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RARE DECAY OF A PSEUDOSCALAR MESON INTO A LEPTON PAIR

A WAY TO DETECT NEW INTERACTIONS ?

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A B S T R A C T

The rare electromagnetic decay of a neutral pseudoscalar meson into a lepton pair is calculated in a bound state quark model. For heavy mesons, the leading QCD diagram is argued to be dominant allowing higher order QCD corrections to be neglected for the branching ratio of this decay to the two-photon decay. The experimentally interesting case of pion decay is treated separately, and the rates for competing processes (weak neutral currents, axions, technicolour...) are estimated. We conclude that existing data may well allow for such contributions.

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1. - INTRODUCTION

Recently, progress has been made in calculating electromagnetic form factors of neutral pseudoscalar mesons (P mesons) : π^o, η, η_c, η_b, etc., in QCD [1),2)] for large (timelike and spacelike) Q^2. Likewise, on the low Q^2 side, potential model calculations have been done for heavy mesons [3)]. A coherent picture of the electromagnetic structure of P mesons for all Q^2 is thus emerging from QCD.

Unfortunately, the processes calculated in Refs. 1) and 2) are hardly feasible to detect experimentally in the near future due to the fast decrease with Q^2 of the production rate in those exclusive channels (e.g., $e^+e^- \rightarrow \pi^o\gamma$), and the high Q^2 required to justify the approximations made (neglecting rest masses, etc.).

On the other hand, for low Q^2 there exist some data, e.g., for Dalitz pair decays $\pi^o \rightarrow e^+e^-\gamma$ [4),5)] and $\eta \rightarrow \mu^+\mu^-\gamma$ [6)] where the form factors have been measured as a function of the invariant mass of the lepton pair. These light mesons are difficult to treat in a quarkonium model due to strong binding effects, but still reasonable agreement with the data is achieved even in rough quark models [3),7)]. We now want to extend these direct investigations of the form factor by looking at a possible indirect measurement, namely, by studying the rare decay into a lepton pair, $P \rightarrow \ell^+\ell^-$.

To lowest order in QED and QCD, this decay is represented by the diagrams of Fig. 1. Due to the photon loop appearing in the diagrams, this decay is sensitive to the form factor for all values of Q^2. Indeed, with a purely pointlike coupling at the $P\gamma\gamma$ vertex (i.e., a constant form factor), this diagram is even logarithmically divergent.

First, we do a bound state calculation, expected to be applicable for the heavy pseudoscalar mesons η_c, η_b, η_t,... The present experimentally interesting case of the pion is then discussed separately. Since the electromagnetic partial width for $\pi^o \rightarrow e^+e^-$ is of the order of just a μeV (micro-electron Volt), experiments sensitive to this decay may also detect contributions from other feeble interactions. Some of these possible contributions will also be discussed.

2. - THE $P\gamma\gamma$ COUPLING

We write the $P\gamma\gamma$ vertex

$$H_{\mu\nu} = e^2 \epsilon_{\mu\nu\alpha\beta} k_1^\alpha k_2^\beta f_{\gamma\gamma} F_P(k_1^2, k_2^2) , \tag{1}$$

where k_1 and k_2 are the four-momenta of the two photons and $f_{\gamma\gamma}$ is the coupling to two real photons thus normalizing F_p such that $F_p(0,0) = 1$. The matrix element $P \to \ell^+\ell^-$ is then

$$M = i e^2 f_{\gamma\gamma} \int \frac{d^4k}{(2\pi)^4} \frac{\epsilon_{\mu\nu\sigma\tau} k^\sigma q^\tau F_P(k^2,(q-k)^2) L^{\mu\nu}}{[k^2+i\epsilon][(q-k)^2+i\epsilon]} , \tag{2}$$

where q is the four-momentum of the P meson and $L^{\mu\nu}$ is the singlet S projection of the final state lepton pair. We find

$$L^{\mu\nu} = i e^2 4\sqrt{2} \left(\frac{m_\ell}{m_P}\right) \epsilon^{\mu\nu\alpha\beta} k_\alpha q_\beta \frac{1}{(k-p)^2 - m_\ell^2 + i\epsilon} , \tag{3}$$

where m_ℓ and m_P are the lepton and P meson masses, and p is the four-momentum of one the leptons. Inserting (3) into (2) we can express the branching ratio $B^P \equiv \Gamma_{P\to\ell^+\ell^-}/\Gamma_{P\to\gamma\gamma}$ in terms of a reduced, dimensionless amplitude R :

$$B^P = 2\sqrt{1 - \frac{4m_\ell^2}{m_P^2}} \left(\frac{\alpha m_\ell}{\pi m_P}\right)^2 |R|^2 , \tag{4}$$

where

$$R = \frac{1}{i\pi^2 m_P^2} \int d^4k \frac{2(q^2k^2 - (k.q)^2) F_P(k^2,(q-k)^2)}{[k^2+i\epsilon][(q-k)^2+i\epsilon][(k-p)^2-m_\ell^2+i\epsilon]} . \tag{5}$$

In earlier work, R has been calculated by making an ad hoc cut-off in F_p [8], by using a vector meson dominance model [9], by calculating a nucleon loop [10] and by choosing a simple form factor with the right analytic properties [11] :

$$F_P(k^2,(q-k)^2) = \frac{\Lambda^2}{\Lambda^2 - k^2 - (q-k)^2} . \tag{6}$$

Recently, F_p has been calculated in a QCD-inspired relativistic bound state quark model [12]. The result is :

$$e^2 f_{\gamma\gamma} F_p(s_1,s_2) =$$

$$\frac{4\pi\alpha e_Q^2}{m_p}\sqrt{\frac{m_p}{\lambda}}\int\frac{d^3p}{(2\pi)^3}\,\tilde{\phi}_s(p)\frac{m_Q^2}{pE}\left\{\ln\frac{m_p s_1-(m_p^2+s_{12})E-\sqrt{\lambda}\,p}{m_p s_1-(m_p^2+s_{12})E+\sqrt{\lambda}\,p}+(s_1\leftrightarrow s_2)\right\}, \quad (7)$$

where e_Q^2 is the effective quark charge squared (including colour),

$$\lambda = m_p^4+s_1^2+s_2^2-2m_p^2 s_1-2m_p^2 s_2-2s_1 s_2\,,\quad s_{12}=s_1-s_2\,,\quad E=\sqrt{p^2+m_Q^2}\,,$$

and $\tilde{\phi}_s(p)$ is the lowest order momentum space wave function which, at least for heavy quarkonium states, can be taken to be a solution of the Schrödinger equation with one of the phenomenologically successful potentials [13),14)]. The normalized form factor F_p turns out to be quite insensitive to the particular potential chosen [cf. the discussion in Ref. 3)]. To make Eq. (7) more tractable, we consider the "static" limit $(p \to 0$ in the integrand) and also the weak binding limit $(m_p \to 2m_Q)$. For the absolute value of the form factor this is certainly not a good approximation [in Ref. 15), corrections on the order of 25% to $f_{\gamma\gamma}$ were derived even for such a heavy meson as the tentative η_b]. For the normalized F_p the errors thus introduced should be tolerable. The result is then simply

$$F_p^{stat}(k^2,(q-k)^2) = \frac{m_p^2}{m_p^2-k^2-(q-k)^2}\,. \quad (8)$$

Interestingly enough, this is exactly of the same form as the form factor (6) of Berman and Geffen, who chose this shape mainly by intuition. However, our form factor corresponds to $\Lambda^2 = m_p^2$, whereas they supposed Λ^2 large and presented results only to lowest order in m_p^2/Λ^2. This expansion is inapplicable in our case ; we have to calculate the amplitude exactly.

The shape of (8) deserves some comments. The pole at $s = m_p^2$ of $F_p^{stat}(s,0)$ is an artifact of the weak-binding approximation of placing the quark-antiquark spinors on-shell. In reality, the poles should correspond to the physical vector mesons coupling to the pseudoscalar meson and a photon. For a heavy bound system which can be well described by a non-relativistic model, the coupling of the lowest pseudoscalar (e.g., η_c) to $V+\gamma$ is almost saturated by its vector partner (J/ψ) which is nearly degenerate in mass. Therefore, (8) should give a good description in this case. However, for the pion (and to some extent for the η) this is not necessarily so, since the ρ, ω poles are rather far away. In this respect, the good results obtained in Ref. 3) also for the light mesons seem somewhat fortuitous *). Indeed, with known $PV\gamma$ couplings a dispersion relation approach may be more rewarding for $F_p(s,0)$:

$$F_p(s,0) = \frac{1}{\pi} \int \frac{\mathrm{Im}\, F_p(s',0)}{s'-s}\, ds' \quad , \tag{9}$$

where $\mathrm{Im}\, F_p(s,0)$ gets contributions from the physical $PV\gamma$ couplings and the continuum. In this case (neglecting the continuum) the vector meson dominance (VMD) model gives the same result. However, for the double form factor $F_p(s_1,s_2)$, matters are more complicated, and the approximation (8) has some nice features besides giving simple, analytical results. First it admits an immediate generalization to two-gluon processes, which may be of relevance for the heavy quarkonium states. [With a trivial addition of colour factors, the diagrams of Fig. 1 may represent the decay of a heavy pseudoscalar particle into quark-antiquark jets. The same type of diagram is also supposed to be responsible for the $\eta(\eta') - \eta_c$ mixing.] The form factor (8) also shows the asymptotic behaviour predicted by current algebra relations and the Deser-Gilbert-Sudarshan representation of the time-ordered product of the two currents [16]. Namely, if we define the pseudoscalar coupling constant f_p by

$$\langle 0 | J_5^\mu | P \rangle = \sqrt{2}\, f_p\, P^\mu \tag{10}$$

where $J_5^\mu = \bar{q}\gamma^\mu\gamma^5 q$ is the axial current, this behaviour is

$$f_{\gamma\gamma} F_p(k^2,(q-k)^2) \xrightarrow[k^2\to\infty]{} \frac{12 \langle e_Q^2 \rangle_{em} f_p}{k^2} \tag{11}$$

For π^0 for instance, $\langle e_Q^2 \rangle_{em} = 1/18$, and the asymptotic limit is $\frac{2}{3} f_\pi / k^2$ where $f_\pi \simeq 93$ MeV (in QCD, higher order effects may renormalize the coefficient ; see next section).

Now, inserting (8) into (5), a straightforward though tedious calculation in the rest frame of the meson gives

$$R = \frac{i\pi}{2v} \ln\left(\frac{1-v}{1+v}\right) + \frac{1}{v}\left\{ \frac{1}{4} \ln^2\left(\frac{1+v}{1-v}\right) - \ln\left(\frac{1+v}{1-v}\right) + \frac{\pi^2}{12} - \Phi\left(\frac{1-v}{1+v}\right) \right\}. \tag{12}$$

Here v is the velocity of one of the final state leptons ; $v = \sqrt{1-4m_\ell^2/m_P^2}$ and $\phi(x)$ is the Spence function

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*) I am grateful to J.D. Jackson for a discussion on this point.

$$\Phi(x) = \int_0^x dt \, \frac{\ln(1+t)}{t} .$$

The imaginary part of (12) is due to the two-photon intermediate state and furnishes a "unitary limit" for the branching ratio [putting $Re(R) = 0$]. For the pion, for example, this is $B^{\pi}_{unit} = 4.7 \cdot 10^{-8}$.

3. - RESULTS FOR HEAVY MESONS

Equation (12) is just the tree-level result of QCD. Up to this point, everything has been equally applicable to pure QED, for instance the decay of a pseudoscalar $\mu^+\mu^-$ bound state : $(\mu^+\mu^-)_{1S_0} \to e^+e^-$. This particular decay will of course hardly ever be observed experimentally. However, as a check of our result, we can consider the virtual process $(e^+e^-)_{1S_0} \to \gamma^*\gamma^* \to (e^+e^-)_{1S_0}$, i.e., the two-photon annihilation contribution to the hyperfine splitting (hfs) between ortho-($^3S_1$) and parapositronium ($^1S_0$). Taking the limit $v \to 0$ in (12) we obtain

$$\lim_{v \to 0} R = -i\pi - 2 + 2\ln 2 , \tag{13}$$

which coincides with the corresponding contribution to the hfs as originally calculated by Karplus and Klein [17] and which is well known to agree with experiment. Alternatively, we could try to calculate this hfs contribution using the analogue of VMD. In QED, this would correspond to approximating the two-photon form factor of parapositronium by an M1 coupling to orthopositronium which would then decay into the other photon (to the relevant order in α this is the only allowed M1 transition). Assuming vector dominance at one or both photon legs, one would customarily [9] have either

$$F_P^{VMD1}(s_1, s_2) = \frac{1}{2}\left(\frac{m_V^2}{m_V^2 - s_1} + \frac{m_V^2}{m_V^2 - s_2}\right) \tag{14}$$

or

$$F_P^{VMD2}(s_1, s_2) = \frac{m_V^4}{(m_V^2 - s_1)(m_V^2 - s_2)} . \tag{15}$$

A similar calculation to that above gives in these cases

$$\lim_{v\to 0} R^{VMD1} = -i\pi - 2 + 2\ln 2 + (3 - 6\ln 2) \tag{16}$$

and

$$\lim_{v\to 0} R^{VMD2} = -i\pi - 2 + 2\ln 2 + (\sqrt{3}\pi + 2 - 10\ln 2) \tag{17}$$

respectively. The extra terms in brackets would give contributions to the hfs of +236 MHz and -104 MHz, respectively, which would ruin the present good agreement between theory and experiment concerning the hfs [for a recent review, see 18)]. Of course, this only tells us that (12) is a good form factor to use in loop calculations when the system is QED-like. One still has to convince oneself that the quarkonium system one is dealing with really has this property.

This analysis also shows that a proper two-photon form factor showing vector meson poles in $F_p(s,0)$ is neither (14) nor (15) as has been previously suggested, but rather $F_p^{VMD3} = m_V^2/(m_V^2 - s_1 - s_2)$. This reproduces the Karplus and Klein result (13) since for positronium $m_V = 1\,^3S_1$ and $m_p = 1\,^1S_o$ are nearly degenerate in mass, making corrections to (13) higher order in α. We also note that F_p^{VMD2} does not fulfil (11) and $F_p^{VMD1}(s,0)$ does not decrease as $s \to \infty$ which is the QCD prediction [1], whereas our suggestion obeys both these conditions. It would be interesting to have an experimental discrimination between these three possibilities for mesons, e.g., by studying exclusive pseudoscalar production in double-tagged $\gamma\gamma$ collisions at the large e^+e^- machines.

We have to discuss how higher order QCD corrections may affect the result. We may write the two-photon form factor

$$F_p(s_1, s_2) = \frac{m_p^2 N(s_1, s_2)}{m_p^2 - s_1 - s_2} , \tag{18}$$

where N is a symmetric function of s_1 and s_2 containing the effects of the QCD evolution which we conjecture to be slow. In fact, it is normalized to $N(0,0) = 1$, and the boundary conditions $N(\infty,0) = N(0,\infty) = 3/2$ have been proved by Lepage and Brodsky [1]. Incidentally, for s not too near $4m_Q^2$, $N(s,0)$ has been calculated exactly to one loop in QCD [2]. It was shown that the approach to the asymptotic limit is very slow, Fig. 2 (observe the logarithmic scale). In the region where (18) gives the main contribution to the integral (5) we therefore feel confident in using $N(s_1,s_2) = 1$ to our accuracy of approximation. [In fact, one can even convince oneself that an $N(k^2, (q-k)^2)$ rising indefinitely as $\ln(k^2)$ does not significantly alter the results ; cf. Ref. 11)].

A second correction comes from the artificial pole of (18) at m_p^2 instead of the nearest vector state m_V^2. However, numerically we find that the dependence of R on the pole position is also very weak, causing such corrections to be of order $(m_V-m_p)/m_p$ which (e.g., for charmonium) amounts only to a few per cent.

The predictions for the branching ratio B^P [Eq. (4)] using (12) are shown in the Table together with the unitary limits. The small branching ratios predicted for η_c and η_b decay indicate that these decay modes will be hard to see. For $\eta(549) \to \mu^+\mu^-$ the experimental result is $B^\eta_{exp} = (1.8 \pm 0.7) \cdot 10^{-5}$ [19]) which is compatible with our result. This must, however, be noted with caution, since the bound state model may not work well for such a light meson.

4. - PION DECAY INTO e^+e^-

To get a realistic estimate of the branching ratio for the electromagnetic decay of π^0 into e^+e^-, we keep the form (8) for $F_\pi(s_1,s_2)$ but do not make the weak binding approximation, i.e., we put

$$f_{\gamma\gamma} F_\pi(s_1,s_2) = 2 f_\pi \frac{1}{m_R^2 - s_1 - s_2} \quad , \qquad (19)$$

which has the correct behaviour given by QCD [1]) for $s_2 = 0$; $s_1 \to \infty$. The parameter m_R is then determined by the current algebra relation (known to reproduce the $\pi^0 \to \gamma\gamma$ width) $F_{\gamma\gamma}F_\pi(0,0) = 1/4\pi^2 f_\pi$, which gives $m_R = \sqrt{8\pi^2}\, f_\pi \approx 825$ MeV. Fortunately, this is consistent with the expected behaviour of $F_\pi(s,0)$ which should have a resonant structure near the physical ρ and ω masses. Consequently, (19) reduces to the F_π^{VMD3} form factor advocated above. We thus feel confident that using (19) we obtain a very accurate estimate of the electromagnetic contribution to B^π. Numerically, we find

$$B^\pi_{e.m.} = 6.2 \cdot 10^{-8} = 1.3 \times (\text{unitarity limit}) \; , \qquad (20)$$

where the unitarity limit $B^\pi_{unit} = 4.7 \cdot 10^{-8}$ in this case is strict (only the two-photon intermediate state can contribute). For pion decay, our result is almost identical to the VMD2 result, Eq. (15), with ρ dominance (Quigg and Jackson [9])) which gives 1.4x(unitary limit). On the whole, it seems to be difficult to get a significantly different result in any reasonable model due to the relatively weak dependence of B^{π} on the shape of the form factor. In particular, if one also wants to have agreement with the new data on $\eta \to \mu^+\mu^-$ [19] indicating a value close to the unitarity limit, it is difficult to obtain a value for B^{π} greater than, say, two times the unitarity limit in any model.

For instance, Pratap and Smith [10], using an old nucleon loop model for the $\pi^0\gamma\gamma$ coupling, obtain a value of 3.0 times the unitarity limit for B^{π}, but then get a value for B^{η} which is a factor of 2 (or 3 standard deviations) larger than the current experimental value. Looking now at the experimental value of B^{π} [20], $B^{\pi}_{exp} = (4.7^{+5.1}_{-2.3})$ times the unitarity limit, we consequently find it difficult to attribute this to electromagnetic contributions only. In particular, if the ongoing CERN experiment measuring this decay [21] would confirm this unexpectedly high value with better statistics, we will have reasons to look for other contributions to $\pi^0 \to e^+e^-$. We will investigate a few of them here.

In the standard model of weak interactions the decay can proceed through a first order weak neutral current. A simple, non-relativistic quarkonium model calculation gives, however,

$$B^{\pi}_{NC} \equiv \frac{\Gamma_{\pi^0 \to Z^0 \to e^+e^-}}{\Gamma_{\pi^0 \to \gamma\gamma}} = 9 \left(\frac{G_F}{4\pi\alpha}\right)^2 (m_e m_\pi)^2 \simeq 10^{-15}, \tag{21}$$

which is negligible compared to the electromagnetic contribution. Here one must keep in mind, however, that the non-relativistic quark model is rather inadequate for the pion. For instance, calculating $B^{\pi}_{cc} \equiv \Gamma(\pi\to\mu\nu)/\Gamma(\pi^0\to\gamma\gamma)$ in the same model, we find that the result corresponding to (21) underestimates the measured value by a factor of ~600. Still, multiplying (21) by a factor of this order does not yield an observable value for the weak contribution. To get more reliable estimates of other contributions we use the semi-relativistic model of Hayne and Isgur [22),23] which, with just a few parameters, reproduces a great deal of data for the light mesons, including the pion decay constant and the $\pi^0 \to \gamma\gamma$ width to within 25% (and B^{π}_{cc} to 12%). All axial vector couplings are suppressed by a helicity factor m_e/m_π [cf. Refs. 24),25)]. A large contribution can only come from an effective pseudoscalar interaction. An effective interaction $g^{eff}_q(\bar{q}\gamma^5 q) \times g_e(\bar{e}\gamma^5 e)$ with g^{eff}_q, taking into account the colour and flavour content of the π^0, gives the contribution

$$B^{\pi}_{Ps} = (g^{eff}_q g_e)^2 \times \left(0.77 \cdot 10^4 \, (\mathrm{GeV})^4\right), \tag{22}$$

where we have used the parameters of Hayne and Isgur ($m_u = m_d$ = 220 MeV ; Gaussian wave function ; width β = 300 MeV). Parametrizing $g_q^{eff} \simeq g_e \sim 1/\Lambda$, we find that (22) implies a contribution of the order of the unitarity limit for $\Lambda \lesssim$ 600 GeV. In grand unified theories such as SU(5), Λ is presumably of the order of 10^{15} GeV and thus gives a truly negligible contribution.

However, there are other schemes where the effective Λ could be of the order of 500-1000 GeV, such as that of Pati and Salam [26)]. It is also interesting to note that in, e.g., the composite model of Fritzsch and Mandelbaum [27)] where the Z and W vector bosons are composite states one also expects pseudoscalar partners to these, where there is no a priori reason that they should be very much heavier. It is obviously of interest to further examine the pseudoscalar content of such theories.

Another obvious candidate for a pseudoscalar coupling between the π^0 and the leptons would be a Higgs-like particle. However, these also tend to have couplings proportional to the lepton mass, so that the suppression factor m_e/m_π still enters. For instance the axion, invented to take care of the strong CP problem [28)], has couplings [29)] (Fig. 3)

$$g_{q,\ell} = \begin{cases} 2^{\frac{1}{4}} \sqrt{G_F}\, m_q\, x & q = u \\ 2^{\frac{1}{4}} \sqrt{G_F}\, m_{q,\ell} \frac{1}{x} & q,\ell = d,e \end{cases} \tag{23}$$

where x is at present an unknown parameter.

Assuming $m_{axion} \ll m_\pi$ so that the propagator is $1/m_\pi^2$, and x = 3 which is favoured by a (still unverified) experimental indication of the axion [30)], we find

$$\left(g_q^{eff}\right)^2 = \frac{3 G_F}{\sqrt{2}\, m_\pi^4} \left(x m_u - \frac{1}{x} m_d\right)^2 ; \quad \left(g_e\right)^2 = \sqrt{2}\, G_F \frac{m_e^2}{x^2}$$

which inserted into formula (22) gives

$$B^{\pi}_{axion} \simeq 0.5 \cdot 10^{-10} \tag{24}$$

which is two orders of magnitude below the minimal electromagnetic contribution (this prediction is essentially independent of the assumed value for x).

In general, the neutral pseudoscalar fields of technicolour theories are believed to have the same type of coupling (proportional to the fermion mass), but here the exact coefficients as well as the particle mass are less certain [31]. Thus, it is not inconceivable to get an extra factor of 10 in the amplitude which would begin to have consequences for the experimental $\pi^0 \to e^+e^-$ rate. A detailed account of the technicolour phenomenology in other processes is given in Ref. 31).

5. - CONCLUSIONS

In conclusion, we have provided predictions for the branching ratio $P \to \ell^+\ell^-/P \to \gamma\gamma$ in a bound state quark model. The results can easily be modified to processes where the two-photon coupling is replaced by two gluons.

Special interest has been given to the process $\pi^0 \to e^+e^-$, where it may be possible to detect "exotic" contributions. We claim that it is extremely difficult to make the electromagnetic contribution account for more than two times the unitarity lower bound in any model consistent with the known data on $\pi^0 \to \gamma\gamma$, $\pi^0 \to e^+e^-\gamma$, $\eta \to \mu^+\mu^-$ and $\eta \to \mu^+\mu^-\gamma$. If experiments persist in giving a higher value, some interesting new interaction may be implied. In the plausible case that this is of an effective pseudoscalar type, a general expression for its contribution to the branching ratio has been given using a successful semi-relativistic model for the pion.

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Decay	$\eta \to e^+e^-$	$\eta \to \mu^+\mu^-$	$\eta_c \to \mu^+\mu^-$	$\eta_b \to \mu^+\mu^-$	$\eta_b \to \tau^+\tau^-$
B^P	$1.7\cdot 10^{-8}$	$1.1\cdot 10^{-5}$	$1.9\cdot 10^{-6}$	$4.7\cdot 10^{-7}$	$1.1\cdot 10^{-5}$
B^P_{unit}	$4.5\cdot 10^{-9}$	$1.1\cdot 10^{-5}$	$1.5\cdot 10^{-6}$	$2.7\cdot 10^{-7}$	$1.1\cdot 10^{-5}$

Table - Branching ratios $B^P \equiv \Gamma(P\to\ell^+\ell^-)/\Gamma(P\to\gamma\gamma)$ calculated from Eqs. (4) and (12) for various pseudoscalar meson decays, together with the unitarity limits for the branching ratios calculated through the imaginary part of the two-photon intermediate state only. We have assumed $m(\eta_b) = 9.4$ GeV.

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FIGURE CAPTIONS

Fig. 1 Lowest order QCD diagrams contributing to the decay pseudoscalar $P \to \ell^+\ell^-$.

Fig. 2 Variation of $N(s,0)$ [Eq. (18)] due to gluonic corrections [based on Ref. 2)]. In the shaded region the calculation is not reliable because of the singularity of the quark propagator.

Fig. 3 Contribution to $\pi^0 \to e^+e^-$ from some "exotic" object x^0 with pseudoscalar coupling to quarks and leptons.

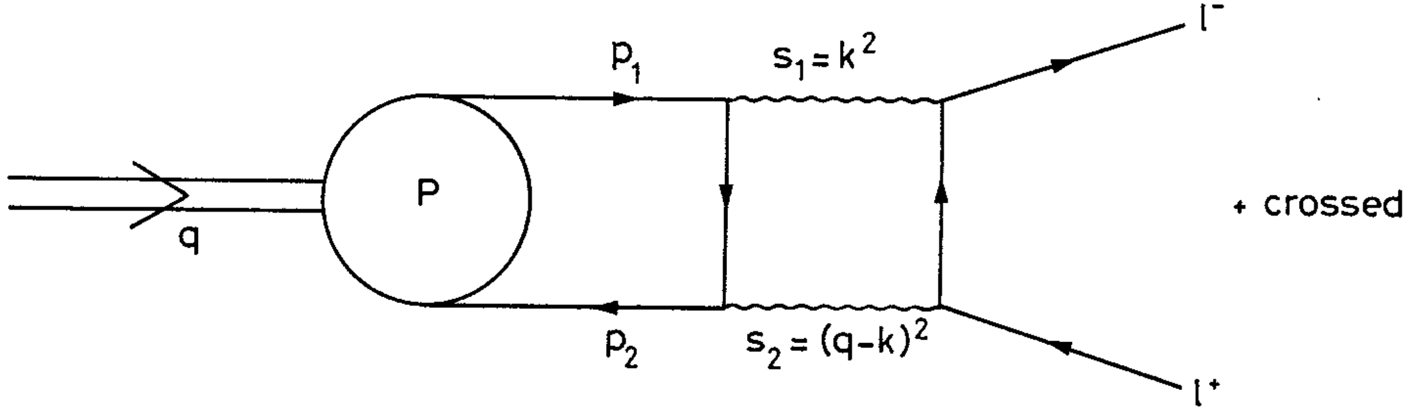

FIG. 1

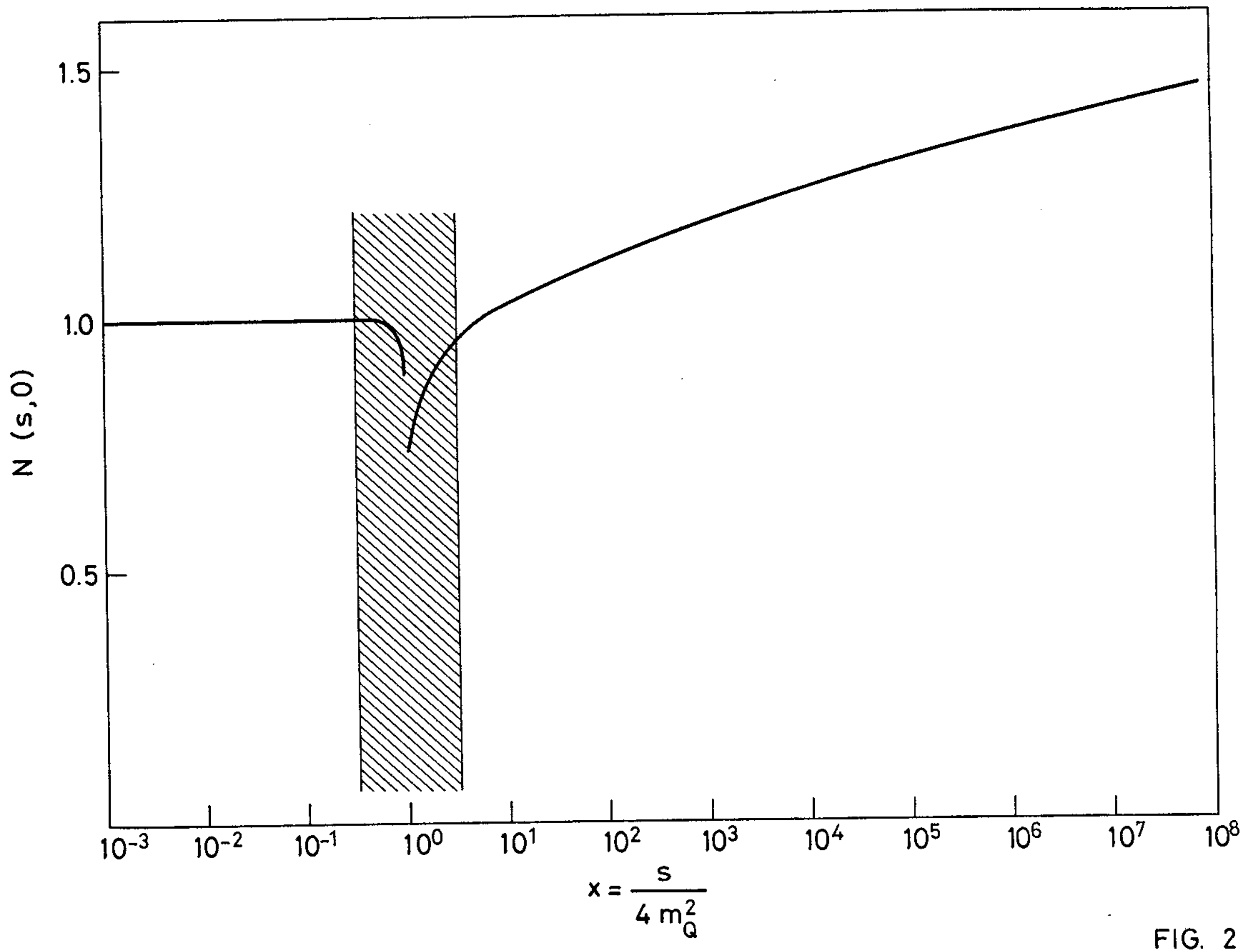
1.5
1.0
0.5
N (s,0)
10^-3
10^-2
10^-1
10^0
10^1
10^2
10^3
10^4
10^5
10^6
10^7
10^8
x = s / 4 m_Q^2

FIG. 2

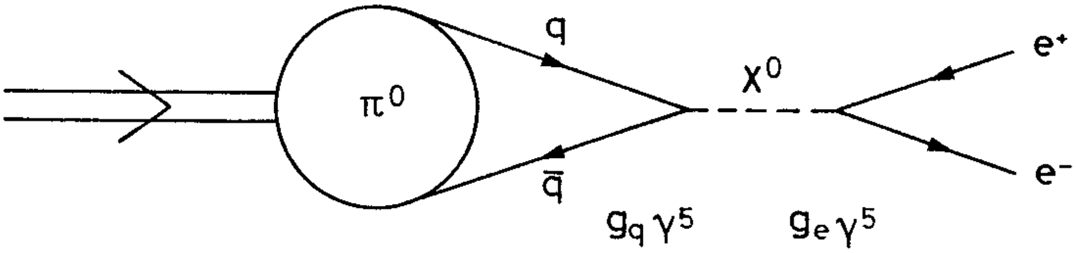

FIG. 3