Rapid Solution of the Nonlinear Step-Drawdown Equation

by Cass T. Miller[a] and Walter J. Weber, Jr.[b]

ABSTRACT

The step-drawdown test is frequently utilized by hydrogeologists as an aid in assessing well efficiency and approximate pumping capacity. Unfortunately, the analysis of step-drawdown data presently requires either a solution to a system of highly nonlinear equations or the application of tenuous assumptions relative to the solution form. A method for solution is derived which does not require limiting assumptions, type-curve methods or extensive computer facilities.

INTRODUCTION

The concept of step-drawdown testing in a water well was first presented by Jacob (1947) as a means to separate the laminar and turbulent components of drawdown. Jacob assumed that the laminar component is directly proportional to the discharge rate and that the turbulent component is a second-order function of well discharge. This assumption leads to the simple analysis schemes still widely used in practice (Bruin and Hudson, 1955).

Rorabaugh (1953) noted that treatment of discharge as a second-order variable in the turbulent component term of the Jacob equation was overrestrictive, and suggested a more general form in which turbulent loss is assigned an nth order dependence on discharge. Both founding theory and experience indicate that n is greater than 1 and usually greater than 2, as well as variable between installations and with respect to time at a given installation. Practical use of the generalized drawdown relationship has been impeded by the need to resort to graphical means to solve the resulting system of nonlinear equations (Rorabaugh, 1953).

A significant contribution was made by Sheahan (1971) with the introduction of a type-curve solution technique. Further contributions by way of computer methods of solution were advanced as the use of digital computers became more common (Labadie and Helweg, 1975; Sheahan, 1975). The solution of Labadie and Helweg, for example, consists of an iterative optimization of a least-squares grid search approach. While producing accurate results, such computer methods require a moderate computational effort and restrict analysis from being practiced in the field, a desirable feature of any solution routine.

This paper presents a solution procedure to the step-drawdown test which does not require dependent-order assumptions, time-consuming graphical means or extensive computing facilities. Further, the procedure is readily usable in field testing situations.

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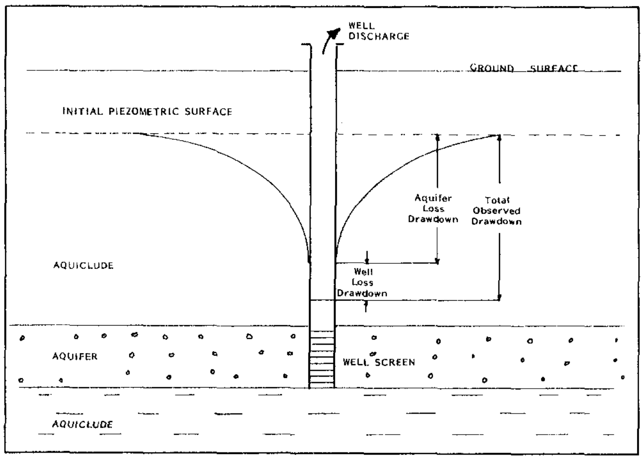

Fig. 1. Well and aquifer loss components of drawdowns.

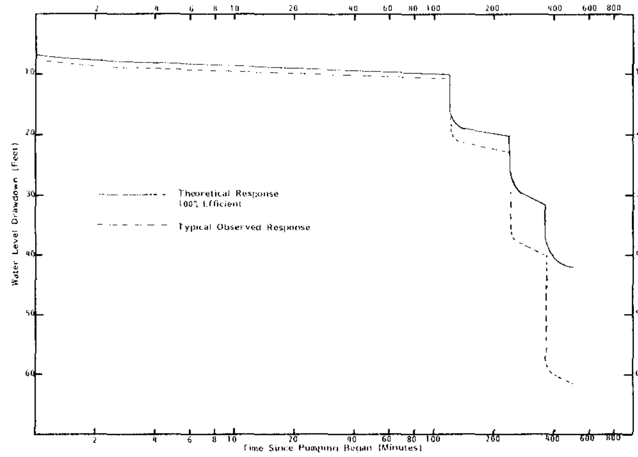

Fig. 2. Typical drawdown-time relationship for a four-step test.

THEORETICAL CONSIDERATIONS

As noted previously, the concept of step-drawdown testing is predicated on the desire to separate the laminar and turbulent well loss components of drawdown. It should be noted that the laminar term may include not only formation loss but a well loss term which is a linear function of pumping rate (Sheahan, 1971). A graphical representation of the two components of drawdown is provided in Figure 1. The step-drawdown relationship given by Rorabaugh (1953) is:

$$s = BQ + CQ^n \qquad (1)$$

where:

- s = drawdown in the well (L);
- B = coefficient in the laminar head loss term (TL^{-2});
- C = coefficient in the turbulent head loss term (arb. dim.);
- n = exponential coefficient in the turbulent head loss term (arb. dim.); and
- Q = discharge rate of well (L^3T^{-1}).

It is apparent that the above relationship does not account for any time dependence of s. Methods of testing and analysis which account for the time dependence of drawdown do exist, but since this factor is secondary to the present discussion, it will not be further considered here. Figure 2 illustrates a typical drawdown-time plot for a step-drawdown test conducted at four different pumping rates. Figure 3 depicts a step-drawdown relationship consistent with drawdown-time data derived from the plot given in Figure 2. It should be noted that the trace through the data presented in Figure 3 is a simple convex curve. If anything but a convex relationship results, then factors such as time of pumping, boundary conditions, local interference, barometric influence or transient well efficiency characteristics are complicating the data collected, and the step-drawdown analysis should not be performed without appropriate correction of the data.

The goal of step-drawdown analysis is to determine the functional coefficients B, C and n for such relationships as that depicted in Figure 3. Determination of three variables requires three

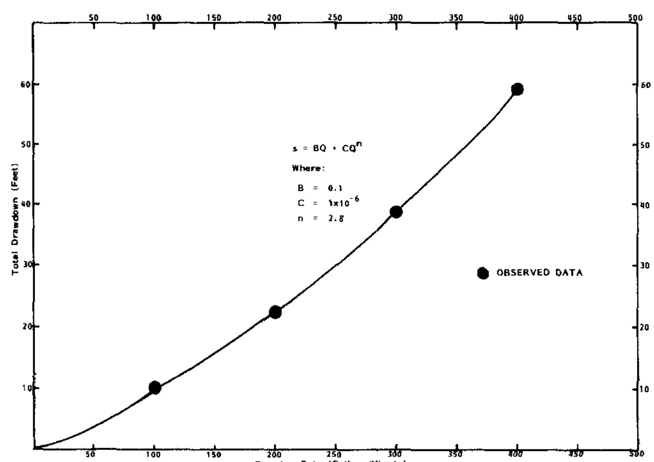

Fig. 3. Step-drawdown pumpage relationship.

independent equations. If it is assumed that loss coefficients are independent of discharge rate, three equations may be written from the general

It may be observed that as n increases in size the residual ϵ becomes smaller; indeed,

of the step-drawdown method. The assumptions suggest that any operational function for step drawdown should have both first and second derivatives of drawdown with respect to pumping rate universally greater than zero. After construction of a suitable operational line, three representative points could be utilized from the line to provide an approximate solution to the problem. An alternate approach would be to select different combinations of three points and solve for all coefficients. Solutions set in close approximation could be used with confidence while wide variations would indicate a deviation from method assumptions.

Utilizing the rapid solution technique presented here for the step-drawdown equation, together with the principle of superposition and the method of determining aquifer coefficients from step-drawdown recovery data outlined by Harrill (1970), a comprehensive well analysis can be accomplished through the performance of a single test. Such an analysis can potentially include transmissivity and storage coefficient, step-drawdown equation coefficients, laminar and turbulent well loss components, and formation losses as a function of time. It is beyond the scope of the present paper to detail such a solution scheme; a later paper will explore this concept in detail.

CONCLUSIONS

The step-drawdown test is an empirical method which has been performed routinely since the late 1940's. This paper develops and presents a simple and straightforward approach for solution of the general form of the equation. The benefit of the solution scheme presented is that accurate results may be obtained quickly without computer facilities or curve-matching exercises. This facet of the analysis scheme reduces solution to the field level, a highly desirable feature of the method.

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