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Randomized Algorithms for Analysis and Control of Uncertain Systems

With 54 Figures



Springer

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to Chicchi and Giulia for their remarkable endurance
R.T.

to Anne, to my family, and to the memory of my grandfather
G.C.

to Paola and my family for their love and support
F.D.

It follows that the Scientist, like the Pilgrim, must wend a straight and narrow path between the Pitfalls of Oversimplification and the Morass of Overcomplication.

Richard Bellman, 1957

Foreword

The subject of control system synthesis, and in particular robust control, has had a long and rich history. Since the 1980s, the topic of robust control has been on a sound mathematical foundation. The principal aim of robust control is to ensure that the performance of a control system is satisfactory, or nearly optimal, even when the system to be controlled is itself not known precisely. To put it another way, the objective of robust control is to assure satisfactory performance even when there is “uncertainty” about the system to be controlled.

During the two past two decades, a great deal of thought has gone into modeling the “plant uncertainty.” Originally the uncertainty was purely “deterministic,” and was captured by the assumption that the “true” system belonged to some sphere centered around a nominal plant model. This nominal plant model was then used as the basis for designing a robust controller. Over time, it became clear that such an approach would often lead to rather conservative designs. The reason is that in this model of uncertainty, every plant in the sphere of uncertainty is deemed to be equally likely to occur, and the controller is therefore obliged to guarantee satisfactory performance for every plant within this sphere of uncertainty. As a result, the controller design will trade off optimal performance at the nominal plant condition to assure satisfactory performance at off-nominal plant conditions.

To avoid this type of overly conservative design, a recent approach has been to assign some notion of probability to the plant uncertainty. Thus, instead of assuring satisfactory performance at every single possible plant, the aim of controller design becomes one of maximizing the expected value of the performance of the controller. With this reformulation, there is reason to believe that the resulting designs will often be much less conservative than those based on deterministic uncertainty models.

A parallel theme has its beginnings in the early 1990s, and is the notion of the complexity of controller design. The tremendous advances in robust control synthesis theory in the 1980s led to very neat-looking problem formulations, based on very advanced concepts from functional analysis, in particular, the theory of Hardy spaces. As the research community began to apply these methods to large-sized practical problems, some researchers began to study the rate at which the computational complexity of robust control synthesis methods grew as a function of the problem size. Somewhat to everyone’s surprise, it was soon established that several problems of practical interest were in fact NP-hard. Thus, if one makes the reasonable assumption

that $P \neq NP$, then there do not exist polynomial-time algorithms for solving many reasonable-looking problems in robust control.

In the mainstream computer science literature, for the past several years researchers have been using the notion of randomization as a means of tackling difficult computational problems. Thus far there has not been any instance of a problem that is intractable using deterministic algorithms, but which becomes tractable when a randomized algorithm is used. However, there are several problems (for example, sorting) whose computational complexity reduces significantly when a randomized algorithm is used instead of a deterministic algorithm. When the idea of randomization is applied to control-theoretic problems, however, there appear to be some NP-hard problems that do indeed become tractable, provided one is willing to accept a somewhat diluted notion of what constitutes a “solution” to the problem at hand.

With all these streams of thought floating around the research community, it is an appropriate time for a book such as this. The central theme of the present work is the application of randomized algorithms to various problems in control system analysis and synthesis. The authors review practically all the important developments in robustness analysis and robust controller synthesis, and show how randomized algorithms can be used effectively in these problems. The treatment is completely self-contained, in that the relevant notions from elementary probability theory are introduced from first principles, and in addition, many advanced results from probability theory and from statistical learning theory are also presented. A unique feature of the book is that it provides a comprehensive treatment of the issue of sample generation. Many papers in this area simply assume that independent identically distributed (iid) samples generated according to a specific distribution are available, and do not bother themselves about the difficulty of generating these samples. The trade-off between the nonstandardness of the distribution and the difficulty of generating iid samples is clearly brought out here. If one wishes to apply randomization to practical problems, the issue of sample generation becomes very significant. At the same time, many of the results presented here on sample generation are not readily accessible to the control theory community. Thus the authors render a signal service to the research community by discussing the topic at the length they do. In addition to traditional problems in robust controller synthesis, the book also contains applications of the theory to network traffic analysis, and the stability of a flexible structure.

All in all, the present book is a very timely contribution to the literature. I have no hesitation in asserting that it will remain a widely cited reference work for many years.

M. Vidyasagar

Hyderabad, India
June 2004

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During the spring semester of the academic year 2002, part of this book was taught as a special-topic graduate course at CSL, University of Illinois at Urbana-Champaign, and during the fall semester of the same year at Politecnico di Milano, Italy. We warmly thank Tamer Başar and Patrizio Colaneri for the invitations to teach at their respective institutions and for the insightful discussions. Seminars on parts of this book were presented at the EECS Department, University of California at Berkeley, during the spring term 2003. We thank Laurent El Ghaoui for his invitation, as well as Elijah Polak and Pravin Varaya for stimulating discussions. Some parts of this book have been utilized for a NATO lecture series delivered during spring 2003 in various countries, and in particular at Università di Bologna, Forlì, Italy, Escola Superior de Tecnologia de Setúbal, Portugal, and University of Southern California, Los Angeles. We thank Constantine Houpis for the direction and supervision of these events.

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Roberto Tempo
Giuseppe Calafiore
Fabrizio Dabbene

Torino, Italy
June 2004

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