Journal of Aircraft, Vol. 47, No. 4, July-August 2010, pp.1216-1228. Also presented at the *2009 Guidance, Navigation, and Control Conference*, August 10-13, Chicago, IL.

Rapid Estimation of Impaired Aircraft Aerodynamic Parameters

Jinwhan Kim*, Karthik Palaniappan†, and P. K. Menon‡
Optimal Synthesis Inc., Los Altos, CA 94022

Estimation of aerodynamic models of impaired aircraft using an innovative differential vortex lattice method tightly coupled with extended Kalman filters is discussed. The approach exploits prior knowledge about the undamaged aircraft to reduce the order of the estimation problem. Three different extended Kalman filter formulations are given, together with a comparative analysis. An approach for designing test maneuvers to improve the observability of the system dynamics is also discussed. Algorithms given in this paper can be used as the basis for online derivation of aircraft performance model, which can then form the basis for designing safe landing guidance laws for damaged aircraft.

I. Introduction

ADaptive control of impaired aircraft is being investigated at NASA and other aerospace research laboratories in the US [1,2]. The focus of these research efforts has been in maintaining control over the attitude dynamics of the impaired aircraft. Assuming that the aircraft remains controllable at its current flight conditions, it is important to be able to predict its performance at other flight conditions in order to derive maneuver constraints that should be enforced to ensure safe transition of the aircraft to landing configuration. The objective of the research discussed in this paper is to develop estimation schemes for rapidly extracting the aerodynamic parameters of d impaired aircraft to enable the assessment of aircraft performance. The performance data of interest include flight envelope boundaries and maneuver limits. This data can form the basis for the design of safe landing guidance laws.

Several innovative concepts have been advanced in this paper. Firstly, a rapid approach for deriving aerodynamic models of impaired aircraft termed as the *Differential Vortex Lattice Method* (DVLM) was developed. This approach recasts the well known Vortex Lattice Method (VLM)[3] to reduce the dimension of the aerodynamic problem. The DVLM formulation exploits prior knowledge about the airframe to create a low-order computational methodology for relating the changes in the vehicle geometry due to damage to its aerodynamic parameters. This low-order method can be implemented in real-time onboard for the aircraft to provide estimates of the aerodynamic parameters for use in the computation of flight envelope and maneuver limits, and for adaptive guidance law synthesis. Approaches for estimating the maneuver limits and structural dynamic characteristics are also outlined.

Secondly, the Extended Kalman filtering (EKF) approach[4-6] is employed for online estimation of impaired aircraft parameters based on the DVLM. Design of maneuvers for enhancing the observability of the impaired aircraft model parameters is also discussed. The model parameters derived from the estimator can be used for computing the flight envelope and the maneuver limits. These can then be used in the synthesis of safe guidance laws for landing the aircraft.

Unlike the airframe stabilization problem, the guidance task is almost entirely based on predictive information about the aircraft dynamics. For instance, landing guidance requires the aircraft to slow down to the approach speeds while descending to the correct altitude at a specified heading. Since impaired aircraft may have a high drag and lower stall angle of attack, the aircraft energy has to be carefully managed to ensure that adequate lift is maintained until flare altitude and touchdown. This will require energy conservative maneuvers and descent strategies. Since impaired aircraft may not be able to employ all its high-lift devices, its speed must be carefully managed to avoid premature loss of lift. These factors make it important to derive a reasonably accurate performance model of the aircraft for the design of a viable guidance system. It may be noted that although most inner-loop flight control systems operate well within the limits of controllability most of the time, the guidance task often involves operating near the edges of the operational envelope.

---

* Research Scientist, 95 First Street, Suite 240, Senor Member AIAA.
† Research Scientist Presently with Amoeba Technologies Inc.
‡ Chief Scientist, 95 First Street, Suite 240, Associate Fellow AIAA.

This motivates the use of the indirect adaptive control framework[7] for the guidance problem, wherein an estimation procedure is used to find the parameters of the model, which then forms the basis for the design of guidance commands. Due to the use of predictive information available in the estimated model, this problem must be formulated carefully to ensure that the model closely approximates the expected dynamics of the impaired aircraft. This is because of the fact that due to their dependence on predictive information, guidance systems are much more susceptible to modeling uncertainties.

The present research assumes that the approximate location and extent of the damage on the airframe are known, perhaps from electro-optical sensors onboard the aircraft. Large modern aircraft may incorporate several cameras in the airframe, allowing rapid detection of any airframe anomalies.

Section II presents the equations of motion and the associated filter formulation. The differential vortex lattice method (DVLM) is described in Section III. The DVLM-based nonlinear filter formulations with various parameterization methods are given in Section II. The performance validation based on numerical simulations is presented in Section V, and the conclusions from the present research are given in Section VI.

II. Equations of Motion and Filter Formulation

The proposed approach for online estimation of impaired aircraft performance uses a nonlinear online estimation algorithm in conjunction with the Differential Vortex Lattice Method (DVLM). This paper focuses on investigating the feasibility of the proposed estimation algorithm. The 3D point-mass model is used for describing the motions of aircraft by assuming that the kinematic states of the aircraft are stabilized by the inner-loop controller. In this section, the equations of motion and the associated filter formulation are presented. Since the system dynamics is nonlinear, an Extended Kalman Filtering (EKF) algorithm is used in the filter formulation.

A. Point-Mass Dynamics of the Aircraft

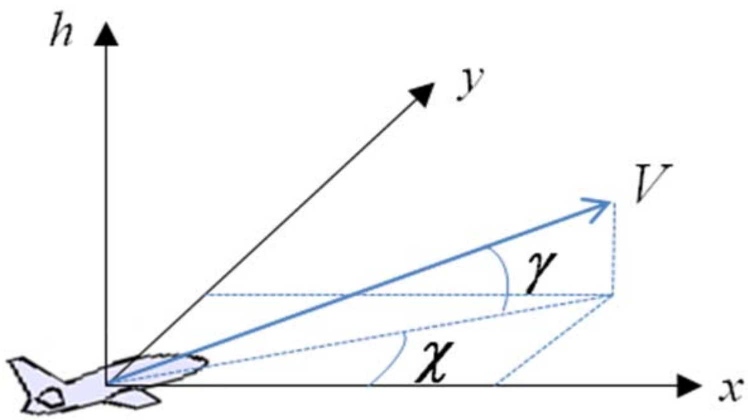

Figure 1. Coordinate system

The 3-D point-mass dynamics of an aircraft is used to describe the motion of aircraft in 3D configuration space. This model assumes that the control variables continuously maintain the aircraft moment equilibrium, such that the aircraft can follow commanded angle of attack α, angle of side slip β, and/or the bank angle ϕ. Based on the coordinate system shown in Figure 1, the equation of motion can be expressed in the form:

$$\dot{x} = \begin{bmatrix} \dot{V} \\ \dot{\gamma} \\ \dot{\chi} \\ \dot{x} \\ \dot{y} \\ \dot{h} \end{bmatrix} = \begin{bmatrix} (\eta T - D)/m - g \sin\gamma \\ (L\cos\phi - S\sin\phi)/(mV) - g\cos\gamma / V \\ (L\sin\phi - S\cos\phi)/(mV\cos\gamma) \\ V\cos\gamma\sin\chi \\ V\cos\gamma\cos\chi \\ V\sin\gamma \end{bmatrix} \quad (1)$$

In Equation (1), L is the lift, D is the drag, S the side force and T is the maximum thrust. The maximum thrust, T, is assumed to be constant. The actual thrust varies linearly with respect to the throttle setting η. Table 1 defines all the symbols used in Equation (1).

Table 1. Symbols used in the Equations of Motion

V	Velocity	Φ	Bank angle
γ	Flight angle	T	Thrust force
χ	Side-slip angle	L	Lift force
x	North	D	Drag force
y	East	S	Side force
h	Altitude	M	Mass
η	Throttle	g	Gravity acc.

B. Computing the Forces on a Impaired Aircraft using DVLM

Under normal operating conditions, the lift, drag, and side force in Equation (1) can be expressed as functions of aircraft geometry and aerodynamic parameters such as aircraft speed, angle of attack, and the angle of sideslip.

If physical damage occurs to the airframe, both the aircraft geometry and the aerodynamic parameters change, and thus the original relationships are no longer valid in describing the aerodynamic forces and the resulting motions. The Vortex Lattice Method (VLM) is capable of describing the aerodynamic forces based on the airframe geometry, and the distribution of circulation over discretized panels defining the geometric airframe can be calculated and integrated into forces at the given flight conditions, i.e.,

$$\begin{aligned} L &= L\left(\rho,\Gamma,A,V_\infty,\alpha,\beta\right) \\ D &= D\left(\rho,\Gamma,A,V_\infty,\alpha,\beta\right) \\ S &= S\left(\rho,\Gamma,A,V_\infty,\alpha,\beta\right) \end{aligned} \tag{2}$$

where ρ is the air density, Γ is the circulation strength distribution, A is the aircraft geometry, V_∞ is the airspeed, α is the angle of attack, and β is the angle of sideslip. Note that the circulation components are also functions of the aircraft geometry and the flight conditions. However, for the conventional VLM, the dimension of Γ is equal to the number of panels approximating the airframe, normally a large number. Designing online recursive estimators based on the VLM is unrealistic due to its high dimension.

The Differential Vortex Lattice Method (DVLM) proposed in this paper allows the calculation of the forces on the impaired aircraft using differential circulation strength components in the vicinity of the impaired section, involving a much smaller set of circulation strengths when compared with the conventional VLM approach. The associated reduction in the dimension allows the DVLM to be used as the basis for the derivation of recursive estimators.

C. EKF-based State-Parameter Estimation

The parameter estimation problem is often called the dual estimation problem since it requires the simultaneous estimation of the system states and the unknown parameters. In general, these problems involve system nonlinearities including nonlinear coupling between the states and the parameters. The extended Kalman filter (EKF) is most widely and commonly used for the formulation of nonlinear dual estimators. The implementation procedure of the EKF proceeds as follows:

For a given nonlinear system with unknown system parameters,

$$\dot{x} = f(x,\theta) + n_w \tag{3}$$

with x being the system state vector, θ the parameter vector to be estimated, and $f(\cdot)$ is the nonlinear function of states and parameters. The vector n_w is the process noise which is assumed to be Gaussian in the EKF development.

The dynamic model of the parameters θ is chosen based on any prior knowledge about its temporal behavior. The simplest model assumption is that θ is piecewise constant or $\dot{\theta} = 0$.

The augmented system dynamics can then be written as:

$$x^a = f^a(x,\theta) = \begin{bmatrix} \dot{x} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} f(x,\theta) \\ 0 \end{bmatrix} + n_w^a \tag{4}$$

Here, x^a is the augmented system state vector.

Although θ is assumed to be piecewise constant, this model allows moderate time variations in the parameters through the artificial process noise vector n_w^a. The remaining filter design procedure is the same as that of the standard EKF implementation procedure outlined in Table 2.

Table 2. Extended Kalman Filter [4]

System model	$\dot{x} = f(x) + w$	$w \sim N(0,\ Q)$
Measurement model	$z_k = h_k(x_k) + v_k$	$v_k \sim N(0,\ R_k)$
Time propagation	$\dot{\hat{x}} = f(\hat{x})$ $\dot{P} = FP + PF^T + Q$	$F = \left[\frac{\partial f}{\partial x}\right]_{x=\hat{x}}$
Measurement update	$\hat{x}_k^+ = \hat{x}_k^- + L_k\left(z_k - h_k(x_k)\right)$ $P_k^+ = (I - L_k H_k) P_k^-$	$H_k = \left[\frac{\partial h_k}{\partial x}\right]_{x=\hat{x}_k^-}$ $L_k = P_k^- H_k^T \left(H_k P_k^- H_k^T + R_k\right)^{-1}$

For the present state-parameter estimation problem, the measurements, assumed to be available for the estimation process, are the aircraft position, velocity and acceleration components.

$$z = \begin{bmatrix} V & \gamma & \chi & x & y & h & \dot{V} & \dot{\gamma} & \dot{\chi} \end{bmatrix}^T + n_v^T \tag{5}$$

where n_v is the measurement noise vector.

III. Differential Vortex Lattice Method

A. Equations of Motion

The vortex lattice method (VLM) is based on inviscid, incompressible, steady and irrotational flow assumptions, and has proven to be highly effective for determining aerodynamic characteristics of the complete aircraft configurations[8-10]. Once the aerodynamic model has been determined, it can be used to predict the performance characteristics of the aircraft.

The first stage in the implementation of the vortex lattice method is the discretization of the vehicle geometry into vortex panels, as shown in Figure 2. The entire wing surface is divided into a number of vortex panels laid out in a lattice like structure. A horseshoe vortex with Circulation Γ is placed at the quarter-chord location of each panel. In the classical implementation of the vortex lattice method, the trailing vortices are assumed to extend to infinity. The wake is assumed to be flat. The control points are assumed to lie at the three quarter point of each panel.

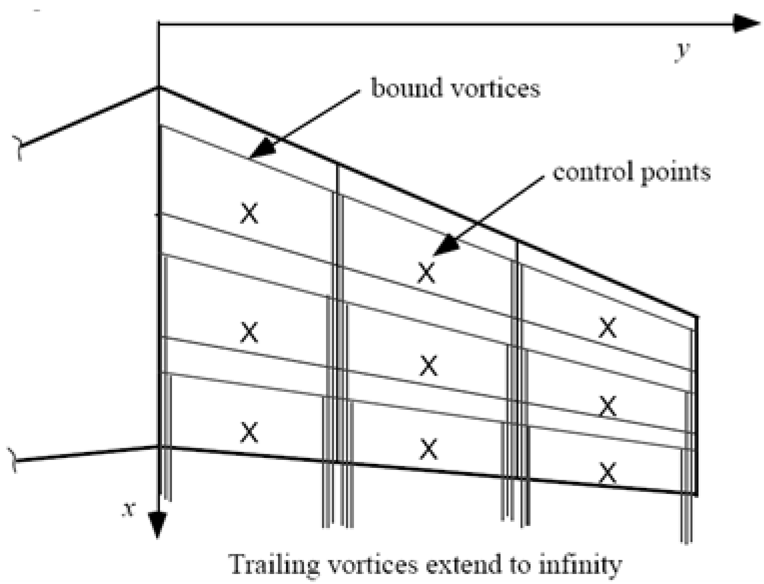

Figure 2. Discretization of a wing surface into vortex panels

The normal velocity at the control point of the m^{th} panel due to the horseshoe vortex at the n^{th} panel is given by

$$V_m = C_{m,n}\Gamma_n \tag{6}$$

Thus the normal velocity at the m^{th} control point due to all the vortices on the wing surface can be written as

$$V_m = \sum_1^N C_{m,n}\Gamma_n \tag{7}$$

The standard boundary condition imposed at any one of these control points ensures that the total normal velocity at any one of these control points is zero (i.e. no penetration boundary condition). This is enforced as follows:

$$V_m + V_\infty sin(\alpha_m + \theta_m) = \sum_1^N C_{m,n}\Gamma_n + V_\infty sin(\alpha_m + \theta_m) = 0 \tag{8}$$

Here, α_m and θ_m are the angle of attack and the local wing twist at the m^{th} control point. The parameters $C_{m,n}$ depend on the geometry of the wing, and can be pre-computed. The above equation can then be written in matrix form as follows:

$$A\Gamma = b \tag{9}$$

Here, the matrix A depends only on the geometry of the wing, and can be pre-computed. The vector of boundary conditions b depends on the free-stream velocity V_∞ , the local angle of attack α_m and the local wing twist θ_m. Γ is the vector of circulation strengths of the horseshoe vortices.

Once the circulation strengths Γ are known, the total velocities $\vec{V}$ at the locations of the control points can be calculated as follows

$$\vec{V}_m = \vec{V}_\infty + \sum_1^N \vec{D}_{m,n}\Gamma_n \tag{10}$$

Bernoulli's equation can then be used to estimate the pressure at the control point locations as:

$$P_m = P_\infty + \frac{1}{2}\rho\left(V_\infty^2 - V_m^2\right) = P_m\left(\Gamma_i\right) \tag{11}$$

The forces along the x, y and z axes are then given respectively by

Journal of Aircraft, Vol. 47, No. 4, July-August 2010, pp.1216-1228. Also presented at the *2009 Guidance, Navigation, and Control Conference*, August 10-13, Chicago, IL.

$$F_x = \sum_1^N P_n(\Gamma_i) dA_{n,x} \qquad F_y = \sum_1^N P_n(\Gamma_i) dA_{n,y} \qquad F_z = \sum_1^N P_n(\Gamma_i) dA_{n,z} \tag{12}$$

B. The Differential Vortex Lattice Method (DVLM)

The differential vortex lattice model investigated under the present research significantly reduces the dimension of the problem by exploiting the knowledge about the circulation over the aircraft structure before the damage. The DVLM problem is formulated in terms of computing the changes in the circulation on the panels in the neighborhood of the damage.

Let the circulation distribution over an unimpaired wing be given by

$$A\Gamma = b \tag{13}$$

Here, Γ is the array containing the circulation strengths in the different panels, arranged in some pre-determined manner. The element a_{ij} of A represents the aerodynamic influence of the j^{th} panel on the i^{th} panel. It should be noted that

$$a_{ij} \propto \frac{1}{r_{ij}^2} \tag{14}$$

When a portion of the airframe is impaired, the circulation strength at that location can be assumed to go to zero. At this point, it is useful to note that the influence of the impaired panels is felt strongly only at the neighboring panels and can be considered insignificant sufficiently far away.

It is important to get a quantitative idea of the decay of the influence of the impaired panel. Consider a wing of span s, divided into n panels of the same size, span-wise. For the purposes of illustration, consider only one panel chord-wise. The size of each panel is then given by s/N. The distance between the control point of a panel and its periphery, where the bound vortex is assumed to be, is given by half this distance $s/(2N)$. The influence on a panel due to its own circulation is then given by

$$a_{i,i} \propto \frac{4N^2}{s^2} \tag{15}$$

The influence on a panel due to another panel, n panels away, is given by

$$a_{i,i+n} \propto \frac{N^2}{n^2 s^2} \tag{16}$$

The ratio of the influences is then given by

$$\frac{a_{i,i+n}}{a_{i,i}} = \frac{1}{4n^2} \tag{17}$$

It can be seen that the above equation is independent of the span s, and the total number of panels chosen N. If n is 3, then the ratio of influences is 2.77 percent, which can be considered quite small. Thus, if an error of 3 percent can be tolerated, it is sufficient to study flow over three panels adjacent to the impaired area on all sides. This has the advantage of reducing the area of computation of circulation strengths to 3 neighboring panels. Thus the elements of Γ_d are the same as those of Γ, except at locations 3 panels away in each direction from the location of damage.

Thus the equation for circulation can be rewritten as

$$\begin{bmatrix} A_{n1} & 0 & A_{n2} \\ 0 & 0 & 0 \\ A_{d1} & 0 & A_{d2} \end{bmatrix} \begin{bmatrix} \Gamma_n \\ 0 \\ \Gamma_d + \delta\Gamma \end{bmatrix} = \begin{bmatrix} b_n \\ 0 \\ b_d \end{bmatrix} \tag{18}$$

where the subscript n represents unimpaired values and d represents impaired values. The vector $\delta\Gamma$ represents the change in circulation between impaired and unimpaired values. The expression for $\delta\Gamma$ then becomes:

Journal of Aircraft, Vol. 47, No. 4, July-August 2010, pp.1216-1228. Also presented at the *2009 Guidance, Navigation, and Control Conference*, August 10-13, Chicago, IL.

$$\delta\Gamma = A_{d2}^{-1}\left(b_d - A_{d1}\Gamma_n\right) - \Gamma_d \tag{19}$$

This is the fundamental hypothesis of the Differential Vortex Lattice Method (DVLM). The process of formulating the differential circulation is illustrated in Figure 3.

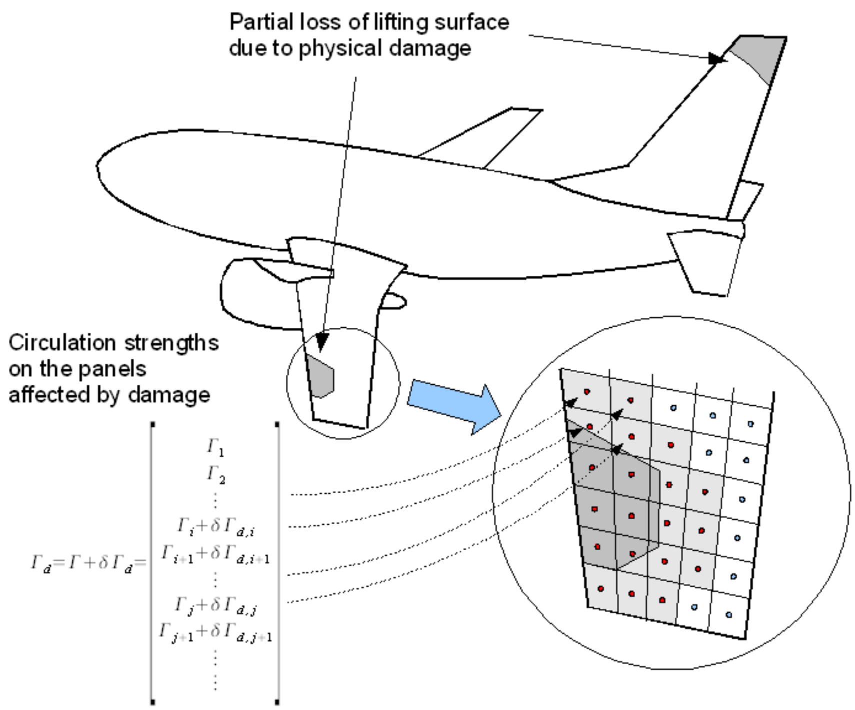

Figure 3. Modeling airframe damage using Differential Vortex Lattice Method

The corrected circulation vector can be used to evaluate the pressure distribution of the impaired airframe. The pressure distribution can be integrated to produce aerodynamic forces. The aerodynamic forces can then be normalized with respect to dynamic pressure and reference area to yield drag, lift and side force coefficients. The Differential Vortex Lattice Method is summarized in Figure 4.

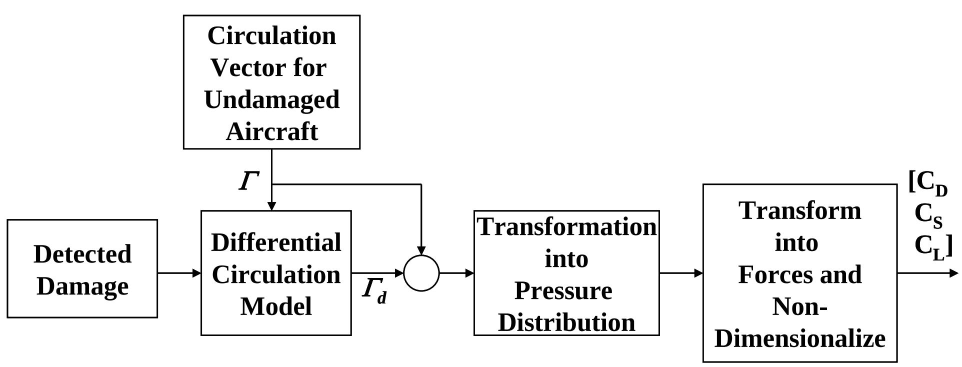

Figure 4. A Summary of the Differential Vortex Lattice Method

IV. DVLM-based EKF Formulations

This section will discuss two DVLM-based EKF formulations with different sets of parameter states. These are:

- *Circulation Strength Estimation* : Circulation strength distributions are introduced as the system parameters to be estimated. Two variations of this estimation problem will be discussed.

- *Parameterized Damage Estimation* : The extent of damage is directly estimated by an appropriate parameterization of the damage.

Details of these two approaches are presented in the following subsections.

A. Formulation 1: Direct Estimation of the Differential Circulation Vector

The filter is initialized with the pre-computed unimpaired aircraft circulation strength vector and the measured state vector. Given the aircraft position, velocity and acceleration vectors derived from GPS/INS system and air data sensors onboard, the EKF will generate estimates of the differential circulation vector and the resulting aerodynamic coefficients. The overall structure of the proposed estimator, together with a potential closed-loop guidance system, is given in Figure 5.

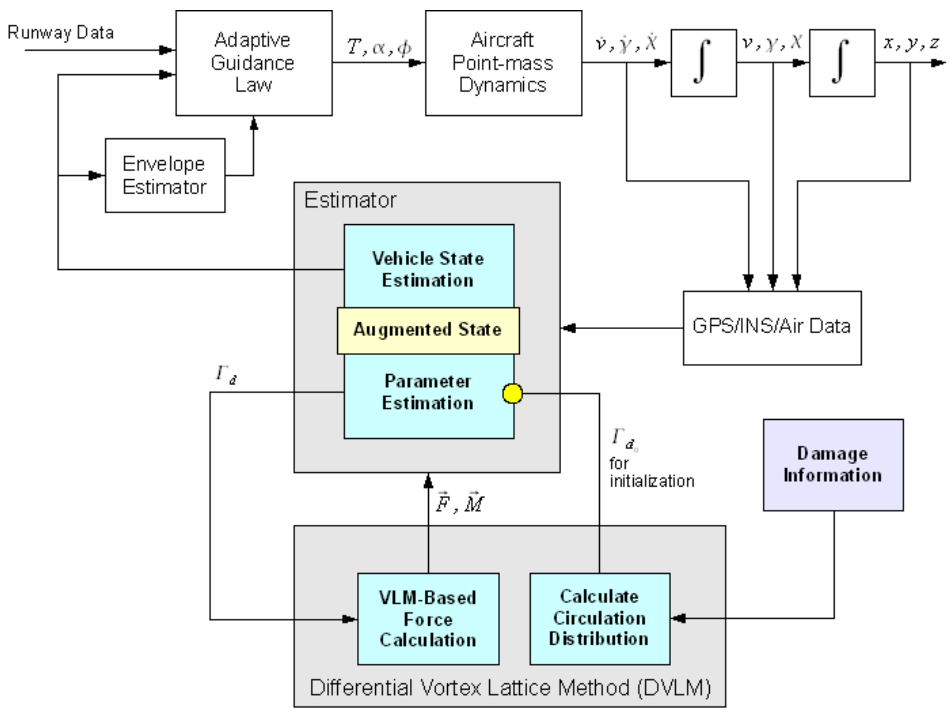

Figure 5. Filter structure for circulation strength estimation

The augmented system dynamics for the state-parameter estimation problem can be written as follows:

$$\dot{x}^a = \begin{bmatrix} \dot{V} \\ \dot{\gamma} \\ \dot{\chi} \\ \dot{x} \\ \dot{y} \\ \dot{h} \\ \delta\dot{\Gamma}_1 \\ \delta\dot{\Gamma}_2 \\ \vdots \\ \delta\dot{\Gamma}_m \end{bmatrix} = \begin{bmatrix} (\eta T - D)/m - g\sin\gamma \\ (L\cos\varphi - S\sin\varphi)/(mV) - g\cos\gamma/V \\ (L\sin\varphi - S\cos\varphi)/(mV\cos\gamma) \\ V\cos\gamma\sin\chi \\ V\cos\gamma\cos\chi \\ V\sin\gamma \\ -\zeta_1\delta\Gamma_1 \\ -\zeta_2\delta\Gamma_2 \\ \vdots \\ -\zeta_m\delta\Gamma_m \end{bmatrix} + n_w^a \tag{20}$$

Here, n_w^a is the process noise vector.

The associated Jacobian matrix for the augmented system dynamics, F^a, can be expressed as:

$$F^a = \frac{\partial f^a}{\partial x^a} = \left[\begin{array}{c|c} A \in R^{6\times 6} & B \in R^{6\times m} \\ \hline 0 \in R^{m\times 6} & diag(\varsigma_i) \in R^{m\times m} \end{array} \right] \tag{21}$$

The partitioned matrix A involves the partial derivatives of force components with respect to the speed, L_V, D_V and S_V which can be calculated by numerical differentiation. The force sensitivities to the changes in circulation components, L_Γ, D_Γ and S_Γ involved in the partitioned matrix B can be evaluated inside the DVLM solver, which reduces the computational load. Two different perturbation models for the differential circulation strength estimator were formulated. Based on the magnitude of expected changes in the estimated differential circulation strengths, these are termed as the heavy and light perturbation models.

Heavy perturbation model

The state vector $\delta\Gamma_i$, is the perturbation term defined as

$$\Gamma_{d,i} = \Gamma_{u,i} + \delta\Gamma_i \tag{22}$$

where Γ_u is the circulation vector for the unimpaired case, and Γ_d is for the impaired case. Note that this formulation attempts to estimate the entire differential circulation strength vector. To achieve a better filter performance, first-order Markov processes are employed for the differential circulation states. A small leakage constant ζ is introduced to improve the condition of the estimation problem. If ζ is set to zero, $\delta\Gamma$ follows a standard Wiener process.

Light perturbation model

The second differential approach attempts to determine the changes in the differential circulation strength vector defined as:

$$\Gamma_{d,i} = \Gamma_{d,i}^{DVLM} + \delta\Gamma_i \tag{23}$$

In this case, the circulation strength vector for the impaired case is calculated using the DVLM, and the perturbation of this circulation strength vector is estimated using the EKF. This reduces the absolute magnitude of the perturbation by exploiting the DVLM more effectively, perhaps leading to a better estimation performance.

This procedure requires the DVLM solver to run at each time step, and is computationally more intensive than the heavy perturbation approach. However, the computational requirements of DVLM approach are modest, enabling its use in the EKF algorithm.

B. Formulation 2: Estimator based on Parameterized Description of the Damage

The second formulation is based on damage parameterization in which the parameter state represents the size or extent of damage. The filter directly estimates the actual size of the damage using sensor measurements, and the DVLM algorithm provides circulation strength solutions and the resulting forces on the airframe. This approach has several advantages over the circulation estimation approach given previously.

Firstly, the filter is robust to initial error and the uncertainty in the initial damage estimate determined from sensors, since the damage size is directly estimated by the filter. Secondly, the damage parameterization generally involves a much smaller number of parameter states when compared with the number of circulation states. This reduces the number of state variables to be estimated, improving the system's observability and estimator performance.

The major limitation of this damage estimation approach is that the DVLM module has to be run at each time step. Moreover, sufficient accuracy is required for the DVLM solver since its output is directly used for the subsequent force evaluation without being updated by sensor measurements. In order to improve the accuracy, the original DVLM solver is modified using the Gauss Seidel smoothing procedure that incurs a slight increase in computational cost.

However, various numerical experiments during the present research have revealed that the proposed approach can provide higher accuracy with improved computational efficiency. Figure 6 gives the structure of the estimator. Note that this is a slightly modified version of the estimator given in Figure 5. The filter employs the sensor measurements in conjunction with the forces computed from the DVLM solver to update the damage parameter state. As stated in Introduction, it is assumed that information required for parameterizing the damage is available from onboard electro-optical sensors.

Journal of Aircraft, Vol. 47, No. 4, July-August 2010, pp.1216-1228. Also presented at the *2009 Guidance, Navigation, and Control Conference*, August 10-13, Chicago, IL.

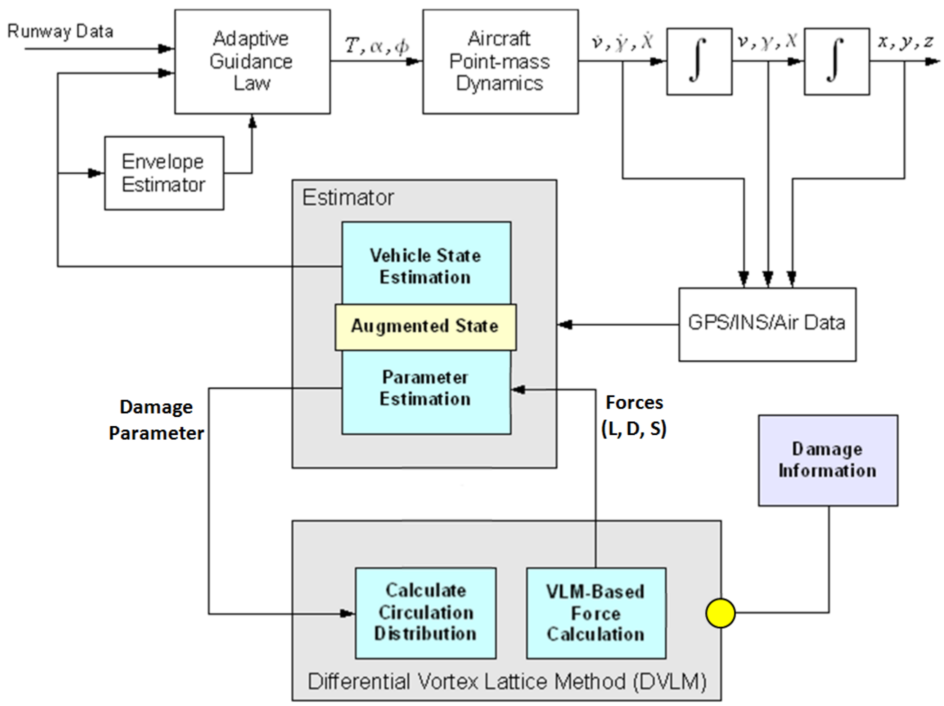

Figure 6. Filter structure for parameterized damage estimation

The augmented system state for state-parameter estimation can be expressed by

$$\dot{x}^a == \begin{bmatrix} \dot{V} \\ \dot{\gamma} \\ \dot{\chi} \\ \dot{x} \\ \dot{y} \\ \dot{h} \\ \dot{\theta}_1^d \\ \vdots \end{bmatrix} = \begin{bmatrix} (\eta T - D)/m - g\sin\gamma \\ (L\cos\phi - S\sin\phi)/(mV) - g\cos\gamma/V \\ (L\sin\phi - S\cos\phi)/(mV\cos\gamma) \\ V\cos\gamma\sin\chi \\ V\cos\gamma\cos\chi \\ V\sin\gamma \\ -\zeta_1\theta_1^d \\ \vdots \end{bmatrix} + n_w^a \tag{24}$$

Here θ^d are the damage parameters. The Jacobian matrix for the system dynamics can be represented as

$$F^a = \frac{\partial f^a}{\partial x^a} = \left[\begin{array}{c|c} A = \dfrac{\partial f}{\partial x} \in R^{6\times 6} & f_\theta = \dfrac{\partial f}{\partial \theta} \in R^{6\times Dim(\theta)} \\ \hline 0 \in R^{Dim(\theta)\times 6} & diag(\varsigma_i) \end{array}\right] \tag{25}$$

The partitioned Jacobian matrix f_θ needs to be evaluated numerically due to the complexity of the relationship between damage parameterization and the system dynamics.

In fact, the damage parameterization can be done in various ways. A parameterization scheme using a single scalar damage parameter is employed in the present research. Specifically, the number of impaired panels, which is a direct measure of the impaired area, is used as a parameter state. Illustrative examples of the parameter numbering schemes depending on damage configurations are shown in Figure 7 and Figure 8.

Journal of Aircraft, Vol. 47, No. 4, July-August 2010, pp.1216-1228. Also presented at the *2009 Guidance, Navigation, and Control Conference*, August 10-13, Chicago, IL.

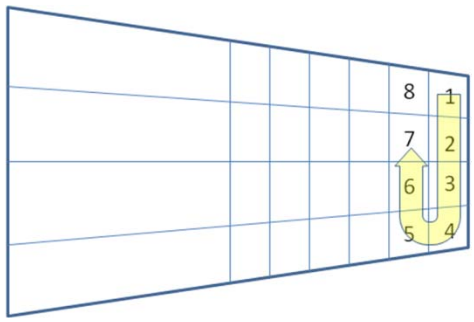

Figure 7. Tip damage parameterization

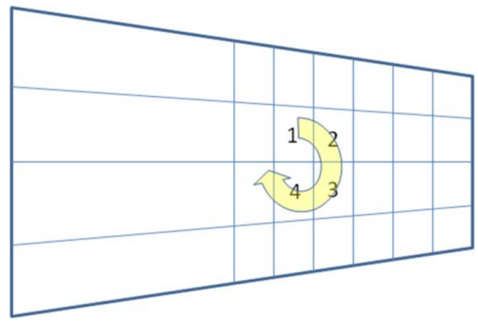

Figure 8. Hole damage parameterization

V. Simulation Results and Performance Analysis

A series of numerical simulations are set up to evaluate the performance of the state-parameter estimation methods discussed in this chapter. The full VLM is used for simulating the actual aircraft dynamics, and the DVLM-based EKF formulations are used for online estimation of aircraft states and parameters.

A. Simulation Scenarios

The two following damage configurations are considered for the present simulations.

Case 1 : Wing-tip damage

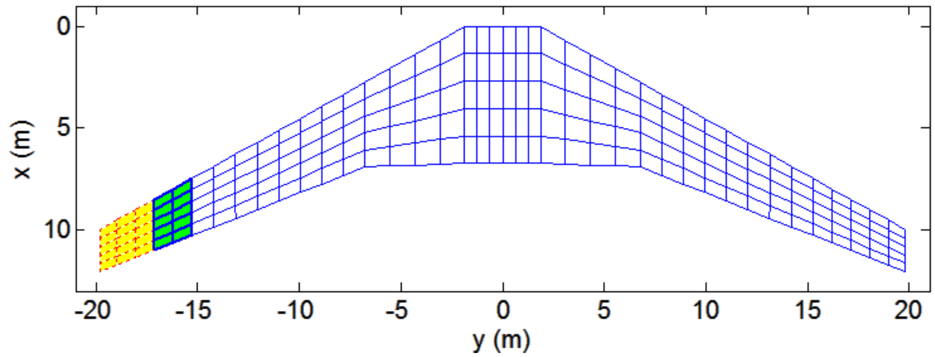

Figure 9. Damage configuration (case 1): Total number of panels = 200, number of impaired panels = 15, number of panels for DVLM analysis = 10

Case 2 : Hole-in-the-wing damage

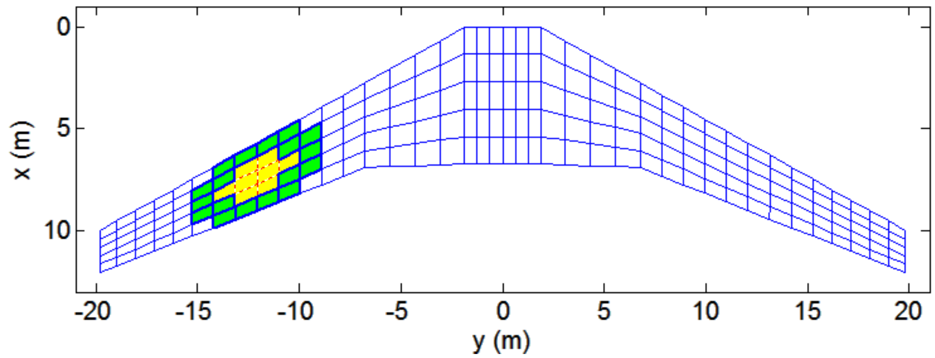

Figure 10. Damage configuration (case 2): Total number of panels = 200, number of impaired panels = 8, number of panels for DVLM analysis = 18

Note that term *damage* in this paper implies the loss of lifting surface panels on which the circulation strengths are zero.

Simulation settings are given in Table 3.

Table 3. Simulation parameter settings

Parameter	Value
Aircraft weight	45941 (kgf)
Nominal thrust	5791 (N)
Nominal angle of attack	6º
Initial Speed	130 (nm/hr)
Initial Altitude	3000 (m)
Process noise covariance	diag(0,0,0,0,0,0,1,…,1)
Measurement noise covariance	diag(0.1^2, 0.00175^2, 0.00175^2, 1.0^2, 1.0^2, 1.0^2, 1.0^2, 0.0175^2, 0.0175^2)

B. Information-Enhancing Input Design

Oscillatory inputs in angle of attack, angle of sideslip and bank angle are used as the information-enhancing inputs. The amplitudes of these inputs are determined based on sensitivity analysis, and the excitation frequency and simulation time span are chosen based on the period of the Phugoid mode [11]. Figure 11 illustrates the control input histories used to generate the results given in this chapter.

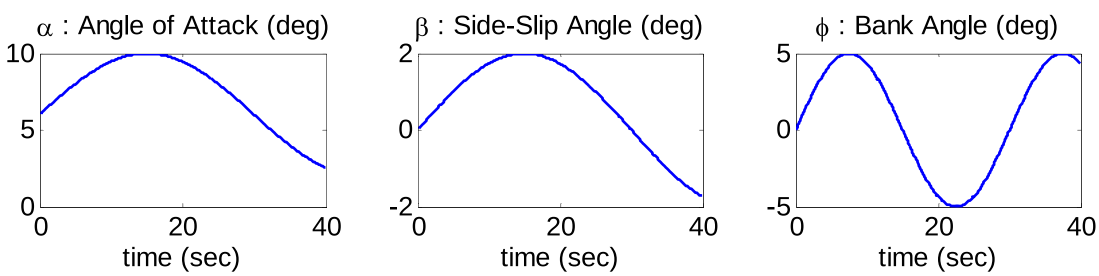

Figure 11. Aircraft inputs used in the simulation

For the present simulations, the angle of attack starts from the nominal equilibrium value of 6º for the unimpaired aircraft.

C. Simulation Results

Perturbation models in Equations (22) and (23) are used in the estimation algorithm for the two damage scenarios. It is assumed here that 15 panels are impaired in the wing-tip damage configuration, and 8 panels in the hole-in-the-wing damage configuration. The dimension of the circulation state vector varies depending on the damage configuration for the filter based on Formulation 1. In order to construct the circulation state vector for this approach, 10 panels are used for the wing-tip damage configuration, and 18 panels for the hole-in-the-wing damage configuration.

Figure 12 and Figure 13 show the motion state estimate errors for the wing-tip damage scenario and the hole-in-the-wing damages scenario, respectively. For each damage scenario, the error histories of the three estimators, two for Formulation 1 and one for Formulation 2, are plotted to enable a quantitative comparison between them. In the figures, the solid blue lines and the dashed green lines correspond to the DVLM estimators based on Formulation 1, and the dotted red lines are the error histories from the parameterized damage estimator based on Formulation 2.

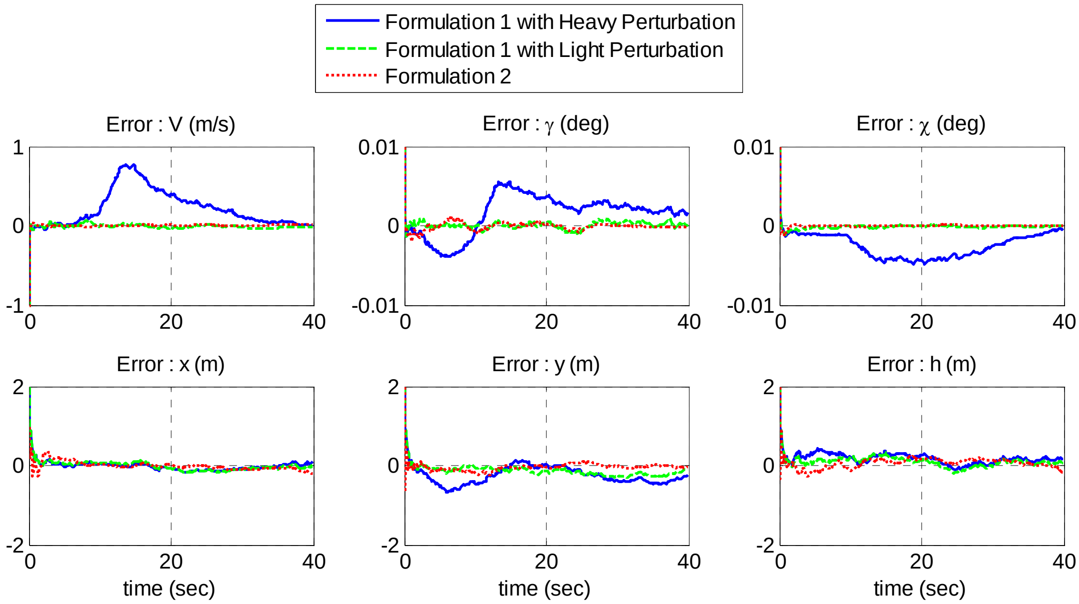

Figure 12. Errors in the aircraft motion state estimates (wing-tip damage)

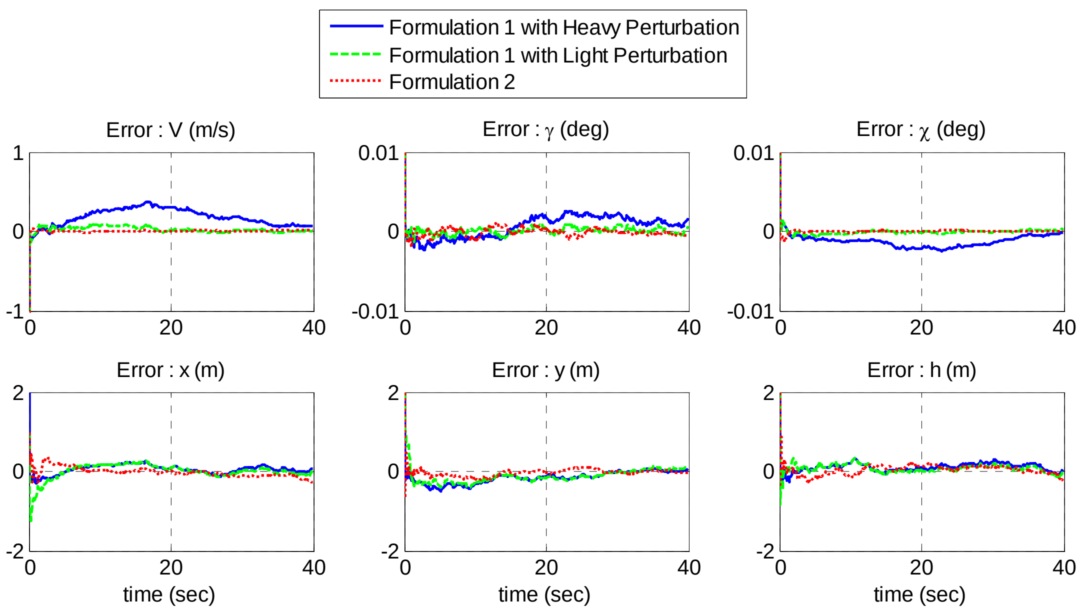

Figure 13. Errors in the aircraft motion state estimates (hole-in-the-wing damage)

All the proposed formulations show satisfactory performance for the aircraft motion state estimates. In particular, excellent filter performance is observed for Formulation 1 with light perturbation and Formulation 2. This is mainly because of the availability of direct measurements for all those motion states in the measurement equation shown in Equation (5).

However, their performances in estimating unmeasured states and resulting aerodynamic forces greatly vary depending on the formulations and perturbation models. The results for the wing-tip damage and the hole-in-the-wing damage scenarios are shown in Figure 14 and Figure 15.

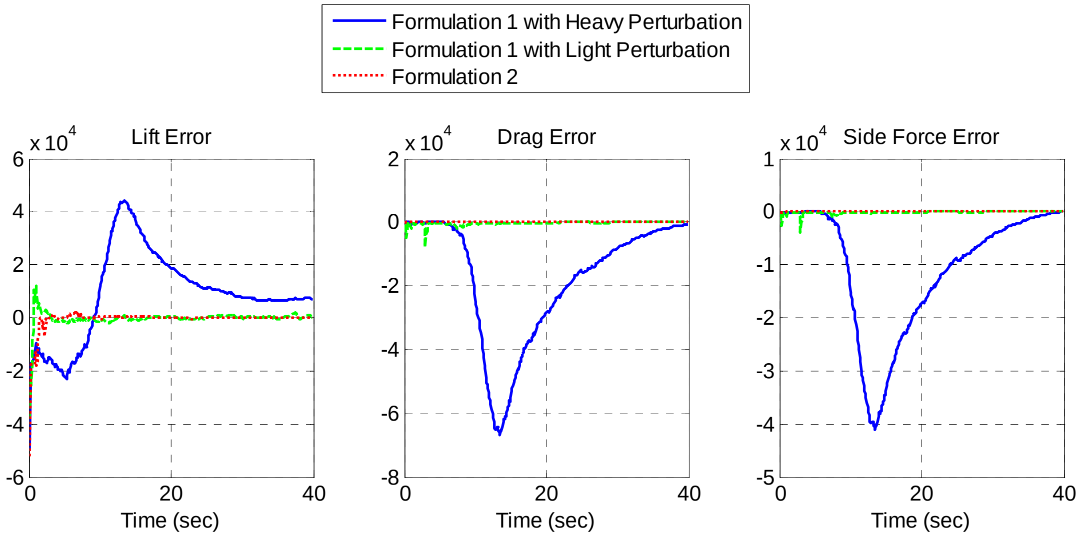

Figure 14. Errors in the aerodynamic force estimates (wing-tip damage)

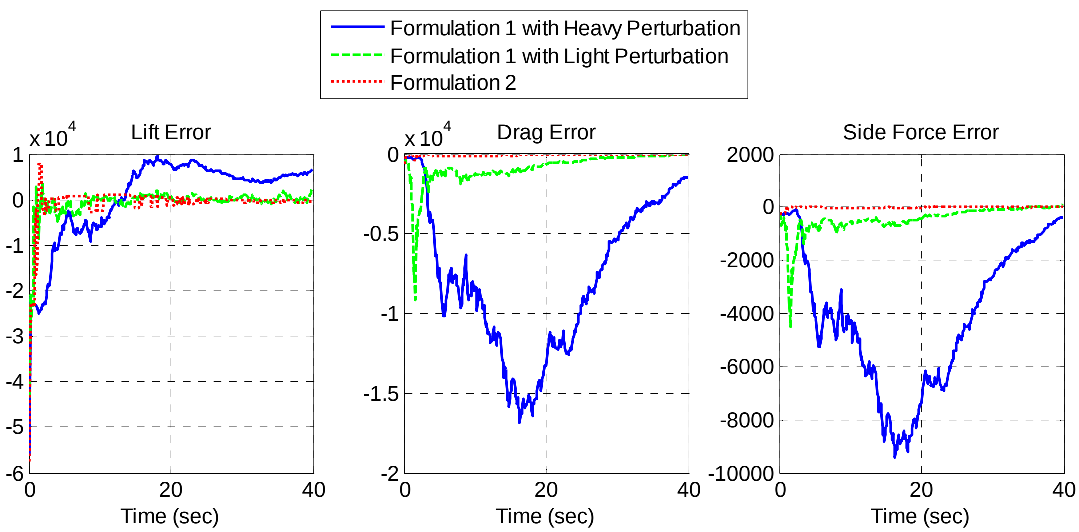

Figure 15. Errors in the aerodynamic force estimates (hole-in-the-wing damage)

Regarding Formulation I, the filter with light perturbation outperforms the heavy perturbation filter. The performance of the heavy perturbation model is inadequate. The leakage constant provides artificial system damping and prevents the filter from diverging, however the filter with heavy perturbation basically lacks numerical stability. The light perturbation model employs smaller corrections and uses more information from the DVLM solver, and shows improved performance considering the system's marginal observability and the minor inaccuracies in the DVLM. In fact, this observation led to the conjecture that using the DVLM as an open loop solver and focusing on improving the accuracy of inputs to the DVLM solver might provide better overall performance, which resulted in the second formulation with direct damage parameterization.

As can be observed in the above figures, the second formulation provides excellent filter performance. In addition to the accurate estimation of the aircraft states, good force estimation results are obtained as shown in Figure 14 and Figure 15. The state estimate for the number of impaired panels for the hole-in-the-wing damage configuration is shown in Figure 16. Figure 17 compares the associated circulation strengths and the results by the full VLM formulation.

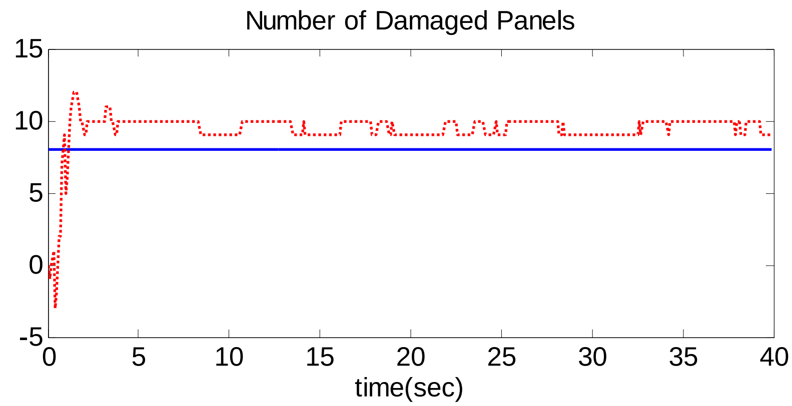

Figure 16. Estimate of the number of impaired panels

(Formulation 2, hole-in-the-wing damage)

Figure 16 shows that the estimate converges within a few seconds, however steady state errors remain. This error is due to the difference between the full VLM and DVLM formulations. The error can be reduced by improving the accuracy and fidelity of the DVLM algorithm.

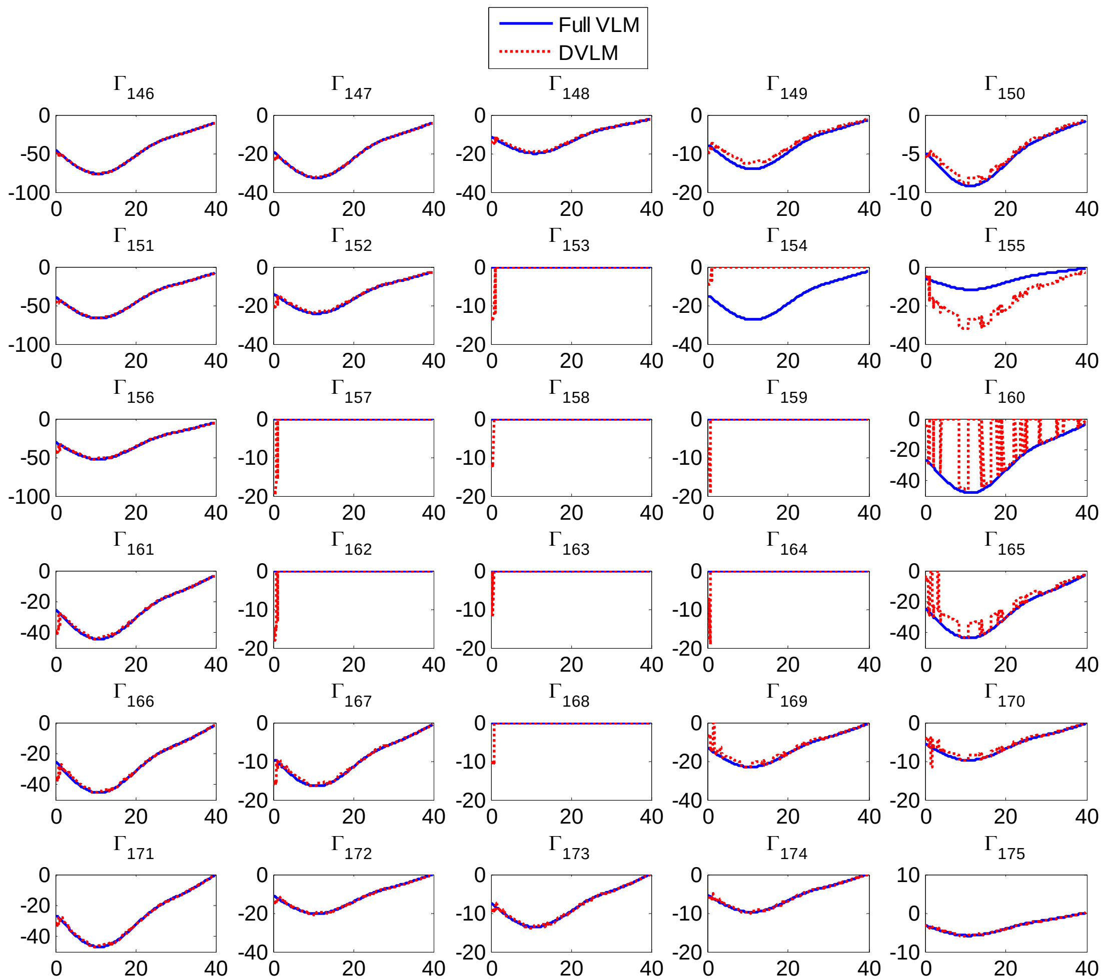

Figure 17. Circulation vector estimates: VLM vs DVLM+EKF

(Formulation 2, hole-in-the-wing damage)

For the hole-damage, the panels with the ID numbers *153*, *157, 158, 159*, *162, 163, 164*, and *168* are the impaired panels on which circulation strengths are zero, and the adjacent panels are the boundary panels. Some errors and ambiguities are observed near the boundaries of the damage due to the errors in the estimation of the number of impaired panels and the approximation algorithm by the DVLM algorithm. However, the overall estimation performance appears to be very satisfactory. In particular, it is observed that the circulation strength prediction errors based on the estimated parameter are quite localized. Another advantage of this formulation is that the filter dimension and its convergence rate are not explicitly dependent on the resolution of the airframe geometric discretization used in formulating the DVLM.

The following observations can be made from these plots:

- The filter using DVLM Formulation 1 with the light perturbation model and the filter using Formulation 2 with the parameterized damage outperform the filter using the DVLM Formulation 1 with the heavy perturbation.
- The filter using Formulation 2 shows the best force estimation performance among the three proposed approaches, especially during the transient phase.

Table 4 summarizes the performance comparison between the three approaches proposed in this paper.

Table 4. Performance Summary

Filter Formulation	**Formulation 1** (Circulation Strength Estimation)		**Formulation 2** (Parameterized Damage Estimation)
	Heavy Perturbation	Light Perturbation	
Parameter State	$\delta\Gamma_i$ $\left(= \Gamma_{u,i} - \Gamma_{d,i}\right)$	$\delta\Gamma_i$ $\left(= \Gamma_{d,i}^{DVLM} - \Gamma_{d,i}\right)$	θ_i (Damage Size)
State-dimension Scalability	Poor	Poor	Good
Dependency on DVLM	Low	Moderate	High
Overall Computational Cost	Low ~ Moderate	Moderate ~ High	Moderate
Estimation Accuracy	Poor	Good	Good
Filter Stability and Robustness	Poor	Fair	Good

VI. Concluding Remarks

This paper described extended Kalman filter algorithms based on a differential vortex lattice method to realize a practical approach for determining the aerodynamic model of a impaired aircraft, and to use it as the basis for estimating flight constraints. The present research was motivated by the desirability of relating the impaired aircraft geometry with its flight dynamics. Central premises involved in the research are that the inner-loop flight control system allows the continued flight of the aircraft, onboard sensors can provide information about the location and qualitative nature of the damage on the airframe, and that the Vortex Lattice method can provide sufficiently accurate aerodynamic characterization of the aircraft.

A novel, computationally efficient algorithm for computing the aerodynamic forces on impaired aircraft, termed as the Differential Vortex Lattice Method (DVLM), was advanced in this paper. This algorithm uses prior knowledge of the aerodynamic model to derive a differential formulation of the well-known Vortex Lattice Method. It was shown that the algorithm is accurate, and can provide several orders of magnitude savings in computational

Journal of Aircraft, Vol. 47, No. 4, July-August 2010, pp.1216-1228. Also presented at the *2009 Guidance, Navigation, and Control Conference*, August 10-13, Chicago, IL.

time when compared with the Vortex Lattice Method. The computationally efficient differential formulations make it possible to rapidly estimate the aerodynamic model of impaired aircraft, and use it eventually for designing safer landing guidance algorithms.

The differential vortex lattice algorithm was then used as the basis for the design of three extended Kalman filters. Using a point-mass dynamic model, these estimators were shown to be capable of extracting the impaired aircraft aerodynamic parameters from the motion measurements. Accurate estimation of the aircraft motion and the aerodynamic parameters using the extended Kalman filters was demonstrated in two distinct damage simulations. Numerical simulations were performed to demonstrate and compare the performance of the proposed filtering algorithms. The results confirm that Formulation 2 based on the direct damage parameterization outperforms the other approaches which estimate circulation strengths.

Acknowledgments

This research was supported under NASA SBIR Contract No. NX08CA50P, with Ms. Diana Acosta of NASA Ames Research Center serving as the Technical Monitor. We thank Dr. K. Krishnakumar and Dr. Nhan Nguyen of NASA for their interest and support of this research.

References

[1] Nguyen, N., Krishnakumar, K., Kaneshige, J., and Nespeca, P., "Flight Dynamics and Hybrid Adaptive Control for Stability Recovery of Damaged Asymmetric Aircraft," Journal of Guidance, Control, and Dynamics, Vol. 31, No. 3, pp. 751-764, 2008.

[2] Rysdyk, R.T. and Calise, A.J., "Fault Tolerant Flight Control via Adaptive Neural Network Augmentation," AIAA Guidance, Navigation, and Control Conference, Boston, MA. 1998.

[3] Kuethe, A. M., and Chow, C-Y, *Foundations of Aerodynamics*, Wiley, New York, NY, 1998.

[4] Gelb, A., *Applied Optimal Estimation*, MIT Press, Cambridge, MA, 1989.

[5] Bar-Shalom, Y., Li X. R. and Kirubarajan, T., *Estimation with Applications to Tracking and Navigation*, John Wiley & Sons, 2001.

[6] Jazwinski, A. H., *Stochastic Processes and Filtering Theory*, Academic Press, New York, NY, 1970.

[7] Astrom, K. J., and Wittenmark, B., *Adaptive Control*, Addison-Wesley, Menlo Park, CA, 1989.

[8] Melin, T., "A Vortex Lattice MATLAB Implementation for Linear Aerodynamic Wing Applications," Master's Thesis, Department of Aeronautics, Royal Institute of Technology (KTH), Stockholm, Sweden, 2000.

[9] Magnus, A. E. and Epton, M. A., *PAN AIR-A Computer Program for Predicting Subsonic or Supersonic Linear Potential Flows About Arbitrary Configurations Using A Higher Order Panel Method*, Vol. 1. Theory Document (Version 1.0), NASA CR-3251, 1980.

[10] Miranda, L. R., Elliott, R. D., and Baker, W. M., "A Generalized Vortex Lattice Method for Subsonic and Supersonic Flow Applications," NASA CR-2865, 1977.

[11] Blakelock, J. H., *Automatic Control of Aircraft and Missiles*, John Wiley & Sons, 1991.