Randomization Inference with Natural Experiments: An Analysis of Ballot Effects in the 2003 California Recall Election

Kosuke Imai

Department of Politics
Princeton University

Joint work with
Daniel E. Ho, Yale Law School

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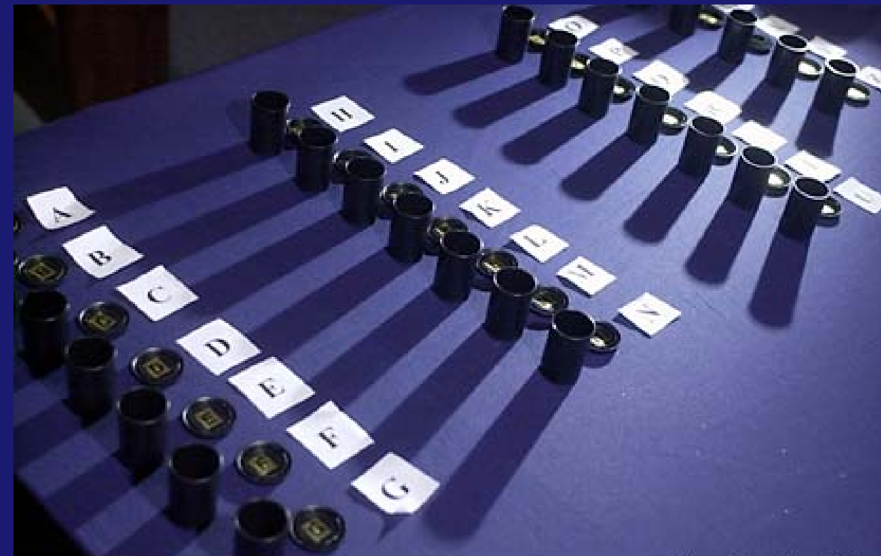

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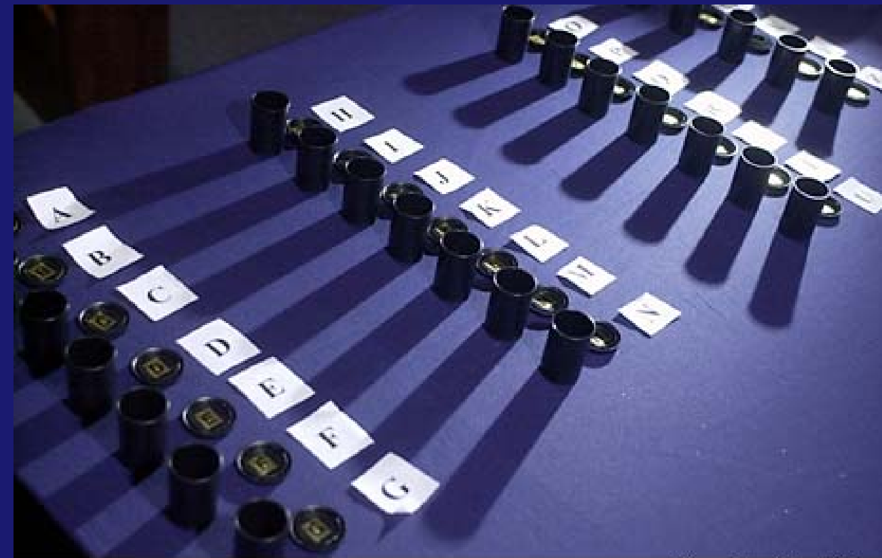

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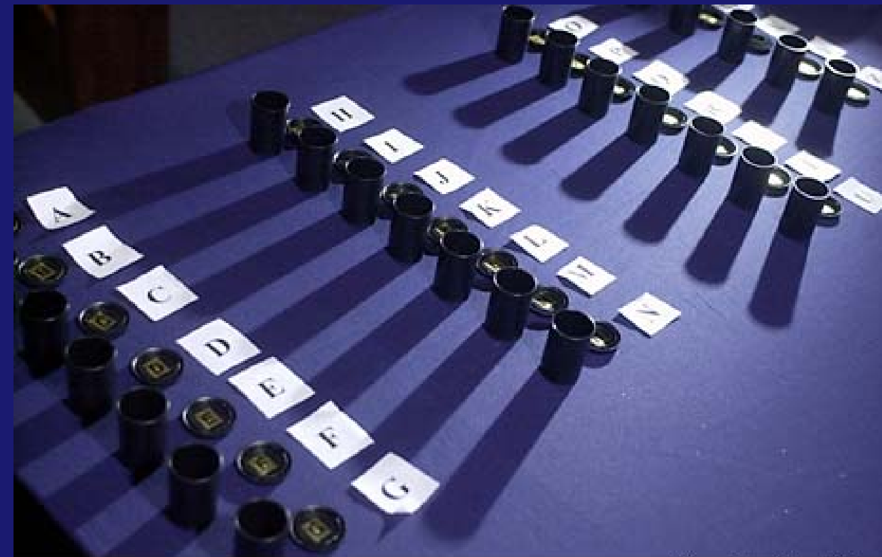
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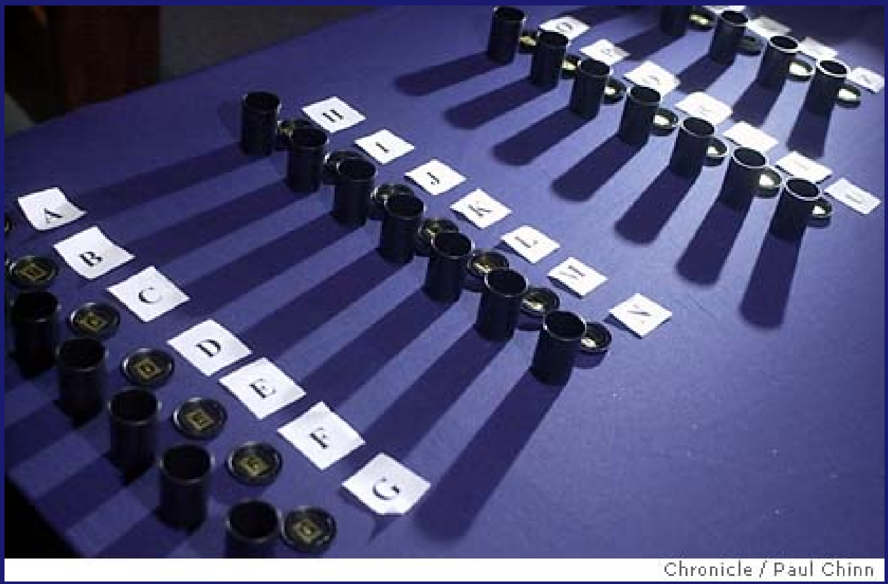

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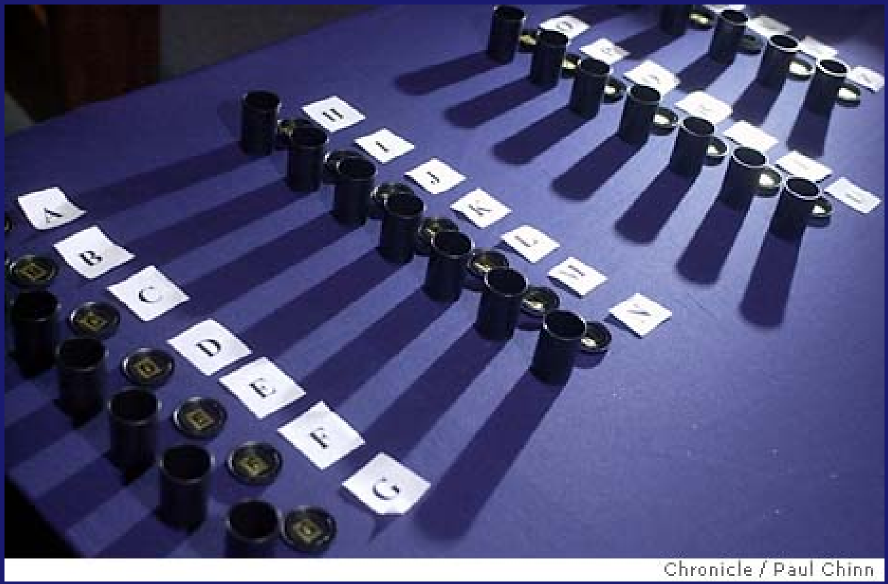

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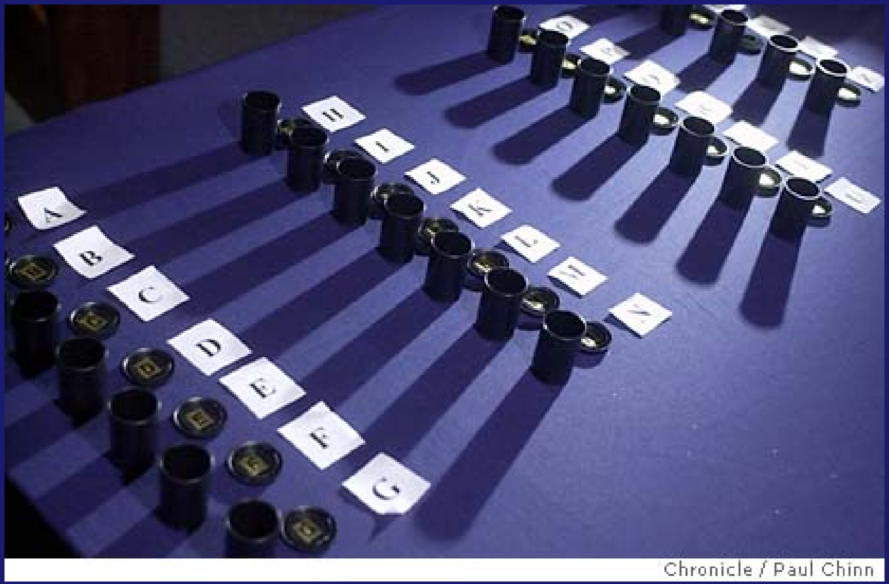

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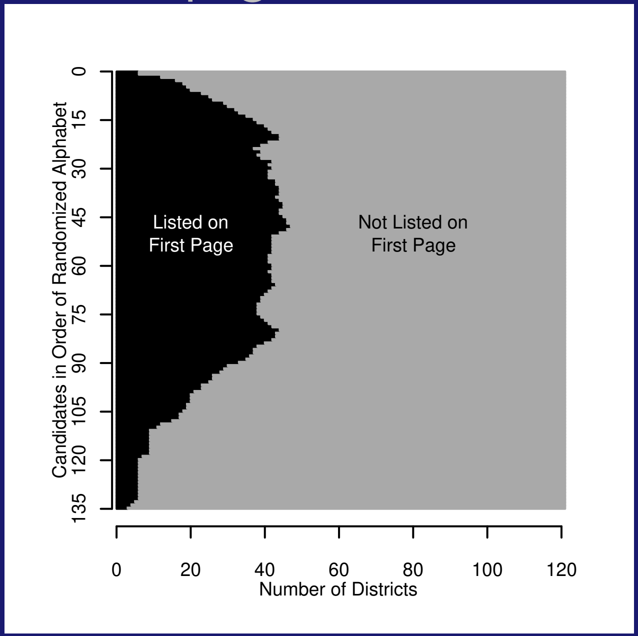

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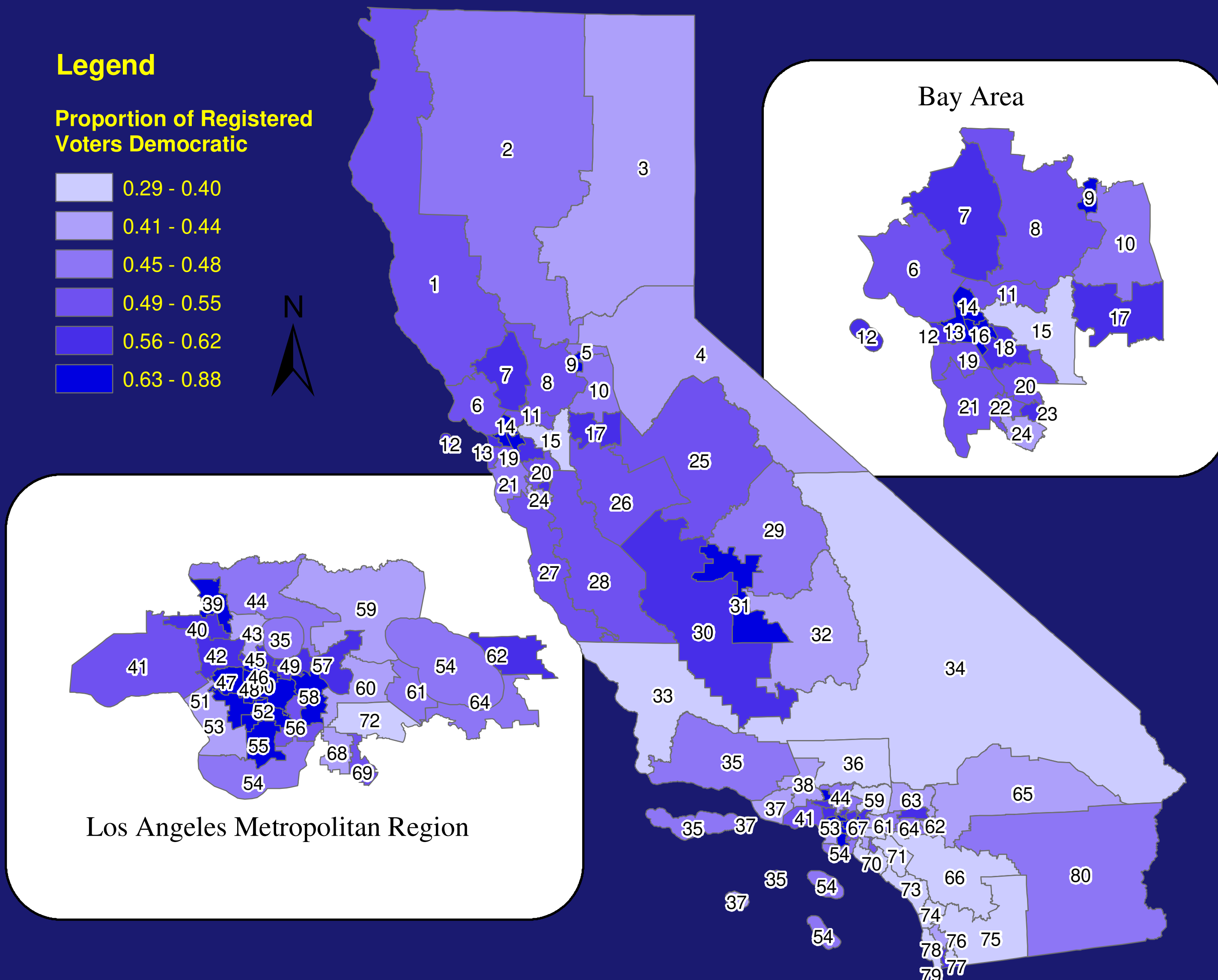
California Assembly Districts by Percentage of Registered Democrats



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2. **Choose a test statistic**:
   - ⋆ Sample average ballot effect:

$$W^D(T) = \frac{\sum_{i=1}^{121} T_i y_i}{N_1} - \frac{\sum_{i=1}^{121} (1 - T_i) y_i}{N_0},$$

corresponding to the difference-in-means estimator.

- ⋆ Covariance-adjusted statistic:

$$W^L(T) = \left(T^\top M T\right)^{-1} T^\top M y$$

corresponding to the linear least squares estimator, where $y = (y_1, y_2, \ldots, y_{121})$, $M = I - X(X^\top X)^{-1}X^\top$, and X is the matrix of the observed pretreatment covariates.

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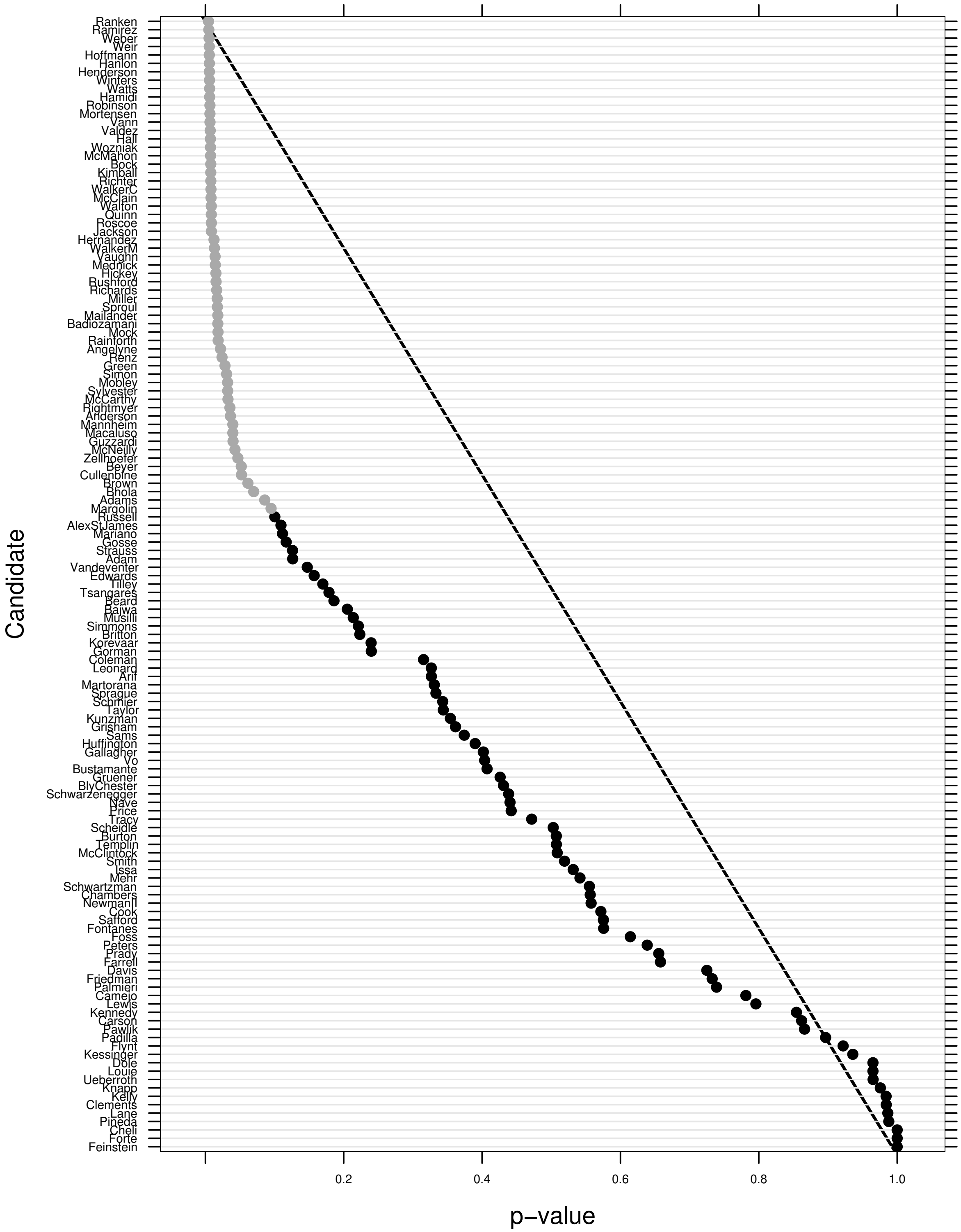
- Since the number of permutations is large, we use Monte Carlo approximation,

$$\Pr(W^D(T) \geq W^D(t)) \approx \frac{1}{m} \sum_{j=1}^{m} I(W^D(T^{(j)}) \geq W^D(t)),$$

with $m = 10,000$.



- No significant effect on major candidates.
- Positive ballot effect on 40% of minor candidates.

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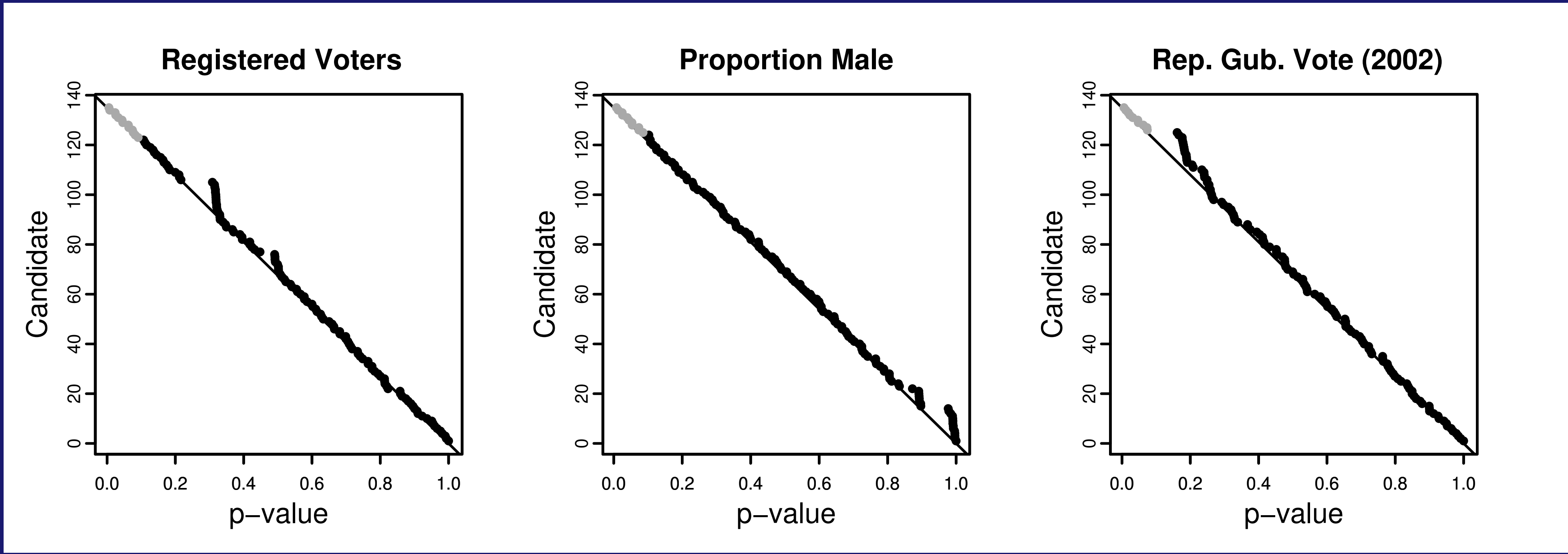

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3. Two-tailed level α test; accept H_0 if

$$t \in A_\alpha(\tau_0) = \left\{u : \frac{\alpha}{2} \leq \Pr(W^D_{\tau_0}(T) \geq W^D(u)) \leq 1 - \frac{\alpha}{2}\right\},$$

   and reject H_0 otherwise.

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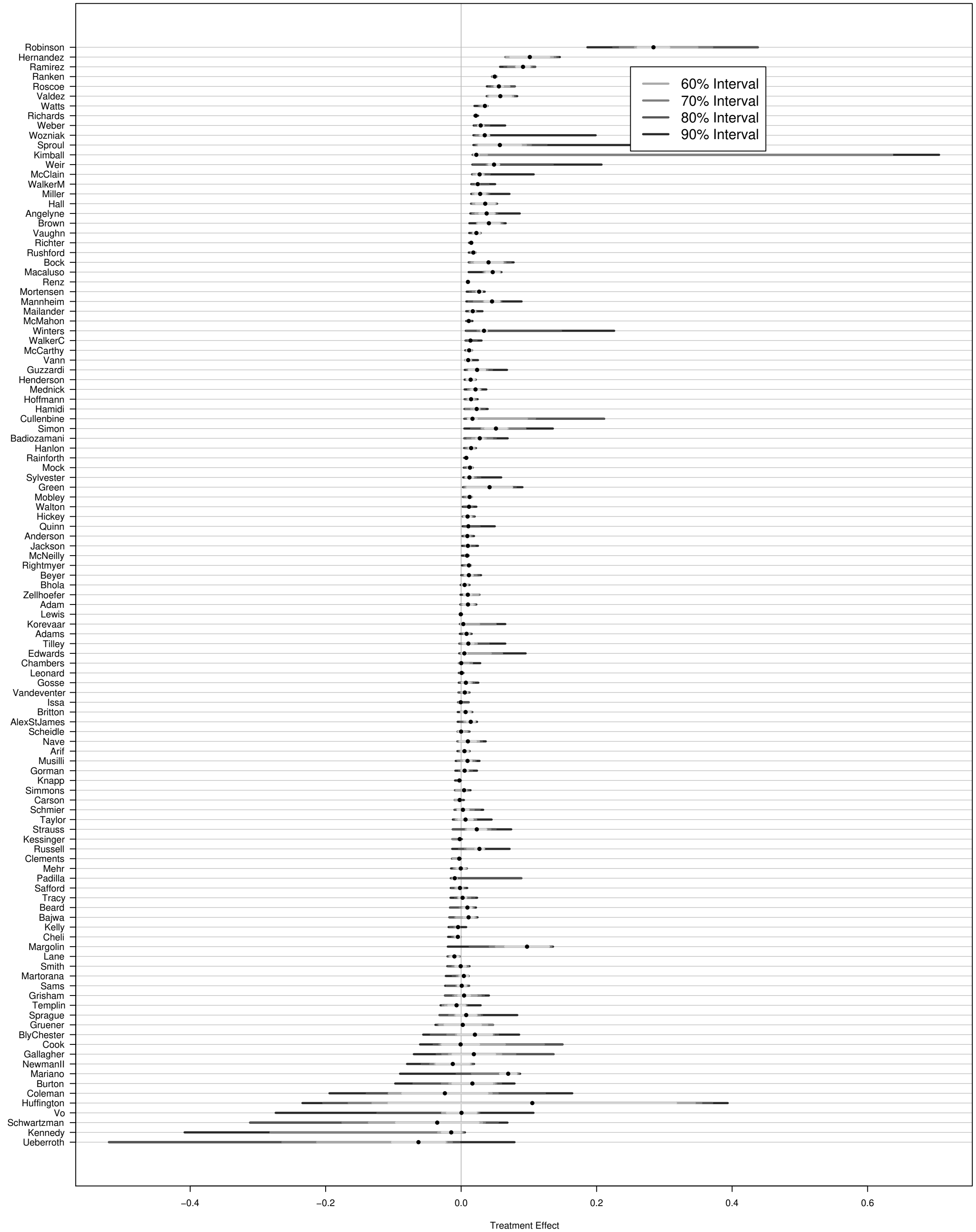
- Nonparametric estimates of CDF (for the sampling distributions of causal effect estimators) can also be obtained by estimating τ_U and τ_L for different values of $\alpha \in [0, 0.5]$.


Robinson
Hernandez
Ramirez
Ranken
Roscoe
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Watts
Richards
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Sproul
Kimball
Weir
McClain
WalkerM
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Richter
Rushford
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Renz
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WalkerC
McCarthy
Vann
Guzzardi
Henderson
Mednick
Hoffmann
Hamidi
Cullenbine
Simon
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Hanlon
Rainforth
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Sylvester
Green
Mobley
Walton
Hickey
Quinn
Anderson
Jackson
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Rightmyer
Beyer
Bhola
Zellhoefer
Adam
Lewis
Korevaar
Adams
Tilley
Edwards
Chambers
Leonard
Gosse
Vandeventer
Issa
Britton
AlexStJames
Scheidle
Nave
Arif
Musilli
Gorman
Knapp
Simmons
Carson
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Taylor
Strauss
Kessinger
Russell
Clements
Mehr
Padilla
Safford
Tracy
Beard
Bajwa
Kelly
Cheli
Margolin
Lane
Smith
Martorana
Sams
Grisham
Templin
Sprague
Gruener
BlyChester
Cook
Gallagher
NewmanII
Mariano
Burton
Coleman
Huffington
Vo
Schwartzman
Kennedy
Ueberroth
60% Interval
70% Interval
80% Interval
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−0.2
0.0
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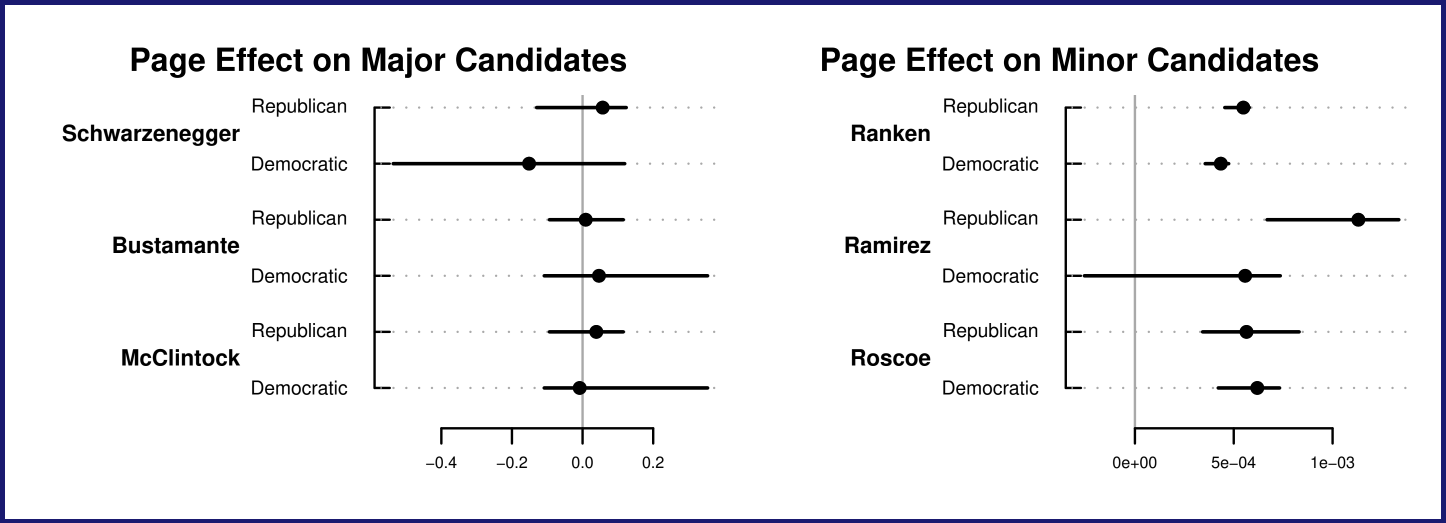

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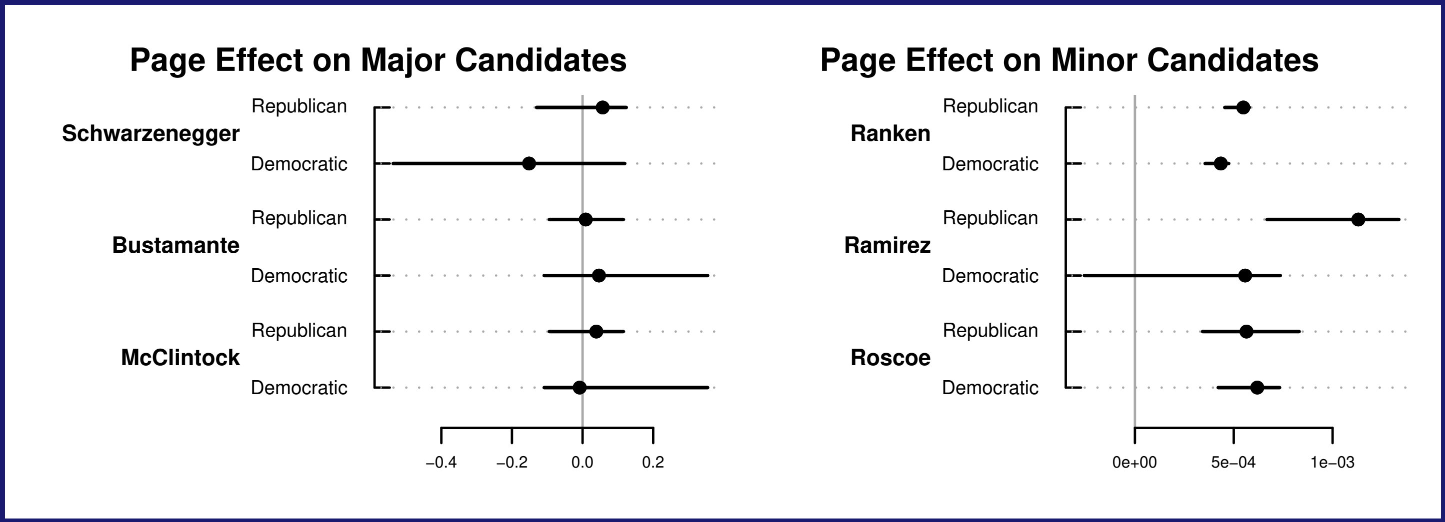

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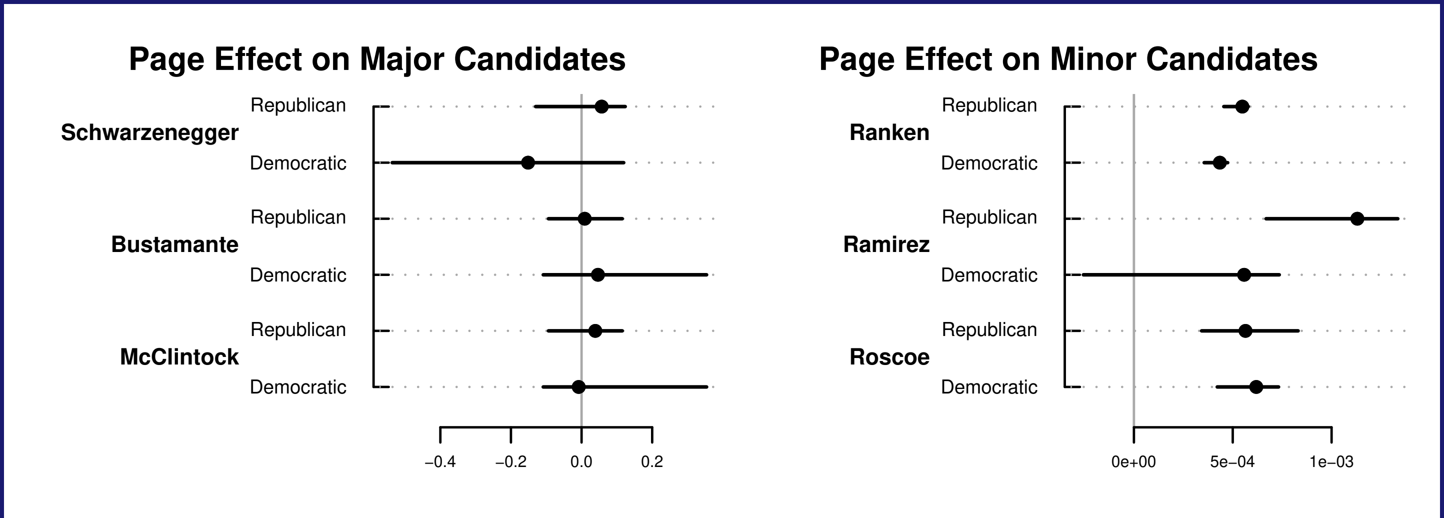

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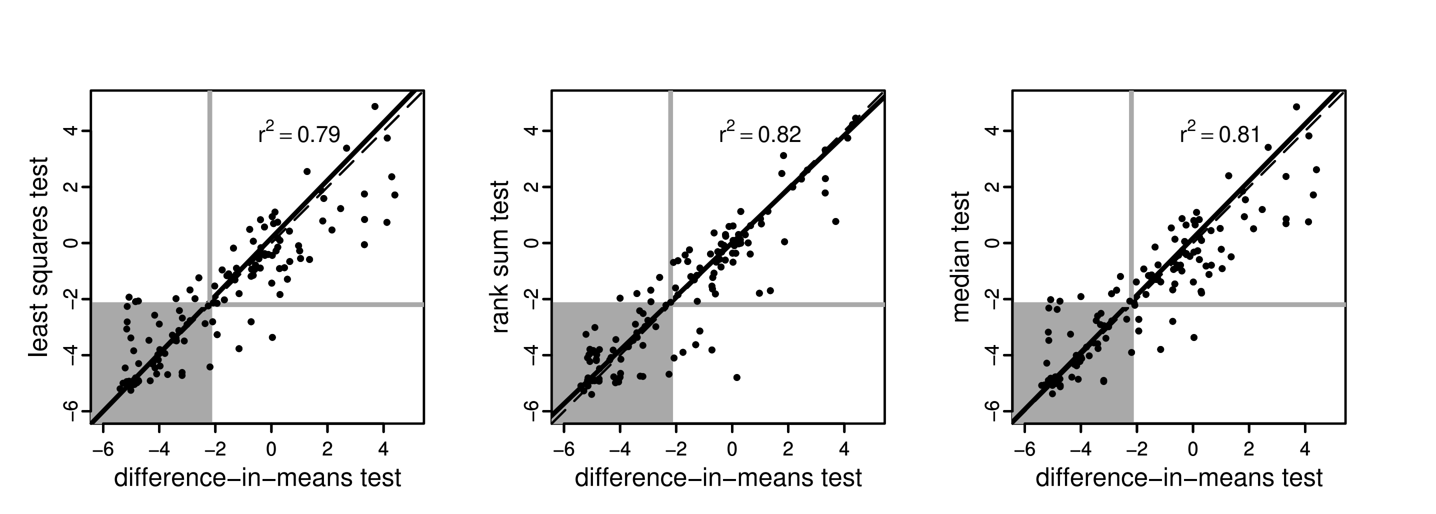

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	Without Covariates						With Covariates					
	OLS		GLM logit		RI Fisher		OLS		GLM logit		RI Fisher	
Major Candidates												
Schwarzenegger	1.09	9.63	−1.23	7.53	−23.72	19.90	−2.97	0.21	−4.98	−2.05	−6.44	6.87
Bustamante	−8.46	0.54	−5.37	4.04	−20.07	20.31	−1.12	1.78	0.96	3.01	−5.86	5.64
McClintock	0.50	3.09	−1.10	1.24	−3.47	6.36	1.56	3.25	0.29	2.05	0.36	3.57
All Candidates												
Positive effects	56(41%)		63(47%)		55(41%)		50(37%)		59(44%)		47(35%)	
Negative effects	11 (8%)		8 (6%)		4 (3%)		8 (6%)		17(13%)		2 (1%)	
Null effects	68(50%)		64(47%)		59(44%)		77(57%)		59(44%)		64(47%)	
Unidentified	0 (0%)		0 (0%)		17(13%)		0 (0%)		0 (0%)		22(16%)	
Comparison with Randomization Inference												
Agreement	89(66%)		87(64%)		108(80%)		88(65%)		74(55%)		108(80%)	

Comparisons of Confidence Intervals and CDF

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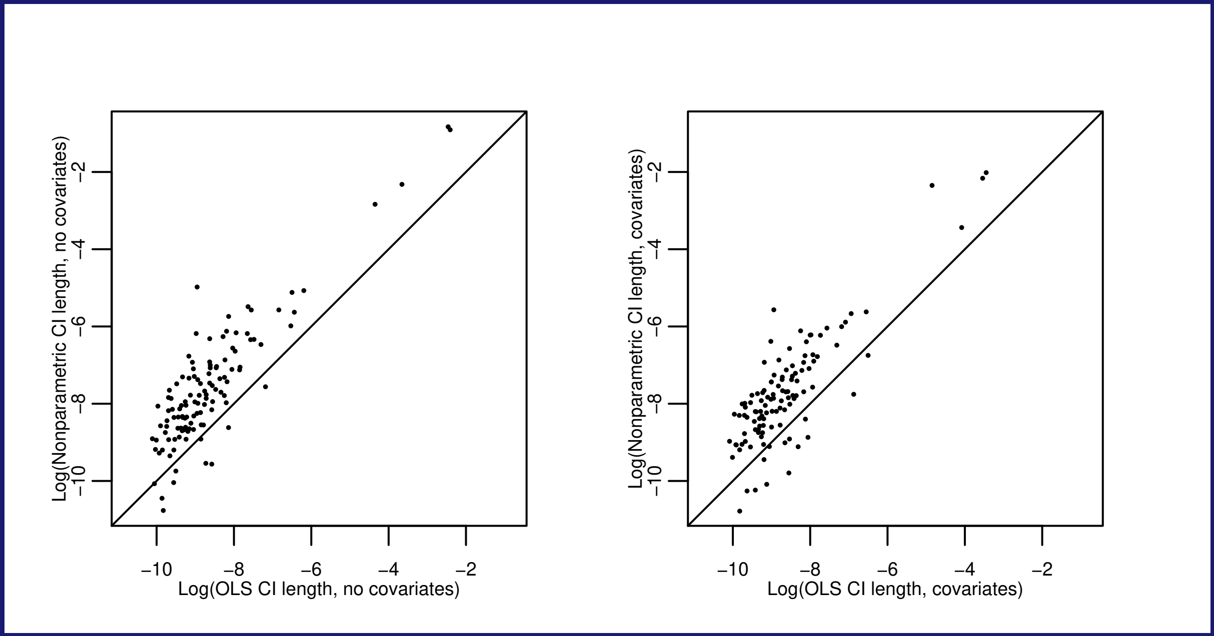

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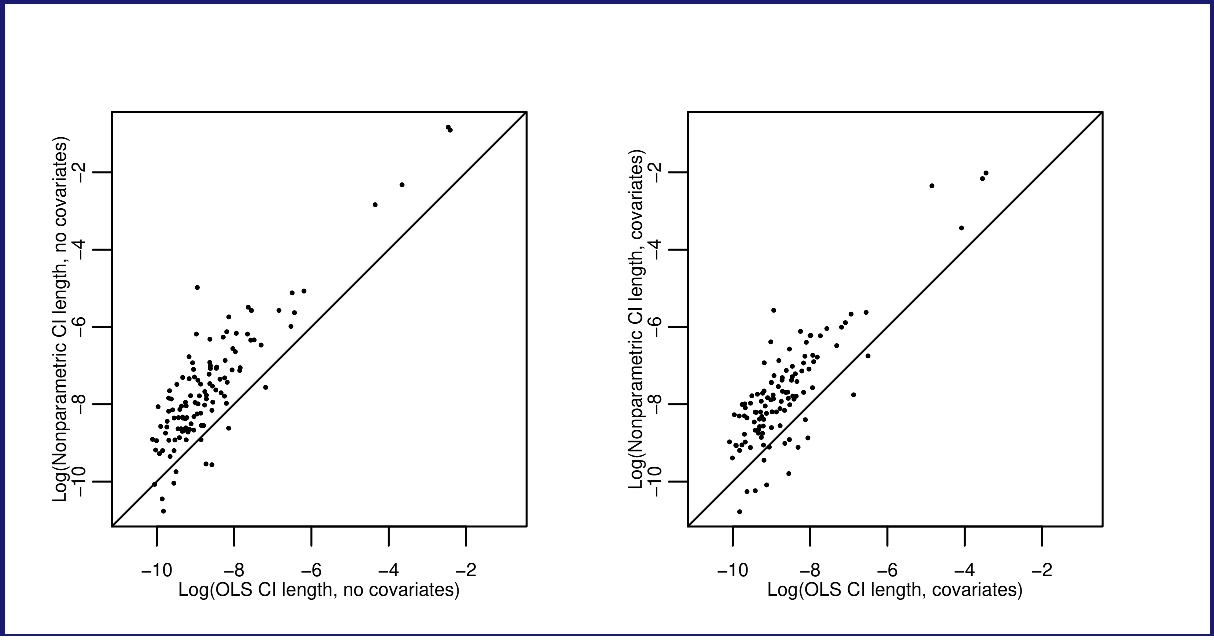

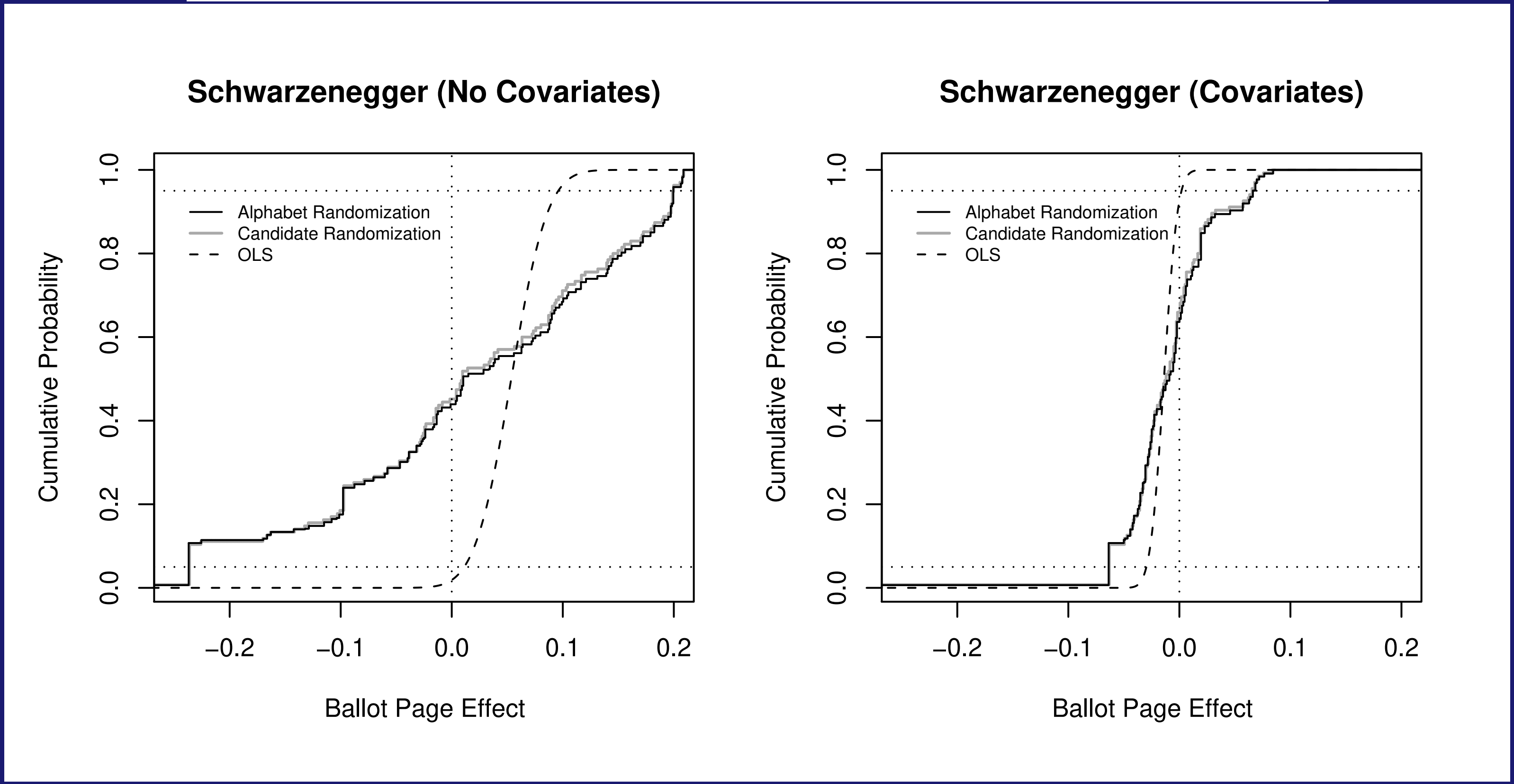

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- Randomization inference provides a general framework for robust causal inference in randomized experiments.
- Parametric inferences can be sensitive to modeling and other assumptions.
- Randomized natural experiments provide social scientists with rare opportunities to draw valid causal inferences.
- The randomization inference framework can directly incorporate complex randomization schemes in natural experiments.