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Rapid merger of binary primordial black holes: An implication for GW150914

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Abstract

We propose a new scenario for the evolution of the binaries of primordial black holes (PBH). We consider dynamical friction by ambient dark matter, scattering of dark matter particles with a highly eccentric orbit besides the standard two-body relaxation process to refill the loss cone, and interaction between the binary and a circumbinary disk, assuming that PBHs do not constitute the bulk of dark matter. Binary PBHs lose the energy and angular momentum by these processes, which could be sufficiently efficient for a typical configuration. Such a binary coalesces due to the gravitational wave emission on a time scale much shorter than the age of the universe. We estimate the density parameter of the resultant gravitational wave background. Astrophysical implications concerning the formation of intermediate-mass to supermassive black holes is also discussed.

Key words: accretion, accretion disks — black hole physics — galaxies: evolution — gravitational waves — quasars: general

1 Introduction

Primordial black holes (PBHs) are hypothetical objects formed due to gravitational collapse of density perturbations in the early phase of the universe (Hawking 1971; Carr & Hawking 1974). A PBH mass larger than $\sim 10^{-18}\,M_{\odot}$, where $M_{\odot}$ is the solar mass, is enough to prohibit evaporation via the Hawking radiation (Hawking 1974). PBHs with such a mass could remain as the dark matter in the universe.

The composition of dark matter has been poorly known, although nowadays people are well aware that it amounts to more than 20% of the whole energy in the universe (Larson et al. 2011; Planck Collaboration 2015). The possibility that PBHs constitute a certain fraction of the whole dark matter has been investigated by many authors (see Carr et al. 2010 and references therein). Some of those massive PBHs may be residing in the Milky Way's halo as massive compact halo objects (MACHOs), but their abundance has been constrained for a certain range of mass by, e.g., gravitational lensing observations (Tisserand et al. 2007; Griest et al. 2014).

Nakamura et al. (1997) considered the formation and evolution of binary PBHs. If PBHs were formed randomly rather than uniformly in space, some pairs of PBHs would have sufficiently small separations to overcome the cosmological expansion and then form a binary. They estimated the amplitude of gravitational waves from coalescing black holes following the binary evolution taking the gravitational wave emission into account. Because the gravitational radiation is not effective in extracting an angular momentum from binary PBHs unless the separation is very small, only a small fraction of the binaries coalesces during the age of the universe. Nonetheless, the cosmological gravitational wave background amounts to $\Omega_{\rm gw} \sim 10^{-10}$ at the Laser Interferometer Space Antenna (LISA) band ($\nu_{\rm gw} \sim 10^{-4}$ Hz), where $\Omega_{\rm gw}$ and $\nu_{\rm gw}$ are the density parameter and frequency of the gravitational wave, respectively (Ioka et al. 1999; Inoue & Tanaka 2003).

Recently, the Advanced Laser Interferometer Gravitational-Wave Observatory (LIGO) has detected the gravitational waves originated from the merger of nearly equal mass binary black holes with $\sim 30\,M_\odot$ at $z \approx 0.09$ (Abbott et al. 2016a). The discovery of this gravitational wave source (GW150914) has now brought to us the new problem about how such an intermediate-mass binary is formed (Abbott et al. 2016b). It remains a matter of debate, although some ideas have already been proposed. Binary PBHs could be one candidate for merging massive binaries.

In this paper, we consider two prominent processes which affect the evolution of binary PBHs. One is the gravitational interaction between the binary and the ambient dark matter. Another is the gravitational interaction between the binary and a gaseous disk surrounding it (i.e., circumbinary disk). Both processes are inevitable and make the binary shrink rapidly. Our scenario predicts that binary PBHs generically coalesce during the age of the universe and gravitational waves are emitted more efficiently compared to the result in Ioka, Tanaka, and Nakamura (1999). Throughout this paper, we adopt the cosmological parameters obtained by the seven-year Wilkinson Microwave Anisotropy Probe (WMAP) (Larson et al. 2011).

In section 2, we summarize the basic elements of the evolution of binary PBHs, assuming that PBHs do not constitute the bulk of the dark matter. We discuss the gravitational interaction of dark matter and of gas with the binary in sections 3 and 4, respectively. Finally, we summarize our results and also give a simple estimation of the gravitational wave background predicted in our scenario in section 5.

2 Binary PBHs

First, we give a summary on the evolution scenario of binary PBHs presented by Nakamura et al. (1997) and Ioka et al. (1998), extending to a case where PBHs do not constitute the bulk of the dark matter.

Density fluctuations of radiation with a wavelength exceeding the Hubble radius in the very early stage of the radiation-dominated era eventually come into the Hubble horizon and, if the amplitude is large enough (and not too large), gravitationally collapse to form PBHs. The black hole mass is comparable to the horizon mass and is written as $M_{\rm bh} \sim 0.5\,\gamma\,(T_{\rm bhf}/{\rm GeV})^{-2}\,M_\odot$, where $T_{\rm bhf}$ is the temperature of the universe at the moment of the PBH formation (bhf) and γ is a numerical factor which, in an analytical treatment, takes a value of around 0.2 (Carr 1975). Hereafter we suppress γ since we do not need very precise figures. Also, Kawasaki, Sugiyama, and Yanagida (1998) showed the mass scale of PBHs is $\sim 1\,M_\odot$ in a certain parameter region of our double inflation model. We, therefore, adopt the typical mass of PBHs as $1\,M_\odot$ in what follows.

For simplicity, we assume a primordial spectrum of density fluctuations which has a sharp peak with a sufficient amplitude to form PBHs that have a monochromatic mass function (see, e.g., Kawaguchi et al. 2008 for such an inflationary model).

We also assume that PBHs constitute a fraction, f, of the dark matter, that is, $\Omega_{\rm bh} = f\Omega_{\rm DM}$ where $\Omega_{\rm bh}$ and $\Omega_{\rm DM}$ are the current density parameter of PBHs and dark matter, respectively. Note that a certain fraction of the dark matter is in the form of particles with mass m much smaller than that of PBHs. The value of f is constrained for a wide range of $M_{\rm bh}$ and can be as large as 0.1 for $M_{\rm bh} \lesssim 0.1\,M_\odot$ while much more severe constraints, $f \lesssim 10^{-8}$, are obtained for $M_{\rm bh} \gtrsim 10^3\,M_\odot$ (Tisserand et al. 2007; Ricotti et al. 2008; Capela et al. 2013a, 2013b; Defillon et al. 2014; Pani & Loeb 2013, 2014; Griest et al. 2014). Henceforth, we assume that $f \ll 1$.

Noting that the energy density of PBHs can be written as $\rho_{\rm bh}(T) = f\rho_{\rm DM}(T) = f\Omega_{\rm DM}(T/T_0)^3(3H_0^2/8\pi G)c^2$ where T_0, H_0, c, and G are the temperature at the present universe, the current Hubble parameter, the light velocity, and the gravitational constant, respectively, and that the average separation of PBHs at the formation, $\bar{r}$, is expressed as

$$\bar{r} \sim \left[\frac{M_{\rm bh}c^2}{\rho_{\rm bh}(T_{\rm bhf})}\right]^{1/3} \sim 8\times10^{-11}\,f^{-1/3}\left(\frac{M_{\rm bh}}{M_\odot}\right)^{5/6}\ {\rm pc}, \qquad (1)$$

where the normalization corresponds to $T_{\rm bhf} = 1\,{\rm GeV}$. Actually, PBHs are randomly distributed in space at formation and we consider a pair of nearest PBHs with the initial separation parametrized as $\alpha\bar{r}$. Statistically, most of the nearest PBH pairs will have $\alpha \lesssim 1$ while only a few would have $\alpha \ll 1$. In fact, the probability density distribution function is $dP/d\alpha \sim 3\alpha^2 e^{-\alpha^3}$ for a random distribution as given by Nakamura et al. (1997).

After the black hole formation, the separation of the pair evolves in proportion to the scale factor due to the cosmic expansion. However, if the local energy density of the pair, $\rho_{\rm pair} = \alpha^{-3}\rho_{\rm bh}$, becomes larger than the radiation energy density, ρ_γ, the pair decouples from the cosmic expansion and forms a gravitationally bound system. We see that the turnaround (ta) occurs at the temperature

$$T_{\rm ta} \approx \left(\frac{\alpha^3}{f}\right)^{-1} T_{\rm eq}, \tag{2}$$

with the temperature at the matter–radiation equality $T_{\rm eq} \sim 1\,$eV. Here it should be noted that for $\alpha > f^{1/3}$ we always have $\rho_{\rm pair} < \rho_{\rm DM}$, so that the turnaround never occurs. Thus we obtain the condition for α that a pair forms a bound system, $\alpha_0 \le \alpha \le f^{1/3}$, where $\alpha_0 \sim 10^{-3} f^{1/3}(M_{\rm bh}/M_\odot)^{1/6}$ and the lower bound comes from the requirement that $\alpha\bar{r}$ should be greater than the Schwarzschild radius of a PBH. Hereafter we consider a typical pair with $\alpha \sim f^{1/3}$, which turns around roughly at the matter–radiation equality. The separation at the turnaround is

$$r_{\rm ta} = \frac{T_{\rm bhf}}{T_{\rm ta}}\alpha\bar{r} \sim 5\times 10^{-2}\left(\frac{\alpha^3}{f}\right)^{4/3}\left(\frac{M_{\rm bh}}{M_\odot}\right)^{1/3} \text{pc}. \tag{3}$$

After the turnaround, if the two black holes had no relative velocity, they would coalesce to form a single black hole. However, as discussed in Nakamura et al. (1997) and Ioka et al. (1998), the tidal force from neighboring black holes would give the pair some angular momentum and prevent the coalescence. Since the semimajor axis of the binary is roughly given by $a \sim r_{\rm ta}$, the acceleration due to the tidal force and free-fall time are estimated as

$$a_{\rm tidal} \sim \frac{GM_{\rm bh}}{R^2}\frac{r_{\rm ta}}{R} \quad \text{and} \quad t_{\rm ff} \sim \sqrt{\frac{r_{\rm ta}^3}{GM_{\rm bh}}}, \tag{4}$$

respectively, where $R \equiv (T_{\rm bhf}/T_{\rm ta})\bar{r}$ is the typical separation between the binary and the third PBH at the turnaround of the pair. Thus the semiminor axis is estimated to be $b \sim a_{\rm tidal} t_{\rm ff}^2 \sim r_{\rm ta}^4/R^3 \sim \alpha^3 r_{\rm ta}$. We can see that the orbital eccentricity can be written as $e = \sqrt{1 - b^2/a^2} \sim \sqrt{1-\alpha^6}$ (the typical value of e is $\sqrt{1-f^2}$).

It is possible that the black hole pair obtains an angular momentum from ambient density fluctuation of cosmic plasma as well as the neighboring black holes. This effect can be estimated as follows. Although the density fluctuations exist for a wide range of length scale, the most effective would be the one with scale of $r_{\rm ta}$. Denoting the amplitude of fluctuation as $\delta \approx 10^{-5}$, we can think that the black hole pair obtains angular momentum from an object with mass $\rho_\gamma r_{\rm ta}^3 \delta$ which is separated from the pair by $r_{\rm ta}$. Here $\rho_\gamma = \Omega_\gamma (T/T_0)^4 (3H_0^2/8\pi G)c^2$, where Ω_γ is the current density parameter of radiation, is the radiation energy density, and the acceleration due to the tidal force from this object is written as

$$a_{\rm tidal,fluc} \sim G\rho_\gamma r_{\rm ta}\delta. \tag{5}$$

Comparing this with the acceleration from a neighboring black hole, equation (4), we see that the effect of the density fluctuations is important only when $f < \delta$ assuming $\alpha = f^{1/3}$. It should be noted that peculiar velocities of PBHs are negligible because density fluctuations already damped at such a small scale as given by equation (3).

Once a binary is formed, it emits gravitational waves and the separation shrinks gradually. The coalescence timescale of a binary PBH is given by Peters (1964),

$$t_{\rm gw} = \frac{5c^5 a^4}{64\,G^3 M_{\rm bh}^3}(1-e^2)^{7/2}$$
$$\sim 1\times 10^{34} f^7 \left(\frac{\alpha^3}{f}\right)^{16/3}\left(\frac{M_{\rm bh}}{M_\odot}\right)^{-5/3}\left(\frac{a}{r_{\rm ta}}\right)^4 \text{yr}, \tag{6}$$

where we assumed that tidal force from a neighboring black hole is dominant compared with that from density fluctuations, $a_{\rm tidal} > a_{\rm tidal,\,fluc}$. Thus we see that the gravitational wave emission is not effective for most of the binaries to merge during the age of the universe, unless f is very small; $f \lesssim 10^{-3.5}(M_{\rm bh}/M_\odot)^{5/21}$. However, as we see below, if we consider two other processes which extract the energy and angular momentum from the binary, its evolution drastically changes. Note that if f is very small, the number of the binaries is very small so that the total amount of gravitational waves is also small even if most of the binaries merge during the age of the universe.

3 Interaction with dark matter

In this section, we describe how moving PBHs interact with the surrounding dark matter. In the case of $f \ll 1$, which we consider here, the dark matter other than PBHs is abundant and will accrete on to PBHs, forming a halo around them.

At the turnaround of a pair of PBHs, each PBH will have a dark halo with the density $\sim\rho_{\rm DM}(T_{\rm ta})/c^2$. It would be reasonable to assume that, at the turnaround, two dark halos around PBHs merge and collapse into a single halo around the binary. The halo would then undergo a violent relaxation and reach the virial equilibrium with the mass $M_{\rm DM} \sim \rho_{\rm DM}(T_{\rm ta})r_{\rm ta}^3/c^2 \sim (\alpha^3/f)M_{\rm bh}$. Adopting a singular isothermal sphere solution, $\rho_{\rm halo}(r) = \sigma^2/(2\pi G r^2)$, for the halo density profile, the velocity dispersion of the dark halo and the virial radius are $\sigma \sim 0.2 \times (\alpha^3/f)^{-1/6}(M_{\rm bh}/M_\odot)^{1/3}\,{\rm km\,s^{-1}}$ and $r_{\rm v} = GM_{\rm DM}/\sigma^2 = 2r_{\rm ta}$, respectively.

In the presence of enveloping dark matter, the separation of the two PBHs decays due to the dynamical friction (df), i.e., the energy of each PBH is lost through the gravitational interaction with the field dark matter particles. Its timescale is given by Binney and Tremaine (1987) as

$$t_{\rm df} \sim \frac{v_{\rm bh}^3}{4\pi G^2 M_{\rm bh}\rho_{\rm halo}(r)\ln\Lambda}\left[{\rm erf}(X) - \frac{2X}{\sqrt{\pi}}e^{-X^2}\right]^{-1}$$
$$\sim 5\times 10^3 \left(\frac{\ln\Lambda}{\ln 10^6}\right)^{-1}\left(\frac{\alpha^3}{f}\right)\,{\rm yr}, \quad (7)$$

where $v_{\rm bh} = \sqrt{GM_{\rm bh}/r_{\rm ta}} \sim 0.3(\alpha^3/f)^{-2/3}(M_{\rm bh}/M_\odot)^{1/3}$, erf is the error function, and $X \equiv v_{\rm bh}/(\sqrt{2}\sigma) = 1$. The Coulomb logarithm, $\ln\Lambda$, is related, as $\ln\Lambda = \ln(r_{\rm ta}\sigma^2/2Gm) = \ln(N/2)$ (where m is the mass of the dark matter particle) to the number of dark matter particles within the halo:

$$N = \frac{M_{\rm DM}}{m} \sim 10^6\left(\frac{\alpha^3}{f}\right)\left(\frac{m}{10^{-6}\,M_\odot}\right)^{-1}\left(\frac{M_{\rm bh}}{M_\odot}\right). \quad (8)$$

The binary orbit decays until the binding energy of the binary becomes comparable to the kinetic energy of the dark halo. Then, the binary becomes hard and the dynamical friction is no longer effective. Equating the total kinetic energy of dark matter in the halo to the binding energy, we obtain the hardening radius: $a_{\rm h} = r_{\rm ta}/2$.

It should be checked here whether or not dynamical friction really works by comparing the timescale of dynamical friction, $t_{\rm df}$, with the Hubble time H^{-1}. In the matter-dominated era, we have (for $\alpha \sim f^{1/3}$)

$$H\,t_{\rm df} \sim \frac{1}{\Gamma}\left(\frac{1+z}{1+z_{\rm eq}}\right)^{3/2}, \quad (9)$$

where $\Gamma = 1/[H(z_{\rm eq})\,t_{\rm df}] = \mathcal{O}(1\text{–}10)$ is a numerical factor and $z_{\rm eq}$ is the redshift at the matter–radiation equality. Thus the dynamical friction could be in fact effective for $1 + z < \Gamma^{2/3}(1 + z_{\rm eq})$, i.e., throughout the matter-dominated era.

After the binary becomes hardened, dark matter particles in the loss cone are depleted (e.g., Merritt & Milosavljevic 2005). The binary orbit can, however, still decay due to the dark matter scattering if dark matter particles refill the loss cone through two-body relaxation (tbr). Its relaxation timescale is given by Spitzer (1987) as

$$t_{\rm tbr}(r) \approx \frac{0.34\sigma^3 N}{G^2\rho_{\rm halo}(r)M_{\rm DM}\ln\Lambda}$$
$$\sim 5\times 10^9\left(\frac{r}{r_{\rm v}}\right)^2\left(\frac{\alpha^3}{f}\right)^{3/2}\left(\frac{N/\ln N}{10^6/\ln 10^6}\right)\,{\rm yr}, \quad (10)$$

This is marginally shorter than the age of the universe, $t_0 \approx 1.37 \times 10^{10}\,{\rm yr}$, for N of the order of 10^6, so there is a possibility for some range of N that the dark matter scattering continues to work to reduce the semimajor axis of the binary. Its orbital decay rate obeys

$$\frac{d}{dt}\left(\frac{1}{a}\right) = -C\frac{G\rho_{\rm halo}}{\sigma}, \quad (11)$$

where C is a numerical factor, the typical value of which is 14.3 (Quinlan 1996). Integrating equation (11) under the assumption of a constant-density core, we obtain

$$a_{\rm ds}(t) = \frac{\sigma}{CG\rho_{\rm halo}(a_{\rm h})t}$$
$$\sim 2\times 10^{-7}\left(\frac{\alpha^3}{f}\right)^{-8/3}\left(\frac{M_{\rm bh}}{M_\odot}\right)^{1/3}\left[\frac{t_{\rm tbr}(r_{\rm v})}{t}\right]\,{\rm pc}. \quad (12)$$

Unfortunately, the above scenario will not be realized in most cases, as we shall explain. Since $t_{\rm tbr}$ is proportional to N and N increases as m falls below $10^{-6}\,M_\odot$ as seen in equation (8), it exceeds the age of the universe for lower masses of the halo dark matter. In particular, if the bulk of dark matter is composed of elementary particles like supersymmetric ones, then binary PBHs are stalled at the hardening radius.

Here we advocate an alternative approach to make the binary orbit decay, supplying the dark matter particles into the loss cone radius. In our cosmological setup, the ambient dark matter continues to accrete on to the binary as its influence radius expands. A binary in the dark halo could, in principle, shrink by scattering dark matter particles. However, to do so, it is necessary that the dark matter particles pass close enough to the binary, that is, they enter the loss cone (Merritt & Milosavljevic 2005). How much the binary shrinks is determined by the dark matter mass supplied into the loss cone.

For simplicity, we assume that a dark matter particle enters the loss cone if the pericenter distance of its orbit is shorter than the semimajor axis of the binary. Let us evaluate the orbital elements of a typical dark matter particle. As in the case of a PBH pair, we suppose that dark matter particles feel a tidal force from nearby PBHs during their infall. Since the turnaround radius of dark matter is given as $r_{\rm ta,dm} \sim 10^{-2}\,(M_{\rm bh}/M_\odot)^{1/3}\,[(1+z)/(1+z_{\rm eq})]^{-4/3}\,{\rm pc}$ (Bertschinger 1985; Mack et al. 2007; Ricotti et al. 2008), the semimajor and semiminor axes are estimated to be $\sim r_{\rm ta,\,dm}$ and $\sim r_{\rm ta,dm}^4/R^3$, respectively, where R is the distance to the third PBH at the moment of infall. Since the dark matter particles come to have highly eccentric orbits, we evaluate the pericenter distance as

$$d \sim \frac{r_{\rm ta,dm}^7}{R^6} \sim 10^{-2}\, f^2 \left(\frac{1+z}{1+z_{\rm eq}}\right)^{-10/3} \left(\frac{M_{\rm bh}}{M_\odot}\right)^{1/3} {\rm pc}\,. \quad (13)$$

Requiring that $d < a \sim r_{\rm ta}$ yields the following condition on the redshift:

$$1+z > (1+z_{\rm eq}) \left(\frac{f}{\sqrt{5}}\right)^{3/5} \left(\frac{\alpha^3}{f}\right)^{-2/5}. \quad (14)$$

Dark matter particles which turn around before this redshift enter the loss cone of the binary in a dynamical time. Since the total mass of the whole dark matter which turns around by a redshift z is given by $\sim M_{\rm bh}(1+z_{\rm eq})/(1+z)$ (Mack et al. 2007), the mass which refills the loss cone, $M_{\rm lc}$, is estimated as

$$M_{\rm lc} \sim \left(\frac{f}{\sqrt{5}}\right)^{-3/5} \left(\frac{\alpha^3}{f}\right)^{2/5} M_{\rm bh}\,. \quad (15)$$

It should be kept in mind that the above calculation is crude, and there were many uncertainties which could cumulatively affect the final expression of $M_{\rm lc}$. A precise evaluation will need some N-body simulations. However, we stress that the primary implications of equation (15), that $M_{\rm lc}$ is proportional to the black hole mass $M_{\rm bh}$ and that it increases as the fraction f decreases, seem robust. We hence believe that this mechanism certainly has the potential to supply a non-negligible amount of dark matter into the loss cone region around binary PBHs.

Now we can evaluate how much the binary could shrink due to the dynamical friction. We parameterize the uncertainties in the above computation by a function η of the parameters f and α: $M_{\rm lc} = \eta(f, \alpha)\, M_{\rm bh}$. A dark matter element of mass dm would extract an energy $\sim G\, M_{\rm bh}\, dm/a$ from a binary (Merritt & Milosavljevic 2005), so the evolution of the orbital radius obeys

$$G\, M_{\rm bh}^2\, d\left(\frac{1}{a}\right) = \frac{G\, M_{\rm bh}}{a}\, dm\,. \quad (16)$$

Thus, the residual separation after the dynamical friction ceases is estimated as

$$a_{\rm df} \sim \exp\left(-\frac{M_{\rm lc}}{M_{\rm bh}}\right) r_{\rm ta} = e^{-\eta(f,\alpha)}\, r_{\rm ta}\,. \quad (17)$$

If the function η acquires a value of around 7, which, if equation (15) applies in this case, requires only a reasonable value of $f \approx 0.1$, then the binary could shrink by about three orders of magnitude within $t_{\rm df}$.

Again we note that the shrinkage factor contains various uncertainties that come from the uncertainties in the determination of $M_{\rm lc}$. Here we have restricted ourselves to proving the potential utility of our scenario, but we will revisit this issue in more realistic setups via both analytical and numerical approaches in the future publications.

4 Interaction with gas

Baryon gas, as well as dark matter, also accretes on to the PBHs. Even if the gas has negligible angular momentum with respect to the center of mass of the binary, the gas cannot fall directly into the black holes but instead would form a rotating disk around the binary (i.e., a circumbinary disk). Many authors have addressed the computational hydrodynamic simulations of interaction between the nearly equal mass binary black holes and the circumbinary disk (Hayasaki et al. 2007, 2008; Cuadra et al. 2009; Roedig et al. 2011; Shi et al. 2012; Farris et al. 2014). The circumbinary disk can play the role of an efficient mechanism to extract the angular momentum from a binary (Artymowicz et al. 1991; Armitage & Natarajan 2005; MacFadyen & Milosavljevic 2008; Hayasaki 2009; Cuadra et al. 2009; D'Orazio et al. 2013). This is mainly because the circumbinary disk and nearly equal mass binary exchange their masses and angular momenta through the tidal-resonant interaction (Artymowicz & Lubow 1994).

Neglecting the angular momentum of the gas, we consider that the Bondi accretion radius, $r_{\rm B}$, is given by

$$r_{\rm B} = \frac{G M_{\rm bh}}{c_\infty^2} \sim 4 \times 10^{-5} \frac{M_{\rm bh}}{M_\odot} \frac{1+z_{\rm eq}}{1+z}\, {\rm pc}, \quad (18)$$

where $c_\infty \sim 10\sqrt{(1+z)/(1+z_{\rm eq})}\,{\rm km\,s^{-1}}$ is the average sound velocity (Ricotti 2007). When the Bondi radius is smaller than the separation of the binary, the gas accretes on to each PBH separately and a circumbinary disk would not form. Because the Bondi radius increases at a time when the binary separation would shrink due to the dynamical friction, the Bondi radius would exceed the inner edge of circumbinary disk, which is typically given by twice the binary separation (Artymowicz & Lubow 1994), sooner or later and a circumbinary disk would form. The circumbinary disk would begin to form at $z \lesssim 400$ when $r_{\rm B} = a_{\rm df}$.

According to Ricotti (2007) and Ricotti et al. (2008), the gas accretion rate, $\dot{M}_{\rm B}$, on to a black hole with $1\,M_\odot$ has a peak around the matter–radiation equality time and the peak accretion rate is several percent of the Eddington rate, $\dot{M}_{\rm Edd} \sim 2 \times 10^{-9} (M_{\rm bh}/M_\odot)\, M_\odot\,{\rm yr}^{-1}$. For $M_{\rm bh} \leq M_\odot$, the mass of the circumbinary disk, $M_{\rm cbd} \sim \dot{M}_{\rm B} H^{-1}$, can be fitted well with $M_{\rm cbd}/M_{\rm bh} \sim 10^{-5}(M_{\rm bh}/M_\odot)$. Considering only the interaction between the binary and the circumbinary disk, that is, neglecting the effects of the respective disks, the time scale of the orbital decay can be estimated in the same way as by Hayasaki (2009) [see also equations (16)–(19)

of Hayasaki et al. (2010) and alternatively equation (38) of Hayasaki et al. (2013)]:

$$t_{\rm cbd} = t_{\rm vis}\frac{M_{\rm bh}}{M_{\rm cbd}} \sim 7\times 10^{6}\left(\frac{a}{10^{-4}\,{\rm pc}}\right)^{1/2}\left(\frac{M_{\rm bh}}{M_{\odot}}\right)^{-1}\,{\rm yr}, \quad (19)$$

where $t_{\rm vis} \sim 7 \times 10^3 (a/{\rm pc})^{1/2}$ yr is the viscous time scale of the circumbinary disk, under the assumption that the temperature of the inner edge of the circumbinary disk is $(1 + z)T_0$ and the Shakura–Sunyaev viscosity parameter is 0.1 (Shakura & Sunyaev 1973).

As the separation decreases, the time scale given by equation (19) also decreases gradually while the timescale of gravitational waves, equation (6), decreases much more rapidly. When the two time scales become comparable, the most effective process of extracting angular momentum from the binary switches to the gravitational wave emission. The critical separation, $a_{\rm c}$, is

$$a_{\rm c} \sim 10^{-9}\left(\frac{M_{\rm bh}}{M_{\odot}}\right)^{4/21}\,{\rm pc}, \quad (20)$$

and the critical redshift, $z_{\rm c}$, which is determined by $H^{-1} = t_{\rm cbd}$, is given by

$$1 + z_{\rm c} \sim 100\left(\frac{M_{\rm bh}}{M_{\odot}}\right)^{2/3}. \quad (21)$$

Here we assumed the binary orbit is circular after the dynamical friction. Substituting equation (20) for equation (6), the time scale of gravitational radiation reduces to $t_{\rm gw} \sim 10^4 (M_{\rm bh}/M_{\odot})^{-19/21}$ yr. Thus, for the typical parameter set, binary PBHs of $M_{\rm bh} \gtrsim 0.01\,M_{\odot}$ generically coalesce by around $z_{\rm c} > 5$. This is to be contrasted with Nakamura et al. (1997) where only a small fraction of binary PBHs coalesces during the age of the universe.

5 Summary and discussions

We have studied the evolution of binary PBHs by considering the gravitational interactions with the ambient dark matter and baryon gas. Let us summarize our scenario in turn. In the early stage of a radiation-dominated era, the PBHs are formed by the gravitational collapse of radiation with a prominent amplitude of the density fluctuations. Some PBH pairs of randomly distributed PBHs decouple from the cosmic expansion and form binary PBHs at the turnaround ($z \sim z_{\rm eq}$).

Each dark halo, which is formed by the dark matter accretion on to each PBH, collapses into a single halo around binary PBHs. The energy and angular momentum of the binary are lost by the dynamical friction against the dark matter particles in the halo. The dynamical friction is not so effective when the binary is hardened at the half radius of the turnaround radius. Even after the hardening, the scattering between binary PBHs and infalling dark matter particles by two-body relaxation becomes effective. The binary orbit then decays and its semi-major axis shortens by about five orders of magnitude of the turnaround radius within the age of universe. This mechanism, however, seems not to work in the realistic case because of the strong constraint on the mass of dark matter particles (should be $m \lesssim 10^{-6}\,M_{\odot}$) according to the recent EROS-2 (Tisserand et al. 2007) and Kepler (Griest et al. 2014) observations.

If the third PBH tidally affects the orbits of dark matter particles around the binary, those orbits could become highly eccentric. A family of dark matter particles with high eccentric orbits makes the binary shrink effectively and rapidly. Although a precise estimate for how much the binary separation is shrinking by is hard analytically, our simple model for $f \approx 0.1$ implies that the binary orbit can decay by three orders of magnitude of the turnaround radius.

Some time after the recombination ($z \lesssim 1000$), the circumbinary disk would form and extract the angular momentum from the binary. At $z \sim O(100)$, the most effective process of extracting the angular momentum from the binary switches to the gravitational wave emission. Finally, the binary would coalesce at around $z \sim 100$.

If our scenario really works, most binary PBHs would coalesce around $z = z_{\rm c}$ and emit gravitational waves. Thus we can expect a substantial amount of gravitational wave background. Below we present an order-of-magnitude estimation simply assuming that all binaries coalesce at $z = z_{\rm c}$. Given the probability distribution function for the initial separation, $dP/d\alpha \sim 3\alpha^2 e^{-\alpha^3}$, we see that the fraction of PBHs which form a binary is $P(\alpha < f^{1/3}) \sim f$, assuming $f \ll 1$. Then, at $z = z_{\rm c}$, the number of binaries in a horizon volume is given by $\sim f^2 H^{-3}\rho_{\rm DM}/M_{\rm bh}$. Noting that the energy of gravitational waves emitted by a binary is $\sim (GM_{\rm bh}\nu)^{2/3} M_{\rm bh}$, where $\nu = \sqrt{GM_{\rm bh}/a^3}$ is the frequency of the gravitational wave determined by the binary separation, the energy density of the gravitational waves at $z = z_{\rm c}$ is $\epsilon_{\rm gw}(\nu, z_{\rm c}) \sim (GM_{\rm bh}\nu)^{2/3} f^2 \rho_{\rm DM}(z_{\rm c})$. Finally, the current density parameter of the gravitational waves is given by

$$\Omega_{\rm gw,0}(\nu) = \frac{\epsilon_{\rm gw}[(1+z_{\rm c})\nu, z_{\rm c}]}{(1+z_{\rm c})^4\rho_{\rm c}} \sim 4\times 10^{-7} f^2\left(\frac{\nu}{\nu_{\rm c,0}}\right)^{2/3}\left(\frac{M_{\rm bh}}{M_{\odot}}\right)^{37/63}, \quad (22)$$

where $\nu_{\rm c,\,0} \sim 5 \times 10^{-2} (M_{\rm bh}/M_{\odot})^{3/14} (a/a_{\rm c})^{-3/2}/(1 + z_{\rm c})$ Hz represents the lowest frequency corresponding to the separation $a_{\rm c}$. The frequency domain is given as

$\nu_{c,0} < \nu < \nu_{mso,0}$ where $\nu_{mso,0} \sim 10^5 (M_{bh}/M_\odot)^{-1}/(1+z_c)$ Hz, which is the frequency corresponding to the marginally stable orbit (mso) of a PBH binary. The gravitational wave background with $f < 0.03$ is estimated to be $\Omega_{gw,0} \sim 10^{-10}$, which is comparable with that estimation by Nakamura et al. (1997) corresponding to $f = 1$, and is large enough to be detected with eLISA (Amaro-Seoane et al. 2013), DECi-hertz Interferometer Gravitational wave Observatory (DECIGO) (Seto et al. 2001), and the Advanced LIGO (Abbott et al. 2016a). Thus the observation of the gravitational wave background would give a strong constraint on the dark matter fraction of PBHs with $M_{bh} \lesssim M_\odot$.

Let us mention the other astrophysical implications of our scenario. Fermi detected gamma-rays with a luminosity of $\sim 10^{49}$ erg s^{-1}, 0.4 s after the black hole coalescence (Connaughton et al. 2016). The probability of chance coincidence is 0.002 or so. Loeb (2016) suggested that this electromagnetic counterpart might be triggerd by the collapse of a single, rapidly rotating massive star surrounding the binary black hole. Alternatively, it would suggest the existence of a circumbinary disk not decoupled from the binary intermediate-mass black holes (e.g., Hayasaki et al. 2013). If so, the short gamma-ray event is caused by the subsequent accretion of the circumbinary disk after the coalescence.

GW150914 finally has $62^{+4}_{-4}\,M_\odot$, after the gravitational wave burst by the merger of two black holes with 36^{+5}_{-4} and 29^{+4}_{-4} solar masses (Abbott et al. 2016a). Such massive black holes are thought to be formed by the metal-poor star. Successive mergers of PBHs discussed in this paper could also explain the existence of the binary black holes with such intermediate masses. Further studies of the formation channel of intermediate-mass black holes are desired.[1]

If the spectrum of density perturbation is broad rather than monochromatic as assumed here, clusters of PBHs can be formed (Chisholm 2006). If this is the case, PBH mergers may frequently occur in clusters to form much larger black holes than the original PBHs (Clesse & Garcia-Bellido 2016; Bird et al. 2016). This may help one explain the presence of a supermassive black hole recently discovered at $z \sim 6$ (Fan et al. 2003; Willott et al. 2003), providing supermassive black holes directly by successive merger of PBHs or smaller "seed" black holes (Kawasaki et al. 2012).

[1] Sasaki et al. (2016) have very recently suggested that the binary black holes having led to GW150914 are a primordial origin by estimating the merger rate of a binary composing of two 30 $M_\odot$ PBHs. They have not, however, taken into account the interaction processes with surrounding dark matter and gasses which we proposed in this paper. We will come back to the issue of how the binary of 30 $M_\odot$ black holes could form after successive mergers of $\gtrsim 1\,M_\odot$ PBHs within our scenario in a forthcoming paper.

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