Rate Performance Objectives of Multi-hop Wireless Networks

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Outline

I. Introduction and problem statement

II. Model of ad-hoc network

III. Our findings

IV. Conclusions

I. Introduction

- Goal: design MAC and routing protocol for given network technology.

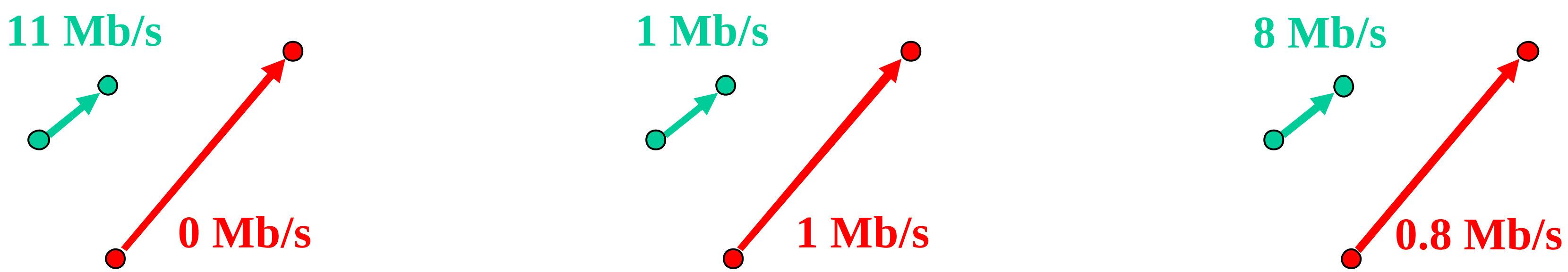
  Q: What performance objective to use?

- Performance objectives in multi-hop wireless networks:
  - Rate based objectives (802.11, UWB, CDMA)
  - Energy based objectives (sensor networks)
  - Combined
- We focus on rate-based objectives

Rate-based Performance Objectives

- **Total capacity:** maximize sum of rates of all flows
- **Max-min fairness:** a rate of a flow cannot be increased at the expense of a flow with an already smaller rate.
- **Proportional fairness:** maximize sum of logs of rates of all flows.
- **Transport rate** of a flow = rate * distance
  All above metrics applicable to transport capacities
- We can also define metrics corresponding to these objectives, when evaluating performance rather than designing network.

Efficiency and Fairness



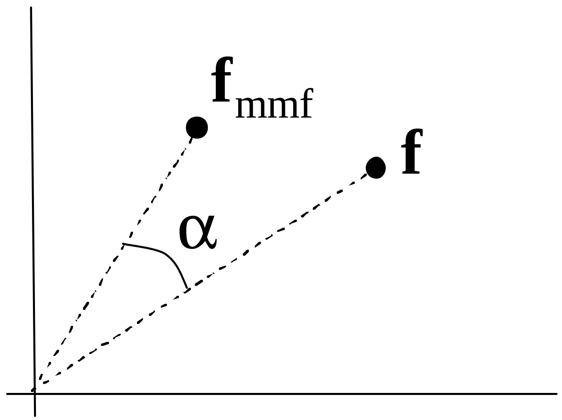
Maximal total capacity *Max-min fairness* *Proportional fairness*

- Antagonism between efficiency and fairness
- Maximizing total capacity is unfair (like in wired networks)
- Max-min fairness is inefficient (unlike wired networks)

Q: Given a network technology, what design objective to use to make a compromise between efficiency and fairness?

Performance Indices

- **Q: How to quantify efficiency and fairness?**
- *Efficiency index of rate allocation* **f**: $\Sigma\, \mathbf{f}_i \,/\, \Sigma\, \mathbf{f}^*_i$
  where $\mathbf{f}^*$ is rate allocation that maximizes total capacity.
- *Fairness index of rate allocation* **f**: $\cos^2(\alpha)$
  where α is angle between **f** and max-min fair allocation $\mathbf{f}_{mmf}$
  when MMF rates are equal, this coincides with Jain fairness index.

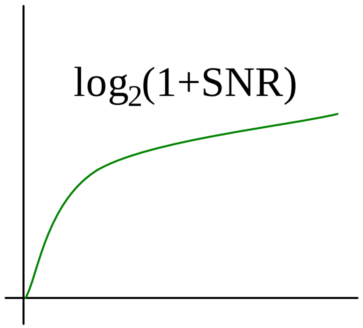


II. Model of Ad-hoc Wireless Network

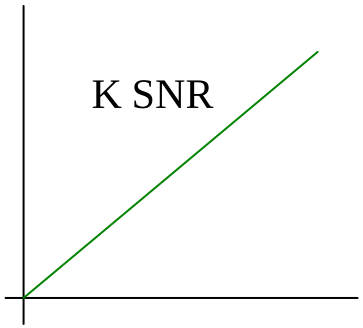
- Physical model properties
- MAC protocol
- Routing protocol and traffic flows
- Power control

Physical Model Properties

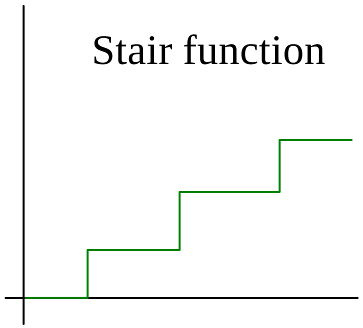
- Point-to-point links: no broadcast, relay channels, multi-user detection
- Constant and positive attenuation h_{ij} between any two points i,j
- Interference allowed, no collisions.
- Signal-to-noise ratio at the receiver of link l: ratio of received power over white noise plus interference of other transmitters.
- Rate r(SNR) is strictly increasing function.



Gaussian channel



Ultra-wide band



Variable rate 802.11
CDMA/HDR

MAC Protocol

- **Schedule** consists of several **slots**, each of length a_n.
  In each slot, nodes have different **power allocations** $\mathbf{p}_n$.

$\alpha_1, \mathbf{p}_1$ | $\alpha_2, \mathbf{p}_2$ | $\alpha_3, \mathbf{p}_3$ | $\alpha_4, \mathbf{p}_4$ | …

- In each slot, node achieves rate $\mathbf{x}_n$
  as a function of SNR and corresponding coding.
- **Long term average rate** is average rate over all slots

$$\bar{\mathbf{x}} = \sum_n a_n \mathbf{x}_n$$

- We assume ideal control plane – no protocol overhead

Routing Protocol and Traffic Flows

- Traffic demand is described by end-to-end flows.
- Each flow is unicast of multicast.
- Each flow is mapped to one path (single-path routing) or more paths (multi-path routing)
- Mathematical formulation of constraints on average rates:

$$\mathbf{f} = F\mathbf{y},\ \mathbf{x} = R\mathbf{y}$$

$F_{f,p}$ = 1 if path *p* belongs to flow *f*, else 0
$R_{p,l}$ = 1 if path *p* uses link *l*, else 0

Power Constraint

- **Peak power constraint**: maximum power of a symbol in a codebook. Integrated in model trough rate function.
- **Transmission power constraint** P^{MAX}: average power of transmission in given slot. Corresponds to average power of codebook used.
- **Long term average transmission power constraint** P^{MAX}_{avg}: average power dissipated over the schedule.
  It corresponds to battery lifetime:

$$T_{lifetime} = E_{battery} / (P^{MAX}_{avg}\ u)$$

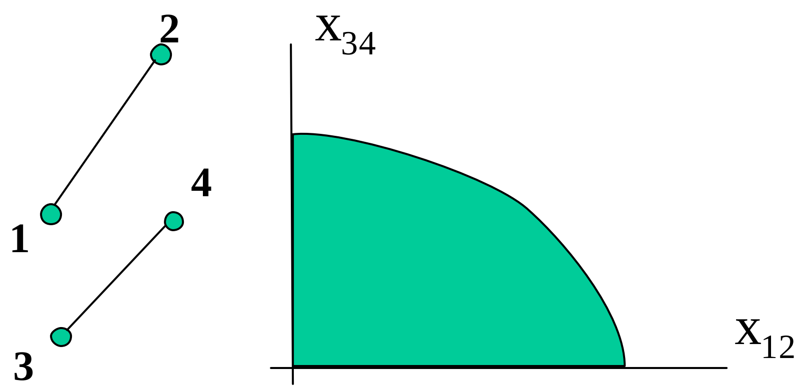
u - fraction of time node has data to send

Optimization Problem

- **Input constraints** (due to technology and user preferences): transmission power constraint, rate function, attenuation
- Given network topology and traffic matrix, we have set of feasible rates and set of feasible transport rates.

Q: for each performance objective, find optimal end-to-end rates on given feasible rates and feasible transport rates set.

- Non-convex optimization – heuristic needed sometimes.

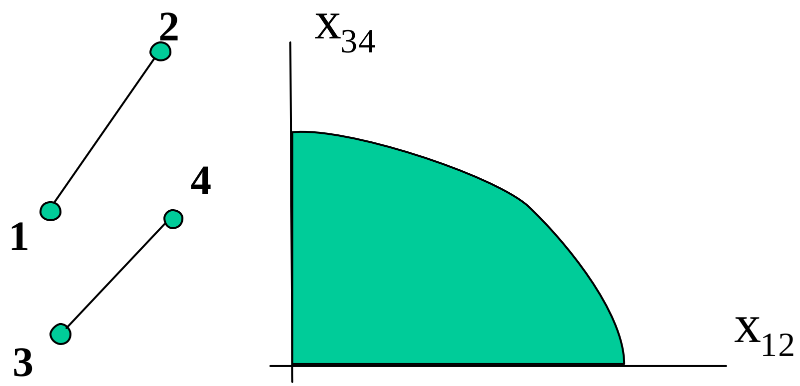


III. Finding 1: Max-min Fairness is inefficient

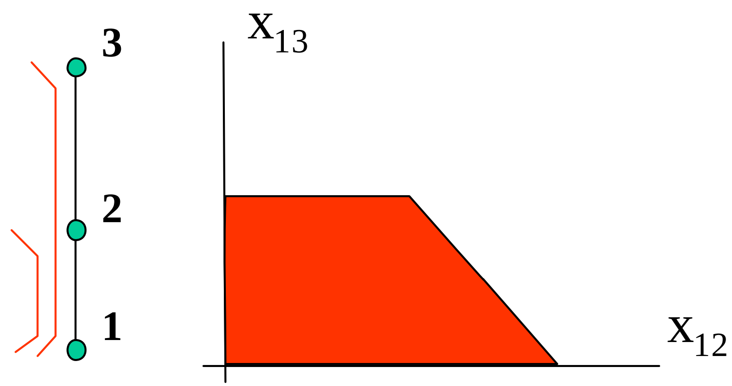
- **Theorem:** Max-min fair rate allocation on arbitrary network, without battery lifetime constraint, has all rates equal.
- **Theorem:** Max-min fair transport rate allocation have all transport rates equal.

Result on Max-Min Fair Allocation is due to Solidarity Property

- *Solidarity property* – a set has solidarity property if one can always trade value of one coordinate for other coordinate.
- MMF allocation on set with solidarity has all coordinates equal.
- Not all convex sets have solidarity property.
- Feasible set of rates of wireless network has solidarity property; Feasible set of transport rates also has solidarity property.

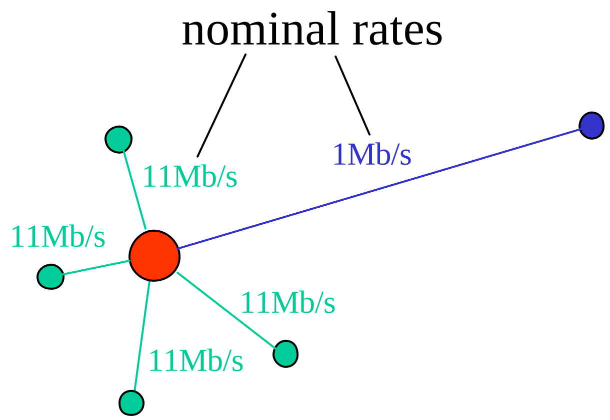


Example **with** solidarity property:
Feasible set of wireless network



Example **without** solidarity property:
Feasible set of wired network

Application to 802.11 Network



Actual rates of all flows in the example: 1 Mb/s!

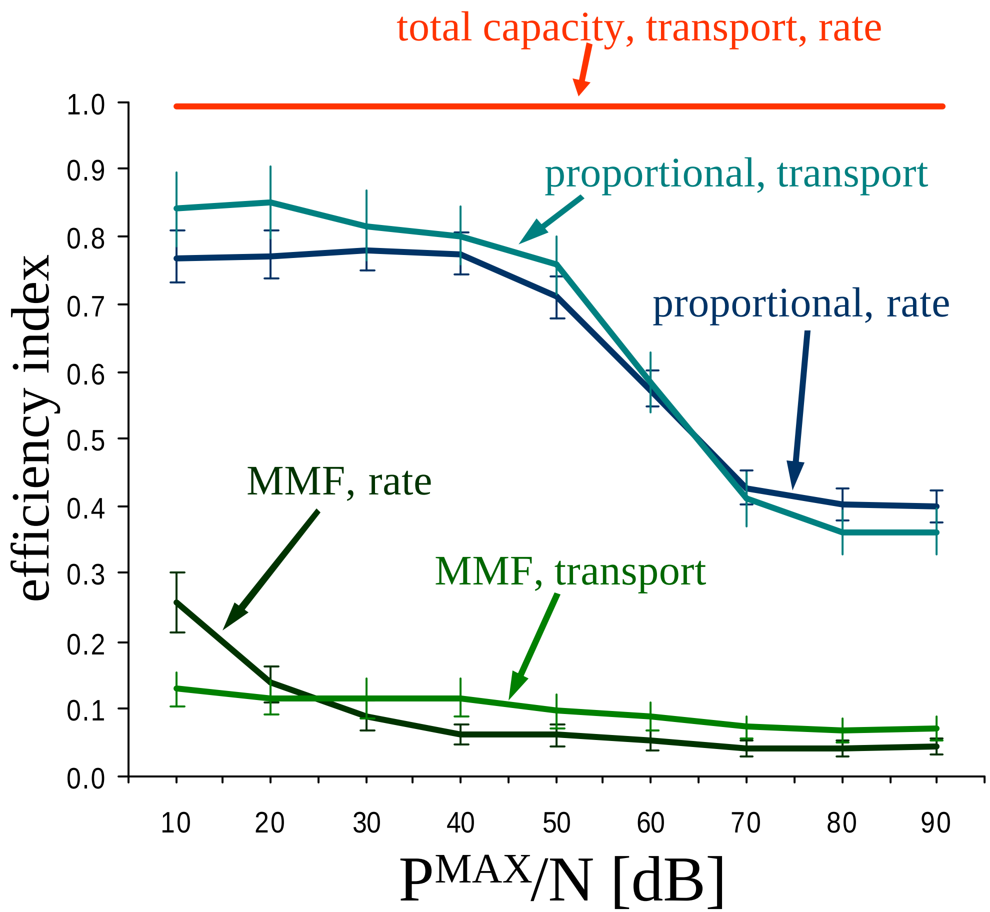
- All nodes have equal probability to gain access to channel
- All nodes have packets of equal sizes: slower nodes take more time to send packet.
- System is essentially max-min fair
- Conclusion: All nodes will have the same average rate, regardless of coding used

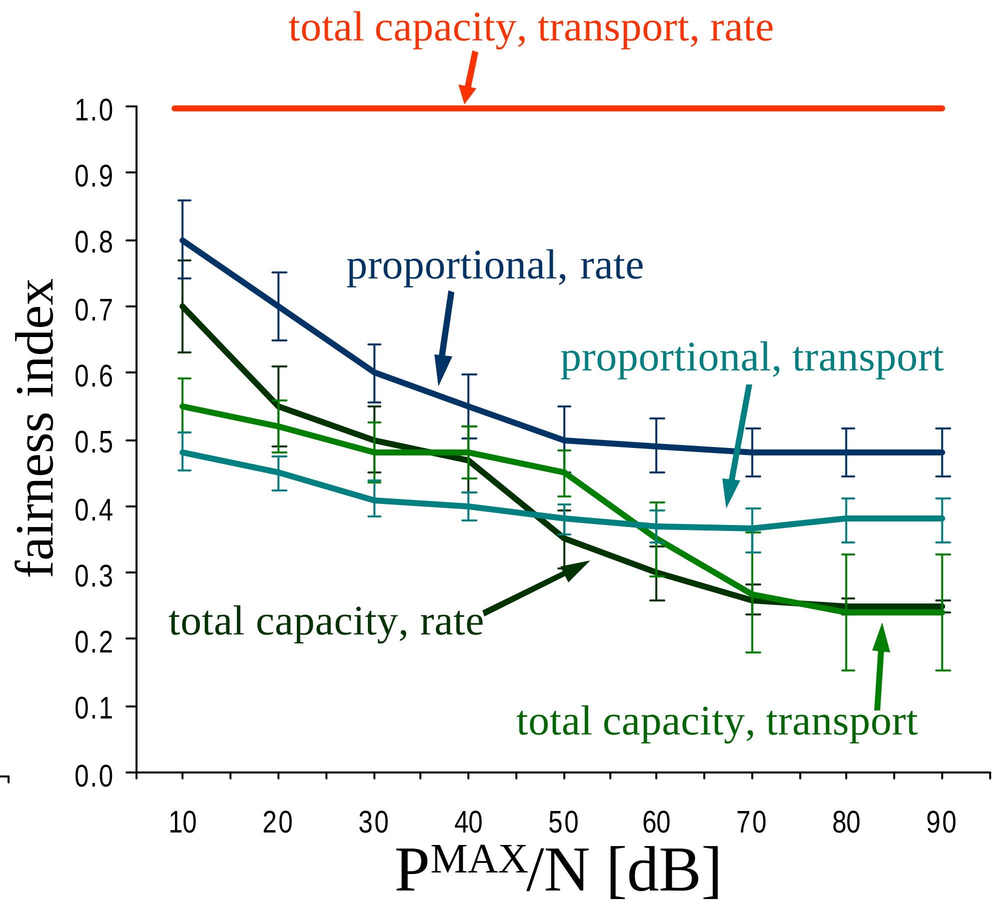
Phenomenon is not due to physical layer choice, but due to choice of design objective.

III. Finding 2: Maximizing Total (Transport) Capacity is Grossly Inefficient

- **Theorem:** Asymptotic results on maximizing (transport) capacity
  - when power constraint P^{MAX} goes to infinity, only the most efficient flows will have positive rate; the rates of other flows will be zero.
  - The same hold for maximizing transport rates – transport rates and rates of inefficient flows will be zero.

III. Finding 3: Proportional Fairness is Good Compromise





IV. Conclusions

- We analyzed three rate-based performance objectives: max. total capacity, max-min fairness, proportional fairness
- We defined a general model of wireless network, that incorporates most of the existing networks.
- Our findings on the general model:
  - *Total capacity* is unfair metric, especially for large power constraints; longer and inefficient flows get small or zero rate.
  - *Max-min fairness* is inefficient metric. Under no battery lifetime constraints, all flows get the same rate, that is the rate of the most inefficient flows.
  - *Proportional fairness* maintains fairness while increasing efficiency. It is robust to changes in power constraints. It is the optimal performance objective.

Future Work

- Incorporate power into the metric, rather then in constraint.
- Inspect influence of random fading on the results.