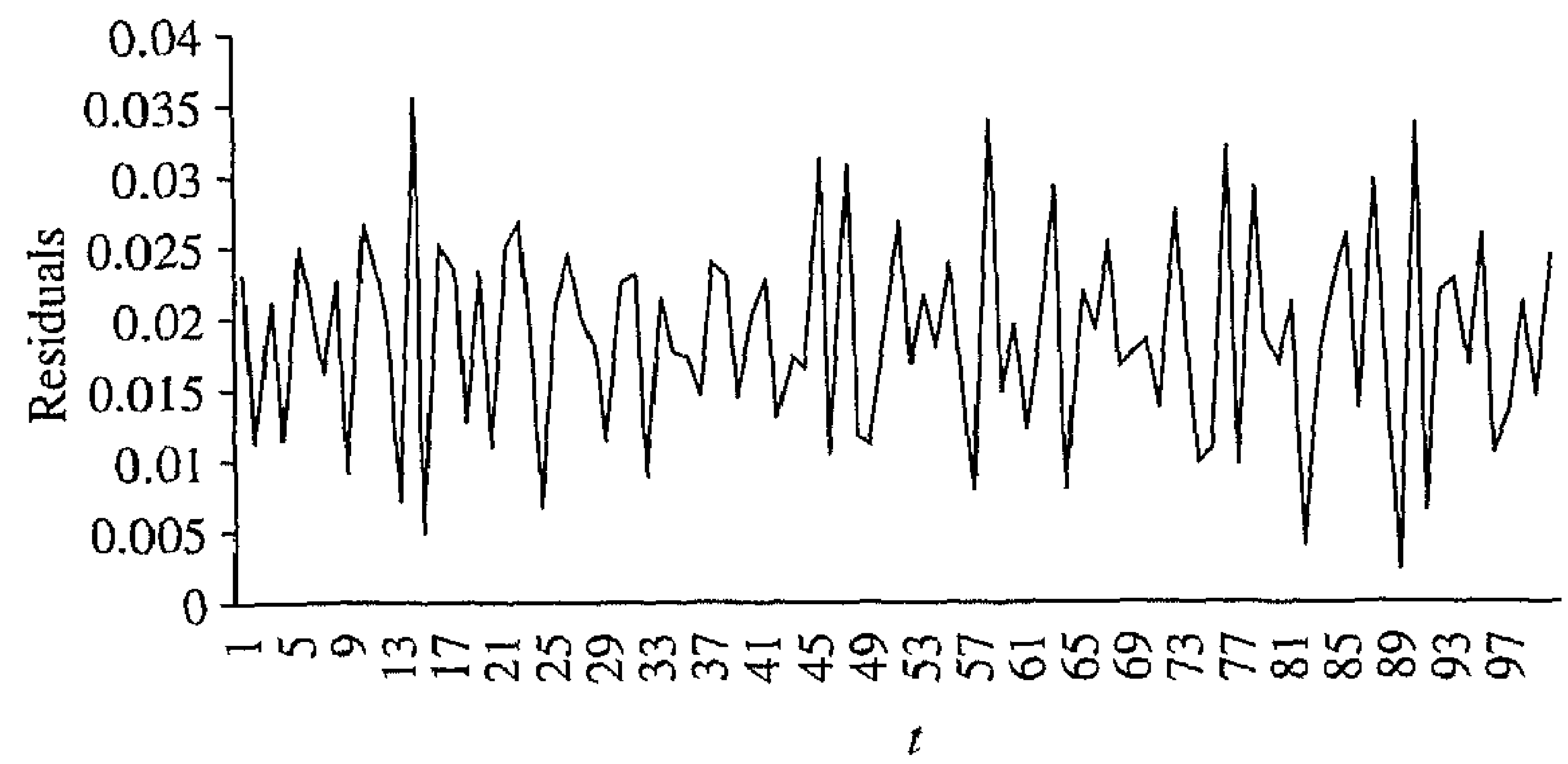

Figure 3.1 *i.i.d. residuals (uniformly distributed)*

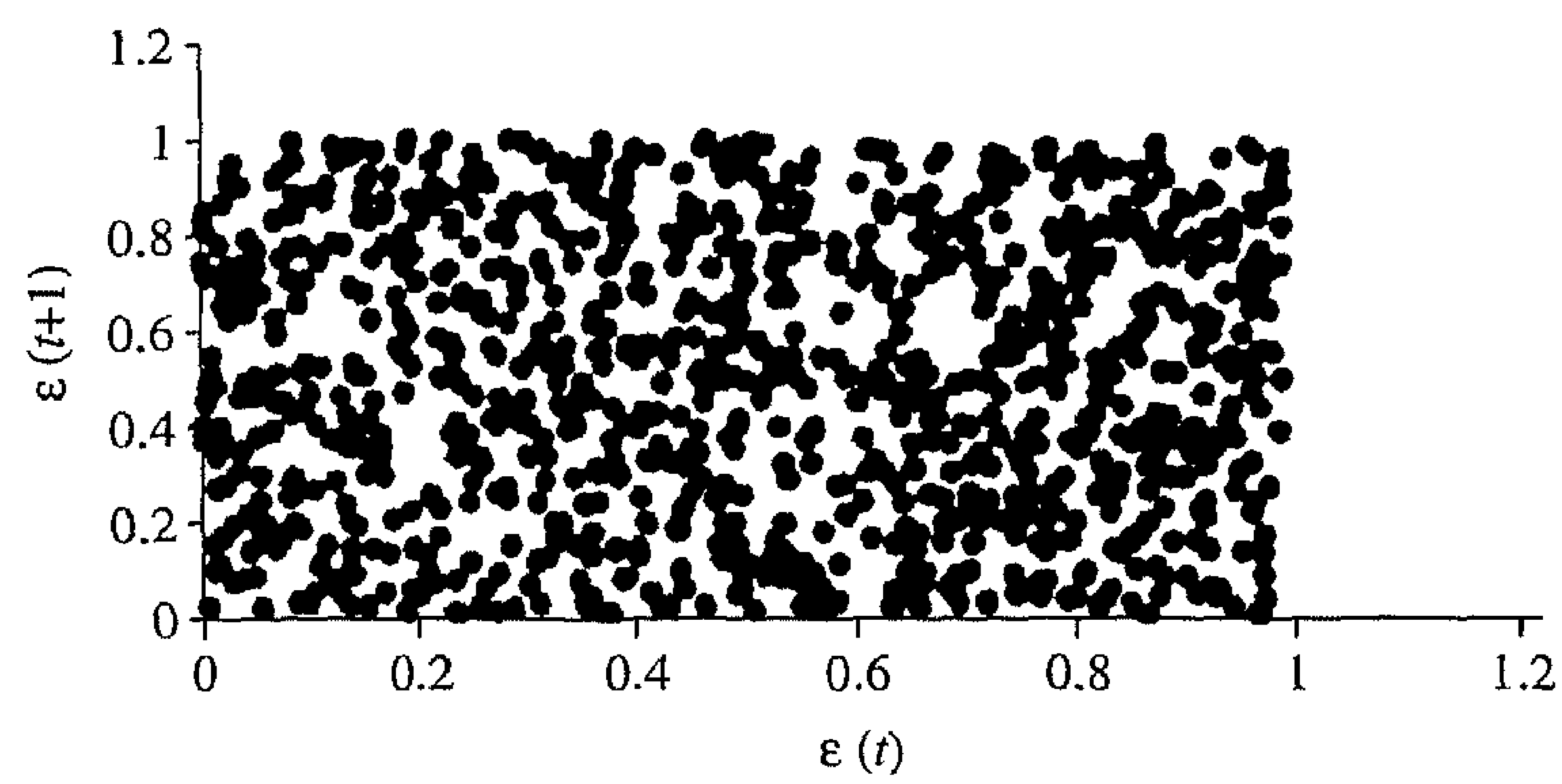

Figure 3.2 *Phase plot of i.i.d. residuals*

There are also qualitative visual devices that allow us to uncover complex structures in data and even to single out exceptional historical events. They are the *phase portraits* and the recurrence plots. The phase portrait is simply a graphical representation that plots the value $x(t)$ against $x(t-h)$. While in Figure 3.1 the residuals $\varepsilon(t)$ are plotted against time, they are plotted against $\varepsilon(t-1)$ in Figure 3.2.[16]

The recurrence plots by Eckmann et al. (1987) are a graphical tool for the qualitative analysis of time series based on phase portraits and allow us to uncover deterministic structures that could not be revealed by phase

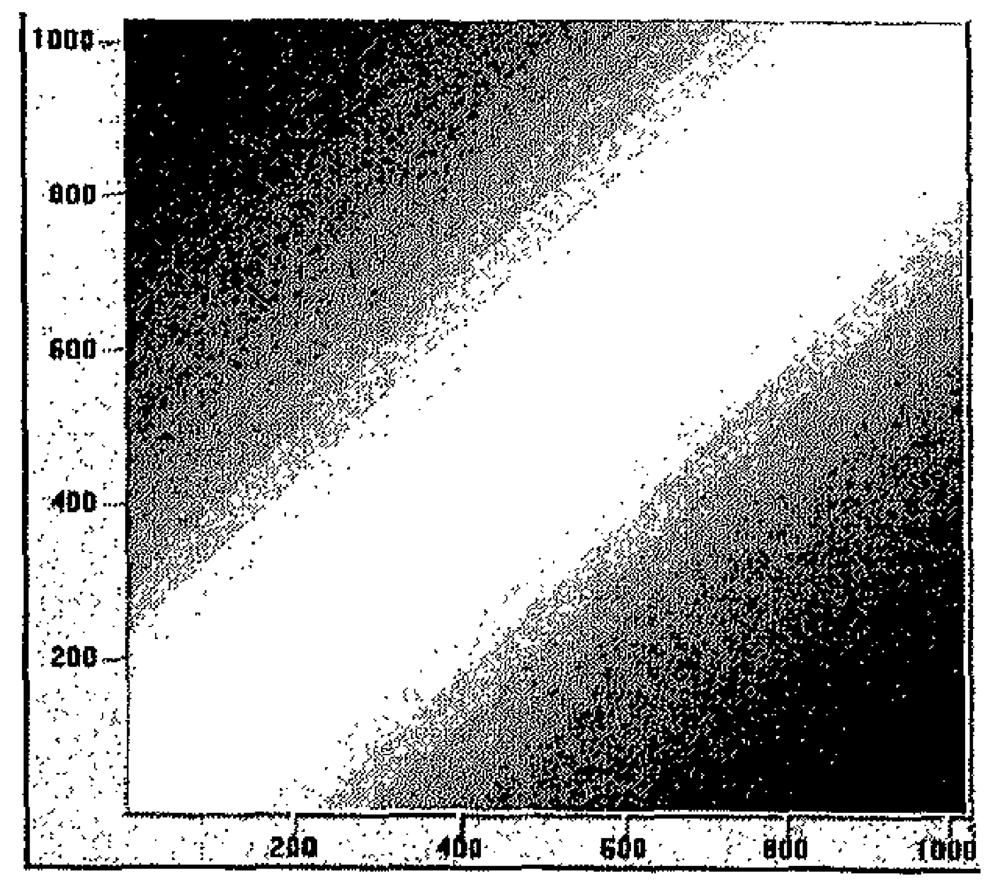

Figure 3.3 Recurrence plot of a linear trend

plots. In the simplest recurrence plots, the distances between observations are measured and marked by a grey tone. On the axis each point corresponds to a dated observation. The diagonal is the locus where $\|x(t)-x(t-h)\|=0$, where $h=0$ and the corresponding tone is white.

In the case of a deterministic trend the distance grows with the temporal distance of observations. The most distant observations are $x(0)$ and $x(T)$; hence the points $[x(0)-x(T)]$ and $[x(T)-x(0)]$ are marked by a black tone (Figure 3.3). The points along the parallels to the 45 degree line are characterized by the same grey tone and this indicates that the couples of observations that keep the same temporal distance are also characterized by the same spatial distance (represented by the same grey tone).

On the contrary, recurrence plots of i.i.d. residuals should neither present any continuous line between points nor particular areas characterized by the same grey tone. The fact that some nearly continuous lines may be noticed (Figure 3.4) is due to the random number generator, which is a mathematical algorithm and therefore does not produce purely unstructured time series. However Figure 3.4 shows much less structure than the Ruelle plot in Figure 3.3 and is close to the one of a purely i. i.d. process.

Actually, Ruelle plots may allow us to single out much more hidden structures when they compare embedded vectors[17] instead of single observations. Ruelle plots mark the distances between points[18] with a grey tone. If we choose $m=1$, we obtain Figures 3.3 and 3.4. If we had chosen different values of m, we would also have graphs similar to Figures 3.3 and 3.4. However, in other cases, especially in the case of chaotic systems, the choice of appropriate values for m allows us to uncover otherwise neglected structures.

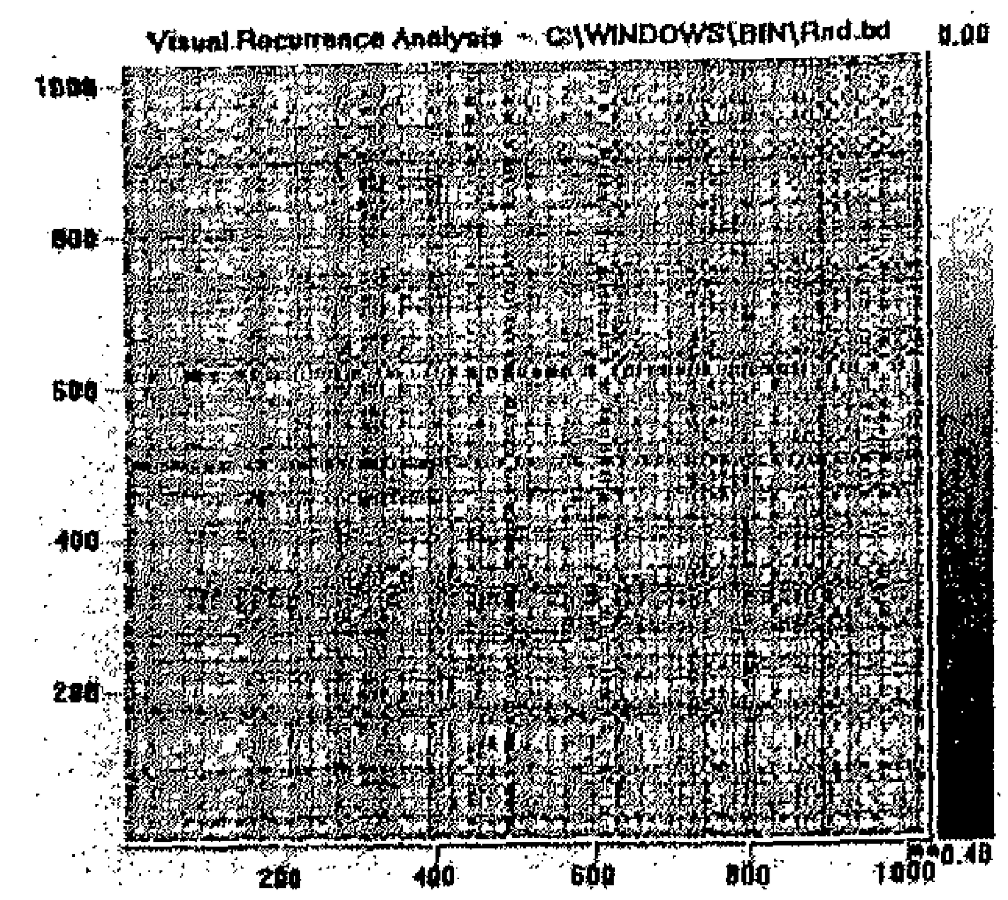

Figure 3.4 *Recurrence plot of i.i.d. residuals*

To discriminate a stochastic process from a process that contains a deterministic structure we apply the BDS test. The null hypothesis is that the time series is characterized by an i.i.d. process, while the alternative hypothesis is that the time series follows a non-linear law. Applying the BDS test to the residuals randomly generated by computer, we have found a value for the BDS function equal to -1.28 and a critical value of 1.96 at the 5 percent significance level. As expected, we accept the null i.i.d. hypothesis.

From this simple exercise we have obtained the following results:

- Using the Dickey–Fuller test, we have correctly concluded that the time series on levels is stationary and follows a deterministic trend.
- The entropy indicates that the time series of levels is stable and the time series of residuals is extremely unstable. The maximal Liapunov exponent of residuals is sharply positive, and this indicates that nearby trajectories diverge over time. Neither the values of entropy nor the maximal Liapunov exponent provide a definitive answer to the question regarding the nature of time series.
- Recurrence plots and phase portraits allow us to identify the existence of structures that are different from those of an i.i.d. process.
- The BDS test allows us to better appreciate the importance of the time order in time series, that is, to detect the existence of deterministic structures in time series. In this case we were not able to detect any deterministic structure in the residuals since there weren't any (except that of the random number generator algorithm).

5.2 Random Walks

We now analyze another limit case: the random walk. The random walk hypothesis is not generally rejected by the unit root literature and it is at the core of real business cycle theory.

In the random walk case, shocks, contrary to what happens in the case of deterministic trends, have persistent effects and accumulate over time, without being reabsorbed even partially in the future. The time series is not stationary, does not follow a linear trend, but can still grow in a quite similar way to the case of the deterministic trend. From a visual comparison between a series that grows like a random walk and a series that grows along a deterministic linear path, it is often not possible to distinguish the two time series. The Dickey–Fuller test serves to single out which of the two time series follows a random walk.

In a random walk process, the value of the variable x_t depends on its lagged value x_{t-1} and an i.i.d. shock ε_t:

$$x_t - x_{t-1} + \varepsilon_t$$

Suppose now that we are interested in the dynamics of a variable y that grows yearly at the average rate of 2 percent, as an effect of the accumulation of shocks:

$$\ln y_t = \ln y_{t-1} + \varepsilon_t \rightarrow \ln y_t - \ln y_{t-1} = \varepsilon_t \rightarrow \ln \frac{\gamma_t}{\gamma_{t-1}} = \varepsilon_t \rightarrow y_t = e^{\varepsilon_t} y_{t-1}$$

Plotting the log series against time, we would see a dynamics similar to the case of the deterministic trend (not shown here). It is not possible to determine which of the two time series is the random walk by a direct visual inspection alone. A growth trend exists, but it is a stochastic one.

To distinguish between a stochastic trend and a deterministic trend we apply the Dickey–Fuller test and, as we expected, we are not able to reject the unit root hypothesis. The value of the test function turned out to be -1.98, while the critical value is -3.41 at the 5 percent significance level (Table 3.3). Residuals turned out not to be serially correlated (the Durbin–Watson statistic is 1.99).

The entropy level, the maximal Liapunov exponent, the BDS test and Ruelle plots of the residuals are exactly the same as those obtained for the deterministic trend case. Inasmuch as the aim of non-linear dynamics is to detect complex structures in residuals, both in the case of stochastic growth and deterministic growth, residuals are stochastic and the tools of non-linear dynamics cannot be used to detect linear determinism. The suitable instrument to detect linear determinism is indeed the Dickey–Fuller test.

5.3 Non-linear Walks

5.3.1 Autoregressive tent map growth

We now apply the Dickey–Fuller test to an artificial time series where the value of the variable depends on its lagged value and a deterministic non-linear shock. We will apply the BDS test and other tools of non-linear dynamics to identify the deterministic structures that the Dickey–Fuller test is not able to detect.

Suppose that a time series is generated by the following deterministic law:

$$x_t = x_{t-1} + 0.04x_{t-1}\varepsilon_t \text{ with } \begin{matrix} \varepsilon_t = 2\varepsilon_{t-1} \text{ for } \varepsilon_{t-1} < 0.5 \\ \varepsilon_t = 2(1-\varepsilon_{t-1}) \text{ for } \varepsilon_{t-1} < 0.5 \end{matrix}$$

This system is known as the *tent map* and it appeared in an *Economic Journal* article by Scheinkman (1990) and in a working paper of the University of Texas by Vastano and Wolf (1986). This peculiar system generates a chaotic time series which has the same statistical properties as a uniform distribution. Similarly to the random walk, $0.04\varepsilon_t$ has an average value equal to 0.02.[19]

A visual inspection of the generated time series x_t may be puzzling because x_t is very similar to a time series with either a deterministic or stochastic trend. In order to find out whether this system follows a stochastic or a deterministic trend we apply the Dickey–Fuller test and the unit root hypothesis cannot be rejected. In fact the value of the test function turned out to be -1.8 (Table 3.3), while the null hypothesis is rejected for values less than -3.4 at the 5 percent confidence level. The time series appears to be similar to the stochastic trend or to the deterministic linear trend. But we know that it is neither. The Durbin–Watson statistic turned out to be exactly equal to 2.00, and this indicates that residuals are not serially correlated. At this stage we would again apply the Dickey-Fuller test to the residuals to see whether they are stationary, and we would conclude that the process is autoregressive of order one with i.i.d. residuals.

This conclusion is only partly valid. The process is autoregressive of order one and therefore there is a unit root, but the residuals (shown in Figures 3.5 and 3.6) are deterministic and, knowing the law that generates the residuals, the process is perfectly predictable. In this case we must be very careful to read the results obtained with the Dickey–Fuller test. It suggests that it is not possible to reject the null hypothesis of the existence of a unit root, that is, the hypothesis of autoregressive process of order one. However, the residuals, as this case shows, can be non-stochastic. Consequently the Dickey–Fuller test is a tool that is not suitable for unveiling whether the series follows a deterministic law, except for the special case that the series follows a deterministic

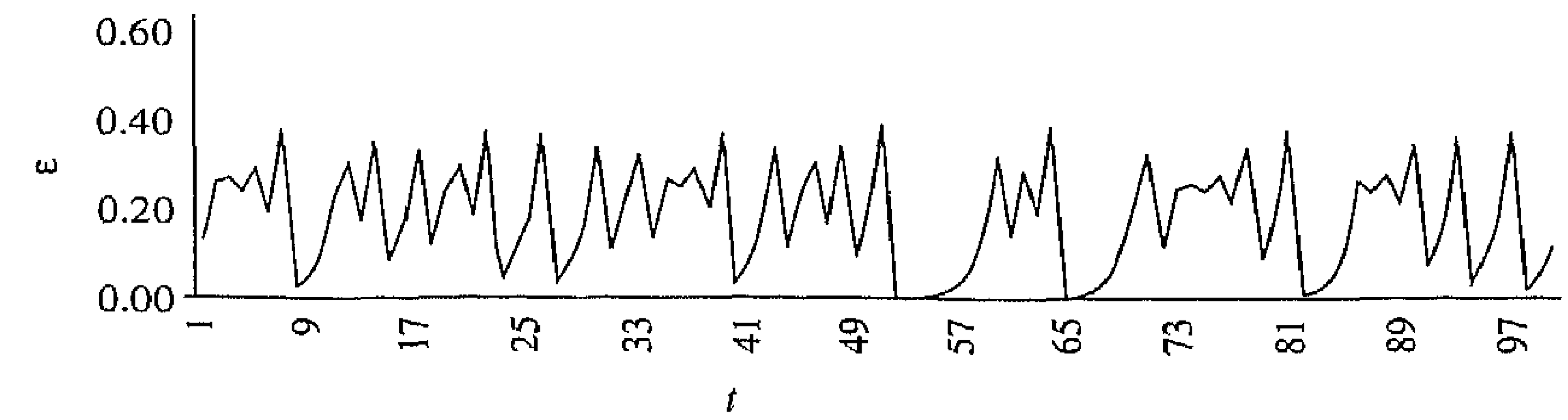

Figure 3.5 *Tent growth residuals*

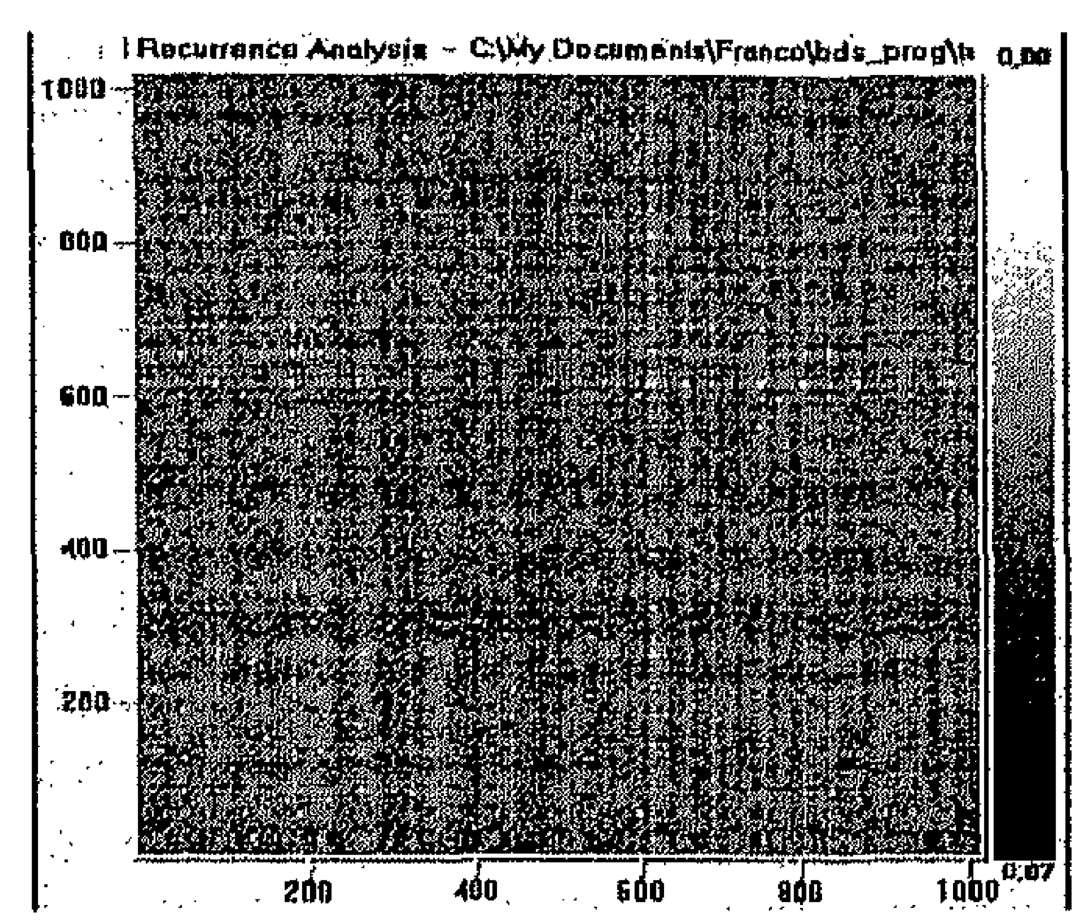

Figure 3.6 *Recurrence plot of the 'tent map'*

linear trend. The acceptance of a unit root hypothesis and the presence of non-serially correlated residuals does not authorize us to take the stochastic origin of the time series for granted.

From the values of entropy (78 percent) and the positive maximal Liapunov exponent we may infer that the system is nearly unpredictable. However, these characteristics are typical of both stochastic and chaotic processes. In order to infer the existence of non-linear structures we have performed the BDS test. The value of the BDS statistic (which asymptotically converges to normality)[20] turned out to be 99.2, and this allows us to reject the null i.i.d. hypothesis with a minimal probability of being mistaken.

5.3.2 Autoregressive Rossler growth

Consider the following system:

$$x_t = x_{t-1} + 0.02x_{t-1}\left(\tfrac{\varepsilon_t}{10} + 1\right),$$

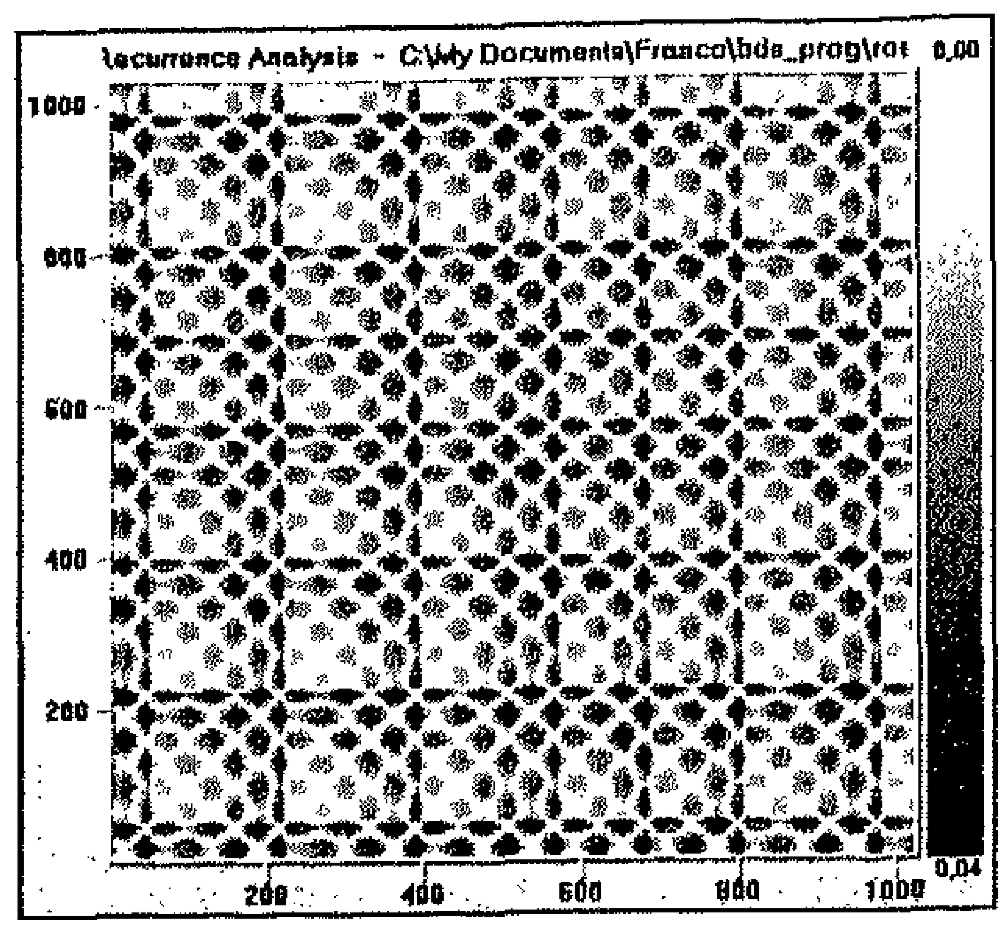

Figure 3.7 Recurrence plot of the 'Rossler' map

where $0.02(\frac{\varepsilon_t}{10} + 1)$ has an average value equal to 0.02, and ε_t is the result of a deterministic non-linear system that generates aperiodic (chaotic) cycles[21] (see Figure 3.8).

Applying the Dickey–Fuller test, we would reject the null hypothesis of autoregressive process of order one and accept the alternative hypothesis of a deterministic trend. The Dickey–Fuller statistic turned out to be -57.52, a value enormously greater than the respective critical value (-3.97 is the corresponding 5 percent critical value) (Table 3.3). The Durbin–Watson statistic turned out to be 0.09, and residuals are indeed serially correlated. Given these results, we would think that the time series follows a deterministic trend and fluctuations are cyclical with reversible effects. However, our model is autoregressive of order one, it does not follow a deterministic trend and the time series is entirely generated by fluctuations ε_t that have persistent effects.

The value of entropy of the residuals is 15 percent, and this low value implies that the system tends to preserve a certain stability over time. The maximal Liapunov exponent is positive and therefore the evolution of the system is sensitive with respect to its initial conditions, but since its value is close to zero, this suggests that the system is also cyclical. In fact it has aperiodic cycles; thus the system is also chaotic. The recurrence plots of the residuals (Figure 3.7), just like a simple graph against time (Figure 3.8), show a cyclical and a periodical dynamical structure.

The support for the existence of non-linear structures in the time series follows from the high value of the BDS statistic (Table 3.3). The null i.i.d. hypothesis is rejected. Although the BDS test was also able to detect correctly the existence of non-linear structures in the data in this case, we may

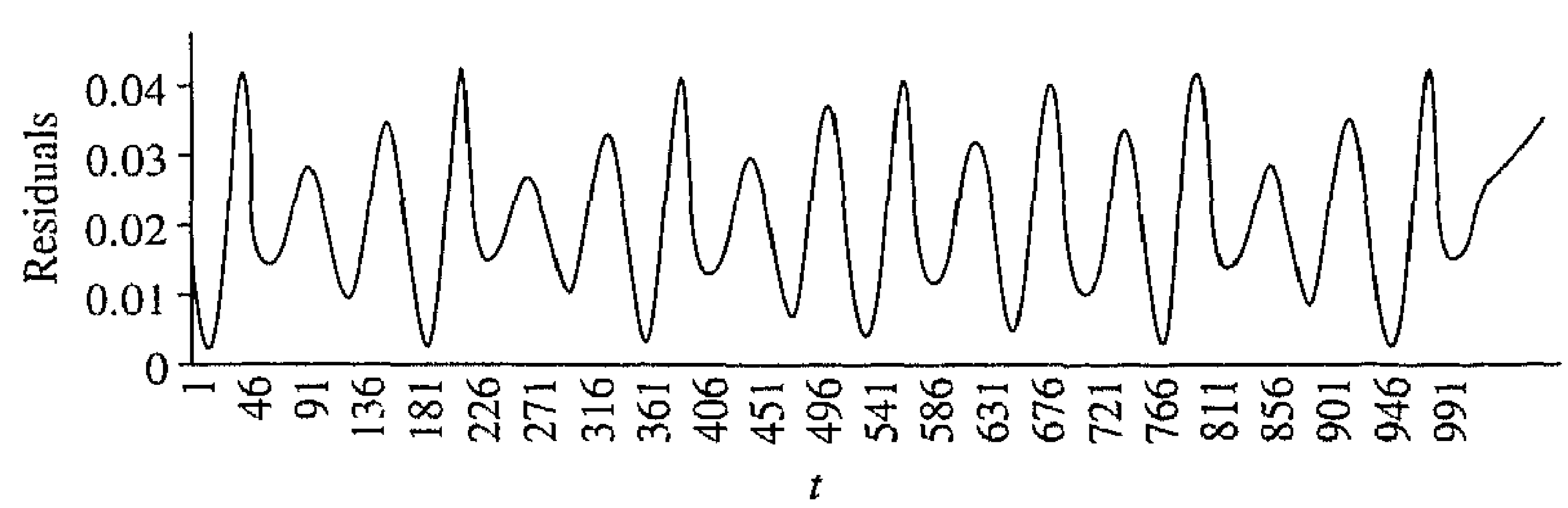

Figure 3.8 *Rossler residuals*

better appreciate its effectiveness when residuals are not serially correlated, as in the cases of the tent map and seasonally adjusted real time series.

6. EMPIRICAL EVIDENCE: THE US TIME SERIES

In the past 15 years the detection of non-linearities in real economic time series has turned out to be a very difficult task. The main problem is to apply the non-linear dynamic tools to time series that contain a sufficient number of observations. In order to reliably calculate the BDS test a quite high number of observations is needed. Around 400 observations are necessary to detect low-dimensional non-linearities. If we wish to discover more complex structures, we need an even higher number of observations. This is because the BDS test has a very low power for small finite samples. The application of the BDS test, as well as all the tools of non-linear dynamics based, as the BDS test, on the correlation dimension, to small samples may produce spurious results. In Section 5.2 the problem of spurious results did not arise since the sample was sufficiently large and consequently the power of the test was also high.

When we have a time series with a very limited number of observations it is necessary to use linear metrics; the use of non-linear dynamics tools would only produce wrong results. For instance, the frequency of observations for GDP is only quarterly and data are available starting from 1959. Although the Bureau of Economic Analysis is going to release these data from 1929, we could only have a maximum of 280 observations and this limitation would not allow us to prove the existence of a non-linear dynamics.[22]

Chavas and Holt (1991) have chosen to analyze a very specific time series which was already known to have a cyclical nature: the *Pork Cycle*. Chavas and Holt have shown the existence of aperiodic cycles in the quarterly time series of the US quantities and prices of pork meat from 1910 until 1984. Chavas and Holt have the great merit to have shown that fluctuations in time series may have a non-linear origin.

In the analysis that follows, we focus on some main real macroeconomic time series. We check whether it is possible to extract signals from the residuals that economic literature has assumed to be stochastic. What we want to ascertain is whether the residuals also contain a non-linear component together with a truly stochastic component. We try to find out whether important temporal linkages are present between residuals. We will attempt to falsify the results of rejection of the null i.i.d. hypothesis. We will proceed to a random shuffle of the time series in order to break any temporal link among data. Afterwards we will apply non-linear dynamics tools to the shuffled time series. If the results of non-linear tests on both the original and the shuffled time series are similar, it means that time linkages are not important and the time series is generated by a stochastic process; otherwise there is evidence that time cannot be ruled out and there exists a non-linear component.

6.1 Industrial Production

The time series for industrial production is certainly one of the most complete available. Data go back to 1919 and the frequency of observation is monthly.

Applying the Dickey–Fuller test[23] to the log of the observed values, we cannot reject the null hypothesis of a unit root (Table 3.4).

Then we estimated the following linear model that best fits the data:

$$Y(t) = 0.02 + 0.99\,Y(t-1) + 0.51[Y(t-1) - Y(t-2)] + 0{,}000029t + \hat{e},$$

where $Y(t)$ are the observed values of the industrial production in terms of value.[24] The Durbin–Watson statistic is 1.95, well within the acceptance range 1.89–2.10. This indicates that the estimated residuals are not serially correlated.

From the original series Y we focused on the estimated residuals $\hat{e}$. The residuals also appear to be characterized by a very complicated dynamics if we look at the entropy level (80 percent).

The calculus of the maximal Liapunov exponent depends on the parameter of the embedding dimension m. There exists a maximal Liapunov exponent for each value of m. The maximal Liapunov exponents are all positive for different values of m and this indicates a high sensitivity of the time series with respect to its initial conditions (Table 3.5).

The existence of a structured dynamics also seems to be corroborated by

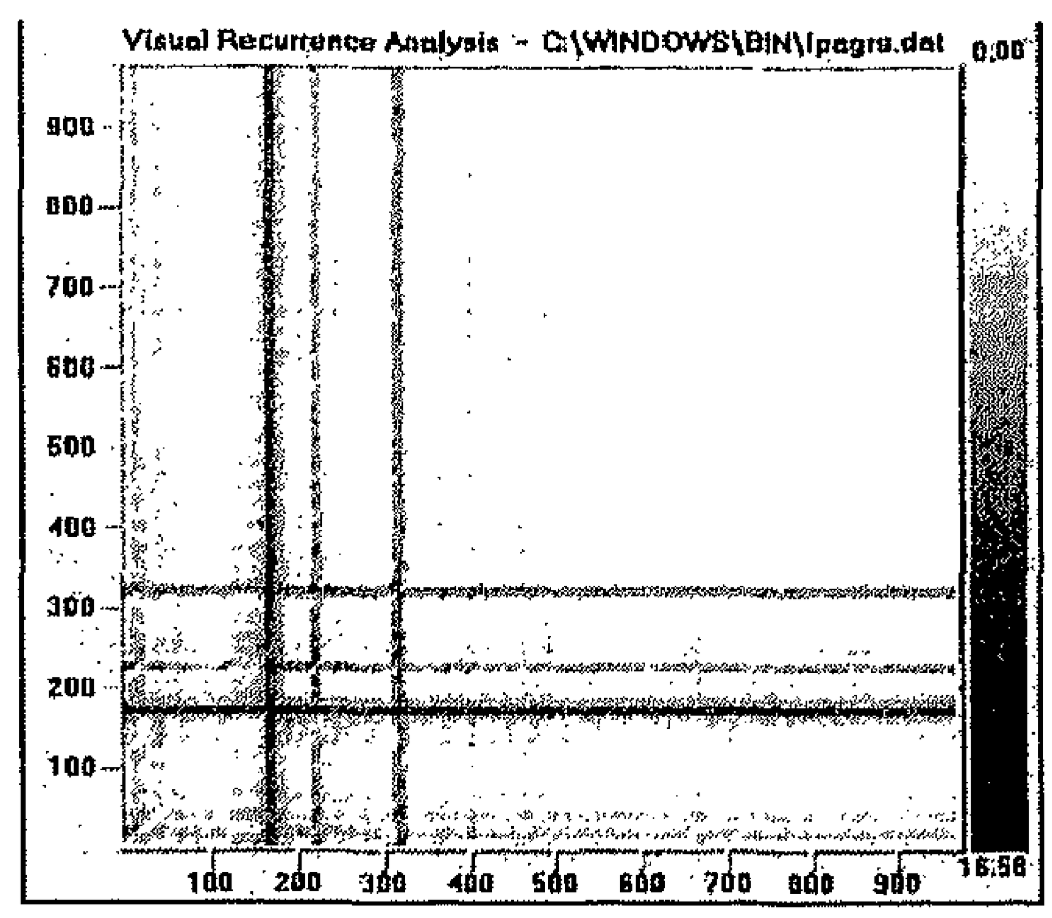

Figure 3.9 *Recurrence plot, residuals, industrial production*

the Ruelle plot[25] (Figure 3.9), where the presence of continuous lines is clear. In Figure 3.9 we can easily detect, without any *a priori* historical knowledge, the periods in which significant historical events have perturbed industrial production. From this recurrence plot we can realize that the first years of the 1920s, the years around 1933 and 1944, have been characterized by an anomalous dynamics. The embedded vectors represented by the single points around those dates show a big distance, marked with a dark shade, compared to nearly all the other vectors. Moreover, we can see that after the 400th embedded vector, the dynamics is more settled and also seems to repeat (see the bright area on the upper right). What is evident in Figure 3.9 is the existence of a structure that differs from a random walk (Figure 3.4).

To ascertain whether the time series is generated by a non-linear deterministic process we have applied the BDS test. The null i.i.d. hypothesis is strongly rejected (Table 3.4, column $W_{m,N}$). A similar test based on the same statistic of the BDS test is the dimension test (Table 3.4, column d_m). The correlation dimension d_m grows very slowly with m and tends to converge to a fixed value. This is typical of a process that is not guided by chance (Hommes 1998).[26]

If we randomize the order of the events of the original time series, we find that the values of the BDS test (column $W_{m,N}^{AF}$, Table 3.4) and the correlation dimension (column d_m^{AF}, Table 3.4) turn out to be very different from the values obtained using the original time series, and we correctly accept the null i.i.d. hypothesis for the shuffled time series. This is evidence that the time order of the residuals of the original time series is not random, and a temporal causality in the fluctuations exists.

We conclude that residuals in industrial production show a structure that cannot come from a mere linear stochastic process and therefore a non-linear explanation is necessary to understand the temporal causality of events. This result shows that there exists a clear non-linear structure in the estimated residuals, which in turn should be considered as truly signals and not as noise.

6.2 Empirical Analysis of Other Macroeconomic Time Series: Industrial Production in the Main US Sectors, Employment, Hourly Wages and the Consumer Price Index

A thorough analysis of each sector would be beyond the scope of this chapter, whose focus is on the existence of deterministic structures in macroeconomic time series. Shortly we summarize the results obtained analyzing some of the main US macroeconomic time series. We have restricted our analysis to the main sectors of the US economy,[27] employment, hourly wages and the consumer price index. Regarding the economic variables characterized by seasonal cycles we analyzed the seasonally adjusted time series. The frequency of observations is monthly. Data go back to 1947 for the transportation sector, industrial machinery and electrical machinery, 1967 for the hybrid high-tech sector (computers, semiconductors and communications), 1939 for employment, 1932 for hourly wages and 1913 for the consumer price index.

All the time series (log transformed), except employment, seem characterized by a unit root, since for most of them we are not able to reject the null i.i.d. hypothesis of the Dickey–Fuller test (Table 3.4) with high confidence levels (higher than 5 percent).[28] These results are qualitatively similar to those obtained by Nelson and Plosser. For all the time series, the estimated residuals of the linear model[29] that best fit the data turn out to be serially uncorrelated (the null hypothesis of the Durbin–Watson test is never rejected, even at a high confidence level for all the time series).

All the time series we analyzed are characterized by high entropy values (generally higher than 70 percent) that are typical of both chaotic and stochastic processes. For all the real time series we found positive values of the corresponding maximal Liapunov exponents (Table 3.5) and this result suggests that nearby trajectories diverge over time at a positive exponential rate. The interesting result is that all the real time series are characterized by a Liapunov exponent that is decidedly lower than that of an i.i.d. process, and lower than that of the tent map. This suggests that even if real time series have to be considered unpredictable in the long run, in the short run they are more predictable than an i.i.d. process and a deterministic process like the tent map.[30]

Table 3.4 Log-transformed time series

	Industrial production	Transportation equipment	Industrial machinery	Electrical machinery
ADF statistic	−3.1	−3.9*	−3.8*	−2.8
Estimated model	$Y_t = 0.02 + 0.99Y_{t-1} + 0.51\Delta Y_{t-1} + \varepsilon$	$Y_t = 0.13 + 0.96Y_{t-1} + \varepsilon$	$Y_t = 0.05 + 0.98Y_{t-1} + 0.09\Delta Y_{t-1} + 0.29\Delta Y_{t-2} + 0.26\Delta Y_{t-3} + \varepsilon$	$Y_t = 0.05 + 0.97Y_{t-1} + 0.17\Delta Y_{t-1} + \varepsilon$
Durbin–Watson stat.	1.95	1.86	2.04	2.06
Entropy of residuals	80%	73%	77%	Not available

Parameters		BDS and dimension tests				BDS and dimension tests				BDS and dimension tests				BDS and dimension tests			
ε	m	$W_{m,N}$	d_m	$W^{AF}_{m,N}$	d^{AF}_m	$W_{m,N}$	d_m	$W^{AF}_{m,N}$	d^{AF}_m	$W_{m,N}$	d_m	$W^{AF}_{m,N}$	d^{AF}_m	$W_{m,N}$	d_m	$W^{AF}_{m,N}$	d^{AF}_m
2σ	2	11.3**	0.13	0.9	0.15	10.4**	0.13	−0.3	0.17	4.1**	0.26	−0.1	0.27	4.1**	0.26	−0.1	0.27
2σ	4	15.1**	0.21	1.4	0.30	10.7**	0.23	0.1	0.33	3.8**	0.51	−0.5	0.55	3.8**	0.51	−0.5	0.55
2σ	6	16.2**	0.28	0.9	0.46	9.5**	0.32	0.1	0.49	4.1**	0.74	−0.7	0.83	4.1**	0.74	−0.7	0.83
σ	2	13.9**	0.21	0.6	0.25	11.8**	0.26	−0.7	0.34	3.5**	0.61	−0.7	0.49	3.5**	0.61	−0.7	0.49
σ	4	19.4**	0.32	0.7	0.49	13.0**	0.46	0.0	0.67	3.3**	1.2	−0.7	0.99	3.3**	1.2	−0.7	0.99
σ	6	22.5**	0.42	0.2	0.74	13.1**	0.65	0.0	1.00	3.8**	1.7	−0.8	1.50	3.8**	1.7	−0.8	1.50
0.5σ	2	15.5**	0.31	0.4	0.37	11.7**	0.45	−1.2	0.55	2.8**	0.9	−0.7	0.7	2.8**	0.9	−0.7	0.7
0.5σ	4	21.5**	0.50	0.4	0.74	13.9**	0.81	−0.4	1.09	2.9**	1.8	−0.7	1.4	2.9**	1.8	−0.7	1.4
0.5σ	6	27.3**	0.66	−0.2	1.12	15.3**	1.17	−0.0	1.63	3.6**	2.6	−0.5	2.1	3.6**	2.6	−0.5	2.1

Table 3.4 (Continued)

		High-tech				Employment				Hourly earnings				Consumer price index			
ADF statistic		0.6				−4.2**				−1.1				−0.8			
Estimated model		$Y_t = 1.00Y_{t-1} + 0.12\Delta Y_{t-1} + \varepsilon$				$Y_t = 0.16 + 0.97Y_{t-1} + 0.27\Delta Y_{t-1} + 0.27\Delta Y_{t-2} + \varepsilon$				$Y_t = 1.00Y_{t-1} + 0.20\Delta Y_{t-1} + 0.24\Delta Y_{t-2} + \varepsilon$				$Y_t = 1.00Y_{t-1} + 0.33\Delta Y_{t-1} + 0.16\Delta Y_{t-2} + 0.13\Delta Y_{t-3} + \varepsilon$			
Durbin–Watson stat		2.05				2.07				2.04				2.05			
Entropy of residuals		Not available				68%				71%				71%			
Parameters		BDS and dimension tests				BDS and dimension tests				BDS and dimension tests				BDS and dimension tests			
ε	m	$W_{m,N}$	d_m	$W_{m,N}^{AF}$	d_m^{AF}	$W_{m,N}$	d_m	$W_{m,N}^{AF}$	d_m^{AF}	$W_{m,N}$	d_m	$W_{m,N}^{AF}$	d_m^{AF}	$W_{m,N}$	d_m	$W_{m,N}^{AF}$	d_m^{AF}
2σ	2	3.7**	0.13	0.0	0.17	10.8**	0.07	−1.0	0.09	12.2**	0.06	−1.6	0.08	8.2**	0.12	−1.4	0.13
2σ	4	4.0**	0.25	1.4	0.30	11.0**	0.13	−0.9	0.19	12.0**	0.11	−1.2	0.16	11.7**	0.21	−0.9	0.27
2σ	6	4.4**	0.36	2.2*	0.43	10.9**	0.18	−0.9	0.28	12.4**	0.15	−1.2	0.25	14.0**	0.27	−1.3	0.41
σ	2	4.7**	0.41	0.1	0.46	10.0**	0.17	−0.9	0.21	12.3**	0.17	−0.4	0.22	12.3**	0.26	−1.4	0.30
σ	4	6.3**	0.75	2.1*	0.87	12.1**	0.29	−0.1	0.41	15.4	0.28	−0.3	0.43	18.6**	0.43	−0.3	0.60
σ	6	7.9**	1.05	2.8**	1.25	14.6**	0.37	0.2	0.62	18.3**	0.37	−0.4	0.65	25.3**	0.56	0.2	0.90
0.5σ	2	4.9**	0.57	−0.1	0.63	9.1**	0.34	−0.3	0.40	12.9**	0.33	0.2	0.40	15.3**	0.43	−1.1	0.50
0.5σ	4	7.2**	1.05	1.9	1.20	14.7**	0.57	0.4	0.79	19.0**	0.56	−0.3	0.82	24.9**	0.72	−0.3	1.00
0.5σ	6	9.5**	1.47	2.4**	1.74	22.2**	0.75	0.2	1.19	26.9**	0.74	−0.5	1.24	41.6**	0.92	0.5	1.48

Note: * and ** denote significance at 5% and 1% levels. $W_{m,N}^{AF}$ and d_m^{AF} are respectively the BDS and the dimension tests after randomization.

Table 3.5 Maximal Liapunov exponents

	$m=1$	$m=2$	$m=4$
Uniform i.i.d.	3.40	1.41	0.77
Tent map	2.93	0.91	0.36
Rossler map	0.67	0.06	0.09
Industrial prod.	2.68	0.75	0.33
Transp. equip.	1.71	0.60	0.36
Industrial mach.	1.75	0.64	0.28
Electrical mach.	1.81	0.49	0.26
High-tech	1.59	0.46	0.21
Employment	1.55	0.67	0.30
Hourly earnings	1.81	0.70	0.39
Consumer price index	1.88	0.93	0.45

The presence of structures different from those typical of an i.i.d. process has been pointed out by the recurrence plots of all the time series. If we compare Figures 3.9–3.16 with Figure 3.4 (Figure 3.4 is typical of an unstructured random process), we can see clearly the existence of structures (repetitive continuous lines over time) in the distances (represented by the intensity of grey) between the embedded vectors (represented by each single point in the coordinates).[31]

The application of the BDS test gives us further information about the existence of determinism in time series. Applying the BDS test to all the time series at our disposal, we are not able to accept the null i.i.d. hypothesis. All the series are characterized by high values of the BDS statistic well beyond their respective critical values (column $W_{m,N}$, Table 3.4). The dimension test,[32] based, as the BDS test, on the calculus of the correlation dimension, allows us in some cases to measure the dimension of the chaotic attractor that characterizes the time series. Without going into the details, the dimension test is based on the fact that a truly stochastic process is characterized by the growth of the correlation dimension with the increase of the embedding dimension, while a truly chaotic process is characterized by the correlation dimension tending to settle to a constant value when the embedding dimension increases (Hommes 1998). This constant value represents the dimension of the chaotic attractor. In all the series we have analyzed the correlation dimension (column d_m in Table 3.4) grows less than proportionally with respect to m, but in many cases we cannot detect a clear tendency of the correlation dimension to settle clearly to a constant value. For all the time series we have analyzed, the BDS test suggests that the time series contains a deterministic structure, but it is not possible to quantify,

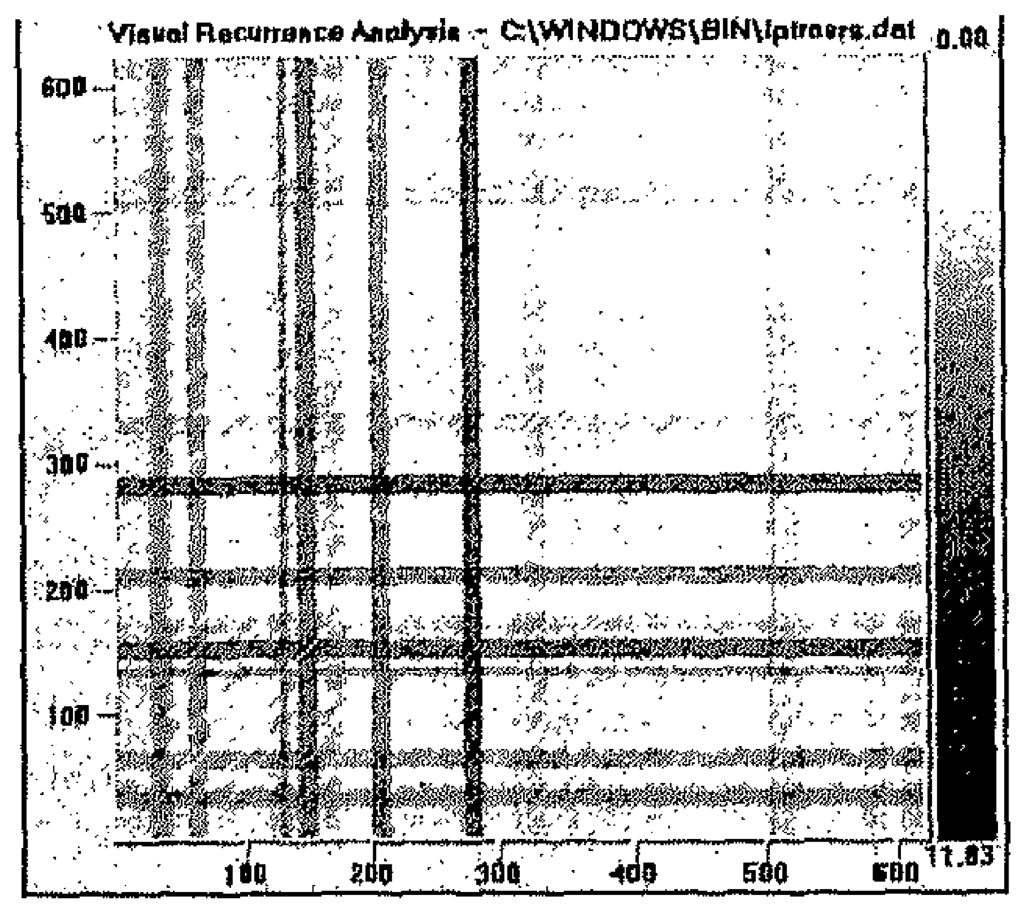

Figure 3.10 *Recurrence plot, residuals, transportation equipment*

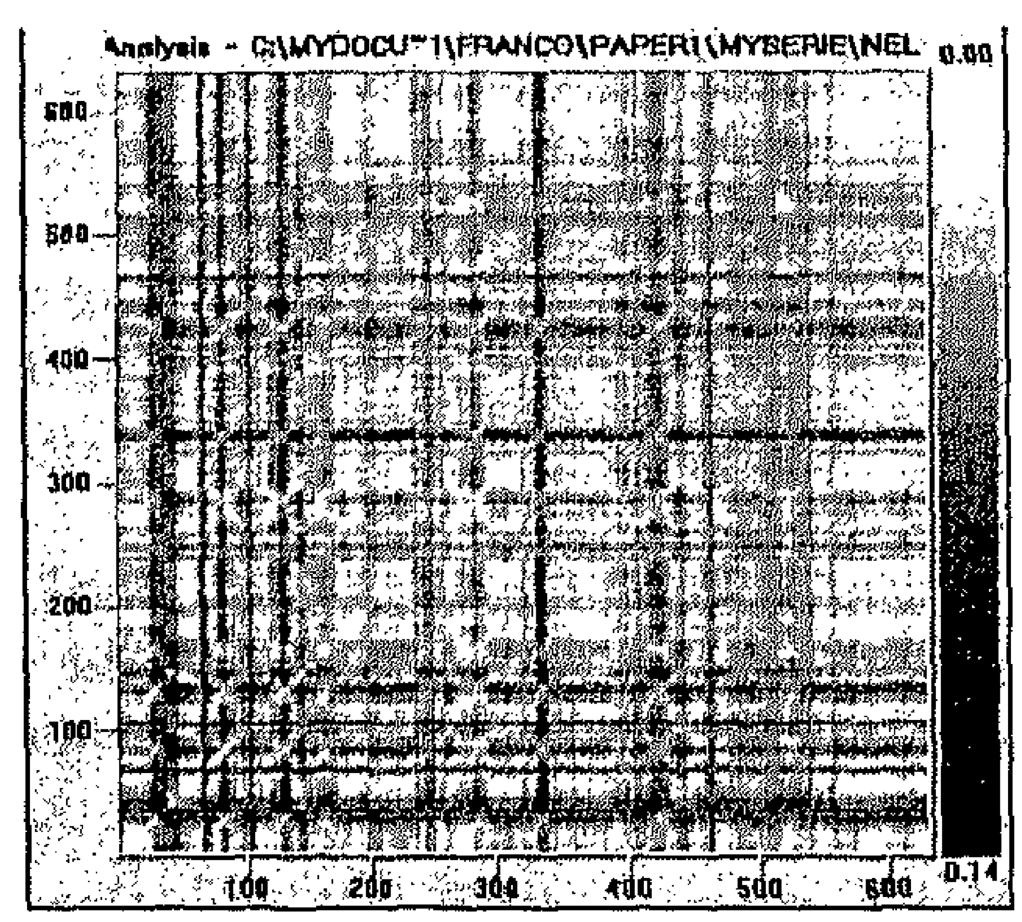

Figure 3.11 *Recurrence plot, residuals, industrial machinery*

via the dimension test, the dimension of the underlying attractor of the time series.[33]

To check further our results we have randomly ordered the real time series, applied BDS and calculated the dimension correlation of the shuffled time series to see whether temporal linkages were relevant. In all the cases the values of the BDS and the dimension tests of the shuffled time series were notably different. We could not reject the null hypothesis of the BDS test for all the shuffled time series (column $W_{m,N}^{AF}$ Table 3.4) and the correlation dimension also was also higher (column d_m^{AF}, Table 3.4) with respect to the original time series. This is confirmation that temporal link-

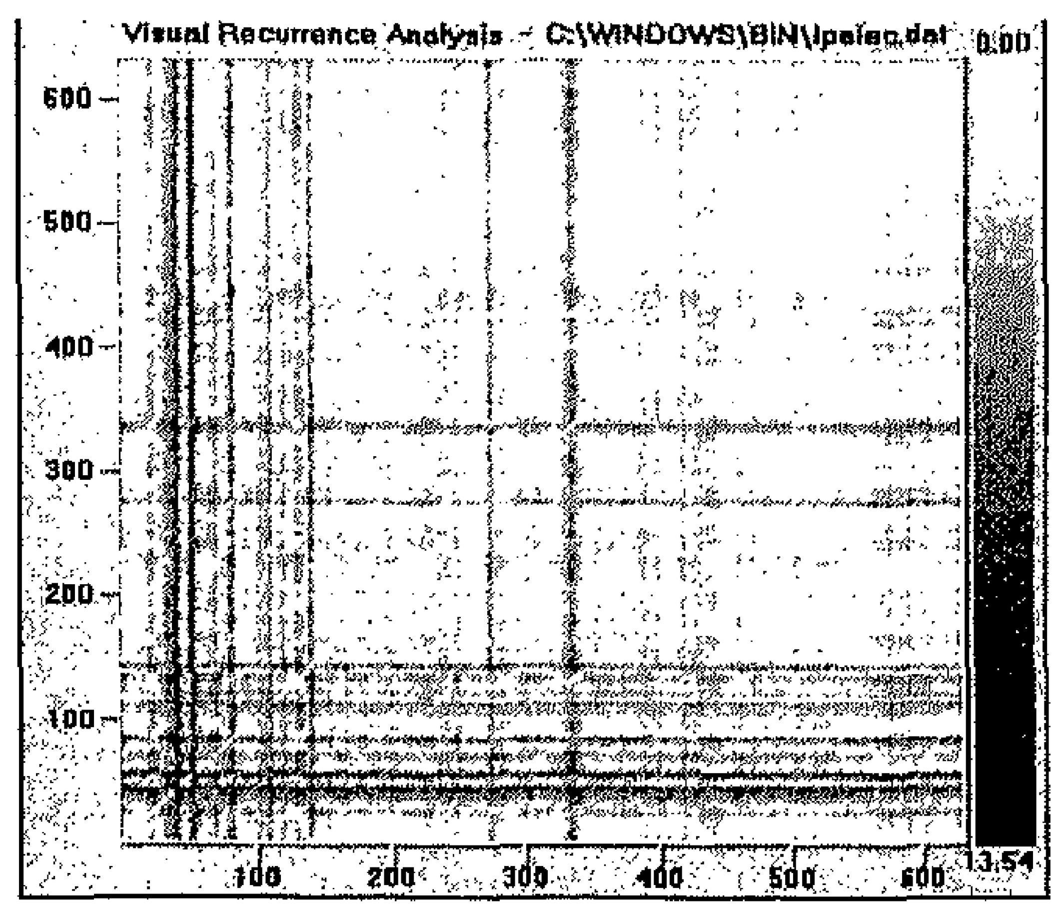

Figure 3.12 *Recurrence plot, residuals, electrical machinery*

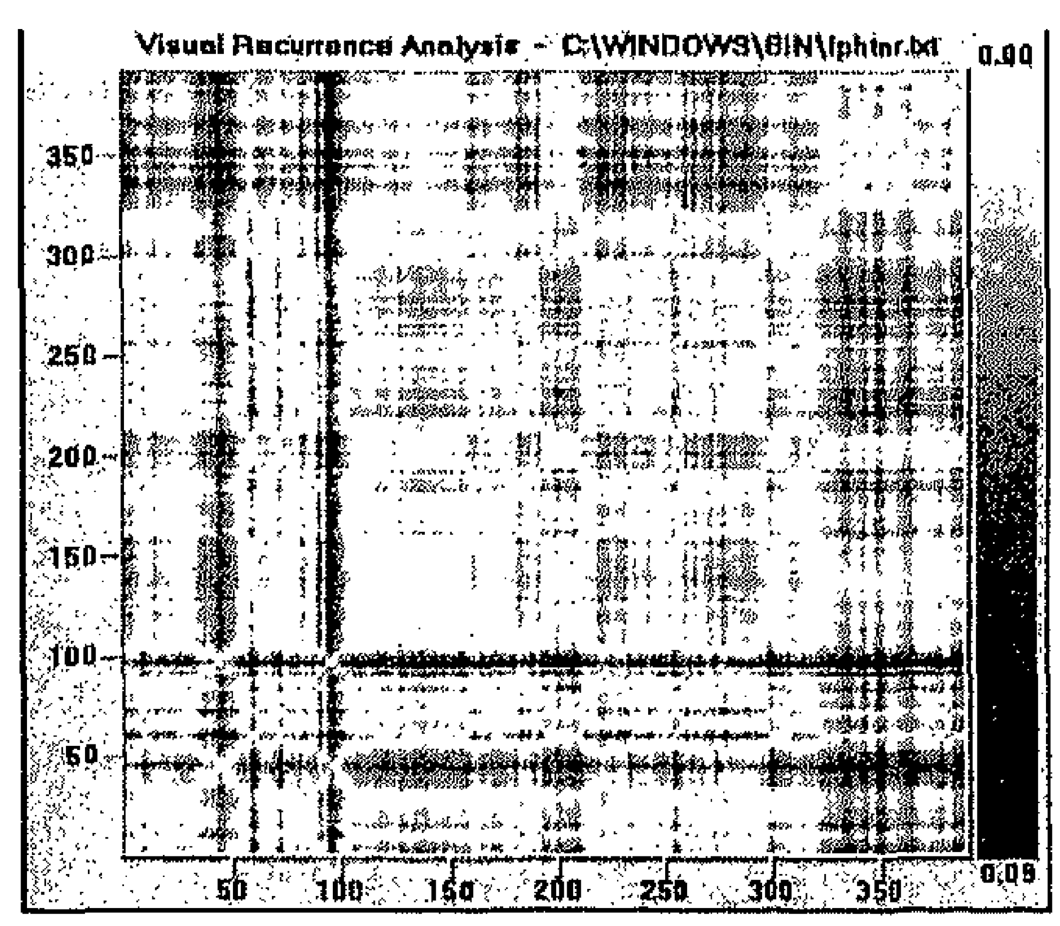

Figure 3.13 *Recurrence plot, residuals, high technology*

ages between residuals are really important and therefore a mere probabilistic hypothesis on the residuals of macroeconomic time series does not have empirical grounds.

7. CONCLUDING REMARKS

We have first shown the theoretical possibility (Sections 4 and 5) and then the empirical evidence (Section 6) that in the serially uncorrelated residuals there are non-linear signals which, in the models with a deterministic

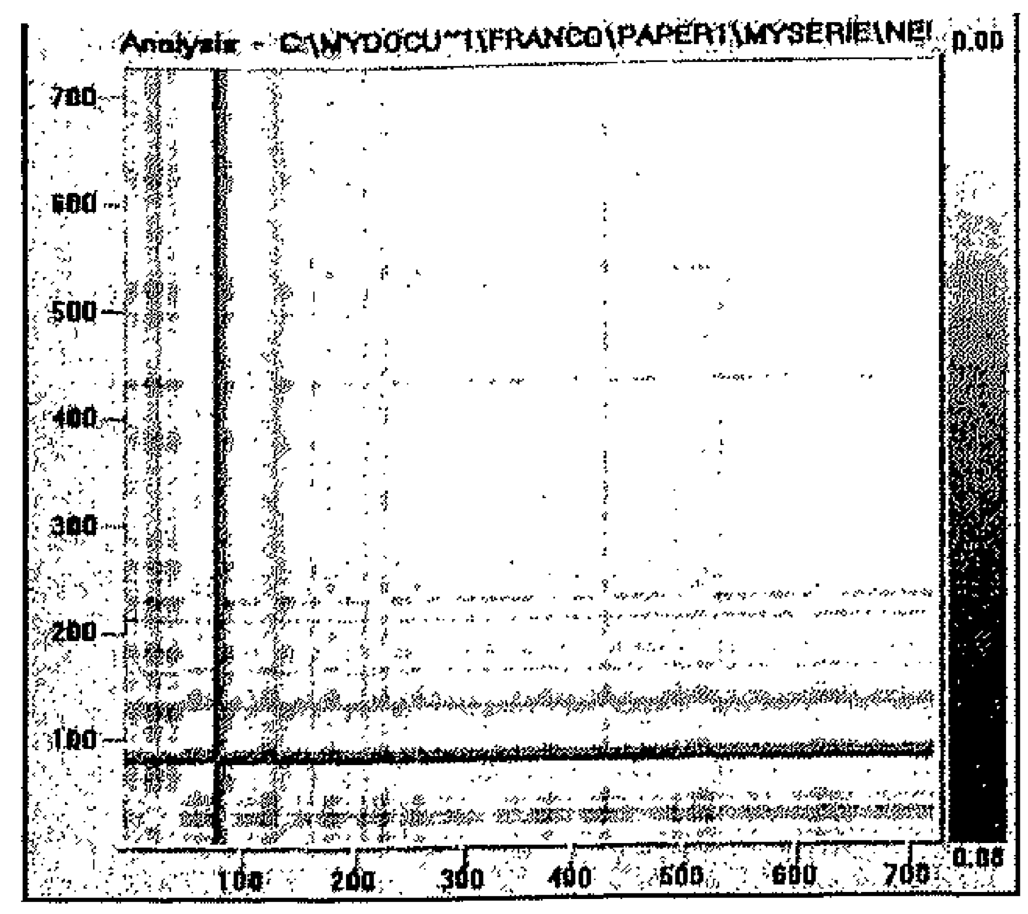

Figure 3.14 Recurrence plot, residuals, employment

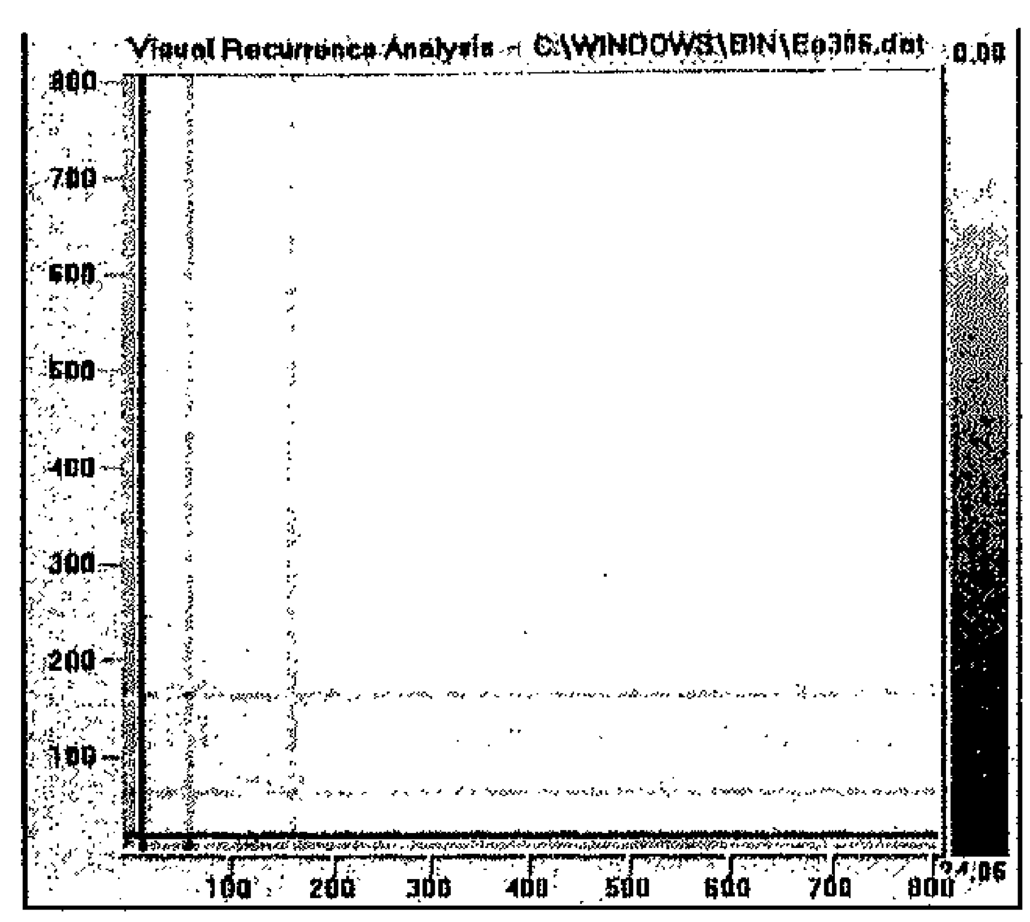

Figure 3.15 Recurrence plot, residuals, hourly earnings

(linear or broken) or stochastic trend, are assumed to be i.i.d., like white noise. The approach that we put forward is to separate the stochastic component (that is indeed present in the residuals) from the deterministic component and study these two components separately. To be successful in this task we need a data filter based on the concepts of non-linear dynamics. In this chapter we have limited our analysis to the detection of the existence of clear non-linearities in the residuals of macroeconomic time series. We have detected non-linearities in all the time series we analyzed. All the time series we have considered are thus characterized by determinism, notwithstanding all the series (except employment) are non-stationary and residu-

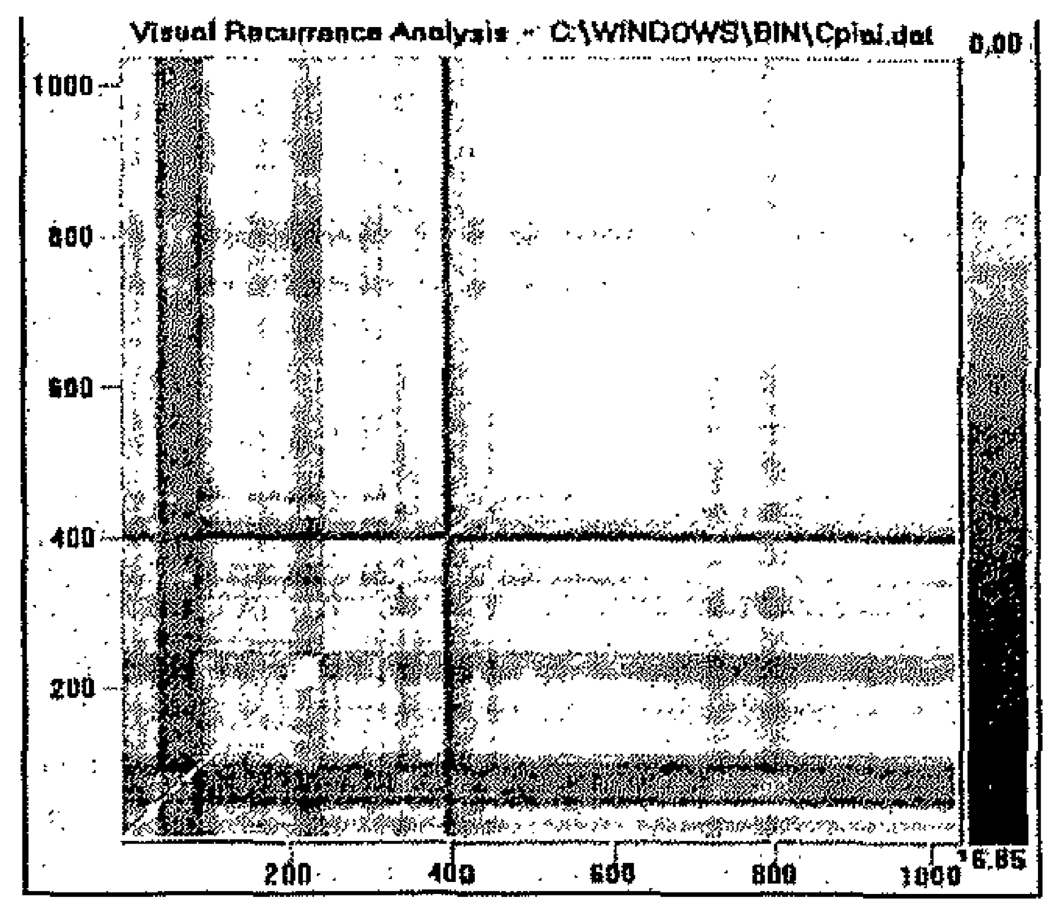

Figure 3.16 *Recurrence plot, residuals, consumer price index*

als are serially uncorrelated. If all this is true, in the short run, we may make better predictors than simple autoregressive models.

The problem of distinguishing between the two alternative hypothesis, deterministic trend or stochastic trend, was at the core of unit root and broken trend literature (Section 2), but for us it was not the most important issue. Our aim was to detect non-linear structures in those components that linear stochastic models have assumed as exogenous factors. In so far as in linear stochastic models noise plays the relevant role to make 'non-stationary' basically stationary processes, it was for us of primary importance, from the theoretical point of view, to check whether a component of what has been so far assumed as noise might have an endogenous explanation. If this is the case, as confirmed in Section 6, economic variables may not follow a stationary path even in the absence of external shocks and the observed non-stationarity may be the consequence of complex relations between the economic variables.

NOTES

1. Nelson and Plosser have analyzed 14 macroeconomic time series for the USA (with starting date between 1860 and 1909 and with final date 1970). Among these there are real GNP, nominal GNP, industrial production, employment, the unemployment rate, the consumer index rate, nominal wages and real wages.
2. In the classical econometric works, time series were considered stationary along a deterministic trend, that is variables are a linear function of time:

$$x_t = \beta t + \alpha + \varepsilon_t$$

with ε_t i.i.d., α and β parameters, t time and x_t a random variable x observed at time t.

In this case the time series of the variable x is stationary along a time trend and each ε_t has only temporary effects. The short-run component may be insulated, regressing x_t against time and assuming the regression line as the abscissa. This procedure was approximately the one that was used in the 1970s to analyze short-run cycles.

3. In *unit root* processes, time series are not stationary and follow a *random walk*, such as: $x_t = \rho x_{t-1} + \varepsilon_t$ with ε_t i.i.d. and $\rho = 1$. This process is called '*unit*' *root* because x_{t-1} is multiplied by a parameter equal to one (or close to one). It is a '*root*' because one is the root of a characteristic equation (see Enders 1995, p. 25). Each ε_t has persistent effects since, as we can see, each fluctuation will not be reabsorbed in the future: $x_t = x_{t-1} + \varepsilon_t = x_{t-2} + \varepsilon_{t-1} + \varepsilon_t = \ldots = \varepsilon_0 + \varepsilon_1 + \ldots + \varepsilon_{t-1} + \varepsilon_t$. The *signal* x_t is therefore generated by the past and present noise ε. Since noise is an i.i.d. and exogenous variable, we conclude that the variable x_t depends entirely on a variable which we don't know anything about.
4. This result also seems not to depend on the frequency of observation: Wells (1997) and Osborn et al. (1999) have found similar results using both quarterly and monthly data.
5. Except Banerjee et al. (1992), Bresson and Celimene (1995), Dolado and Lopez (1996).
6. The consumer price index and nominal wages for instance were found to follow a random walk.
7. Where the consumption decisions are based on a well-behaved utility function

$$U = \sum_{t=0}^{\infty} \beta^t u(C_t, L_t) \text{ with } \beta < 1,$$

where L_t is the leisure at time t, u the utility. ∞ indicates that the individual is the infinitely lived representative.
8. A short but detailed description of all the methods used in this chapter can be found in Bevilacqua (2001).
9. We exclude the possibility of analyzing any time series of GDP and GNP because of the dearth of data, since these time series are at most quarterly.
10. Links to the files concerning monthly seasonally adjusted and in real terms for industry productions were found at: http://www.bog.frb.fed.us/releases/G17/download2.htm

    Indices of industrial production go back to 1919 and the respective base year is 1992.

    A table showing the historical consumer price index for all urban consumers beginning in 1913 was available from the Bureau of Labor and Statistics at: ftp://ftp.bls.gov/pub/special.requests/cpi/cpiai.txt.

    This table refers to all urban consumers, with 1982 as the base year.

    The seasonally adjusted 'hourly wages' time series in this chapter refers to manufacturing industry with data of the type 'average hourly earnings of production workers'.
11. As calculated by E. Kononov (1999), VRA 4.2 program.
12. See Section 5.3.1: the case of the *tent map*.
13. Such as, for instance, the Rossler map in Section 5.3.2.
14. This step is also sometimes called 'shuffle diagnostic' (see Lorentz 1989) via 'surrogate time series' (Kantz and Schreiber 1997). A 'surrogate' time series is essentially the shuffle of the original time series preserving all the linear properties of the time series like frequencies, amplitudes and eventual linear autocorrelations. We have derived the surrogate time series for all the economic time series we have analyzed, but we called them the more general and less specialistic term of 'shuffled time series'.
15. Note that ε in all our experiments is distributed as a uniform distribution. Similar results can be obtained using other distributions, such as the normal. However, what is important is that ε is i.i.d. whatever its distribution. We have chosen to use the uniform distribution because in Section 5.3 we show the deterministic case of the tent map which produces ε that are uniformly distributed.
16. We could obviously plot residuals $\varepsilon(t)$ against the residuals of any preceding period, for example $\varepsilon(t-4)$. Knowing *ex ante* that ε is the result of a random number generator, the $\varepsilon(t) - \varepsilon(t-4)$ plot is qualitatively equivalent to the $\varepsilon(t) - \varepsilon(t-1)$ plot.

17. The embedded vectors are simply defined as:

$$\mathbf{x}_i = \{x_i - (m-1), x_i - (m-2), \ldots, x_i\}$$

where x_i is the observed value at a certain point at time and m is called embedding dimension.

For example, suppose we have a series of ten observed values of a certain variable x:

$$x = \{8, 5, 6, 9, 4, 4, 1, 7, 3, 2, 7\},$$

we obtain the following embedded vectors:

$$\mathbf{x}_2 = \{x_{2-(2-1)}, x_{2-(2-2)}\} = \{x_1, x_2\} = \{8,5\}$$
$$\mathbf{x}_3 = \{x_{3-(2-1)}, x_{3-(2-2)}\} = \{x_2, x_3\} = \{5,6\}$$
$$\mathbf{x}_4 = \{x_{4-(2-1)}, x_{4-(2-2)}\} = \{x_3, x_4\} = \{6, 9\}$$
$$\ldots$$
$$\mathbf{x}_{10} = \{x_{10-(2-1)}, x_{10-(2-2)}\} = \{x_9, x_{10}\} = \{3,7\}$$
for $m = 2$,

and $\mathbf{x} = \{x_2, x_3, x_4, \ldots, x_{10},\}$ is the embedded time series for $m = 2$.

The embedded time series are of great importance in non-linear dynamics because, thanks to them, as has been shown by Takens (1981), we may uncover some properties such as the correlation dimension of an unknown underlying motion law that generated the time series itself from the observed values of the process.

18. That is, between vectors $\mathbf{x}_i$.
19. Because of the finite approximation of the program we used, we could not obtain more than 50 observations. Consequently we have added a very small ratio of white noise to each ε_t so that the system does not repeat itself even in the long run. We have added 0.000001 * $U(0.5, 1)$ noise.
20. See Bevilacqua (2001) or the original work by Brock et al. (1991) for the size and power of the BDS test.
21. For a detailed description of the Rossler process see Lorentz (1989) or Gandolfo (1997).
22. A generally accepted result is that the GDP time series, as pointed out by the vast literature on unit roots and co-integration, is characterized by a stochastic trend, but it cannot be reliably tested with the non-linear numerical tools because of a paucity of observations. Hence we cannot ascertain whether the GDP is really characterized by a non-linear dynamics.
23. Since some time series were autocorrelated in the residuals, we have used for all the real time series the 'augmented' form of the Dickey–Fuller test including more lags, trend and intercept. The number of lags we have considered is the minimal that allows us to obtain uncorrelated residuals. See Harris (1995) for more details.
24. All the sector time series we have considered are in terms of value.
25. Obtained setting $m = 5$.
26. Similar results were also obtained by adding a small percentage of noise (5 percent of the variance). We added noise to the time series simply because, when the non-linear structure is well defined, adding a small stochastic component should not change significantly the result of the test. Even if there were small i.i.d. measure errors, these should not call in question the obtained results.
27. Those that are the most important with respect to the value added.
28. However, for transportation equipment production and industrial machinery production we are not able to reject the null hypothesis at the 1 percent significance level.
29. See the estimated equations within Table 3.4.
30. It is worth mentioning that in Section 6.1 we found a maximal Liapunov exponent for industrial production close to zero, indicating the presence of cycles.
31. The presence of continuous lines in the recurrence plots indicates that the embedded vectors represented by each point keep approximately the same distance with respect to

all the vectors that belong to the continuous line. In a normal i.i.d. process, each vector is randomly distant from any other vector and the probability that nearby vectors have similar distances is very low. Thus in a normal i.i.d. process we should not notice any continuous line in the recurrence plots.

32. Note that the 'dimension test', contrary to the BDS test, is not really a statistical test since critical values are not specified. It's a numerical tool that suggests the existence of a deterministic dynamics when the calculated correlation dimension tends towards a fixed value when the embedding dimension grows.
33. This phenomenon may be due to the presence of a stochastic component in the time series. It should therefore be important to filter our data in order to separately analyze the deterministic component and to quantify the dimension of the chaotic attractor. The future application of filters that allow us to reduce and, it is hoped, remove the stochastic component may allow us to detect the dimension of chaos for all the real time series for which we have already uncovered the presence of chaos.

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