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RARE KAON DECAYS IN CHIRAL PERTURBATION THEORY

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ABSTRACT

Chiral Perturbation Theory provides a useful framework to analyze rare kaon decays, where long-distance effects are expected to play an important rôle. Some theoretical predictions obtained within this framework for radiative kaon decays are reviewed, together with the present experimental status. Special consideration is given to the $K_L \to \pi^0 e^+ e^-$ decay, which appears as an ideal candidate to look for new signals of CP-violation within the standard model.

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Chiral Perturbation Theory provides a useful framework to analyze rare kaon decays, where long-distance effects are expected to play an important rôle. Some theoretical predictions obtained within this framework for radiative kaon decays are reviewed, together with the present experimental status. Special consideration is given to the $K_L \to \pi^0 e^+ e^-$ decay, which appears as an ideal candidate to look for new signals of CP-violation within the standard model.

1. INTRODUCTION

High precision experiments on rare kaon decays offer the exciting possibility of unravelling new physics beyond the standard model. Searching for forbidden flavour-changing processes at the 10^{-10} level , one is actually exploring energy-scales above the 10 TeV region. The study of allowed (but highly suppressed) decay modes provides, at the same time, very interesting tests of the standard model itself. Electromagnetic-induced non-leptonic weak transitions and higher-order weak processes are a useful tool to improve our understanding of the interplay among electromagnetic, weak and strong interactions. In addition, new signals of CP-violation, which would help to elucidate the source of CP-violating phenomena, can be looked for.

Since the kaon mass is a very low scale, the standard short-distance approach to weak transitions is unable, at the moment, to make accurate predictions for non-leptonic K-decays. Using renormalization group techniques, one gets an effective $\Delta S = 1$ Hamiltonian which is a sum of local four-quark operators , constructed with the light (u,d,s) quark fields only, modulated by Wilson coefficients which are functions of the heavy (W,t,b,c) masses and an overall renormalization scale μ. In order to predict the physical amplitudes, one is still confronted with the calculation of the matrix elements of the quark operators between on-shell pseudoscalar states. This is a difficult problem due to the fact that these hadronic matrix elements are governed by the long-distance behaviour of the strong interactions, i.e., the confinement regime of QCD. That becomes even more involved in the case of radiative modes, due to the additional presence of the electromagnetic interactions.

These difficulties can be avoided by following a completely different approach, which takes advantage of the fact that the pseudoscalar mesons are the lowest energy modes of the hadronic spectrum; they correspond to the octet of Goldstone bosons associated with the spontaneous chiral symmetry breaking of QCD, $SU(3)_L \otimes SU(3)_R \to SU(3)_V$. The effective chiral perturbation theory formulation (CHPT) of the standard model is an ideal framework to describe kaon decays. This is because in K-decays the only physical states which appear are pseudoscalar mesons, photons and leptons, and because the characteristic momenta involved are small compared to the natural scale of chiral symmetry breaking ($\sim 1GeV$).

In this paper I will review some of the results obtained ,within this framework, for radiative decay modes. Sect. 2 contains a brief introduction to the effective lagrangian techniques . The decay $K^+ \to \pi^+ e^+ e^-$ is discussed in sect. 3, and the neutral modes $K_s \to \gamma\gamma$ and $K_L \to \pi^0 \gamma\gamma$ are considered in sect. 4. The present status of the $K_L \to \pi^0 e^+ e^-$ mode, which appears to be an ideal candidate to look for new signals of CP-violation within the standard model, is given in sect. 5. Sect. 6 summarizes finally the main conclusions.

2. CHIRAL PERTURBATION THEORY

In the absence of quark masses, the QCD lagrangian is invariant under independent $SU(N_f)_L \otimes SU(N_f)_R$ transformations of the left and right handed quarks in flavour space ($q_L \rightarrow g_L q_L$, $q_R \rightarrow g_R q_R$). This chiral symmetry, which should be approximately good for the light quark sector (u,d,s), is however not seen in the hadronic spectrum: Although hadrons can be nicely classified in $SU(3)_V$ representations, degenerate multiplets with opposite parity do not exist. To be consistent with this experimental fact, the ground state of the theory (the vacuum) should not be symmetric under the chiral group. The $SU(3)_L \otimes SU(3)_R$ symmetry spontaneously breaks down to $SU(3)_{L+R}$ and, according to Goldstone's theorem, an octet of pseudoscalar massless bosons appears in the theory. The eight lightest hadronic states ($\pi^+, \pi^-, \pi^0, \eta, K^+, K^-, K^0 and \bar{K}^0$) are then identified with the Goldstone bosons of chiral symmetry, their small masses being generated by the quark mass matrix, which explicitely breaks the global symmetry of the QCD lagrangian.

The Goldstone nature of the pseudoscalar mesons implies strong constraints on their interactions, which can be analyzed on the basis of an effective lagrangian. The quark and gluon fields of QCD are replaced by a unitary 3x3 matrix $U(x) \equiv \exp(-i\sqrt{2}\Phi(x)/f)$, incorporating the pseudoscalar octet fields $\Phi(x) \equiv \frac{\vec{\lambda}}{\sqrt{2}}\vec{\phi}$. $U^{ij}(x)$ parametrizes the Goldstone excitacions over the vacuum quark condensate $< \bar{q}^j_L q^i_R >$. Under the chiral group, it transforms as $U \rightarrow g_R U g_L^+$ ($g_{R,L} \in SU(3)_{R,L}$).

At low energies, it is possible to work out the consequences of the chiral symmetry properties of the underlying constituent QCD-theory, by writing the most general effective lagrangian involving the matrix U, which is consistent with the $SU(3)_L \otimes SU(3)_R$ symmetry. Moreover, we can organize the lagrangian in terms of increasing powers of momentum or, equivalently, in terms of increasing number of derivatives. In the low energy domain we are interested in, the terms with a minimum number of derivatives will dominate.

The lowest-dimensional effective chiral lagrangian is uniquely given by

$$L_2 = \frac{f^2}{4}(< D_\mu U D^\mu U^+ > +2B_0 < MU^+ + UM >), \tag{2.1}$$

where the second term is an explicit breaking of chiral symmetry due to the presence of the quark mass matrix $M = diag(m_u, m_d, m_s)$ in the QCD lagrangian. The parameter B_0 ($\simeq - < \bar{u}u > /f^2$) relates the squares of the pseudoscalar meson masses to the quark masses,

$$B_0 = \frac{m^2_{\pi^+}}{m_u + m_d} = \frac{m^2_{K^+}}{m_u + m_s} = \frac{m^2_{K^0}}{m_d + m_s}, \tag{2.2}$$

$f \simeq f_\pi = 93.3 MeV$ is the pion decay constant (to lowest order), $< >$ denotes the trace of the corresponding matrix and the covariant derivative

$$D_\mu U = \partial_\mu U - ieA_\mu[Q, U], \tag{2.3}$$

accounts for the coupling to electromagnetism with the charge matrix $Q = \frac{1}{3}diag(2, -1, -1)$.

At the same order p^2, the effect of strangeness changing non-leptonic weak interactions with $\Delta S = 1$ is incorporated as a perturbation to the strong interaction lagrangian (2.1), which is dominated by a term transforming as a $8_L \otimes 1_R$ operator under chiral $SU(3)$ rotations,

$$L_2^{\Delta S=1} = \frac{G_F}{\sqrt{2}} s_1 c_1 c_3 g_8 (L_\mu L^\mu)_{23} + h.c. + nonoctet\ terms. \tag{2.4}$$

The matrix $L_\mu = if^2 U^+ D_\mu U$ represents the octet of $V - A$ currents, and g_8 is a dimensionless coupling constant which can be extracted from $K \rightarrow 2\pi$ decays, $|g_8| \simeq 5.1$.

Using the lagrangians (2.1) and (2.4), the rates for decays like $K \rightarrow 3\pi$ or $K \rightarrow \pi\pi\gamma$ can be predicted at $O(p^2)$ through a trivial tree level calculation. However, the data are already accurate enough for the next-order corrections to be sizeable. Moreover, due to a mismatch between the minimum number of powers of momenta required by gauge invariance and the powers of momenta that the lowest-order effective lagrangian can provide, the amplitude for any non-leptonic radiative K-decay with at most one pion in the final state ($K \to \gamma\gamma, K \to \gamma l^+l^-, K \to \pi\gamma\gamma, K \to \pi l^+l^-$, ...) vanishes to lowest order in CHPT [1] , i.e., $O(p^2)$. These decays are then sensitive to the non-trivial quantum field theory aspects of CHPT.

At the one-loop level, corresponding to $O(p^4)$, we need to add to the effective lagrangian all possible terms with four powers of momenta, satisfying the symmetry constraints. Each term will introduce an additional coupling constant, not fixed by chiral symmetry. These constants can be seen as remnants of the fundamental theory after quarks and gluons have been integrated out; they contain both long- and short-distance information, and some of them (like g_8) have in addition a CP-violating imaginary part. Since the one-loop divergences are reabsorbed by the $O(p^4)$ couplings, these constants will depend, in general, on an arbitrary renormalization scale.

The complete list of $O(p^4)$ terms describing strong and electromagnetic interactions can be found in ref. [2] , where the numerical values of the corresponding couplings have been determined using experimental information . Two of those terms are relevant for our purposes; in the notation of Gasser and Leutwyler they read

$$L_4^{em} = -ieL_9F^{\mu\nu} < QD_\mu UD_\nu U^+ + QD_\mu U^+ D_\nu U > + e^2 L_{10}F^{\mu\nu}F_{\mu\nu} < UQU^+Q > . \qquad (2.5)$$

When combined with the lowest order $\Delta S = 1$ lagrangian (2.4), the term (2.5) gives rise to physical contributions to the various K-decays we are going to consider here.

Another source of $O(p^4)$ contributions comes from direct $\Delta S = 1$ terms. Although the complete list of possible chiral structures is rather long [3] , only a few terms are relevant for the kind of processes we are going to discuss here (radiative K-decays with at most one pion in the final state) [4] [5] [1]:

$$\begin{aligned} L_4^{\Delta S=1,em} = & -\frac{ieg_8}{2f^2}F^{\mu\nu}\{w_1 < Q\lambda_{6-i7}L_\mu L_\nu > \\ & + w_2 < QL_\mu\lambda_{6-i7}L_\nu >\} \\ & + \frac{1}{2}e^2f^2g_8w_4F^{\mu\nu}F_{\mu\nu} < \lambda_{6-i7}QUQU^+ > \\ & + h.c. . \end{aligned} \qquad (2.6)$$

3. THE DECAY $K^+ \to \pi^+e^+e^-$

In the standard model, $K \to \pi l^+l^-$ transitions ($l = e$ or μ) are allowed as electromagnetically induced non-leptonic weak processes. Their amplitudes should then normally exhibit the typical enhancement factor of a non-leptonic $\Delta I = \frac{1}{2}$ transition. It is instructive to compare the measured $K^+ \to \pi^+e^+e^-$ branching ratio [6] ,

$$Br(K^+ \to \pi^+e^+e^-) = (2.7 \pm 0.5) \times 10^{-7}, \qquad (3.1)$$

with the corresponding branching ratio of the semileptonic decay mode $K^+ \to \pi^0 e^+\nu_e$:

$$\frac{\Gamma(K^+ \to \pi^+e^+e^-)}{\Gamma(K^+ \to \pi^0e^+\nu_e)} = (5.6 \pm 1.1) \times 10^{-6}. \qquad (3.2)$$

This ratio is three orders of magnitude smaller than the characteristic electromagnetic factor $e^4 = (4\pi\alpha)^2$, showing clear evidence for a substantial suppression of the expected $\Delta I = \frac{1}{2}$ enhancement.

It is quite easy to check that the $K^+ \to \pi^+\gamma^*$ transition is in fact forbidden to lowest order in CHPT. The relevant diagrams are shown in fig.1, where the boxes indicate an $O(p^2)$ weak vertex from eq. (2.4) and the circles an $O(p^2)$ electromagnetic vertex from eq. (2.1). Although each diagram gives a nonzero contribution, the sum of them is exactely zero for all q^2 as long as $p^2 = M_K^2, p' = m_\pi^2$ (on-shell mesons).

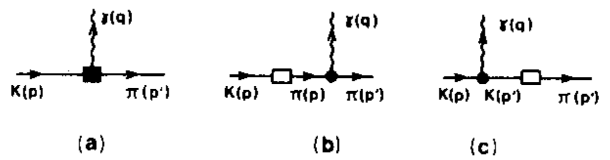

FIGURE 1
Tree level diagrams for $K \to \pi\gamma^*$. Strong (and strong + electromagnetic) vertices are denoted by circles, weak (and weak + electromagnetic) vertices by boxes. A full box or circle indicates a coupling to the photon.

There is a simple reason for this cancellation, which is due to the combined requirements of gauge invariance and chiral symmetry embodied in the effective lagrangian [4]. Gauge invariance requires the $K \to \pi\gamma^*$ amplitude to be of the general form ($G_8 \equiv \frac{G_F}{\sqrt{2}} s_1 c_1 c_3 g_8$)

$$T = G_8 e \epsilon^\mu [q^2 (p+p')_\mu - (M_K^2 - m_\pi^2) q_\mu] \Phi(q^2/M_K^2), \tag{3.3}$$

i.e. at least three powers of external momenta are needed. However the effective chiral lagrangians (2.1) and (2.4) can only produce an amplitude linear in the momenta. In order to get the powers of momenta required by gauge invariance, we have to go at least to $O(p^4)$ in the chiral expansion. The resulting amplitude will contain then a characteristic chiral suppression factor $q^2/(16\pi^2 f_\pi^2) \leq M_K^2/(16\pi^2 f_\pi^2) \simeq 0.18$. As already mentioned in the introduction, the same suppression is present in any radiative K-decay with at most one pion in the final state [1].

The $O(p^4)$ amplitude gets tree level contributions from the same three diagrams shown in fig.1, but now with an $O(p^4)$ vertex from eq. (2.6) in diagram 1.a, and $O(p^2)$ weak (eq. (2.4)) and $O(p^4)$ electromagnetic (eq.(2.5)) couplings in the vertices appearing in diagrams 1.b,c. In addition, one needs to compute all possible one-loop graphs constructed with the lowest-order lagrangians. The result can be written as

$$\Phi(q^2/M_K^2) = \frac{1}{(16\pi^2)} [\Phi_{loop} + \omega_+], \tag{3.4}$$

where Φ_{loop} denotes the loop contribution, which is a known function of q^2/M_K^2, and ω_+ is a combination of chiral couplings,

$$\omega_+ = \frac{-1}{3}(4\pi)^2[w_1^r + 2w_2^r - 12L_9^r] - \frac{1}{3}\log(M_K m_\pi/\mu^2), \tag{3.5}$$

which is expected to be of order one by naive power counting arguments. The upper index r in the chiral couplings indicates the corresponding renormalized coupling evaluated at the renormalization scale μ; measurable quantities like ω_+ are of course μ-independent.

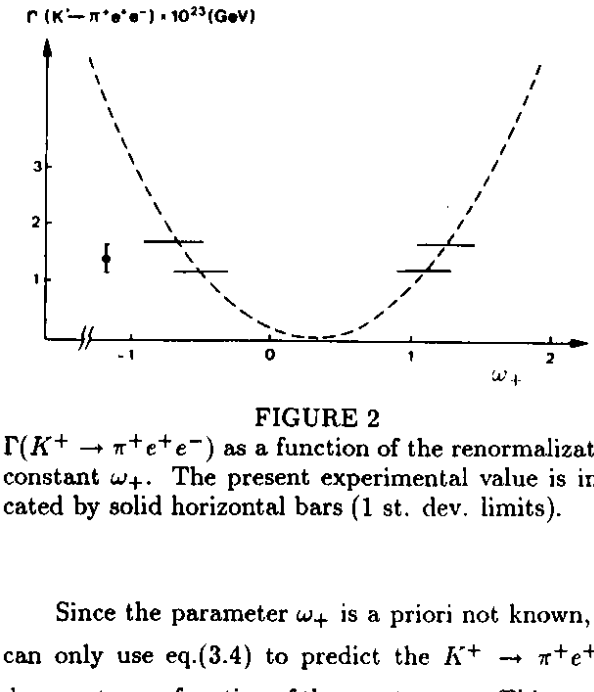

FIGURE 2
$\Gamma(K^+ \to \pi^+ e^+ e^-)$ as a function of the renormalization constant ω_+. The present experimental value is indicated by solid horizontal bars (1 st. dev. limits).

Since the parameter ω_+ is a priori not known, we can only use eq.(3.4) to predict the $K^+ \to \pi^+ e^+ e^-$ decay rate as a function of the constant ω_+. This results in the parabola shown in fig.2. The experimental result (3.1) determines two possible solutions for ω_+ [4]

$$\omega_+^{(1)} = 1.16 \pm 0.08\ , \tag{3.6a}$$

$$\omega_+^{(2)} = -0.57 \pm 0.08\ . \tag{3.6b}$$

Once ω_+ has been fixed, we can predict [4] the rates and q^2 distributions for the related decay modes $K^+ \to \pi^+ \mu^+ \mu^-$,$K_S \to \pi^0 e^+ e^-$ and $K_S \to \pi^0 \mu^+ \mu^-$. The resulting branching ratios are shown in table 1, for the two possible solutions for the constant ω_+. The predicted q^2-distributions can be found in ref. [4]. Additional experimental information on these modes would allow to resolve the remaining twofold ambiguity.

ω_+	1.16 ± 0.08	-0.57 ± 0.08
$Br(K^+ \to \pi^+\mu^+\mu^-)$	61 ± 11	45 ± 10
$Br(K_S \to \pi^0 e^+ e^-)$	0.48 ± 0.20	4.9 ± 0.6
$Br(K_S \to \pi^0\mu^+\mu^-)$	0.10 ± 0.04	1.0 ± 0.1

TABLE 1
Predictions (in units of 10^{-9}) for $Br(K^+ \to \pi^+\mu^+\mu^-)$, $Br(K_S \to \pi^0 e^+ e^-)$ and $Br(K_S \to \pi^0\mu^+\mu^-)$ for the two possible solutions for the coupling constant ω_+.

The value of ω_+ can also be estimated theoretically (but with large uncertainties) by using additional dinamical input . The explicit models investigated so far [7] tend to favour the positive solution for this coupling, i.e. $\omega_+^{(1)}$.

4. THE DECAYS $K_S \to \gamma\gamma$ AND $K_L \to \pi^0\gamma\gamma$

For some processes,like $K_S \to \gamma\gamma$ and $K_L \to \pi^0\gamma\gamma$, the symmetry constraints do not allow any tree level contribution from $O(p^4)$ terms in the chiral lagrangian. Therefore, these processes share the remarkable property of having a finite decay amplitude at the one-loop level (there are no possible counterterms to renormalize divergences!). Both the rates and the spectra are then unambiguosly predicted at $O(p^4)$ [8] [9] [5][10] :

$$Br(K_S \to \gamma\gamma) \simeq 2.0 \times 10^{-6} \ , \qquad (4.1a)$$

$$Br(K_L \to \pi^0\gamma\gamma) \simeq 6.7 \times 10^{-7} \ . \qquad (4.1b)$$

The prediction for $K_S \to \gamma\gamma$ [8][9] nicely agrees with the experimental value [11]

$$Br(K_S \to \gamma\gamma)_{exp.} = (2.4 \pm 1.2) \times 10^{-6} \ . \qquad (4.2)$$

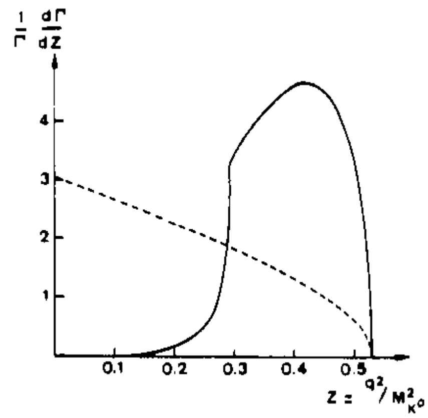

FIGURE 3
Normalized q^2-distribution for $K_L \to \pi^0\gamma\gamma$ (full curve). Also shown for comparison is the phase-space distribution (dashed curve). q^2 is the invariant mass-squared of the photon pair.

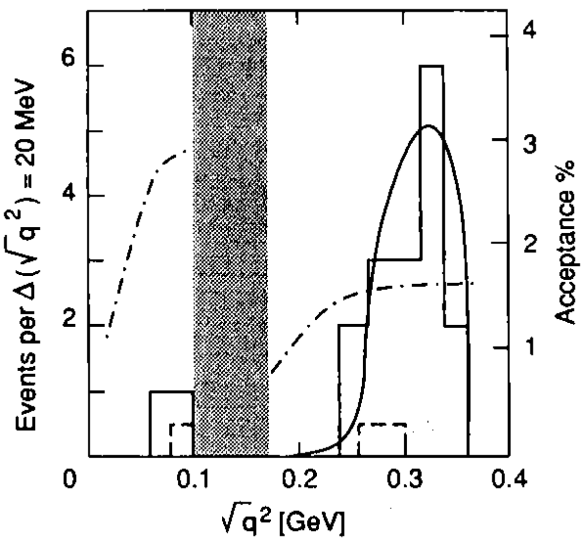

FIGURE 4
Measured invariant $\gamma\gamma$ mass distribution for $K_L \to \pi^0\gamma\gamma$ (solid histogram). The shaded area indicates the region where the NA31 experiment is insensitive. The experimental acceptance is given by the dash-dotted line. The dashed histogram corresponds to the estimated remaining background. The shape of the $O(p^4)$ CHPT distribution is given by the continous curve.

For $K_L \to \pi^0\gamma\gamma$, the spectrum in the invariant mass of the two photons is predicted [5] to have a very characteristic behaviour as shown in fig.3 (full curve). For the sake of comparison, the spectrum expected from phase-space alone has also been plotted (dashed curve). Fig.4 compares the $O(p^4)$ CHPT result with the distribution recently measured by NA31 [12] . The agreement is remarkably good. The same experiment has also measured the $K_L \to \pi^0\gamma\gamma$ branching ratio obtaining a number somewhat higher than (but not inconsistent with) the chiral prediction. This number is, however, very close to the upper bound obtained by the E731 experiment [13] at Fermilab:

$$Br(K_L \to \pi^0\gamma\gamma) = (2.1 \pm 0.6) \times 10^{-6} \quad [12] \quad (4.3a)$$

$$Br(K_L \to \pi^0\gamma\gamma) < 2.7 \times 10^{-6} (90\% C.L.) \; [13]. \quad (4.3b)$$

5. $K_L \to \pi^0 e^+e^-$ AND CP-VIOLATION

The rare decay $K_L \to \pi^0 e^+e^-$ is an interesting process in looking for new CP-violating signatures. If CP were an exact symmetry, only the CP-even state K_1^0 could decay via one-photon emission, while the decay of the CP-odd state K_2^0 would proceed through a two-photon intermediate state and, therefore, its decay amplitude would be suppressed by an additional power of α . When CP-violation is taken into account, however, an $O(\alpha)$ $K_L \to \pi^0 e^+e^-$ decay amplitude is induced , both through the small K_1^0 component of the K_L (ε effect) and through direct CP-violation in the $K_2^0 \to \pi^0 e^+e^-$ transition. The electromagnetic suppression of the CP-conserving amplitude, makes then it plausible than this decay is dominated by the CP-violating contributions.

The short-distance analysis of the product of weak and electromagnetic currents, allows a reliable estimate of the direct CP-violating $K_2^0 \to \pi^0 e^+e^-$ amplitude. Including the effect of "W-box" and "Z^0-penguin" diagrams, which become important for large m_t, the corresponding branching ratio induced by this amplitude has been estimated [14] [15] to be around

$$Br(K_L \to \pi^0 e^+e^-)_{Direct} \simeq 10^{-11} , \qquad (5.1)$$

the exact number depending on the values of m_t and the quark mixing angles.

The indirect CP-violating amplitude induced by the K_1^0 component of the K_L, is given by the $K_S \to \pi^0 e^+e^-$ amplitude times the CP-mixing parameter ε. CHPT allows to relate the $K_S \to \pi^0 e^+e^-$ amplitude to the measured $K^+ \to \pi^+ e^+e^-$ branching ratio. Two possible solutions are then found [4][1], corresponding to the two possible values for the parameter ω_+ discussed in section 3,

$$Br(K_L \to \pi^0 e^+e^-)_{Indirect} \simeq \begin{cases} 1.5 \times 10^{-12} & (\omega_+^{(1)}\ ) \\ 1.5 \times 10^{-11} & (\omega_+^{(2)}\ ) \, . \end{cases} \qquad (5.2)$$

Comparing these values with eq. (5.1), we see that the interesting direct CP-violating contribution is of the same order or even bigger than the indirect one. This is very different from the situation in $K \to \pi\pi$, where the contribution due to mixing completely dominates.

The present experimental upper bound [16] [17] (90 % C.L.)

$$Br(K_L \to \pi^0 e^+e^-)_{Exp.} < 5.5 \times 10^{-9} , \qquad (5.3)$$

is still far away from the expected standard model signal, but the prospects for getting the needed sensitivity of around 10^{-11} in the next few years are rather encouraging [18] . In order to be able to interpret a future experimental measurement of this decay as a CP-violating signature, it is first necessary, however, to pin down the actual size of the two-photon exchange CP-conserving amplitude.

With CP invariance assumed, the most general form ot the amplitude for $K_2^0 \to \pi^0\gamma\gamma$ depends on two independent invariant amplitudes A and B,

$$M(K(p) \to \pi(p')\gamma(q_1)\gamma(q_2)) = \varepsilon_\mu(q_1)\varepsilon_\nu(q_2) M^{\mu\nu}(p, q_1, q_2) \qquad (5.4a)$$

$$M^{\mu\nu}(p, q_1, q_2) = \frac{A(y,z)}{M_K^2}(q_2^\mu q_1^\nu - q_1 q_2 \; g^{\mu\nu}) + \frac{2B(y,z)}{M_K^4}(-pq_1 \; pq_2 \; g^{\mu\nu} - q_1 q_2 \; p^\mu p^\nu + pq_1 \; q_2^\mu p^\nu + pq_2 \; p^\mu q_1^\nu) , \qquad (5.4b)$$

where $y = |p(q_1 - q_2)|/M_K^2$ and $z = (q_1 + q_2)^2/M_K^2$.

Only the amplitude $A(y,z)$ is nonvanishing to lowest notrivial order $O(p^4)$ in CHPT,

$$B(y,z)|_{O(p^4)} = 0 . \qquad (5.5)$$

This has an interesting implication. Because of its helicity structure, the contribution from A(y,z) to the $K_L \to \pi^0 e^+ e^-$ amplitude is strongly suppressed by a factor m_e/M_K [19] , suggesting the dominance of the CP-violating one-photon exchange amplitude discussed before. Taking the absortive part due to the two photon discontinuity as an educated guess of the actual size of the complete amplitude, one gets [4][1]

$$Br(K_L \to \pi^0\gamma^*\gamma^* \to \pi^0 e^+ e^-)|_{O(p^4)} \sim 5 \times 10^{-15} . \qquad (5.6)$$

However, this helicity suppression is not a general property of the two-photon exchange contribution to $K_2^0 \to \pi^0 e^+ e^-$. In general, the amplitude B(y,z) in (5.4b) does not have to vanish and it contributes to $K_2^0 \to \pi^0 e^+ e^-$ even in the limit $m_e = 0$. A constant B(y,z) amplitude is in fact generated at $O(p^6)$ in CHPT [1][7]

$$B(y,z) = -2a_V \frac{G_8 M_K^2 \alpha}{\pi}, \qquad (5.7)$$

where a_V is some unknown coupling. Disregarding the helicity suppressed contribution due to amplitude A(y,z), one gets then

$$Br(K_L \to \pi^0\gamma^*\gamma^* \to \pi^0 e^+ e^-)|_{abs.} = 4.4 a_V^2 \times 10^{-12}. \qquad (5.8)$$

Naive chiral power counting would suggest [1] than a_V is quite small ($\sim 10^{-2}$) and therefore the contribution of this $O(p^6)$ amplitude to the branching ratio is well below the corresponding one-photon exchange result. However it was pointed out [20] [21] [22] that there is a potentially large vector meson exchange diagram shown in fig.5.

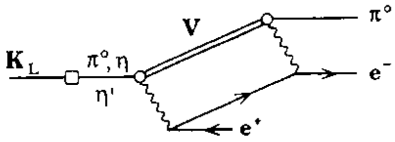

FIGURE 5
Vector meson exchange diagram for $K_L \to \pi^0 e^+ e^-$.

It has been demostrated recently [23] [24] [25] , that the most important coupling constants of the $O(p^4)$ strong lagrangian are in fact dominated by vector meson exchange contributions. One could try then to use Vector Meson Dominance to estimate the $O(p^6)$ $\Phi^2\gamma\gamma$ electromagnetic coupling; inserting the $O(p^2)$ weak vertex (2.4) in the external pseudoscalar legs (fig.6b,c), that would result in a value for the unknown parameter a_V. The number obtained in this way turns out to be quite big [7], $a_V \simeq 0.32$. That is, however, a very dangerous procedure, since direct $O(p^6)$ weak vertices (fig.6a) are neglected.

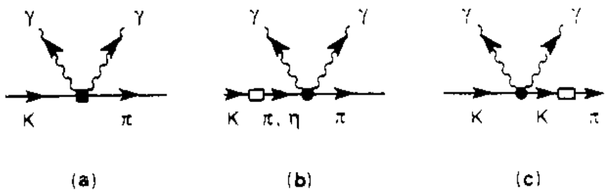

FIGURE 6
Tree level diagrams for $K \to \pi\gamma^*\gamma^*$. The notation is the same as in fig.1 .

In section 3, we have already seen that for the analogous $K \to \pi\gamma^*$ transition, diagram 1a exactely cancels the contribution of diagrams 1b and 1c at $O(p^2)$. At $O(p^4)$, a very strong cancellation still persists; ignoring the contribution of diagram 1a, one would predict a $K^+ \to \pi^+ e^+ e^-$ decay width 30 times bigger than the experimental one [26] ! The existence of similar cancellations in the $K \to \pi\gamma\gamma$ transition has been corroborated [7] by the study of explicit models based in the geometry of the coset space $SU(3)_L \otimes SU(3)_R/SU(3)_V$. It seems then safe to conclude [26][7] that the sum of diagrams 6b and 6c can be taken as a generous upper bound on the actual amplitude, i.e. $|a_V| \leq 0.32$.

The experimental study of the decay $K_L \to \pi^0\gamma\gamma$ can provide a direct measurement of the parameter a_V. A big value of a_V, in addition to change the predicted branching ratio (4.1b), would produce a strong distortion of the photon spectrum, especially at low q^2 values. Moreover, while for $a_V = 0$ the Dalitz-distribution of the two photons does not depend on the second kinematical variable y, a non-negligible a_V-contribution would result in a strong $(y^2 - \bar{y}^2)^2$ dependence [7][$\bar{y} \equiv \frac{1}{2}\lambda^{1/2}(1, q^2/m_K^2, m_\pi^2/m_K^2)$]. To exhibit the effect, the y-spectra is displayed in fig.7 for the extreme (and according to CHPT very unlikely) values $a_V = \pm 1.5$, corresponding to $Br(K_L \to \pi^0 e^+ e^-)|_{abs.} = 10^{-11}$. To enhance the effects, a cut $z \leq 0.3$ has been applied. In comparison the y-spectrum for the pure $O(p^4)$ amplitude ($a_V = 0$) is flat up to $y \simeq 0.28$ and the differential decay rate is much smaller (in fig.7 the spectrum for $a_V = 0$ is scaled up by a factor 10). Fig.8 shows the corresponding spectra in the z-variable. The resulting branching ratios for different values of a_V are given in table 2.

a_V	$Br(K_L \to \pi^0\gamma\gamma)$	$Br(K_L \to \pi^0 e^+ e^-)_{abs.}$
0	0.67	0.08
0.32	0.60	4.5
−0.32	0.89	4.5
1.5	1.6	100
−1.5	3.0	100

TABLE 2
Predictions for $Br(K_L \to \pi^0\gamma\gamma)$ (in units of 10^{-6}) and $Br(K_L \to \pi^0 e^+ e^-)_{abs.}$ (in units of 10^{-13}) for various values of the coupling a_V.

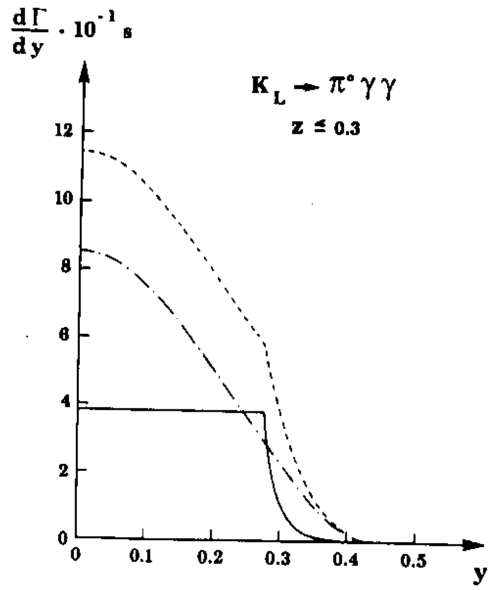

FIGURE 7
y-distributions with a cut $z \leq 0.3$ for $a_V = 1.5$ (dash-dotted curve) and $a_V = -1.5$ (dotted curve). The $O(p^4)$ spectrum ($a_V = 0$, full curve) is scaled up by an order of magnitude.

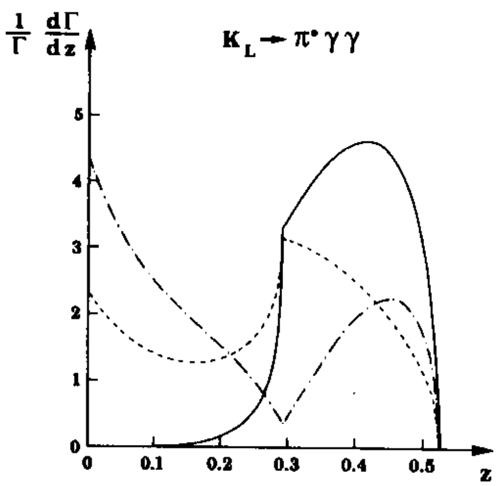

FIGURE 8
z-distributions for $a_V = 1.5$ (dash-dotted) and $a_V = -1.5$ (dotted) compared to $a_V = 0$ (full curve).

It is clear that the recent NA31 measurement [12] of the photon spectrum, shown in figure 4, already rules out the possibility of a big a_V-value. The present data allows to deduce the limits [12]

$$-0.3 < a_V < 0.5 \ \ (90\% C.L.) \ , \qquad (5.9)$$

in good agreement with the chiral expectations. One can therefore conclude that

$$Br(K_L \to \pi^0 e^+ e^-)_{CP-conserving} < 10^{-12}, \qquad (5.10)$$

implying that this decay is in fact dominated by the CP-violating contributions .

6. SUMMARY

CHPT is a very powerful tool to study rare K-decays. This effective lagrangian framework incorporates all the constraints implied by the chiral symmetry of the underlying lagrangian at the quark level, allowing for a clear distinction between genuine aspects of the standard model and additional assumptions of variable credibility usually related to the problem of long-distance dynamics. The low-energy amplitudes of the standard model are calculable in CHPT, except for some coupling constants which are not restricted by chiral symmetry. Those constants reflect our lack of understanding of the QCD confinement mechanism and must be determined experimentally for the time being. Further progress in QCD can only improve our knowledge of those constants, but it cannot modify the low energy structure of the amplitudes.

K-decays are ideally suited for the effective lagrangian approach because the characteristic momenta are small compared to the scale of chiral symmetry breaking and because the particles involved are pseudoscalar mesons, photons and leptons.

The symmetry constraints imply relations among different processes, which can be systematically worked out at a given order in the momentum expansion. In same favourable cases, like the decays $K_S \to \gamma\gamma$ and $K_L \to \pi^0\gamma\gamma$, the rates are unambigously calculable at the one loop level. The recent experimental measurement [11][12] of these two processes, in good agreement with the chiral predictions, has provided an encouraging check of the CHPT techniques.

CHPT provides a convenient language to improve our understanding of the long-distance dynamics. Once the chiral couplings are experimentally known, one can test different dynamical models, by comparing the predictions they give for these couplings with their phenomenologically determined values. The final goal would be, of course, to derive these low energy constants from the QCD lagrangian itself. Although this is a very difficult problem, the recent attempts done in this direction [27] [28] look quite promising.

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