International Mathematical Forum, 2, 2007, no. 9, 429 - 440

Ranking Bank Branches with Interval Data The Application of DEA

F. Hosseinzadeh Lotfi[1]

Department of Mathematics, Science and Research Branch,
Islamic Azad University, Tehran, Iran

M. Navabakhs

Department of Society, Science and Research Branch,
Islamic Azad University, Tehran, Iran

A. Tehranian

Department of Mathematics, Science and Research Branch,
Islamic Azad University, Tehran, Iran

M. Rostamy-Malkhalifeh

Department of Mathematics, Science and Research Branch,
Islamic Azad University, Tehran, Iran

R. Shahverdi

Department of Mathematics, Science and Research Branch,
Islamic Azad University, Tehran, Iran

Abstract

DEA (Data Envelopment Analysis) evaluates the relative efficiency of a set of DMUs (Decision Making Units). The relative efficiency of a DMU is the result of comparing the inputs and outputs of the DMU and those of other DMUs in the PPS (Production Possibility Set). If the inputs and outputs each vary in intervals, the DMUs cannot be easily evaluated and ranked using the obtained efficiency scores. In this paper, presenting a new idea for computing the efficiency of DMUs with

[1] Corresponding author:Farhad Hosseinzadeh Lotfi, E-mail:hosseinzadeh_lotfi@yahoo.com

interval data, an interval will be defined for the efficiency score of each unit. And finally, a method for ranking DMUs by the obtained efficiency interval is presented. And, the new technique will be applied to a set of real data.

Keywords: Data Envelopment Analysis; Interval data; Ranking

1 Introduction

Data Envelopment Analysis technique was presented in the CCR paper by Charnes et al. (1997), and since then was developed by various researchers. In this method, the relative efficiency of a set of DMUs which use similar types of (multiple) resources to produce similar types of (multiple) outputs is computed. Finally, DMUs are divided into two groups of efficient and inefficient DMUs. In ordinary DEA models, the input and output values are assumed to be definite. In recent year, in different applications of DEA, inputs and outputs have been observed whose values are indefinite. Such data are called "inaccurate". Inaccurate data can be probabilistic, interval, ordinal, qualitative, or fuzzy. Therefore, some papers were presented on the theoretical development of this technique whit interval data, of which we can name Despotis et. al(2002)and Jahanshahloo et al. (2004). In the above-mentioned papers, all the DMUs are divided into three groups which are defined according to the interval obtained for the efficiency value of DMUs. This paper consists of the following sections: In section 2, DEA is discussed. DEA with interval data is presented in section3. And, the method for ranking DMUs with interval data is put forward in section4. Finally, an example with real interval data will be given.

2 DEA

Suppose that we have n DMUs, $\text{DMU}_j\ : j = 1, \ldots, n$, to be evaluated, each DMU using m inputs to produce s outputs. $X_j = (x_{1j}, \ldots, x_{mj})$ and $Y_j = (y_{1j}, \ldots, y_{sj})$ are the input and output vectors of DMU_j, respectively, in which $X_j,\ Y_j \geq 0, X_j \neq 0$ and $Y_j \neq 0$. The input-oriented CCR model to evaluate the relative efficiency of DMUp is as follows:

$$
\begin{aligned}
e_p = \quad & \max \sum_{r=1}^{s} u_r y_{rp} \\
& s.t. \quad \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0, \quad j = 1, \ldots, n, \\
& \qquad \sum_{i=1}^{m} v_i x_{ip} = 1 \\
& \qquad u_i, v_r \geq \varepsilon \qquad\qquad i = 1, \ldots, m, \quad r = 1, \ldots, s,
\end{aligned} \tag{1}
$$

where U= $(u_1, \ldots, u_s)$ and $V = (v_1, \ldots, v_m)$ are output and input weight vectors, respectively, and they are unknown, and should be computed by solving problem (1). Assume that (U^*, V^*) is the optimal solution to problem (1). Then, the DMU_p under evaluation is relative efficient if and only if $e_p^* = 1$.Otherwise, DMU is said to be inefficient. In problem (1), it is assumed that all inputs and outputs of any DMU are of known values.

3 Interval DEA

Let input and output values of any DMU be located in a certain interval, where $x_{ij}^L$and $x_{ij}^U$are the lower and upper bounds of the ith input of the$j$th DMU, respectively, and y_{rj}^L and$y_{rj}^U$are the lower and upper bounds of the rth output of the jth DMU, respectively; that is to say,$x_{ij}^L \leq x_{ij} \leq x_{ij}^U$ and$y_{ij}^L \leq y_{ij} \leq y_{ij}^U$. Such data are called interval data, because they are located in intervals. Note that always $x_{ij}^L \leq x_{ij}^U$ and $y_{ij}^L \leq y_{ij}^U$. If $x_{ij}^L = x_{ij}^U$, then the i th input of the j th DMU has a definite value. Interval problems are those whose parameter values are located in intervals, their exact values being unable to be identified. The CCR model for evaluating$DMU_p$with interval data is as follows:

$$
\begin{aligned}
e_p = \quad & \max \sum_{r=1}^{s} u_r [y_{rp}^L, y_{rp}^U] \\
& s.t. \quad \sum_{r=1}^{s} u_r [y_{rj}^L, y_{rj}^U] - \sum_{i=1}^{m} v_i [x_{ij}^L, x_{ij}^U] \leq 0, \quad j = 1, \ldots, n, \\
& \qquad \sum_{i=1}^{m} v_i [x_{ip}^L, x_{ip}^U] = 1 \\
& \qquad u_i, v_r \geq \varepsilon \qquad\qquad i = 1, \ldots, m, \quad r = 1, \ldots, s,
\end{aligned} \tag{2}
$$

In problem (2), seeing that all parameters of the problem are in intervals and do not have definite values, the relative efficiency ofDMU_p is also located in an interval. The upper and lower bounds of the relative efficiency of DMU_p are obtained by solving the following problems, respectively.

$$\begin{aligned} e_p^U = \quad & \max \sum_{r=1}^{s} u_r y_{rp}^U && (3)\\ & s.t. \sum_{r=1}^{s} u_r y_{rj}^L - \sum_{i=1}^{m} v_i x_{ij}^U \leq 0, \quad j = 1, \ldots, n, j \neq p \\ & \sum_{r=1}^{s} u_r y_{rp}^U - \sum_{i=1}^{m} v_i x_{ip}^L \leq 0, \\ & \sum_{i=1}^{m} v_i x_{ip}^L = 1 \\ & u_i, v_r \geq \varepsilon \qquad i = 1, \ldots, m, \quad r = 1, \ldots, s, \end{aligned}$$

$$\begin{aligned} e_p^L = \quad & \max \sum_{r=1}^{s} u_r y_{rp}^L && (4)\\ & s.t. \sum_{r=1}^{s} u_r y_{rj}^U - \sum_{i=1}^{m} v_i x_{ij}^L \leq 0, \quad j = 1, \ldots, n, j \neq p \\ & \sum_{r=1}^{s} u_r y_{rp}^L - \sum_{i=1}^{m} v_i x_{ip}^U \leq 0, \\ & \sum_{i=1}^{m} v_i x_{ip}^U = 1 \\ & u_i, v_r \geq \varepsilon \qquad i = 1, \ldots, m, \quad r = 1, \ldots, s, \end{aligned}$$

In problem (3), the DMU under evaluation is in its best condition, and the other DMUs are in their worst condition. So, $e_p \leq e_p^U$ always holds. Whereas, in problem (4), the DMU under evaluation is in its worst condition and the other DMUs are in their best condition. Therefore, the obtained efficiency will be the worst possible efficiency for the DMU under evaluation, so $e_p \geq e_p^L$. According to what has been mentioned, we can say, $e_p \in [e_p^L, e_p^U]$. Considering that the efficiency of any DMU lies in an interval, all DMUs can be divided into one of the three following classes:
Class 1 includes all DMUs which are efficient both in their best and worst

conditions, that is to say, $E^{++} = \{DMUj : e_j^L = e_j^U = 1\}$.
Class 2 consists of all DMUs which are efficient in their best condition, but inefficient in their worst condition, in other words, $E^{+} = \{DMUj : e_j^L < 1, e_j^U = 1\ \}$.
And, class 3 contains all DMUs which are inefficient in their best condition. It goes without saying that such DMUs are, also, inefficient in their best condition, that is to say, $E^{-} = \{DMUj : e_j^L < 1, e_j^U < 1\ \}$. All DMUs in E^{++} are as efficient DMUs, since they are efficient both in their best and worst conditions. And, all DMUs in E^{-} are inefficient, as they are inefficient both in their best and worst conditions. But, in case of DMUs in E^{+}, one cannot determine their being efficient or inefficient, because they are efficient in some condition and inefficient in some other condition. In the next section, a method will be presented for ranking DMUs with interval data.

4 Ranking DMUs with interval data

In the previous section, a method was presented by which interval efficiency was obtained for every DMU with interval data. Ranking interval efficiency seems a little difficult. In other words, if two DMUs are located in E^{++}, how can we comment on one of them being better than the other? The same holds true for DMUs in E^{+} or in E^{-}. In this section, a method is proposed for ranking the DMUs in each class. To being with, suppose that any DMU in E^{+} has a better rank than DMUs in E^{-}. Therefore, the following proposed method can be applied to each one of the classes E^{++}, E^{+} and E^{-}, separately.
For each DMU, four types of efficiency can be computed:
a) the DMUs under evaluation in its best condition, and the other DMUs in their worst condition,

$$
\begin{aligned}
e_p^1 = \quad & \max \ \sum_{r=1}^{s} u_r y_{rp}^U && (5)\\
& s.t. \ \ \sum_{r=1}^{s} u_r y_{rj}^L - \sum_{i=1}^{m} v_i x_{ij}^U \le 0, \quad j = 1, \dots, n, j \ne p \\
& \qquad \sum_{r=1}^{s} u_r y_{rp}^U - \sum_{i=1}^{m} v_i x_{ip}^L \le 0, \\
& \qquad \sum_{i=1}^{m} v_i x_{ip}^L = 1 \\
& \qquad u_i, v_r \ge \varepsilon \qquad\qquad\qquad i = 1, \dots, m, \quad r = 1, \dots, s,
\end{aligned}
$$

b) the DMU under evaluation in its best condition, and the other DMUs, also, in their best condition,

$$\begin{aligned} e_p^2 = \quad & \max \sum_{r=1}^{s} u_r y_{rp}^U && (6) \\ & s.t. \sum_{r=1}^{s} u_r y_{rj}^U - \sum_{i=1}^{m} v_i x_{ij}^L \leq 0, \quad j = 1, \dots, n, \\ & \sum_{i=1}^{m} v_i x_{ip}^L = 1 \\ & u_i, v_r \geq \varepsilon \qquad i = 1, \dots, m, \quad r = 1, \dots, s, \end{aligned}$$

c) the DMU under evaluation in its worst condition , and the other DMUs, also, in their worst condition,

$$\begin{aligned} e_p^3 = \quad & \max \sum_{r=1}^{s} u_r y_{rp}^L && (7) \\ & s.t. \sum_{r=1}^{s} u_r y_{rj}^L - \sum_{i=1}^{m} v_i x_{ij}^U \leq 0, \quad j = 1, \dots, n, \\ & \sum_{i=1}^{m} v_i x_{ip}^U = 1 \\ & u_i, v_r \geq \varepsilon \qquad i = 1, \dots, m, \quad r = 1, \dots, s, \end{aligned}$$

d) the DMU under evaluation in its worst condition, and the other DMUs in their best condition,

$$\begin{aligned} e_p^4 = \quad & \max \sum_{r=1}^{s} u_r y_{rp}^L && (8) \\ & s.t. \sum_{r=1}^{s} u_r y_{rj}^U - \sum_{i=1}^{m} v_i x_{ij}^L \leq 0, \quad j = 1, \dots, n, j \neq p \\ & \sum_{r=1}^{s} u_r y_{rp}^L - \sum_{i=1}^{m} v_i x_{ip}^U \leq 0, \\ & \sum_{i=1}^{m} v_i x_{ip}^U = 1 \\ & u_i, v_r \geq \varepsilon \qquad i = 1, \dots, m, \quad r = 1, \dots, s, \end{aligned}$$

Considering problems (5), (6), (7) and (8), we can state the following: $e_p^1 = e_p^U, e_p^4 = e_p^L, e_p^4 \leq e_p^2 \leq e_p^1, e_p^4 \leq e_p^3, e_p^3 \leq e_p^1$. The following method is proposed for ranking DMUs in each class E^{++}, E^+ and E^-. First, solve the four following models which correspond each DMU, respectively. The models presented below are the AP models corresponding problems (5), (6), (7) and (8).

$$
\begin{aligned}
\theta_p^1 = \quad & \max \sum_{r=1}^{s} u_r y_{rp}^U \\
& s.t. \ \sum_{r=1}^{s} u_r y_{rj}^L - \sum_{i=1}^{m} v_i x_{ij}^U \leq 0, \quad j = 1, \ldots, n, j \neq p \\
& \sum_{i=1}^{m} v_i x_{ip}^L = 1 \\
& u_i, v_r \geq \varepsilon \qquad\qquad i = 1, \ldots, m, \quad r = 1, \ldots, s,
\end{aligned}
\tag{9}
$$

$$
\begin{aligned}
\theta_p^2 = \quad & \max \sum_{r=1}^{s} u_r y_{rp}^U \\
& s.t. \ \sum_{r=1}^{s} u_r y_{rj}^U - \sum_{i=1}^{m} v_i x_{ij}^L \leq 0, \quad j = 1, \ldots, n, j \neq p \\
& \sum_{i=1}^{m} v_i x_{ip}^L = 1 \\
& u_i, v_r \geq \varepsilon \qquad\qquad i = 1, \ldots, m, \quad r = 1, \ldots, s,
\end{aligned}
\tag{10}
$$

$$\begin{aligned}\theta_p^3 = \quad & \max \ \sum_{r=1}^{s} u_r y_{rp}^L \\ & s.t. \ \sum_{r=1}^{s} u_r y_{rj}^L - \sum_{i=1}^{m} v_i x_{ij}^U \le 0, \quad j = 1, \dots, n, j \neq p \\ & \sum_{i=1}^{m} v_i x_{ip}^U = 1 \\ & u_i, v_r \ge \varepsilon \qquad i = 1, \dots, m, \quad r = 1, \dots, s,\end{aligned} \tag{11}$$

$$\begin{aligned}\theta_p^4 = \quad & \max \ \sum_{r=1}^{s} u_r y_{rp}^L \\ & s.t. \ \sum_{r=1}^{s} u_r y_{rj}^U - \sum_{i=1}^{m} v_i x_{ij}^L \le 0, \quad j = 1, \dots, n, j \neq p \\ & \sum_{i=1}^{m} v_i x_{ip}^U = 1 \\ & u_i, v_r \ge \varepsilon \qquad i = 1, \dots, m, \quad r = 1, \dots, s,\end{aligned} \tag{12}$$

Problems (9), (10), (11) and (12) are the very problems (5), (6), (7) and (8), except that the boundedness condition of the objective function has been excluded from the constraints. Therefore, the values of $\theta_p^1, \theta_p^2, \theta_p^3$ and θ_p^4 may exceed unity. Now the following criterion can be suggested for ranking the DMUs of each class.

$$\theta_p = \frac{1}{\sum_{i=1}^{4} \alpha_i}(\alpha_1\theta_p^1 + \alpha_2\theta_p^2 + \alpha_3\theta_p^3 + \alpha_4\theta_p^4).$$

In a certain class, any DMU having a higher value will have a better ranking. The application of the above method for ranking should be carried out separately to each class, and ranking should be done separately in each class.

5 Ranking Bank Branches

We now apply our approach to some commercial bank branches in Iran. There are 20 branches in this district. Each branch uses 3 inputs to produce 5 outputs. Table 1 shows these inputs and outputs.

Inputs	Outputs
Payable interest	The total sum of four main deposits
Personnel	Other deposits
Non-Performing loans	Loans granted
	Received interest
	Fee

Table 1 *Inputs and Outputs*

In Tables 2 and 3 the interval inputs and interval outputs for these DMUs are given. Also in Table 4 the efficiencies of these DMUs are presented.

DMU_j	x^L_{1j}	x^U_{2j}	x^L_{2j}	x^U_{2j}	x^L_{3j}	x^U_{3j}
1	5007.37	9613.37	36.29	36.86	87243	87243
2	2926.81	5961.55	18.8	2016	9945	12120
3	8732.7	17752.5	25.74	27.17	47575	50013
4	945.93	1966.39	20.81	22.54	19292	19753
5	8487.07	17521.66	14.16	14.8	3428	3911
6	13759.35	27359.36	19.46	19.46	13929	15657
7	587.69	1205.47	27.29	27.48	27827	29005
8	4646.39	9559.61	24.52	25.07	9070	9983
9	1554.29	3427.89	20.47	21.59	412036	413902
10	17528.31	36297.54	14.84	15.05	8638	10229
11	2444.34	4955.78	20.42	20.54	500	937
12	7303.27	14178.11	22.87	23.19	16148	21353
13	9852.15	19742.89	18.47	21.83	17163	17290
14	4540.75	9312.24	22.83	23.96	17918	17964
15	3039.58	6304.01	39.32	39.86	51582	55136
16	6585.81	13453.58	25.57	26.52	20975	23992
17	4209.18	8603.79	27.59	27.95	41960	43103
18	1015.52	2037.82	13.63	13.93	18641	19354
19	5800.38	11875.39	27.12	27.26	19500	19569
20	1445.68	2922.15	28.96	28.96	31700	32061

$Table\ 2\ Input - data\ for\ the\ 20\ bank\ branches$

DMU_j	y_{1j}^L	y_{1j}^U	y_{2j}^L	y_{2j}^U	y_{3j}^L	y_{3j}^U	y_{4j}^L	y_{4j}^U	y_{5j}^L	y_{5j}^U
1	2696995	3126798	263643	382545	1675519	1853365	108634.76	125740.28	965.97	6957.33
2	340377	440355	95978	117659	377309	390203	32396.65	37836.56	304.67	749.4
3	1027546	1061260	37911	503089	1233548	1822028	96842.33	108080.01	2285.03	3174
4	1145235	1213541	229646	268460	468520	542101	32362.8	39273.37	207.98	510.93
5	390902	395241	4924	12136	129751	142873	12662.71	14165.44	63.32	92.3
6	988115	1087392	74133	111324	507502	574355	53591.3	72257.28	480.16	869.52
7	144906	165818	180530	180617	288513	323721	40507.97	45847.48	176.58	370.81
8	408163	416416	405396	486431	1044221	1071812	56260.09	73948.09	4654.71	5882.53
9	335070	410427	337971	449336	1584722	1802942	176436.81	189006.12	560.26	2506.67
10	700842	768593	14378	15192	2290745	2573512	662725.21	791463.08	58.89	86.86
11	641680	696338	114183	241081	1579961	2285079	17527.58	20773.91	1070.81	2283.08
12	453170	481943	27196	29553	245726	275717	35757.83	42790.14	375.07	559.85
13	553167	574989	21298	23043	425886	431815	45652.24	50255.75	438.43	836.82
14	309670	342598	20168	26172	124188	126930	8143.79	11948.04	936.62	1468.45
15	286149	317186	149183	270708	787959	810088	106798.63	111962.3	1203.79	4335.24
16	321435	347848	66169	80453	360880	379488	89971.47	165524.22	200.36	399.8
17	618105	835839	244250	404579	9136507	9136507	33036.79	41826.51	2781.24	4555.42
18	248125	320974	3063	6330	26687	29173	9525.6	10877.78	240.04	274.7
19	640890	679916	490508	684372	2946797	3985900	66097.16	95329.87	961.56	1914.25
20	119948	120208	14943	17495	297674	308012	21991.53	27934.19	282.73	471.22

Table 3 Output – data for the 20 bank branches

DMU_j	e_j^1	e_j^2	e_j^3	e_j^4
1	1	1	1	1
2	1	0.657024	0.455349	0.371424
3	1	1	0.83911	0.523443
4	1	1	1	1
5	0.763021	0.666492	0.71051	0.618831
6	1	1	1	0.917738
7	1	1	1	0.728914
8	1	1	1	1
9	1	1	1	1
10	1	1	1	1
11	1	1	1	1
12	0.496069	0.396102	0.403706	0.328345
13	0.701987	0.53513	0.538834	0.449969
14	0.724067	0.353113	0.368022	0.26395
15	1	1	0.930397	0.413252
16	1	0.550914	0.372671	0.221643
17	1	1	1	1
18	0.952539	0.351603	0.400584	0.263737
19	1	1	1	0.991215
20	1	0.399693	0.403382	0.183825

Table 4 Efficiencies of DMUs

In Table 5, the classifications and ranking of these DMUs are presented.

DMU_j	$\bar{\theta_j}^1$	$\bar{\theta_j}^2$	$\bar{\theta_j}^3$	$\bar{\theta_j}^4$	$\bar{\theta_j}^5$	Classification	Rank
1	3.523761	1.846914	1.482492	1.259058	2.028056	E++	7
2	1.020757	0.506559	0.455339	0.207883	0.547634	E+	14
3	1.359532	0.971804	0.83911	0.52135	0.922949	E+	12
4	5.224134	2.418946	2.518259	1.259423	2.85519	E++	5
5	0.763021	0.666492	0.71051	0.618831	0.689714	E-	16
6	1.245606	1.025448	1.059813	0.917738	1.062151	E+	11
7	3.172361	1.463782	1.544707	0.728914	1.727441	E+	8
8	4.812406	1.869024	2.825858	1.433282	2.735142	E++	6
9	5.745185	2.703864	2.451013	1.238175	3.034559	E++	4
10	19.90213	11.50958	15.19096	7.955503	13.63954	E++	2
11	26.44258	20.38311	10.35878	8.209075	16.34839	E++	1
12	0.496069	0.396102	0.403706	0.328345	0.406055	E-	20
13	0.701987	0.53513	0.538834	0.449969	0.55648	E-	17
14	0.724067	0.353113	0.368022	0.26395	0.427288	E-	19
15	3.181636	1.168753	0.930397	0.413252	1.423509	E+	10
16	1.297599	0.550288	0.372671	0.221643	0.61055	E+	13
17	6.536229	2.65356	3.672324	2.158379	3.755123	E++	3
18	0.952539	0.351603	0.400584	0.263676	0.492101	E-	18
19	2.218415	1.349766	1.227325	0.991215	1.44668	E+	9
20	0.977855	0.399693	0.403382	0.183825	0.491189	E+	15

Table 5 Classification and Ranking of DMUs

Regarding Table 4and table 5, it can be seen that branches 1, 4, 8, 9, 10, 11and 17 are efficient in their worst condition there for they are placed in E++, and among these braches branch number 11 has the best rank, and branches 10, 17, 9, 4, 8 and 1 lie after it. It is also seen that branches 2, 3, 6, 7, 15, 16, 19 and 20 are put in E+, because they are efficient in their best condition, but they are inefficient in their worst condition. Among the branches in E+, branch number 7 has the best rank, and after that lie branches 19, 15, 6, 3, 16, 2 and 20. And finally, since branches 5, 12, 13, 14 and 18 are inefficient in their best condition, they are placed in E-, and among them, branch number 5 has the best rank and branches 13, 18, 14 and 12 lie after it. Considering the fact that DMUs in E++, and those in E+ have a better rank that those in E-, the ranking of DMUs has been shown in table 5.

6 Conclusion

Regarding table 5, we can observe that $DMU_{20} \in E^+$ and $DMU_5 \in E^-$ but $\theta_5 > \theta_{20}$, and this is due to the fact that the data of DMU_5 are in a smaller interval than that of the data of DMU_{20} ande^4_{20} in much less than e^4_5 ; i.e.,$e^4_5 - e^4_{20} > e^1_{20} - e^1_5$.

Now we can rank the branches, according to the director,s opinion, in such a way that the higher the θ corresponding to a branch, the better its rank, regardless of which classes of E^{++}, E^+ and E^- the given are placed in. It is obvious that if $DMU_j \in E^{++}$ *and* $DMU_i \in E^{+}$ *or* $DMU_i \in E^{-}$ *then* $\theta_j > \theta_i$. According to what was mentioned, it can be observed that DMUs in E^{++} have better ranks than other DMUs. Thus, the branches in our example can be ranked as follows: 11, 10, 17, 9, 4, 8, 1, 7, 19, 15, 6, 3, 5, 16, 13, 2, 18, 20, 14 and 12

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Received: March 18, 2006