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Ranking Function Methods For Solving Fuzzy Linear Programming Problems

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Abstract:
In this paper, we concentrate on linear programming problems in which the coefficients of objective function are fuzzy numbers, the right-hand side are fuzzy numbers too, and both the coefficients of objective function as well as right-hand side are fuzzy numbers. Then solving these fuzzy linear programming problems by using many linear ranking functions. After that develop six numerical examples to illustrates the steps of solutions for all these type of linear programming problems which studying in this paper.
Keywords: Fuzzy set theory, fuzzy linear programming, linear ranking function, trapezoidal membership.

1- Introduction:

A lot of application problems can be modeled as mathematical problem which may be formulated with uncertainty.
The concept of fuzzy linear programming was first proposed by Zimmerann [9]. Vansant and else [7] applied linear programming with fuzzy parameters for decision making in industrial production planning. Maleki [5] introduced fuzzy variables in linear programming problems and proposed a new method for solving these problems using ranking function. Pandian and Jayalakskmi [6] proposed a new method for solving integer linear programming problems with fuzzy variables. Singh[4] proposed a new method for solving fully fuzzy linear programming problems using ranking function. Dheyab[2] solved the fuzzy fractional linear programming problems by using ranking function. Hashem[3] introduced the decision maker in the form of nonsymmetrical trapezoidal fuzzy numbers and solve it by using ranking function.
In this paper, we solving the fuzzy linear programming problems, when the coefficients of objective function are fuzzy numbers, as well as right-hand side are fuzzy numbers too, and both the coefficients of objective function as well as right-hand side are fuzzy numbers by using Maleki linear ranking function when $(\alpha = \beta)$ and $(\alpha \neq \beta)$ and using Yager linear ranking function when $(\alpha = \beta)$ and $(\alpha \neq \beta)$.
This paper is outlined as follows. In section 2, we study fuzzy set theory. In section 3, we presented trapezoidal fuzzy numbers. In section 4, we interested in ranking functions, but in section 5, we study fuzzy linear programming problems with fuzzy coefficients objective function and fuzzy right-hand side, then both of them. In section 6, we illustrate all type of fuzzy linear programming problems in numerical examples. Finally in section 7 we make conclusion for this study.

2- Fuzzy set theory: [1]

Definition 2.1: If X is a collection of objects denoted generically by x, then fuzzy set $\tilde{A}$ in X is defined to be a set of ordered pairs $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) \mid x \in X\}$, where $\mu_{\tilde{A}}(x)$ is called the membership function for the fuzzy set. The membership function maps each element of X value between [0, 1].

Assuming that X is the real line R.

Definition 2.2: The support of a fuzzy set $\tilde{A}$ is the set of point x in$X$ with $\mu_{\tilde{A}}(x) > 0$. The core of a fuzzy set $\tilde{A}$ is the set of point x in$X$ with $\mu_{\tilde{A}}(x) = 1$.
Definition 2.3: A fuzzy set $\tilde{A}$ is called Normal if its core is nonempty. In other words, there is at least one point $x \in X$ with$\mu_{\tilde{A}}(x) = 1$.
Definition 2.4: The λ-level (or λ-cut) set of a fuzzy set $\tilde{A}$ is a crisp set defined by

$A_\lambda = \{x \in X | \mu_{\tilde{A}}(x) \geq \lambda\}$, the strong λ-cut is defined to be $\bar{A}_\lambda = \{x \in X | \mu_{\tilde{A}}(x) > \lambda\}$.

Definition 2.5: A fuzzy set $\tilde{A}$ on R is convex if for any $x, y \in X$ and any $\lambda \in [0, 1]$, then

$$\mu_{\tilde{A}}(\lambda x + (1-\lambda)y) \geq Min\{\mu_{\tilde{A}}(x), \mu_{\tilde{A}}(y)\}.$$

Remark 2.1: A fuzzy set is convex if and only if all its λ-cuts are convex.

Definition 2.6: A fuzzy number $\tilde{A}$ is a fuzzy set on the real line that satisfies the conditions of normality and convexity.

3- Trapezoidal fuzzy numbers: [1]

There are various types of fuzzy numbers, but the triangular and trapezoidal are the most important fuzzy memberships. In this research we use the trapezoidal fuzzy numbers .In fact, the fuzzy number is defined by its corresponding membership function. The trapezoidal membership function for fuzzy number $\tilde{a}$ as follows:

$$\mu_{\tilde{a}}(x) == \begin{cases} 1-\frac{a^l-x}{\alpha} \text{ ,when } a^l-\alpha \leq x < a^l \\ 1 \text{ ,when } a^l \leq x \leq a^u \\ 1-\frac{x-a^u}{\beta} \text{ ,when } a^u \leq x \leq a^u+\beta \\ 0 \text{ , otherwise} \end{cases} \quad \ldots (1)$$

The figure 1 shown the trapezoidal membership function for fuzzy number which is as follows:

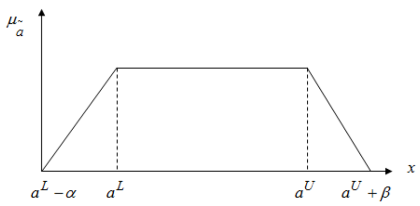

Figure 1 Trapezoidal fuzzy number

A trapezoidal fuzzy number can be shown by $\tilde{a} = (a^l, a^u, \alpha, \beta)$, the support of $\tilde{a}$ *is* $(a^l - \alpha, a^u + \beta)$, and the core of $\tilde{a}$ is $[a^l, a^u]$, where $F(R)$ denoted the set of all trapezoidal fuzzy number.

Assume that $\tilde{a} = (a^l, a^u, \alpha, \beta)$ and $\tilde{b} = (b^l, b^u, \gamma, \theta)$ be two trapezoidal fuzzy numbers and $x \in R$. the arithmetic trapezoidal fuzzy numbers are shown as follows:

Image of $\tilde{a}$:is $-\tilde{a} = (-a^u, -a^l, \beta, \alpha)$

Addition : $\tilde{a} + \tilde{b} = (a^l + b^l, a^u + b^u, \alpha + \gamma, \beta + \theta)$

Subtraction: $\tilde{a} - \tilde{b} = (a^l - b^u, a^u - b^l, \alpha + \theta, \beta + \gamma)$

Scalar Multiplication: if $x \geq 0, x\tilde{a} = (xa^l, xa^u, x\alpha, x\beta)$

if $x < 0, x\tilde{a} = (xa^u, xa^l, -x\beta, -x\alpha)$

As an illustration for the arithmetic trapezoidal fuzzy numbers, let $\tilde{a} = (-2, 1, 1, 2)$,

$\tilde{b} = (2, 4, 1, 1)$ and $x = 2, -2$, then

$-\tilde{a} = (-1, 2, 2, 1)$, $-\tilde{b} = (-4, -2, 1, 1)$

$\tilde{a} + \tilde{b} = (0, 5, 2, 3)$, $\tilde{a} - \tilde{b} = (-6, -1, 2, 3)$

$2\tilde{a} = (-4, 2, 2, 4)$, $-2\tilde{a} = (2, -4, -4, -2)$

4- Ranking function:

The ranking function is approach of ordering fuzzy numbers which is an efficient. Various types of ranking function have been introduced which are used for solving linear programming problems with fuzzy parameters. The ranking function is denoted by $F(\Re)$, where $\Re\colon \mathrm{F}(\Re) \to \Re$, and $\mathrm{F}(\Re)$ is the set of fuzzy numbers defined on a real line, where a natural order exist.

When using ranking function for comparison of fuzzy linear programming problem. Usually define a crisp model which is equivalent to the fuzzy linear programming problem, then using the optimal solution of this model as the optimal solution for fuzzy linear programming problem. Suppose that $\tilde{a}\ and\ \tilde{b}$ be two trapezoidal fuzzy numbers, then the ranking function of $F(\Re)$ is as following:

$if\quad \tilde{a} \geq \tilde{b}$ then $\Re(\tilde{\mathrm{a}}) \geq \Re(\tilde{\mathrm{b}})$
$if\quad \tilde{a} > \tilde{b}$ then $\Re(\tilde{\mathrm{a}}) > \Re(\tilde{\mathrm{b}})$
$if\quad \tilde{a} = \tilde{b}$ then $\Re(\tilde{\mathrm{a}}) = \Re(\tilde{\mathrm{b}})$
Where $\tilde{a}$ and $\tilde{b}$ are in $F(\Re)$, also in the same way we can write $\tilde{a} \leq \tilde{b}$ iff $\tilde{b} \geq \tilde{a}$.

Lemma: let $\Re$ be any ranking function, then:
1- $\tilde{a} \geq \tilde{b}$ iff $\tilde{a} - \tilde{b} \geq 0$ iff $-\tilde{b} \geq -\tilde{a}$.
2- if $\tilde{a} \geq \tilde{b}$ and $\tilde{c} \geq \tilde{d}$, then $\tilde{a} + \tilde{c} \geq \tilde{b} + \tilde{d}$.

In this research we use the form of linear ranking function, such that $\Re\big(\mathrm{k}\tilde{\mathrm{a}} + \tilde{\mathrm{b}}\big) = \mathrm{k}\Re(\tilde{\mathrm{a}}) + \Re(\tilde{\mathrm{b}})$, where $k \in R$.

a- Maleki ranking function: [5]
let $\tilde{a} = (a^l, a^u, \alpha, \beta)$ be a fuzzy numbers ,then the ranking function is:

$$\Re(\tilde{\mathrm{a}}) = \int_0^1 (\inf \tilde{\mathrm{a}}_\lambda + \sup \tilde{\mathrm{a}}_\lambda) \mathrm{d}\lambda$$
$$\Re(\tilde{\mathrm{a}}) = a^l + a^u + \frac{1}{2}(\beta - \alpha), \quad \dots (2)$$
Where $\alpha = \beta$ or $\alpha \neq \beta$.

b- Yager ranking function: [8]
assume that $\tilde{a} = (a^l, a^u, \alpha, \beta)$ be a fuzzy numbers ,then the ranking function is:

$$\Re(\tilde{\mathrm{a}}) = \frac{1}{2}[\int_0^1 [a^l - \alpha L^{-1}(\lambda)]\, d\lambda + \int_0^1 [\,a^u + \beta R^{-1}(\lambda)] d\lambda]$$
$$\Re(\tilde{\mathrm{a}}) = \frac{1}{2}\left[a^l + a^u - \frac{4}{5}\alpha + \frac{2}{3}\beta\right] \quad \dots (3)$$
Where $\alpha = \beta$ or $\alpha \neq \beta$.

5- Fuzzy linear programming:

The crisp linear programming problem defined as follows:

$$Max\ orMin\ z = \sum_{j=1}^{n} c_j x_j$$
$$s.to:$$
$$\sum_{j=1}^{n} a_{ij} x_j \leq b_i \quad , i = 1,2, \dots, m \qquad \dots . (4)$$
$$x_j \geq 0$$
Where $c_j \in R^n, b_i \in R^m, a_{ij} \in R^{n\times m}$.
The parameters in the above model are crisp. But if some or all parameters are fuzzy number then the crisp became fuzzy linear programming model.

In this paper we study three states of fuzzy linear programming problem. The first state are fuzzy numbers in the objective function coefficients, the second state are fuzzy numbers in the right-hand side coefficients, the third state are fuzzy numbers for both in the objective function coefficients and the right-hand side coefficients.
Now, we formulated the above three fuzzy linear programming problem states, as follows:

First state: in this state, we make the objective function coefficients as trapezoidal fuzzy numbers which is as follows:

$$Max\ \ z = \sum_{j=1}^{n} \tilde{c_j} x_j$$
$$s.to:$$
$$\sum_{j=1}^{n} a_{ij} x_j \leq b_i \quad , i = 1,2,\dots,m \qquad \dots.(5)$$
$$x_j \geq 0$$

Where c_j:fuzzy coefficients of objective function, $b_i \in R^m, a_{ij} \in R^{n\times m}$. then we solve the fuzzy linear programming by using two ranking function:
a- the linear programming problem when using Maleki ranking function:

$$Max\ \ z = \sum_{j=1}^{n} [c_j{}^l + c_j{}^u + \frac{1}{2}(\beta - \alpha)] x_j$$
$$s.to:$$
$$\sum_{j=1}^{n} a_{ij} x_j \leq b_i \quad , i = 1,2,\dots,m \qquad \dots.(6)$$
$$x_j \geq 0$$

b- the linear programming problem when using Yager ranking function:

$$Max\ \ z = \sum_{j=1}^{n} \frac{1}{2}\left[c_j{}^l + c_j{}^u - \frac{4}{5}\alpha + \frac{2}{3}\beta\right] x_j$$
$$s.to:$$
$$\sum_{j=1}^{n} a_{ij} x_j \leq b_i \quad , i = 1,2,\dots,m \qquad \dots.(7)$$
$$x_j \geq 0$$

Second state: in this state, we make the right-hand side coefficients as trapezoidal fuzzy numbers which is as follows:

$$Max\ \ z = \sum_{j=1}^{n} c_j x_j$$
$$s.to:$$
$$\sum_{j=1}^{n} a_{ij} x_j \leq \tilde{b_i} \quad , i = 1,2,\dots,m \qquad \dots.(8)$$
$$x_j \geq 0$$

Where b_i:fuzzy right-hand side coefficient, $c_j \in R^n, a_{ij} \in R^{n\times m}$. then we solve the fuzzy linear programming by using two ranking function:
a- the linear programming problem when using Maleki ranking function:

$$Max\ \ z = \sum_{j=1}^{n} c_j x_j$$
$$s.to:$$
$$\sum_{j=1}^{n} a_{ij} x_j \leq [b_i{}^l + b_i{}^u + \frac{1}{2}(\beta - \alpha)] \quad , i = 1,2,\dots,m \qquad \dots.(9)$$
$$x_j \geq 0$$

b- the linear programming problem when using Yager ranking function:

$$Max\ z = \sum_{j=1}^{n} c_j x_j$$
$$s.to:$$
$$\sum_{j=1}^{n} a_{ij} x_j \leq \frac{1}{2}\left[b_i{}^l + b_i{}^u - \frac{4}{5}\alpha + \frac{2}{3}\beta\right] \quad ,i = 1,2,\dots,m \qquad \dots.(10)$$
$$x_j \geq 0$$

Third state: in this state, we make both the objective function coefficients and right-hand side coefficients as trapezoidal fuzzy numbers which is as follows:

$$Max\ \ z = \sum_{j=1}^{n} \widetilde{c_j} x_j$$
$$s.to:$$
$$\sum_{j=1}^{n} a_{ij} x_j \leq \widetilde{b_\iota} \quad ,i = 1,2,\dots,m \qquad \dots.(11)$$
$$x_j \geq 0$$

Where $b_i\ and\ c_j$ are trapezoidal fuzzy numbers, $a_{ij} \in R^{n\times m}$. then we solve the fuzzy linear programming by using two ranking function:

a- The linear programming problem when using Maleki ranking function:

$$Max\ \ z = \sum_{j=1}^{n} [c_j{}^l + c_j{}^u + \frac{1}{2}(\beta - \alpha)] x_j$$
$$s.to:$$
$$\sum_{j=1}^{n} a_{ij} x_j \leq [b_i{}^l + b_i{}^u + \frac{1}{2}(\beta - \alpha)] \quad ,i = 1,2,\dots,m \qquad \dots.(12)$$
$$x_j \geq 0$$

b- The linear programming problem when using Yager ranking function:

$$Max\ \ z = \sum_{j=1}^{n} \frac{1}{2}\left[c_j{}^l + c_j{}^u - \frac{4}{5}\alpha + \frac{2}{3}\beta\right] x_j$$
$$s.to:$$
$$\sum_{j=1}^{n} a_{ij} x_j \leq \frac{1}{2}\left[b_i{}^l + b_i{}^u - \frac{4}{5}\alpha + \frac{2}{3}\beta\right] \quad ,i = 1,2,\dots,m \qquad \dots.(13)$$
$$x_j \geq 0$$

6- Numerical examples:

In this section we explain all states by the following examples which is suggest by the researchers in case of$\alpha = \beta$ or $\alpha \neq \beta$.

Example -1- the following example is suggested by the researchers in case$\alpha = \beta$.

a-

$$Max\ \tilde{z} = (4,8,2,2)\widetilde{x_1} + (1,3,1,1)\widetilde{x_2} + (1,5,2,2)\widetilde{x_3}$$
$$s.to:$$
$$2\widetilde{x_1} - \widetilde{x_2} + 2\widetilde{x_3} \leq 4$$
$$\widetilde{x_1} + 4\widetilde{x_3} \leq 4 \qquad \dots\ (14)$$
$$\widetilde{x_1} + 3\widetilde{x_2} + 2\widetilde{x_3} \leq 7$$
$$\widetilde{x_1}, \widetilde{x_2}, \widetilde{x_3} \geq 0$$

Solution: by using Maleki ranking function we get

$$Max\ z = 12x_1 + 4x_2 + 6x_3$$
$$s.to:$$
$$2x_1 - x_2 + 2x_3 \leq 4$$
$$x_1 + 4x_3 \leq 4 \qquad \dots(15)$$
$$x_1 + 3x_2 + 2x_3 \leq 7$$

$$x_1, x_2, x_3 \geq 0$$

We solve the above crisp linear programming by using simplex method through utilizing win.QSB program, we get the following solution.

$x_1 = 2.71\ , x_2 = 1.43, x_3 = 0, z = 38.3.$

By using Yager ranking function we get

$$\begin{aligned} &Max\ z = 5.86x_1 + 1.93x_2 + 2.867x_3 \\ &s.to: \\ &2x_1 - x_2 + 2x_3 \leq 4 \\ &x_1 + 4x_3 \leq 4 \qquad \text{... (16)} \\ &x_1 + 3x_2 + 2x_3 \leq 7 \\ &x_1, x_2, x_3 \geq 0 \end{aligned}$$

Solving the crisp linear programming (16) by using simplex method through utilizing win.QSB program, we get the following solution.

$x_1 = 2.7143\ , x_2 = 1.428, x_3 = 0, z = 18.681.$

b-

$$\begin{aligned} &Max\ z = 6x_1 + 2x_2 + 3x_3 \\ &s.to: \\ &2x_1 - x_2 + 2x_3 \leq (1,7,3,3) \\ &x_1 + 4x_3 \leq (2,6,2,2) \qquad \text{... (17)} \\ &x_1 + 3x_2 + 2x_3 \leq (5,9,2,2) \\ &x_1, x_2, x_3 \geq 0 \end{aligned}$$

Solution: by using Maleki ranking function we get

$$\begin{aligned} &Max\ z = 6x_1 + 2x_2 + 3x_3 \\ &s.to: \\ &2x_1 - x_2 + 2x_3 \leq 8 \\ &x_1 + 4x_3 \leq 8 \qquad \text{... (18)} \\ &x_1 + 3x_2 + 2x_3 \leq 14 \\ &x_1, x_2, x_3 \geq 0 \end{aligned}$$

Then the solution for linear programming (18) is as follows

$x_1 = 5.43, x_2 = 2.86, x_3 = 0, z = 38.3.$

By using Yager ranking function we get:

$$\begin{aligned} &Max\ z = 6x_1 + 2x_2 + 3x_3 \\ &s.to: \\ &2x_1 - x_2 + 2x_3 \leq 3.8 \\ &x_1 + 4x_3 \leq 3.866 \qquad \text{... (19)} \\ &x_1 + 3x_2 + 2x_3 \leq 6.867 \\ &x_1, x_2, x_3 \geq 0 \end{aligned}$$

Then the solution for linear programming (19) is as follows$x_1 = 2.609, x_2 = 1.419, x_3 = 0, z = 18.495.$

C-

$$\begin{aligned} &Max\ \tilde{z} = (4,8,2,2)\widetilde{x_1} + (1,3,1,1)\widetilde{x_2} + (1,5,2,2)\widetilde{x_3} \\ &s.to: \\ &2\widetilde{x_1} - \widetilde{x_2} + 2\widetilde{x_3} \leq (1,7,3,3) \\ &\widetilde{x_1} + 4\widetilde{x_3} \leq (2,6,2,2) \qquad \text{... (20)} \\ &\widetilde{x_1} + 3\widetilde{x_2} + 2\widetilde{x_3} \leq (5,9,2,2) \\ &\widetilde{x_1}, \widetilde{x_2}, \widetilde{x_3} \geq 0 \end{aligned}$$

Solution: by using Maleki ranking function we get:

$$\begin{aligned} &Max\ z = 12x_1 + 4x_2 + 6x_3 \\ &s.to: \\ &2x_1 - x_2 + 2x_3 \leq 8 \\ &x_1 + 4x_3 \leq 8 \qquad \text{... (21)} \\ &x_1 + 3x_2 + 2x_3 \leq 14 \\ &x_1, x_2, x_3 \geq 0 \end{aligned}$$

Then the solution for linear programming (21) is as follows:

$x_1 = 5.428, x_2 = 2.857, x_3 = 0, z = 76.57.$

By using Yager ranking function we get:

$$Max\ z = 5.86x_1 + 1.93x_2 + 2.867x_3$$
$$s.to:$$
$$2x_1 - x_2 + 2x_3 \le 3.8$$
$$x_1 + 4x_3 \le 3.866 \qquad \text{... (22)}$$
$$x_1 + 3x_2 + 2x_3 \le 6.867$$
$$x_1, x_2, x_3 \ge 0$$

Then the solution for the above linear programming (22) is: $x_1 = 2.609, x_2 = 1.419, x_3 = 0, z = 18.049$.

Example -2- the following example is suggested by the researchers in case $\alpha \ne \beta$.

a-

$$Max\ \tilde{z} = (4,8,3,1)\widetilde{x_1} + (1,3,1,1)\widetilde{x_2} + (1,5,3,1)\widetilde{x_3}$$
$$s.to:$$
$$2\widetilde{x_1} - \widetilde{x_2} + 2\widetilde{x_3} \le 4$$
$$\widetilde{x_1} + 4\widetilde{x_3} \le 4 \qquad \text{... (23)}$$
$$\widetilde{x_1} + 3\widetilde{x_2} + 2\widetilde{x_3} \le 7$$
$$\widetilde{x_1}, \widetilde{x_2}, \widetilde{x_3} \ge 0$$

Solution: by using Maleki ranking function we get

$$Max\ z = 11x_1 + 4x_2 + 5x_3$$
$$s.to:$$
$$2x_1 - x_2 + 2x_3 \le 4$$
$$x_1 + 4x_3 \le 4 \qquad \text{.... (24)}$$
$$x_1 + 3x_2 + 2x_3 \le 7$$
$$x_1, x_2, x_3 \ge 0$$

We solve the above crisp linear programming by using simplex method through utilizing win.QSB program, we get the following solution.

$x_1 = 2.714$, $x_2 = 1.428, x_3 = 0, z = 35.571$

By using Yager ranking function we get

$$Max\ z = 5.13x_1 + 1.93x_2 + 2.13x_3$$
$$s.to:$$
$$2x_1 - x_2 + 2x_3 \le 4$$
$$x_1 + 4x_3 \le 4 \qquad \text{... (25)}$$
$$x_1 + 3x_2 + 2x_3 \le 7$$
$$x_1, x_2, x_3 \ge 0$$

Solving the crisp linear programming (25) by using simplex method through utilizing win.QSB program, we get the following solution.

$x_1 = 2.714$, $x_2 = 1.428, x_3 = 0, z = 16.681$.

b-

$$Max\ z = 6x_1 + 2x_2 + 3x_3$$
$$s.to:$$
$$2x_1 - x_2 + 2x_3 \le (1,7,4,2)$$
$$x_1 + 4x_3 \le (2,6,1,3) \qquad \text{... (26)}$$
$$x_1 + 3x_2 + 2x_3 \le (5,9,1,3)$$
$$x_1, x_2, x_3 \ge 0$$

Solution: by using Maleki ranking function we get

$$Max\ z = 6x_1 + 2x_2 + 3x_3$$
$$s.to:$$
$$2x_1 - x_2 + 2x_3 \le 7$$
$$x_1 + 4x_3 \le 9 \qquad \text{... (27)}$$
$$x_1 + 3x_2 + 2x_3 \le 15$$
$$x_1, x_2, x_3 \ge 0$$

Then the solution for linear programming (27) is as follows $x_1 = 5.143, x_2 = 3.286, x_3 = 0, z = 37.429$.
By using Yager ranking function we get:

$$Max\ z = 6x_1 + 2x_2 + 3x_3$$
$$s.to:$$
$$2x_1 - x_2 + 2x_3 \le 3.067$$

$$x_1 + 4x_3 \le 4.6 \quad \ldots (28)$$
$$x_1 + 3x_2 + 2x_3 \le 7.6$$
$$x_1, x_2, x_3 \ge 0$$

Then the solution for linear programming (28) is as follows$x_1 = 2.4001, x_2 = 1.733, x_3 = 0, z = 17.867$.

c-

$$Max\ \tilde{z} = (4,8,3,1)\widetilde{x_1} + (1,3,1,1)\widetilde{x_2} + (1,5,3,1)\widetilde{x_3}$$
$$s.to:$$
$$2x_1 - x_2 + 2x_3 \le (1,7,4,2)$$
$$x_1 + 4x_3 \le (2,6,1,3) \quad \ldots (29)$$
$$x_1 + 3x_2 + 2x_3 \le (5,9,1,3)$$
$$x_1, x_2, x_3 \ge 0$$

Solution: by using Maleki ranking function we get

$$Max\ z = 11x_1 + 4x_2 + 5x_3$$
$$s.to:$$
$$2x_1 - x_2 + 2x_3 \le 7$$
$$x_1 + 4x_3 \le 9 \quad \ldots (30)$$
$$x_1 + 3x_2 + 2x_3 \le 15$$
$$x_1, x_2, x_3 \ge 0$$

Then the solution for linear programming (30) is as follows$x_1 = 5.143, x_2 = 3.286, x_3 = 0, z = 69.714$.

By using Yager ranking function we get:

$$Max\ z = 5.13x_1 + 1.93x_2 + 2.13x_3$$
$$s.to:$$
$$2x_1 - x_2 + 2x_3 \le 3.067$$
$$x_1 + 4x_3 \le 4.6 \quad \ldots (31)$$
$$x_1 + 3x_2 + 2x_3 \le 7.6$$
$$x_1, x_2, x_3 \ge 0$$

Then the solution for linear programming (31) is as follows$x_1 = 2.4001, x_2 = 1.733, x_3 = 0, z = 15.658$.

7- conclusions:

The coefficients of objective function and the right-hand side with fuzzy numbers are ranked with two special ranking functions for Maleki and Yager linear ranking function. For all twelve states which studying in the paper, noting that $x_3 = 0$, but x_1 and x_2 are not. for the three states with $(\alpha = \beta)$ and $(\alpha \ne \beta)$, we prefer the following states:

1- When the objective function coefficients are fuzzy numbers
$x_1 = 2.71\ , x_2 = 1.43, x_3 = 0, z = 38.3$.

2- When the right-hand side coefficients are fuzzy numbers
$x_1 = 5.43, x_2 = 2.86, x_3 = 0, z = 38.3$.

3- When the objective function coefficients and the right-hand side coefficients are fuzzy numbers
$x_1 = 5.428, x_2 = 2.857, x_3 = 0, z = 76.57$.

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