Rank Image Processing Using Spatially Adaptive Neighborhoods[1]

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Abstract—A new approach to the design of rank-order filters based on the effective use of spatial relations between image elements is proposed. Many rank-order processing techniques can benefit from this approach, such as noise suppression, local contrast enhancement, and local detail extraction. An extension of the approach to rank-order filtering of three-dimensional signals is also discussed. The performance of the proposed rank-order filters in suppressing mixed additive and impulse noise in a test image is compared to that of conventional rank-order algorithms. The comparisons are made using criteria of a mean square error, a mean absolute error, and a subjective human visual error.

1. INTRODUCTION

In recent years, the use of nonlinear filters based on the calculation of rank-order statistics [1] has been increasing in computer and optical research [2–31]. There are several classes of nonlinear filters which incorporate rank-order operations in one way or another. A particular list of the filters includes median filters [2–4], multistage and multilevel median filters [5–7], stack filters [8, 9], alpha-trimmed mean filters [10–12], order-statistics filters [13–17], morphological filters [18–20], and rank-order filters [21–31]. These filters have proven to be very effective for removing additive and impulse noise and image enhancing and restoring. Moreover, they exhibit excellent robustness and provide solutions when linear filters are inappropriate. Perhaps their success in image processing is caused by their ability to suppress noise without destroying important image details, such as edges and fine lines.

In the design of rank-order filters, image elements of a moving window are sorted in ascending order that is called the variational row. The output of the rank-order filter is a function over elements of the variational row built around the central element of the window. Since rank-order filters take into account local image content (local statistics), the rank-order filtering is locally adaptive. A drawback of conventional rank-order filters is that they inadequately use spatial relations between image elements, because they reorder the elements of a two-dimensional moving window into a one-dimensional sequence (variational row).

In this paper, we suggest to design rank-order filters with the use of spatial relations between image elements. The filters make use of spatial and rank information of the input image within a moving window to produce the output. We introduce a path between two arbitrary elements in the window. Two elements of the window are spatially connected if there is a path between them. A set of elements possesses the property of spatial connectivity if all elements of the set are spatially connected. The output of the proposed filters is a function over spatially connected elements of the variational row of a moving window. By using spatial connectivity of elements, various rank-order filters can be designed for edge preservation, detail retention, and additive and impulse noise reduction.

The paper is organized as follows. Section 2 provides a review of rank-order filters using structural and estimation approaches for the filter design. Section 3 introduces a new approach to the design of rank-order filters using spatial relations between image elements. The rank-order filtering of three-dimensional signals is also considered. In Section 4, we illustrate the performance of the proposed rank-order filters comparing them with conventional rank algorithms for suppressing mixed additive and impulse noise. The comparisons are made in terms of objective and subjective criteria. Section 5 summarizes our conclusions.

2. RANK-ORDER FILTERS

Two different approaches to designing rank-order filters can be used. They may be called the structural approach and the estimation approach [9, 22]. The first

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approach relies on structural descriptions of the image and the process, which alters it, while the second relies on statistical descriptions. Rank-order filters are considered in the context of these two approaches. We use the notion of neighborhood in order to define various structures in the image. The structures are local and global grayscale signal variations which are known to be present in the original image. They can be degraded by noise. The goal is to design a rank-order filter that preserves or modifies in a desired way the structures of the original image by eliminating noise and undesirable structures.

The rank filtering is a locally adaptive processing of the signal in a moving window. First, using different neighborhoods, we define desirable structures in the window. Next, the estimation approach can be applied to the elements of the neighborhood structures to compute an estimate of the central pixel of the window with respect to different criteria.

Let us introduce some useful notation and definitions: $\mathbf{s} = \{s_{n,m}\}$ is a vector of pixels of the image to be processed that has Q grayscale levels of quantization; n, m are coordinates of the pixels, $n = 1, 2, \ldots, N$ and $m = 1, 2, \ldots, M$; $L = N \times M$ is the size of image matrix; $\mathbf{v} = \{v_{n,m}\}$ is a vector of pixels of the noise-free (original) image; $\hat{\mathbf{v}} = \{\hat{v}_{n,m}\}$ is a vector of pixels of the resulting image. For each image pixel, a spatial neighborhood of an arbitrary size can be defined as a set of pixels that geometrically surround the given one. Such a neighborhood consisting of pixels nearest to the given one is referred to as the S-neighborhood. Actually, pixels of the S-neighborhood often coincide with those of the moving window. Weighted order statistics filters are successfully applied in various areas of image processing. They often outperform conventional rank-order filters in terms of noise attenuation capability. We introduce the weighted spatial neighborhood (S_w-neighborhood) as a generalization of the S-neighborhood. Let $\{w_{n,m}\}$ be positive integer weights with an odd sum. The S_w-neighborhood centered at each image pixel is a set obtained from the S-neighborhood by duplicating its pixels $w_{n,m}$ times. In the cases of nonstationary additive noise and time-varying data, it is preferable to keep the size of the S-neighborhood sufficiently small so that the signal and noise can be considered approximately stationary over the window area.

An important notion in order statistics is a variational row. It is defined as the one-dimensional sequence $\{V(r)\}$ of K pixels whose elements are sorted in ascending order with respect to their values: $\{V(r)\colon V(r) \le V(r+1), r = 1, 2, \ldots, K\}$. Here, $V(r)$ and $r(V)$ are called the rth order statistics and the rank of the value V, respectively. Both the rank and the order statistics can be computed from the local histogram $\{h(q), q = 0, \ldots, Q-1\}$ of the signal distribution over the S-neighborhood (S_w-neighborhood) centered at each pixel as follows:

$$r(V) = \sum_{q=0}^{V} h(q). \tag{1}$$

In addition, all the parameters of rank-order filters are functions of local (or short-time) histograms computed over pixels of the spatial neighborhoods. Therefore, the computational complexity of rank-order processing depends on the calculation of local histograms. In the digital calculation of local histograms, the shape of a moving window should be rectangular, because only in this case do fast recursive algorithms exist. However, for a large window size the calculation is troublesome. Recently, parallel optical–digital methods of local histogram calculation over the S-neighborhood [27] and the S_w-neighborhood [28] have been proposed. These methods consist of a time-sequential threshold decomposition of an image followed by convolutions of the resulting binary slices with a kernel and element-wise operations that are digitally performed on the convolution results.

To describe different structures in the image, we define the following subsets over the S-neighborhood or the S_w-neighborhood [22, 28, 31]:

EV-neighborhood is a subset of pixels $\{v_{n,m}\}$ whose values deviate from the value of the central pixel $v_{k,l}$ by no more than predetermined quantities $-\varepsilon_v$ and $+\varepsilon_v$; i.e.,

$$EV(v_{k,l}) = \{v_{n,m}\colon v_{k,l} - \varepsilon_v \le v_{n,m} \le v_{k,l} + \varepsilon_v\}. \tag{2}$$

A *KNV*-neighborhood is a subset of K pixels $\{v_{n,m}\}$ whose values are nearest to the value of the central pixel $v_{k,l}$; i.e.,

$$KNV(v_{k,l}) = \left\{V(r)\colon \sum_{r=p}^{p+K-1} |v_{k,l} - V(r)| = \underset{p}{MIN}\right\}. \tag{3}$$

A subset of pixels $\{v_{n,m}\}$ whose ranks deviate from that of the central pixel by no more than predetermined quantities $-\varepsilon_r$ and $+\varepsilon_r$ is called the *ER*-neighborhood; i.e.,

$$\begin{aligned} &ER(v_{k,l}) \\ &= \{v_{n,m}\colon r(v_{k,l}) - \varepsilon_r \le r(v_{n,m}) \le r(v_{k,l}) + \varepsilon_r\}. \end{aligned} \tag{4}$$

The choice of neighborhood (NBH) is defined by the available *a priori* information on the processed image. For example, if *a priori* information about the geometrical size K of the details to be preserved is known, then the *KNV*-neighborhood can be used. The parameter K is chosen of the order of the detailed area to be preserved after further processing. The choice of the *EV*-neighborhood helps us to take into account *a priori* information about either the spread of the signal to be preserved or the noise fluctuation to be suppressed. The *ER*-neighborhood is often used in the edge extraction

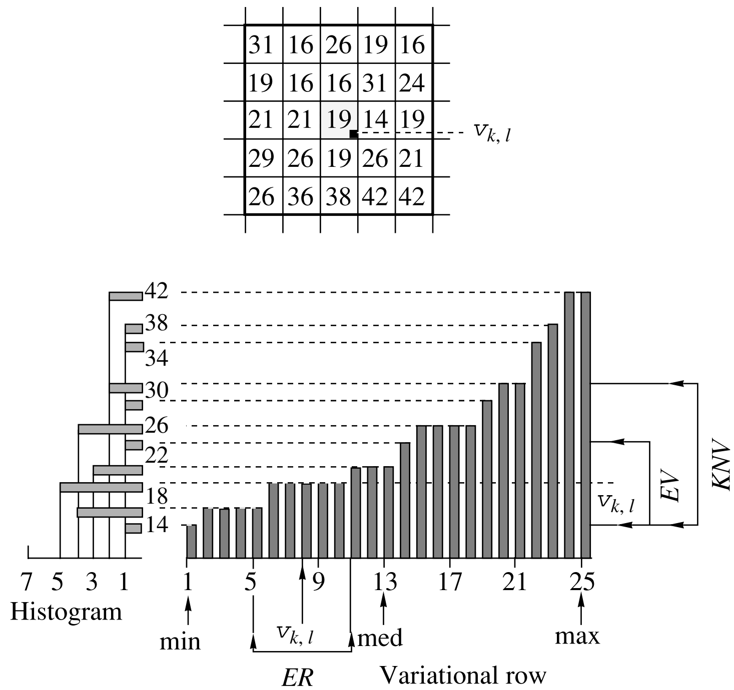

Fig. 1. An example of local neighborhoods.

algorithms and in the algorithms for suppressing a mixture of additive Gaussian noise and noise with a distribution having heavy tails. The size of the *ER*-neighborhood is determined by the part of the outliers in the distribution. Finally, note that the size of the *S*-neighborhood should be nearly twice as big as the size of the minimal structure to be preserved. All the introduced neighborhoods are presented in Fig. 1.

Three types of estimation borrowed from the theory of robust estimation of location parameters [32, 33] may be used to compute an estimate of the central pixel of the neighborhoods: (i) the L-estimator based on linear combination of order statistics; (ii) the R-estimator derived from rank tests; and (iii) the M-estimator or the maximum likelihood estimator. All three types of estimation can be implemented using a few basic operations over the introduced neighborhoods. The operations are defined as follows: *SIZE (NBH)* is the quantity of pixels forming the neighborhood, *MEAN(NBH)* is the sample mean over the neighborhood, *MED(NBH)* is the median value over the neighborhood, *MIN(NBH)* is the minimum over the neighborhood, *MAX(NBH)* is the maximum over the neighborhood, and *CUT(NBH)* is the cross cut through the neighborhood; i.e.,

$$CUT(NBH) = \begin{cases} MN & \text{if} \quad v_{k,l} < MN \\ v_{k,l} & \text{if} \quad MN \le v_{k,l} \le MX \\ MX & \text{if} \quad v_{k,l} > MX, \end{cases} \quad (5)$$

where $MN < MX$, MN and MX are the predefined values for the cross-cut operation.

The output of L-type filters can be expressed as a fixed linear combination of the order statistics over the pixels of a chosen NBH subset of the *S*- or S_w-neighborhoods. The L-filter treats the input image as an image with the locally constant mean. Suppose that the original image is corrupted by zero-mean independent identically distributed (i.i.d.) additive noise. The L-filters have a well-designed methodology, as they are the estimators that minimize the mean-squared error between the filter output and the noise-free signal. In this case, the optimal estimate is the solution to the following implicit equation:

$$\hat{v}_{n,m} = MEAN(NBH\lfloor v_{n,m}\rfloor). \quad (6)$$

In general, Eq. (6) can be solved iteratively,

$$\hat{v}^{i+1}_{n,m} = MEAN(NBH\lfloor \hat{v}^{i}_{n,m}\rfloor), \quad (7)$$

where i is the iteration number and at the first iteration $\{\hat{v}^{1}_{n,m}\} = \{s_{n,m}\}$.

The output of R-type filters is rather affected by relative ranks of data than by the actual values of data. This class of filters has its basis in the rank estimate of statistical theory. The output is an arbitrary order statistics, e.g., the rth order statistics $V(r)$, of a sequence obtained by linear transformation of the input pixels of a chosen NBH subset of the *S*- or S_w-neghborhoods; i.e.,

$$\hat{v}_{n,m} = V(r\lfloor FUN(NBH\lfloor s_{n,m}\rfloor)\rfloor), \quad (8)$$

where *FUN*(x) is an arbitrary linear function of the input neighborhood pixels. Different linear functions yield different structures of R-filters. For example, let us consider the signal model used for the L-filter. If the mean absolute error between the filter output and the noise-free signal is used, the optimal estimate is the median value over the pixels of a chosen NBH subset of the *S*- or S_w-neghborhoods,

$$\hat{v}_{n,m} = MED(NBH\lfloor v_{n,m} \rfloor), \quad (9)$$

and its iterative solution is given by

$$\hat{v}^{i+1}_{n,m} = MED(NBH\lfloor \hat{v}^{i}_{n,m} \rfloor), \quad (10)$$

where $\{\hat{v}^1_{n,m}\} = \{s_{n,m}\}$. Obviously, if the order statistics $V(r)$ is the median value and the linear function *FUN*(x) is the identical transformation, then the filter in Eq. (10) is a particular case of the R-filter in Eq. (8).

The output of M-type filters is defined as a solution of the equation

$$MEAN(FUN(NBH\lfloor s_{n,m} - \hat{v}_{n,m} \rfloor)) = 0, \quad (11)$$

where *FUN*(x) is an odd, continuous, and sign-preserving function. Suppose that the original image is corrupted by zero-mean i.i.d. additive noise. Then, for an even function $p(x)$ which is nondecreasing in x, we can define an estimate $\hat{v}_{n,m}$ of the central pixel v that minimizes

$$\underset{\hat{v}_{n,m}}{MIN}[MEAN(p(NBH[s_{n,m} - \hat{v}_{n,m}]))]. \quad (12)$$

If $FUN(x) = dp(x)/dx$, it equivalently satisfies Eq. (11). Such estimates are generalized forms of maximum-likelihood estimates, and the estimator is called an M-estimator. Let us consider an example of M-type filters. Using the definition of the M-type filter, suppose that

$$FUN(x) = \begin{cases} At, & x \ge t \\ Ax, & |x| < t \\ -At, & x \le t, \end{cases} \quad (13)$$

where t and A are positive constants. The output of the filter is the optimal maximum-likelihood estimate. It can be written as the sample mean over the pixels of a chosen NBH subset, which has a limited signal range

$$\hat{v}_{n,m} = MEAN(CUT(NBH\lfloor s_{n,m} \rfloor)), \quad (14)$$

and its iterative processing is given by

$$\hat{v}^{i+1}_{n,m} = MEAN(CUT(NBH\lfloor \hat{v}^{i}_{n,m} \rfloor)), \quad (15)$$

where $\{\hat{v}^1_{n,m}\} = \{s_{n,m}\}$ is initial condition.

Note that, in practice, the size of the moving window is limited, and, in general, real-life images do not have locally constant means. Therefore, all the mentioned estimates should be computed iteratively. It also has to be noted that each iteration can change the properties of noise. This implies that one should change the type of the neighborhood and operation over pixels of this neighborhood at each iteration.

The choice of the neighborhoods for rank-order algorithms is very important: it should be statistically meaningful. In particular, one should take into account the types of deviations of the actual model from the ideal one. Therefore, we treat the signal model as an approximation for the majority of image data. The minority of the image data such as outliers or deviating signal substructures are not taken into consideration. In general, if the noise distribution is Gaussian, a better estimate is provided by the sample mean operation; if the noise distribution has heavy tails, either the median or other order-statistic gives a better result. In the case of one-sided distribution, the minimum/maximum operations are appropriate.

Now we introduce several rank-order algorithms constructed with the use of basic neighborhoods and operations. The alpha-trimmed filter (α-TM filter) is an example of L-type filters [10]

$$\hat{v}^{i+1}_{n,m} = MEAN(ER(MED\lfloor \hat{v}^{i}_{n,m} \rfloor)). \quad (16)$$

The output of the filter is the trimmed mean value. The number of data values dropped from the average is controlled by the quantities $-\varepsilon_r$ and $+\varepsilon_r$. It is obvious that the extreme values, both low and high, are removed at each end of the variational row. The algorithms in Eqs. (15) and (16) are similar. The only difference is that the α-TM filter rejects certain data values in the window, whereas the filter in Eq. (15) only limits the influence of some data values. The modification of the α-TM filter called the MTM filter can be written as [11]

$$\hat{v}^{i+1}_{n,m} = MEAN(EV\lfloor MED(\hat{v}^{i}_{n,m}) \rfloor). \quad (17)$$

Here, the number of pixels used in averaging is not fixed as it was in the previous case. The algorithm yields the arithmetic mean of order statistics values of the *EV*-neighborhood which is formed at the median value of each image pixel of the *S*- or S_w-neghborhoods. The median value can be computed over pixels of a small moving window. This filter is referred to as the double window MTM filter [11]. Another noise cleaning algorithm using the *KNV*-neighborhood is written as [23]

$$\hat{v}^{i+1}_{n,m} = MEAN(KNV\lfloor \hat{v}^{i}_{n,m} \rfloor). \quad (18)$$

The filter takes the average of a fixed number of values in the window closest to the central pixel, including the pixel itself.

An example of a smoothing algorithm based on estimation over the *EV*-neighborhood is given by [24, 25]

$$\hat{v}^{i+1}_{n,m} = MEAN(EV\lfloor \hat{v}^{i}_{n,m} \rfloor), \quad (19)$$

where it is recommended to choose $\varepsilon_v = 1.5\sigma$ assuming that the standard deviation of additive Gaussian noise is known. The purpose of this processing is to automati-

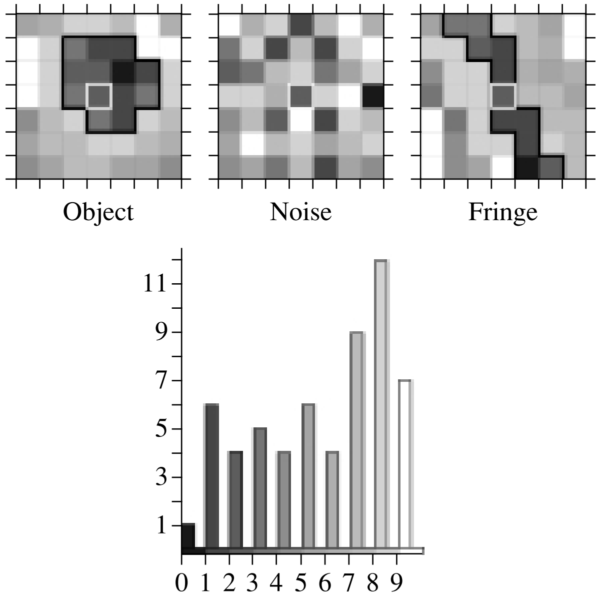

Fig. 2. Three different images having the same histogram.

cally adjust the window size depending on the place of processing—an edge or a smooth region.

An edge-detection algorithm can be defined as follows:

$$\hat{v}_{n,m} = (V(r = R) - V(r = L)) \times F, \quad (20)$$

where F is a gain coefficient of the difference between values of pixels of the two chosen ranks, R and L.

A local-contrast enhancement algorithm has the form

$$\hat{v}_{n,m} = Ar(v_{n,m}) + B, \quad (21)$$

where A and B are normalization constants. This algorithm is also called local histogram equalization.

An algorithm for suppressing mixed additive and impulse noise is as follows [30]:

$$\hat{v}^{i+1}_{n,m} = \begin{cases} MEAN(EV[\hat{v}^i_{n,m}]), \\ \text{if} \quad SIZE(EV[\hat{v}^i_{n,m}]) \ge Thld^i \\ MED(S[\hat{v}^i_{n,m}] - EV[\hat{v}^i_{n,m}]), \quad \text{otherwise,} \end{cases} \quad (22)$$

where $Thld^i$ is a threshold value of outlier detection at the ith iteration and "–" denotes the set difference operation. The algorithm yields either the sample mean of pixels of the *EV*-neighborhood or the medium value of pixels of the *S*-neighborhood. The size of the *S*-neighborhood is usually chosen smaller than that of the moving window. Moreover, the pixels associated with outliers are excluded from the *S*-neighborhood. Thus, the additive noise is suppressed by the arithmetic averaging inside the *EV*-neighborhood of each image pixel, while the pulse noise is removed by the median operation. This algorithm and its modifications are used in our computer experiments.

3. RANK-ORDER FILTERS WITH SPATIALLY ADAPTIVE NEIGHBORHOODS

The use of spatial neighborhoods in image processing reflects the fact that the pixels that are geometrically close to each other belong to the same structure or detail. It is also assumed that the pixels that belong to the same details of an image are highly correlated and fall into the same cluster of the local histogram where the central pixel of the *S*-neighborhood falls. Therefore, by using the introduced neighborhoods, we can easily extract these pixels from the local histograms. However, the spatial proximity does not always form spatial clusters. Figure 2 demonstrates that the pixels of the neighborhoods defined in Section 2 are not necessarily spatially connected to the central pixel of the neighborhood (the spatial connectivity is defined here by their belonging to one image detail). In Fig. 2, the three test images have the same one-dimensional histogram but differ greatly in the content of the images. It seems to contradict the natural assumption that all the pixels from the same neighborhood belong to the same detail of the image. Therefore, one can conclude that conventional rank-order filters designed with the use of one-dimensional local histograms barely exploit spatial relations between image elements, because they reorder the pixels of two-dimensional spatial neighborhoods into one-dimensional variational rows or local histograms. In other words, the representations of spatial relations in local histograms and variational rows may poorly describe image structures and detail orientations. To overcome this drawback, we supplement the neighborhood definitions introduced in Section 2 by requirements for all the pixels of the neighborhood to be spatially connected to each other.

First, we introduce some important definitions for the pixels of the neighborhood.

Definition 1. Two different pixels $v_{k,l}$ and $v_{m,n}$ are spatial neighbors if their coordinates satisfy the following condition: $|k - m| + |l - n| = \Delta$, where Δ is a positive constant called an order of connectivity.

Definition 2. A path from the pixel $v_{k,l}$ to the pixel $v_{m,n}$ ($k \le m$ and $l \le n$) is a sequence of pixels $A_1, A_2, \ldots, A_h$ of the neighborhood, where $A_1 = v_{k,l}$, $A_h = v_{m,n}$ and A_{i+1} is a spatial neighbor of A_i ($i = 1, 2, \ldots, h - 1$).

Definition 3. Two pixels are called spatially connected if there is a path between them in the neighborhood.

Definition 4. A neighborhood region is spatially connected if all of its pixels are spatially connected.

We denote a spatially connected region of the order of connectivity Δ which is formed from the set X, as

$CON_\Delta(X)$. The parameter Δ is suitable to describe connected regions of images corrupted with impulse noise. In this case, Δ can be determined by the probability of impulse noise. In the algorithms used in our computer simulations, Δ is equal to 1 or 2.

Using the definitions, we introduce the concept of an adaptive neighborhood (ANBH). The size and shape of an adaptive neighborhood depend on characteristics of image data and on parameters, which define measures of homogeneity of pixel sets. Thus, an adaptive neighborhood is a spatially connected region constructed for each pixel; it consists of the spatially connected pixels that satisfy the property of similarity with the central pixel. This property can be described by using the *EV*-, *ER*-, and *KNV*-neighborhoods introduced in Section 2. First, we form the *EV*-, *ER*-, and *KNV*-neighborhoods from the pixels of the moving window; then, from these neighborhoods, we construct spatially connected regions including the central pixel. New sets are adaptive neighborhoods, referred to as *AEV*-, *AER*-, and *AKNV*-neighborhoods, respectively. These adaptive neighborhoods are defined as follows.

An *EV*-neighborhood is a subset of pixels $\{v_{n,m}\}$ of the *S*-neighborhood, which are spatially connected with the central pixel $v_{k,l}$ and whose values deviate from the value of the central pixel by no more than the predetermined quantities $-\varepsilon_v$ and $+\varepsilon_v$; i.e.,

$$AEV(v_{k,l}) = CON_\Delta(\{v_{n,m}: v_{k,l} - \varepsilon_v \le v_{n,m} \le v_{k,l} + \varepsilon_v\}). \tag{23}$$

An *AKNV*-neighborhood is a subset of a specified number *K* of pixels $\{v_{n,m}\}$ of the *S*-neighborhood, which are spatially connected with the central pixel $v_{k,l}$ and whose values are nearest to the value of the central pixel $v_{k,l}$; i.e.,

$$AKNV(v_{k,l}) = CON_\Delta\left\{V(r): \sum_{r=p}^{p+K-1} |v_{k,l} - V(r)| = \underset{p}{MIN}\right\}. \tag{24}$$

An *AER*-neighborhood is a subset of pixels $\{v_{n,m}\}$ of the *S*-neighborhood, which are spatially connected with the central pixel $v_{k,l}$ and whose ranks deviate from that of the central pixel by no more than the predetermined quantities $-\varepsilon_r$ and $+\varepsilon_r$; i.e.,

$$AER(v_{k,l}) = CON_\Delta \times \{v_{n,m}: r(v_{k,l}) - \varepsilon_e \le r(v_{n,m}) \le r(v_{k,l}) + \varepsilon_r\}. \tag{25}$$

The choice of an adaptive neighborhood is defined by the available *a priori* information on the processed image. The output of filtering is a value computed as the basic operations *MEAN(ANBH)*, *MED(ANBH)*, *SIZE(ANBH)*, *MIN(ANBH)*, *MAX(ANBH)*, and *CUT(ANBH)* at all possible pixels in the adaptive neighborhood. The operations may be iteratively applied several times. Note, that the adaptive neighborhoods are not formed across region boundaries; therefore, noise suppression will not blur image edges as often happens with other techniques.

Most of the applications of the rank-order filters are limited to one- or two-dimensional signals. However, applications with three-dimensional signals are already numerous and still growing. For example, most of the data collected by satellites, by computer vision systems, and by medical imaging systems (transmission, reflection, and emission tomography) are three-dimensional signals. A common way to process three-dimensional signals is to consider them as multichannel signals and apply rank algorithms developed for signals of lower dimension to each channel. This approach is especially reasonable for applications, where the correlation of the values from different channels is very low. If the pixels of the three-dimensional signal are correlated to each other, then it is preferable to compute the parameters of rank-order filters from pixels of a three-dimensional window moving across the signal. Figure 3 shows a curve (a fine line) in a three-dimensional space. Typical examples of such signals are vessels in a position emission tomography (PET) image. Note that in impulse noise environments, only the proposed approach guarantees a correct signal processing. Noise smoothing based on one- or two-dimensional rank-order filters applied along different axes will remove the signal. An extension of the concept of spatially adaptive neighborhoods from two-dimensional to *P*-dimensional signals is straightforward.

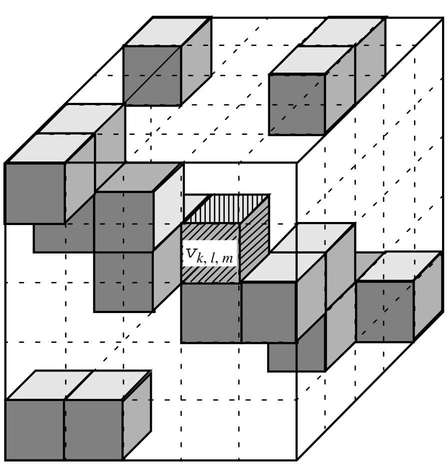

Fig. 3. Spatial connectivity of samples of a three-dimensional signal.

Let $\{\mathbf{v}(\mathbf{k})\} = \{v(k_1, k_2, \ldots, k_P)\}$ be a discrete *P*-dimensional signal, where $\mathbf{k} \in \mathbf{Z^P}$, and $\mathbf{Z}$ is a set of integers. The spatial neighborhood for every multidimensional sample is defined as a set of samples which surround the given one with respect to the distance chosen as

$$l_P(\mathbf{v}(\mathbf{k}), \mathbf{v}(\mathbf{n})) = \sum_{i=1}^{P} |k_i - n_i|. \quad (26)$$

Conventional rank-order filters map all the samples of the P-dimensional neighborhood into one-dimensional variational row. The output of the filters is a function over the elements of the variational row. During processing multidimensional signals with conventional rank-order filters signal structures and detail orientations are not taken into account. For this reason, one can expect their poor performance on multidimensional signals. In a similar manner, we supplement the neighborhood definitions by requiring that all the samples of the neighborhood should be spatially connected to each other in a P-dimensional space.

Definition 5. Two different samples $\mathbf{v}(\mathbf{k})$ and $\mathbf{v}(\mathbf{l})$ are spatial neighbors in a P-dimensional space with the order of connectivity Δ if the distance between them is $l_P(\mathbf{v}(\mathbf{k}), \mathbf{v}(\mathbf{l})) = \Delta$.

Using Definitions 2–4, one can define the adaptive neighborhoods for multidimensional signals and then apply the basic operations over the neighborhoods. Many rank-order processing techniques (such as noise suppression, local contrast enhancement, and detail extraction) may be implemented by applying the concept of adaptive neighborhood to multidimensional signals.

In our computer simulation, we use rank-order filters based on the *AEV*-neighborhood. The adaptive neighborhood can be constructed from the *EV*-neighborhood using a region-growing algorithm [34]. In Section 4, we illustrate the ability of *AEV*-neighborhood to tune itself to contextual details and fine structures of test images.

4. COMPUTER EXPERIMENTS

Signal processing of an image degraded by additive and impulse noises is of interest in a variety of applications. Computer experiments were carried out to illustrate and compare the performance of conventional and proposed algorithms. The question was how well, relative to other filters, did each algorithm remove the noise and preserve fine structures. However, it is difficult to define an error criterion for accurate estimation of image distortion. In this paper, we base our comparisons on the mean square error (MSE), the mean absolute error (MAE), and the subjective visual criterion. The empirical normalized mean square error is given by

$$MSE = \frac{\sum_{n=1}^{N}\sum_{m=1}^{M} |v_{n,m} - \hat{v}_{n,m}|^2}{\sum_{n=1}^{N}\sum_{m=1}^{M} v_{n,m}^2}, \quad (27)$$

where $\{v_{n,m}\}$ and $\{\hat{v}_{n,m}\}$ are the original image and its estimate (filtered image), respectively. In our simulations, N = M = 512 (512 × 512 image resolution) and each pixel has 256 levels of quantization. The empirical normalized mean absolute error is defined as

$$MEA = \frac{\sum_{n=1}^{N}\sum_{m=1}^{M} |v_{n,m} - \hat{v}_{n,m}|}{\sum_{n=1}^{N}\sum_{m=1}^{M} |v_{n,m}|}. \quad (28)$$

Finally, we use an enhanced difference visual display to assess the error in a human visual error criterion. If there is no error in pixel location between the original image and the filtered image, this pixel is displayed in gray. When the error is maximum, the pixel is displayed either black or white. This difference image is the base of our subjective error criterion, and it provides us with information about the distortions introduced by a filter as well as the noise suppression capability of the algorithm. These error measures allow us to evaluate the performance of each filter.

Figure 4a shows the original test image. The image contains piecewise solid regions and fine details. The signal values of different regions in the image are as follows: the background value is 148, the “tree” value is 133, the “cloud” value is 165, the “sun” value is 230, the “house” value is 189, and the “windowpane” values are 181, 175, 165, and 165. In order to better illustrate the significance of the spatial pixel connectivity, we insert the thin dark lines in the picture separating adjacent solid areas.

4.1. Test Image Corrupted by Additive Noise

Figure 4b shows the test image corrupted by zero-mean additive Gaussian noise. The standard deviation of the noise is $\sigma = 15$. Note that the signal difference between adjacent regions in the picture is often of the order of σ. Table 1 shows the difference between the original and noisy images in terms of the MSE and MAE.

Let us compare two rank-order algorithms. RA_0 is the rank algorithm given in Eq. (22). The number of iterations is 2. The filter parameters at each iteration are as follows: the sizes of the moving windows are 25 × 25

Table 1. Results of suppressing additive noise with rank-order filters

Type of filters	Measured errors	
	MSE	MAE
Noisy image	0.0112	0.0871
RA_0	0.0005	0.0088
RA_1_2	0.0001	0.0014

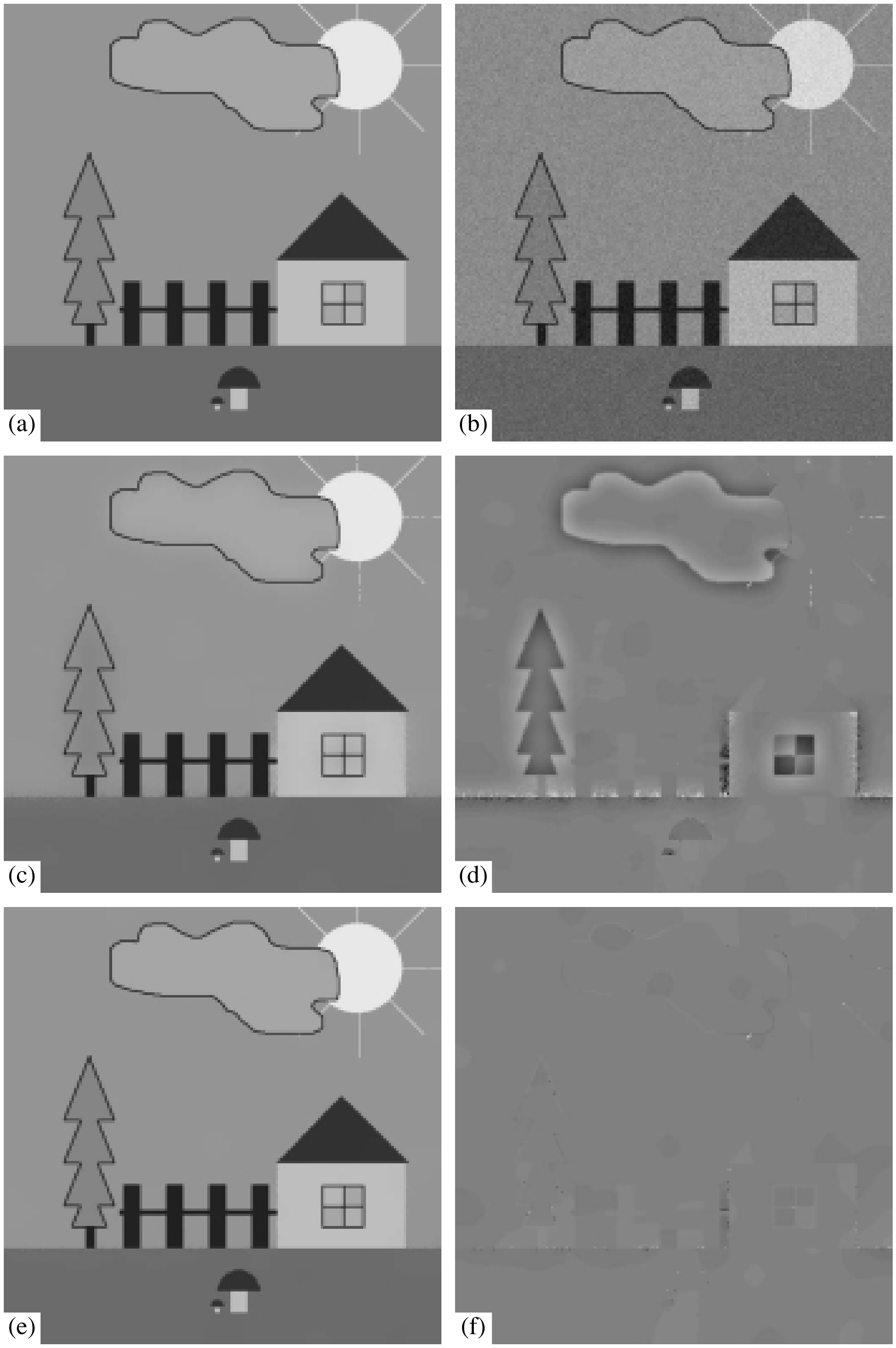

Fig. 4. (a) Original test image, (b) noisy image (additive noise), (c) image processed with conventional rank algorithm RA_0, (d) enhanced difference between the original image and image filtered by RA_0, (e) image processed with the proposed algorithm RA_1_2, and (f) enhanced difference between the original image and image filtered by RA_1_2.

and 41 × 41; the value ε_v of the *EV*-neighborhood is always equal to 30; the size of the *S*-neighborhood for removing noise outliers is 3×3; the threshold values of outlier detection are 4 and 0. The proposed algorithm RA_1_2 is the same rank algorithm as RA_0; however, it uses the *AEV*-neighborhood with the connectivity order of 1 or 2. The number of iterations is 4, and the size of the moving window in all iterations is 35×35. The value ε_v of the *AEV*-neighborhood is always equal to 30. The size of the *S*-neighborhood for removing impulse noise is 5×5. The threshold values are 0, 0, 3, and 3. The orders of connectivity are 1, 1, 2, and 2.

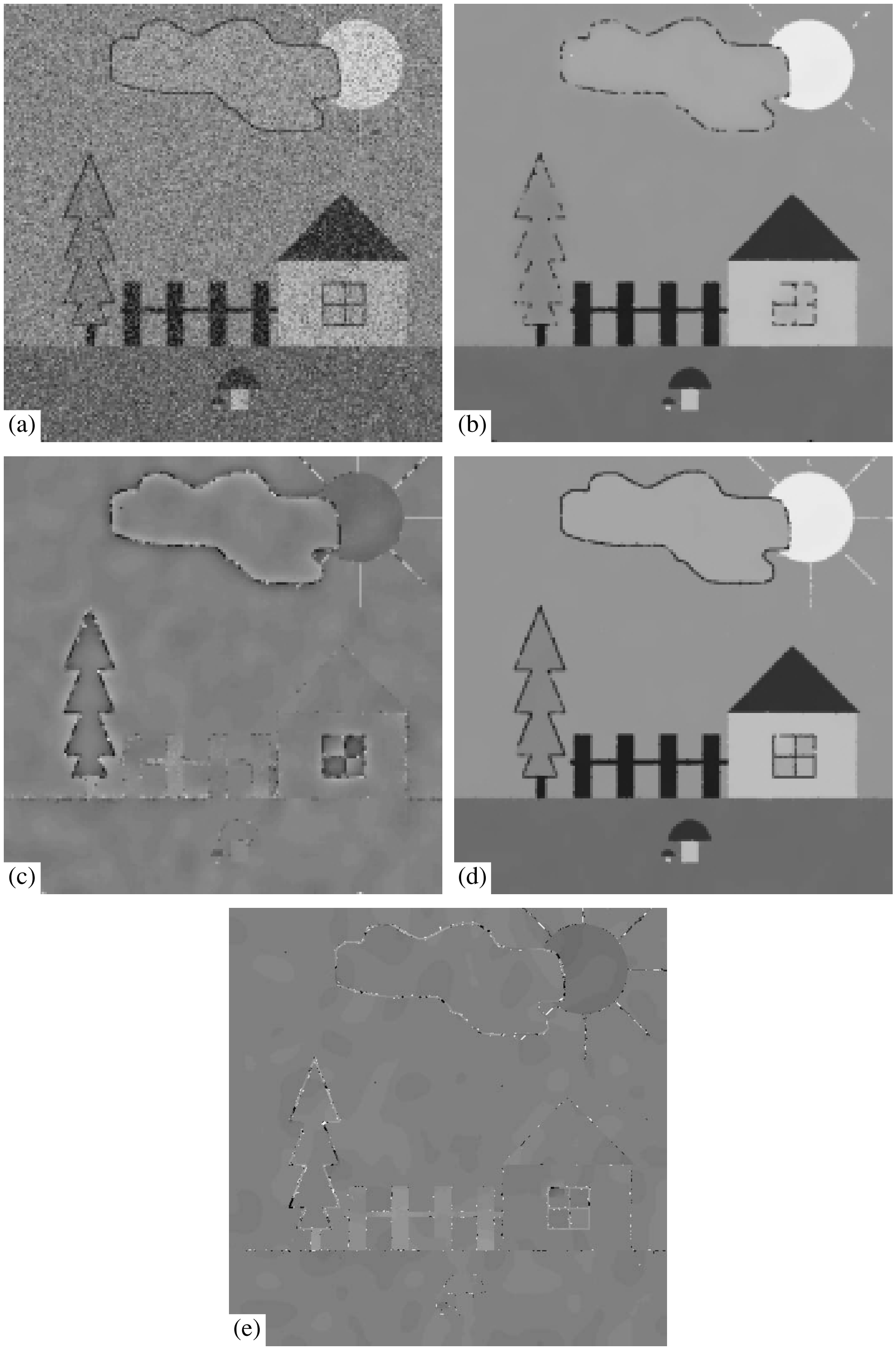

Fig. 5. (a) Noisy image (mixed additive and impulse noise), (b) image processed with rank filter RM_0, (c) enhanced difference between the original image and the image filtered with RM_0, (d) image processed with rank filter RM_1_2, and (e) enhanced difference between the original image and the image filtered with RM_1_2.

Figures 4c and 4e show the images processed with conventional rank algorithm RA_0 and with the proposed algorithm RA_1_2, respectively. Figures 4d and 4f show enhanced differences between (d) the original image and the image filtered with RA_0 and (f) between the original image and the image filtered with RA_1_2. We observe that the noise reduction capability of both filters is very good on the smooth regions. However, the conventional rank filter RA_0 has a poor performance on edges. The distortions are caused by the pixels of different adjacent signal areas that are used in smoothing. Since the filter RA_1_2 only works with spatially connected regions, it yields a very good result on the edge of adjacent regions. Table 1 shows the error retained and introduced by each of the filters according to the MSE and MAE error criteria.

4.2. Test Image Corrupted by Mixed Additive and Impulse Noise

Figure 5a shows the test image with additive zero-mean Gaussian noise degraded by the impulse noise. The standard deviation of the additive noise is $\sigma = 15$. The probability of noise impulse is 0.2 and, if it occurs, it can be positive or negative with equal probability 0.1. In the simulations, the values of the impulse were set to 0 or 255. Table 2 shows the difference between the original and noisy images in terms of the MSE and MAE.

In our computer experiments, we compared two following algorithms. RM_0 is the conventional rank algorithm given in Eq. (22). The number of iterations is 4. The filter parameters at each iteration are as follows: the sizes of moving windows are 25×25, 5×5, 7×7, and 25×25; the values ε_v of the *EV*-neighborhood are 30, 12, 12, and 30; the size of the *S*- neighborhood for removing noise outliers is 3×3; the threshold values of outlier detection are 0, 3, 6, and 9. RM_1_2 is the same rank algorithm as RM_0; however, it uses the *AEV*-neighborhood with the connectivity order of 1 or 2. The number of iterations is 6. The size of the moving window in all iterations is 35×35. The values ε_v of the *AEV*-neighborhood are 30, 15, 15, 21, 25, and 30. The size of the *S*- neighborhood for removing impulse noise is 5×5. The threshold values are 1, 3, 4, 7, 11, and 0. The orders of connectivity are 1, 1, 1, 2, 2, and 1.

Figures 5b and 5d show the processed images with (b) rank filter RM_0 and (d) rank filter RM_1_2. Figures 5c and 5e show enhanced differences between (c) the original image and the image filtered with RM_0 and (e) the original image and the image filtered with RM_1_2. Table 2 shows errors according to MSE and MAE error criteria. It is obvious, that the conventional rank filter has a poor performance in this case also. It has the same drawbacks as in the case of additive noise. Moreover, impulse noise detection goes often wrong at thin lines under the noise, which leads to the erroneous removal of these lines. The proposed filter RA_1_2 uses spatial pixel connectivity and, thus, efficiently reduces mixed (additive and impulse) noise while preserving fine structures and details.

Table 2. Results of suppressing mixed additive noise with rank-order filters

Type of filters	Measured errors	
	MSE	MAE
Noisy image	0.172	0.245
RA_0	0.006	0.015
RA_1_2	0.002	0.006

5. CONCLUSION

In this paper, we present a new approach to designing the rank-order filters for suppressing additive and impulse noise, local contrast enhancement, and local detail extraction. The approach makes explicit use of spatial relations between image elements. An extension of this approach to rank-order filtering of three-dimensional signals is also presented. The proposed rank filters are very attractive for image-processing applications. Extensive testing has shown that, when the input image is degraded by mixed additive and impulse noise, the proposed rank-order filters outperform the conventional rank-order filters in terms of the MSE, MAE, and the subjective visual criterion.

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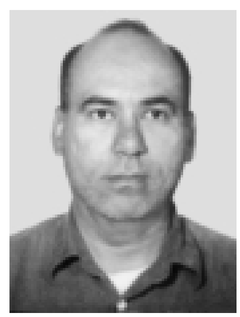

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