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Rate-equation model for multi-mode semiconductor lasers with spatial hole burning

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Abstract: We present a set of rate equations for the modal amplitudes and carrier-inversion moments that describe the deterministic multi-mode dynamics of a semiconductor laser due to spatial hole burning. Mutual interactions among the lasing modes, induced by high-frequency modulations of the carrier distribution, are included by carrier-inversion moments for which rate equations are given as well. We derive the Bogatov effect of asymmetric gain suppression in semiconductor lasers and illustrate the potential of the model for a two and three-mode laser by numerical and analytical methods.

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1. Introduction

Numerical analysis of multi-mode operation in a semiconductor laser would benefit greatly from a set of coupled rate equations capable of describing the full deterministic dynamics of the lasing modes as well as their mutual interactions. Such description in terms of first-order ordinary differential equations (ODEs) would be perfectly suited for an efficient bifurcation

analysis using existing standard tools. Moreover, the inclusion of spontaneous-emission noise in the modes and recombination noise in the carriers in such frame would be straightforward and could easily be carried through. In fact, in a multi-mode semiconductor diode laser several phenomena have their origin in the deterministic nonlinear dynamical evolution of the lasing modes [1,2].

In this paper we report on the results obtained from the numerical simulation of our new model, whose derivation will be published in [3]. The multi-mode rate equations for the longitudinal modes of a semiconductor laser in this model are expressed in terms of ODEs for the complex modal field amplitudes, the overall population inversion and the various lasing-induced population-inversion gratings (spatial hole burning [4]), described by carrier-inversion moments. This is a big advantage over existing models, which are based on complicated partial-differential and/or integral equations [1,2,5,6]. The model takes into account two interaction mechanisms among modes, i.e. (i) carrier sharing and (ii) mutual coherent optical injection (wave mixing). Different modes are identified by their spatial profiles in the laser cavity and their excitations described by complex amplitudes. The full electron-hole density in the active medium is expanded in a complete set of base functions, which allow for the introduction of inversion moments that not only describe the formation of gratings burned by the modal fields, but also provide the mechanism for mutual injection among the modal fields. Thus, we reproduce the Bogatov-effect on asymmetric side-mode suppression [7] and demonstrate the occurrence of a peculiar periodic multi-mode switching scenario similar to what has been reported and explained in [2] and [8]. We note that other nonlinear gain effects, such as carrier heating and two-photon absorption are neglected. A rough estimate based on the non-linear gain parameter value given in Ryan *et al.* [9] leads to the requirement that the laser should not operate at higher injection current than a few times above threshold.

Our model is partly in line with the theory by Yamada [6], where a coupled system of ODEs for the mode intensities and a partial differential equation (PDE) for the carriers is introduced for the description of the laser which takes into account the material polarization derived from a density-matrix analysis. However, in this theory no full optical spectrum can be derived and, although the spatial interference pattern burned in the carrier distribution due to mode interaction is taken into account (by the “beating vibration on the injected carrier density”), no explicit rate equations for the corresponding oscillating inversion gratings are formulated. The spatio-temporal gain function where upon our model is based [3] is similar to that derived in Yamada [6].

Traveling-wave models based on partial differential (wave) equations for the counter-propagating optical fields in a laser cavity were developed, first by Homar *et al.* [10] and later by Serrat and Masoller [5]. There, the decomposition of the field in laser modes is not explicitly included, but is extracted from the resulting field distributions. The paper by Born *et al.* [11] develops a multimode theory much in the spirit of Yamada [6], where all nonlinear gain terms are dealt with in an effective way, not related to inversion moments as in our model. On the other hand, in the paper by Lariontsev [12], a formulation in terms of coupled rate equations is presented for two counter propagating field amplitudes, the overall inversion and the first inversion moment, similar to our approach.

Other approaches such as the Semiconductor Bloch Equations or the Effective Bloch Equations [13] rely on microscopic many-body theory and Hartree-Fock assumption to describe the semiconductor material. These models require a large set of equations and high computation time. In general, most available models are either “too complicated” to be used when describing a device in a dynamical regime or “too simple” meaning that they lack mode-coupling mechanisms responsible for certain dynamical features.

2. Multi-mode rate equations

The electrical field in the laser cavity is denoted by $E(z, t) = \sum_{j=1, \dots, M} \psi_j(z) E_j(t) e^{i\omega_j t}$, where a slowly varying envelope approximation will be assumed. The rate equations for a semiconductor laser operating in M longitudinal modes are given by

$$\frac{d}{dt} E_k(t) = -\frac{1}{2}(\gamma_k - g_k) E_k(t) + \frac{1}{2} \sum_j \sum_n f_{kj;n} \theta_j (1 + i\alpha_j) N_n E_j(t) e^{i\omega_k t}, \quad (1)$$

$$\begin{aligned} \frac{d}{dt} N_n(t) = \Delta J_n - \frac{N_n}{T_n} - Re \left\{ \sum_{jk} g_j f_{jk;n} E_j^* E_k e^{i\omega_j t} \right\} \\ - Re \left\{ \sum_{jkm} \theta_j (1 + i\alpha_j) f_{jk;nm} N_m E_j^* E_k e^{i\omega_j t} \right\}, \end{aligned} \quad (2)$$

for $j, k = p, p+1, \dots, p+M$ and $m, n = 0, 1, 2, \dots$ and where the meaning of the symbols is summarized in Table 1. Note that the optical mode is a high order mode of the laser cavity, therefore $p \sim 1000$. If we use sine functions for the spatial profiles of the longitudinal modes and expand the population inversion in cosine functions, the f -coefficients are given by:

$$f_{jk;0} = \delta_{jk}; f_{jk;0m} = f_{jk;m0} = f_{jk;m}, (\forall m); f_{jk;m} = \frac{1}{\sqrt{2}} [\delta_{m, |k-j|} - \delta_{m, k+j}], (m \geq 1) \quad (3)$$

$$f_{jk;nm} = \frac{1}{2} [\delta_{|j-k|, |n-m|} + \delta_{|j-k|, n+m} - \delta_{j+k, n+m} - \delta_{j+k, |n-m|}], \quad (n, m \geq 1); \quad (4)$$

The moments N_m describe the formation of inversion gratings. The zero moment, i.e. N_0 , is the average inversion available for lasing and for $m \geq 1$ the inversion moments correspond to inversion gratings, which are induced by the existence of multiple lasing modes in the laser cavity. These gratings are regulated by the spatial diffusion of carriers in the semiconductor gain medium. For this reason, the diffusion of carriers has to be considered in the dynamics of multimode semiconductor lasers and results in various carrier lifetimes T_m . The grating formation is responsible for an asymmetric nonlinear gain contribution favouring longer wavelengths over shorter ones in case of a positive alpha parameter (and the other way around for negative alpha); this is known as the Bogatov effect [7].

3. Results

3.1 Two-mode laser

Figure 1 displays an interesting consequence of mode-mode interaction through grating formation. Here, two modes are considered, 80 GHz apart and with all other parameters equal. The only asymmetry between the two modes is the detuning, where mode 1 is on the red side of mode 2 in the optical spectrum. In the absence of hole burning, i.e. no carrier induced grating ($N_m = 0, \forall m = 1, 2, \dots$), the system is indifferent as to which mode will be excited: the total laser intensity is distributed over the two modes on a one-dimensional manifold and the choice of lasing mode is determined by initial conditions.

When the carrier grating is taken into account ($N_1 \neq 0$), the system has two steady states (equilibria) in the sense that either one of the modes can be lasing, the other being off. However, only one of the equilibrium states is stable and for positive alpha this is the long-wavelength mode. This effect, named Bogatov effect [7], is responsible for the asymmetric

Table 1. Explanation and meaning of the various symbols in Eqs. (1) and (2)

symbol	unit	Name	Value in Figs. 1, 2 and 3
E_k	1	(complex) field amplitude for mode k	-
N_o	1	Overall population inversion w.r.t. lasing threshold	-
N_m	1	m -th population inversion moment	-
k, j	1	Optical mode number	1000, 1001, 1002
m, n	1	Inversion-moment number	0, 1, 2
α_k	1	Linewidth parameter for mode k	3
γ_k	1/s	Cavity loss rate for mode k	1/ps
g_k	1/s	Linear gain for mode k	$g_1 = 0.9996 \text{ ps}^{-1}$, $g_2 = 0.9999 \text{ ps}^{-1}$, $g_3 = 1.0 \text{ ps}^{-1}$
θ_k	1/s	Differential gain coefficient for mode k	5000 s^{-1}
$f_{kj:n}$	1	Coupling coefficient of modes k and j via n -th moment	See Eq. (3)
$f_{kj:nm}$	1	Coupling coefficient of moments n and m via modes j and k	See Eq. (4)
$\omega_{jk} \equiv \omega_j - \omega_k$	Rad/s	Angular mode-frequency difference	$\Delta\omega = 2\pi \times (80 \text{ or } 12 \text{ GHz})$
ω_k	Rad/s	Optical angular frequency of mode k	$\omega_3 > \omega_2 > \omega_1$
ΔJ_k	1/s	k -th injection-current moment w.r.t. threshold current	$\Delta J_o = 1.0 \cdot 10^{17} \text{ s}^{-1}$
T_m	s	lifetime for moment m	1 ns

side-mode suppression observed in multi-mode semiconductor lasers. It is also sometimes referred to as a four-wave mixing process. The qualitative explanation for this phenomenon is that if two modes are lasing simultaneously, their fields create an oscillating grating in the inversion (N_1). This grating causes the waves in each mode to be scattered into the other mode, thus creating additional gain or loss in that other mode. If mode 2 is lasing, i.e. $P_2 > 0$ and mode 1 is off, i.e. $P_1 = 0$, the steady-state solution of Eqs. (1) and (2) leads to an explicit expression for the grating-induced gain for mode 1, i.e.

$$g_{Bogatov} = \frac{1}{4} \frac{\theta_2 (g_1 + g_2) \left(a_2 \Delta\omega - \frac{1}{T_1} - \theta_2 P_2 \right)}{\Delta\omega^2 + \left(\frac{1}{T_1} + \theta_2 P_2 \right)^2} P_2, \quad (5)$$

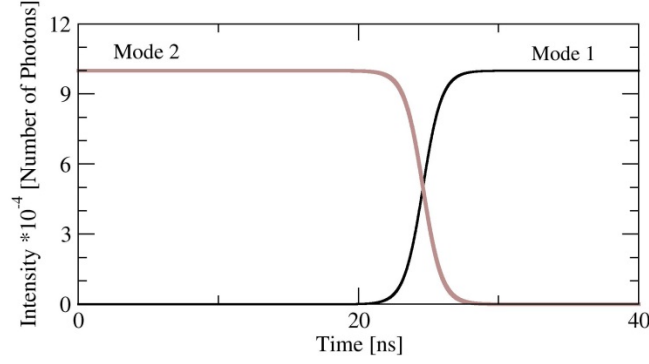


Fig. 1. Transition of lasing mode due to modal interaction: the frequency spacing between the two modes is chosen as 80GHz. The system is prepared in the short wavelength mode and gradually evolves to the long wavelength mode. The time is measured in nanoseconds and the intensity is given in number of photons. Parameters are as in Table 1. The laser is pumped twice above threshold.

where $\Delta\omega = \omega_2 - \omega_1$. The switch-on of mode 1 in Fig. 1 is driven by the Bogatov gain Eq. (5). For the parameter values taken in Fig. 1 we easily calculate $g_{Bogatov} \approx 0.0015 ps^{-1}$, which is consistent with the observed switching time of $\sim 5ns$.

As another application of our model, we will study side-mode suppression (SMS) in case of two-mode operation, where one mode is below threshold such that it cannot lase, while the other mode is operating in a CW steady state. For a realistic treatment of SMS one must consider the effect of spontaneous emission (SE) in the model. The simplest way to do that is by deriving from Eq. (1) the equations for the mode intensities $P_k = |E_k|^2$, and adding the SE-rate R to the right hand sides. Then keeping in Eq. (2) only the average inversion N_0 , this leads to

$$\frac{dP_1}{dt} = R - (\gamma_2 - g)P_1 + \theta N_0 P_1 + g_{Bogatov} P_1 \quad (6)$$

$$\frac{dP_2}{dt} = R + \theta N_0 P_2 \quad (7)$$

$$\frac{dN_0}{dt} = \Delta J_0 - \frac{N_0}{T} - \theta(P_1 + P_2)N_0 - g(P_1 + P_2) \quad (8)$$

Solving for the steady-state solution yields for the ratio of the two intensities we find for the side mode suppression ratio

$$10 \log_{10} \left(\frac{P_1}{P_2} \right) = -10 \log_{10} \left(1 + \frac{P_2}{R} (\gamma_2 - g - g_{Bogatov}) \right) \approx -30dB, \quad (9)$$

for parameter values as in Table 2. The quantitative result Eq. (9) is very reasonable and in agreement with known values for side mode suppression. Here, we assumed positive Bogatov gain for mode 1 (taken as the red-shifted mode). Note that, if the Bogatov gain were sufficiently high to compensate the effective loss $\gamma_2 - g$ of mode 1, the intensity ratio in Eq. (9) would become of order one, corresponding to a situation where mode 1 survives at the expense of mode 2. In that case the considerations leading to Eq. (9) nor the equation itself do not apply anymore.

Table 2. Parameter values taken in Eq. (9)

g	Linear gain (both modes)	1 ps^{-1}
γ_2	Linear loss (mode 2)	1.02 ps^{-1}
P_2	Intensity (mode 2)	10^5
R	Spont. Emission Rate (both modes)	1 ps^{-1}
g_{Bogotov}	Bogotov gain (mode 1)	1.0015 ps^{-1}

3.2 Three-mode laser

Figure 2 shows the simulation result for a 3-mode laser with mode spacing 12 GHz. With this appreciably smaller mode spacing the Bogotov gain Eq. (5) is ~ 6.7 times larger, i.e. $\sim 0.01 \text{ ps}^{-1}$, enough to overcome the small differences in linear gains as indicated in Table 1. Once there is intensity in mode 3 (dashed line in Fig. 2), it automatically feeds the other modes through the effective dynamical (Bogotov) gain effect (see the above-given discussion). The reddest mode (1) takes the most advantage of this gain as it is most favored by the dynamical gain and starts building up intensity. This decreases the amount of gain available for operation in mode 3. While the intensity builds up in mode 1, mode 2 experiences two effects, i.e. suppression by mode 1 and enhancement by mode 3. Apparently, mode 2 survives only during the short time interval, where mode 1 is decreasing under influence of its higher loss and mode 3 is recovering. As soon mode 1 starts to grow, it effectively suppresses mode 2. This cycle then repeats itself. We note in Fig. 2 that the relaxation oscillation (~ 3.8 GHz) plays an important intermingling role in the above-described scenario and that the slow dynamics-induced oscillation corresponds to ~ 760 MHz, which seems to define a period-5 limit cycle.

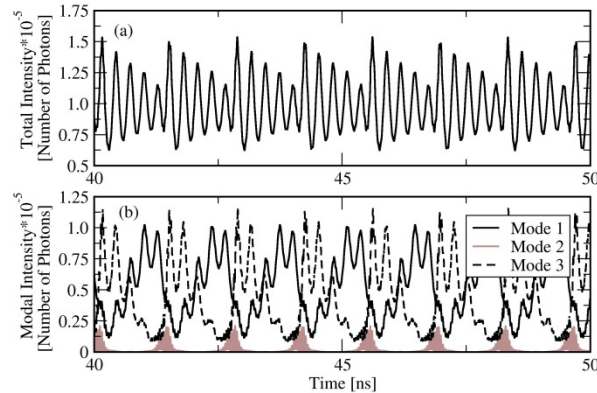


Fig. 2. Sequential on-off switching of modes in the mode-resolved time series. (a) shows the total photon number and (b) the intensity of the individual modes. The parameters are as in Table 1. The mode spacing is $\Delta\omega = 2\pi \times 12$ GHz. Note the period-5 dynamics.

The dynamics shown in Fig. 2 is for one particular set of parameters and we do this in order to give an example of the complex dynamics that our model can show. The intention of Fig. 2 is therefore that of a demonstration, rather than an exact reproduction of an experimental observation. The peculiar periodic multi-mode switching scenario is similar to what has been reported and explained in [2] and [8].

Another interesting aspect of these dynamics becomes evident once we look at the optical spectra in Fig. 3 for the same case as Fig. 2. Each mode contains spectral components of itself and other modes. Interestingly, dynamics at the frequency of the middle mode (mode 2) are

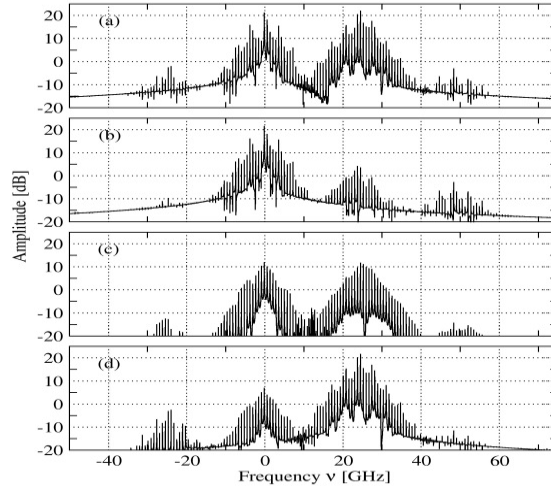


Fig. 3. Optical spectra of the dynamics depicted in Fig. 2. Plot (a) shows the spectrum of the total field $\sum E_j(t)e^{i\omega_j t}$; plots (b), (c) and (d) show modes 1, 2 and 3, respectively. The relaxation oscillation frequency is ~ 3.6 GHz and the ~ 720 MHz peak fine structure corresponds to the period-5 oscillation in Fig. 2 and is caused by the system dynamics and mode competition. Note that each mode 1 and 3 contains spectral components of itself and the other, while the content of mode 2 is at the frequencies of the two side modes.

rather suppressed as can be concluded from the absence of any substantial spectral content at ~ 12 GHz. Clearly, mode 2 is only driven by the injection fields of the side modes and generates no frequency content at its “own” frequency. Mode 2 is used rather as ‘stepping stone’ while the energy bounces between mode 1 and mode 3. Although there are three active (spatial) modes inside the laser, a diffraction-grating-resolved spectrum taken from the laser output would show only two dominant frequencies separated by ~ 24 GHz.

4. Conclusion

We have proposed a new rate-equation model for a multi-mode semiconductor lasers and demonstrated its applicability to two and three-mode lasers. Complex dynamical behavior has been demonstrated as the result of two main parametric interaction mechanisms among different modes, i) competition for the same gain and ii) coherent injection through self-induced gain gratings. For a positive value of the α -parameter, side modes with longer wavelengths compared to the dominant mode are shown to experience enhanced gain, whereas side modes with shorter wavelengths have reduced gain (Bogatov effect). We have also derived an expression for the side mode suppression ratio, which predicts reasonable values. The model can easily be extended with Langevin-noise terms in order to account for the effects of spontaneous-emission noise into the lasing modes as well as carrier recombination shot noise as in [14]. This will be published separately.

The simulated spectrum shows that the spectral contents belonging to one mode shows features of other modes as well. This is apparently due to parametric interactions. Therefore, an observation of the output field will not provide full information on each modal amplitude inside the laser. This is clearly visible in the spectrum shown in Fig. 3.

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