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Rayleigh wave propagation in transversely isotropic magneto-thermoelastic medium with three-phase-lag heat transfer and diffusion

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Abstract

The present research deals with the propagation of Rayleigh wave in transversely isotropic magneto-thermoelastic homogeneous medium in the presence of mass diffusion and three-phase-lag heat transfer. The wave characteristics such as phase velocity, attenuation coefficients, specific loss, and penetration depths are computed numerically and depicted graphically. The normal stress, tangential stress components, temperature change, and mass concentration are computed and drawn graphically. The effects of three-phase-lag heat transfer, GN type-III, and LS theory of heat transfer are depicted on the various quantities. Some particular cases are also deduced from the present investigation.

Keywords: Transversely isotropic, magneto-thermoelastic, three-phase-lag heat transfer, Wave propagation

Introduction

There are two types of surface waves namely Rayleigh wave and Love wave. These waves have primary importance in earthquake engineering. Rayleigh (1885) first investigated the waves that exist near the surface of a homogeneous elastic half-space and named it as Rayleigh waves. Rayleigh wave exists in a homogeneous, elastic half-space whereas Love wave requires a surficial layer of lowers wave velocity than the underlying half-space. The propagation of waves in thermoelastic materials has numerous applications in various fields of science and technology, earthquake engineering, seismology, nuclear reactors, aerospace, submarine structures, and in the non-destructive evaluation in material process control and fabrication.

Green and Naghdi (1992, 1993) dealt with the linear and the nonlinear theories of thermoelastic body with and without energy dissipation. Three new thermoelastic theories were proposed by them, based on entropy equality. Their theories are known as thermoelasticity theory of type I, the thermoelasticity theory of type II (i.e., thermoelasticity without energy dissipation), and the thermoelasticity theory of type III (i.e., thermoelasticity with energy dissipation). On linearization, type I becomes the classical heat equation whereas on linearization type-II as well as type-III, theories give a finite speed of thermal wave propagation.

The effects of heat conduction upon the propagation of Rayleigh surface waves in a semi-infinite elastic solid is studied for transversely isotropic thermoelastic (TIT) materials by Sharma, Pal, and Chand (2005) and Sharma and Singh (1985). Marin (1997) had proved the Cesaro means of the kinetic and strain energies of dipolar bodies with finite energy. Ting (2004) explored a surface wave propagation in an anisotropic rotating medium. Othman and Song (2006, 2008) presented different hypotheses about magneto-thermoelastic waves in a homogeneous and isotropic medium. Kumar and Kansal (2008a) investigated the effect of rotation on the characteristics of Rayleigh wave propagation in a homogeneous, isotropic, thermoelastic diffusive half-space in the context of different theories of thermoelastic diffusion, including the Coriolis and Centrifugal forces. Sharma and Kaur (2010) considered Rayleigh waves in rotating thermoelastic solids with the void. Mahmoud (2011)

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investigated the Rayleigh wave velocity under the effect of rotation, initial stress, magnetic field, and gravity field in a granular medium. Abouelregal (2011) studied Rayleigh wave propagation in thermoelastic half-space in the context of dual-phase-lag mode. Abd-Alla, Abo-Dahab, and Hammad (2011); Abd-Alla, Abo-Dahab, Hammad, and Mahmoud (2011); and Abd-Alla and Ahmed (1996) studied Rayleigh waves in an orthotropic thermoelastic medium under the influence of gravity, magnetic field, and initial stress.

Marin, Baleanu, and Vlase (2017) have discussed the effect of micro-temperatures for micropolar thermoelastic bodies. Mahmoud (2014) studied the effect of the magnetic field, gravity field, and rotation on the propagation of Rayleigh waves in an initially stressed non-homogeneous orthotropic medium. Singh, Kumari, and Singh (2014) solved the basic equations for the Rayleigh wave on the surface of TIT dual-phase-lag material under magnetic field. Kumar and Kansal (2013) investigated the propagation of Rayleigh waves in a TIT diffusive solid half-space. Kumar and Gupta (2015) investigated the effect of phase lags on Rayleigh wave propagation in the thermoelastic medium. Biswas, Mukhopadhyay, and Shaw (2017) dealt with the propagation of Rayleigh surface waves in a homogeneous, orthotropic thermoelastic half-space in the context of three-phase-lag models of thermoelasticity. Kumar, Sharma, Lata, and Abo-Dahab (2017) and Lata, Kumar, and Sharma (2016) investigated the Rayleigh waves in a homogeneous transversely isotropic magneto-thermoelastic (TIM) medium with two temperatures, Hall current, and rotation. Despite this, several researchers worked on a different theory of thermoelasticity as Chauthale and Khobragade (2017); Ezzat and AI-Bary (2016, 2017); Ezzat, El-Karamany, and El-Bary (2017); Ezzat, El-Karamany, and Ezzat (2012); Hassan, Marin, Ellahi, and Alamri (2018); Kumar, Kaushal, and Sharma (2018); Kumar, Sharma, and Lata (2016a, 2016b, 2016c); Lata and Kaur (2019a, 2019b, 2019c, 2019d, 2019e); Lata et al. (2016); Marin (2009, 2010); Marin and Craciun (2017); Marin, Ellahi, and Chirilă (2017); Marin and Nicaise (2016); and Othman and Marin (2017).

Inspite of these, not much work has been carried out in the study of the Rayleigh wave propagation in a transversely isotropic magneto-thermoelastic medium with fractional order three-phase-lag heat transfer. In this paper, we have attempted to study the Rayleigh wave propagation with fractional order three-phase-lag heat transfer in a transversely isotropic magneto-thermoelastic medium.

Basic equations

The basic governing equations for homogeneous, anisotropic, generalized thermodiffusive elastic solids in the absence of body forces, heat and mass diffusion sources following Kumar and Kansal (2008b) are

$$t_{ij} = c_{ijkl}e_{kl} + a_{ij}T + b_{ij}C, \tag{1}$$

$$\left(1+\tau_q\frac{\partial}{\partial t}+\tau_q^2\frac{\partial^2}{\partial t^2}\right)\dot{q}_i = -\left[K_{ij}\left(1+\tau_T\frac{\partial}{\partial t}\right)\dot{T}_{,j} + K_{ij}^*\left(1+\tau_\nu\frac{\partial}{\partial t}\right)T_{,j}\right], \tag{2}$$

$$\rho ST_0 = \rho C_E T + aT_0C - a_{ij}e_{ij}T_0, \tag{3}$$

$$\mathrm{P} = b_{kl}e_{kl} + bC - aT \tag{4}$$

$$\eta_i = -\alpha_{ij}^* P_{,j} \tag{5}$$

Here, C_{ijkl} are elastic parameters and having symmetry ($C_{ijkl} = C_{klij} = C_{jikl} = C_{ijlk}$). The basis of these symmetries of C_{ijkl} is due to

1. The stress tensor is symmetric, which is only possible if ($C_{ijkl} = C_{jikl}$)
2. If a strain energy density exists for the material, the elastic stiffness tensor must satisfy $C_{ijkl} = C_{klij}$
3. From stress tensor and elastic stiffness, tensor symmetries infer ($C_{ijkl} = C_{ijlk}$) and $C_{ijkl} = C_{klij} = C_{jikl} = C_{ijlk}$

The simplified Maxwell's linear equation (Rafiq et al. 2019) of electrodynamics for a slowly moving and perfectly conducting elastic solid are

$$\operatorname{curl}\vec{h} = \vec{j} + \varepsilon_0\frac{\partial\vec{E}}{\partial t};\ \operatorname{curl}\vec{E} = -\mu_0\frac{\partial\vec{h}}{\partial t};\ \vec{E} = -\mu_0\left(\frac{\partial\vec{u}}{\partial t}\times\vec{H}\right);\ \operatorname{div}\vec{h} = 0. \tag{6}$$

From Eq. (6), we obtain

$$\vec{E} = \mu_0H_0(\dot{w}, 0, -\dot{u}) \tag{7}$$

$$\vec{h} = (0, -H_0e, 0), \tag{8}$$

$$\vec{j} = \left(-h_{,z} - \varepsilon_0\mu_0H_0\ddot{w}, 0, -h_{,x} - \varepsilon_0\mu_0H_0\ddot{u}\right) \tag{9}$$

The equation of motion, entropy equation, and mass conservation equation following Kumar and Kansal (2009) are

$$t_{ij,j} + F_i = \rho\ddot{u}_i, \tag{10}$$

$$q_{i,i} + \rho T_0\dot{S} - \rho M + P\eta_{i,i} = 0, \tag{11}$$

$$\eta_{i,i} = \dot{C} + \rho N \tag{12}$$

where

$$F_i = \mu_0\left(\overrightarrow{j} \times \overrightarrow{H}\right)_i$$
$$\overrightarrow{F} = (F_x, F_y, F_z) = \left(\mu_0 H_0^2 e_{,x} - \varepsilon_0 \mu_0^2 H_0^2 \ddot{u}, 0, \mu_0 H_0^2 e_{,z} - \varepsilon_0 \mu_0^2 H_0^2 \ddot{w}\right)$$

are the components of the Lorentz force that appeared due to initially applied a magnetic field, the total magnetic field is given by $\overrightarrow{H} = \overrightarrow{H}_0 + \overrightarrow{h}$, $\overrightarrow{H}_0$ is the external applied magnetic field intensity vector, and M and N are the strengths of the heat source and mass diffusion source per unit mass.

The medium is supposed to be perfectly electrically conducting and is half-space $(x, 0, z)$ such that all the variables are independent of the dimension y. Let $\overrightarrow{H}_0 = (0, H_0, 0)$.

The heat conduction equation following Othman and Said (2018), we have

$$K_{ij}\left(1 + \tau_t \frac{\partial}{\partial t}\right) \dot{T}_{,ji} + K_{ij}^*\left(1 + \tau_v \frac{\partial}{\partial t}\right) T_{,ji} = \left(1 + \tau_q \frac{\partial}{\partial t} + (\tau_q)^2 \frac{\partial^2}{\partial t^2}\right)\left[\rho C_E \ddot{T} + a_{ij} T_0 \ddot{e}_{ij} + a T_0 \ddot{C}\right], \tag{13}$$

where

$$a_{ij} = -a_i \delta_{ij}, \quad b_{ij} = -b_i \delta_{ij}, \quad \alpha_{ij}^* = \alpha_i^* \delta_{ij}, \quad K_{ij}^* = K_i^* \delta_{ij}, \quad K_{ij} = K_i \delta_{ij}$$

Method and solution of the problem

We consider a perfectly conducting homogeneous transversely isotropic magneto-thermoelastic medium in the context of the three-phase-lag model of thermoelasticity initially at a uniform temperature T_0, an initial magnetic field $\overrightarrow{H}_0 = (0, H_0, 0)$ towards y-axis. Moreover, we considered x, y, z taking origin on the surface $(z = 0)$ along the z-axis directing vertically downwards inside the medium. For the 2D problem in the xz-plane, we take

$$\boldsymbol{u} = (u, 0, w)$$

Now using the transformation on Eqs. (7–9) following Slaughter (2002) is as under:

$$C_{11}\frac{\partial^2 u}{\partial x^2} + C_{13}\frac{\partial^2 w}{\partial x \partial z} + C_{44}\left(\frac{\partial^2 u}{\partial z^2} + \frac{\partial^2 w}{\partial x \partial z}\right) - a_1 \frac{\partial T}{\partial x} - b_1 \frac{\partial C}{\partial x} + \left(\mu_0 H_0^2 \frac{\partial e}{\partial x} - \epsilon_0 \mu_0^2 H_0^2 \frac{\partial^2 u}{\partial t^2}\right) = \rho\left(\frac{\partial^2 u}{\partial t^2}\right), \tag{14}$$

$$(C_{13} + C_{44})\frac{\partial^2 u}{\partial x \partial z} + C_{44}\frac{\partial^2 w}{\partial x^2} + C_{33}\frac{\partial^2 w}{\partial z^2} - a_3 \frac{\partial T}{\partial z} - b_3 \frac{\partial C}{\partial z} + \left(\mu_0 H_0^2 \frac{\partial e}{\partial z} - \epsilon_0 \mu_0^2 H_0^2 \frac{\partial^2 w}{\partial t^2}\right) = \rho\left(\frac{\partial^2 w}{\partial t^2}\right), \tag{15}$$

$$K_1\left(1 + \tau_t \frac{\partial}{\partial t}\right)\frac{\partial^2 \dot{T}}{\partial x^2} + K_3\left(1 + \tau_t \frac{\partial}{\partial t}\right)\frac{\partial^2 \dot{T}}{\partial z^2} + K_1^*\left(1 + \tau_v \frac{\partial}{\partial t}\right)\frac{\partial^2 T}{\partial x^2} + K_3^*\left(1 + \tau_v \frac{\partial}{\partial t}\right)\frac{\partial^2 T}{\partial z^2} = \left(1 + \tau_q \frac{\partial}{\partial t} + (\tau_q)^2 \frac{\partial^2}{\partial t^2}\right)\left[\rho C_E \frac{\partial^2 T}{\partial t^2} + T_0\left\{a_1 \frac{\partial \ddot{u}}{\partial x} + a_1 \frac{\partial \ddot{w}}{\partial z}\right\} + a T_0 \ddot{C}\right], \tag{16}$$

$$\alpha_1^*\left[b_1 \frac{\partial^3 u}{\partial x^3} + b_3 \frac{\partial^3 w}{\partial x^2 \partial z}\right] + \alpha_3^*\left[b_1 \frac{\partial^3 u}{\partial x \partial z^2} + b_3 \frac{\partial^3 w}{\partial z^3}\right] - \alpha_1^* b \frac{\partial^2 C}{\partial x^2} - \alpha_3^* b \frac{\partial^2 C}{\partial z^2} + \alpha_1^* a \frac{\partial^2 T}{\partial x^2} + \alpha_3^* a \frac{\partial^2 T}{\partial z^2} = -(\dot{C}). \tag{17}$$

and

$$t_{xx} = C_{11} e_{xx} + C_{13} e_{xz} - a_1 T, \tag{18}$$

$$t_{zz} = C_{13} e_{xx} + C_{33} e_{zz} - a_3 T, \tag{19}$$

$$t_{xz} = 2 C_{44} e_{xz}, \tag{20}$$

where

$$a_1 = (C_{11} + C_{12})\alpha_1 + C_{13}\alpha_{3,} a_3 = 2C_{13}\alpha_1 + C_{33}\alpha_3, b_1 = (C_{11} + C_{12})\alpha_{1c} + C_{13}\alpha_{3c,}.$$

Using dimensionless quantities,

$$\left(x', z', u', w'\right) = \frac{\omega_1^*}{C_1}(x, z, u, w), \rho C_1^2 = C_{11}, \omega_1^* = \frac{\rho C_1^2 C_E}{K_1} T' = \frac{a_1 T}{\rho C_1^2}, C' = \frac{b_1 C}{\rho C_1^2}, \left(t', \tau_0', \tau^{0'}, \tau_T', \tau_v', \tau_q'\right) = \omega_1^*\left(t, \tau_0, \tau^0, \tau_T, \tau_v, \tau_q\right). \tag{21}$$

Making use of (21) in Eqs. (14–17), after suppressing the primes, yield

$$(1+\delta_4)\frac{\partial^2 u}{\partial x^2}+(\delta_1+\delta_4)\frac{\partial^2 w}{\partial x\partial z}+\delta_2\frac{\partial^2 u}{\partial z^2}-\frac{\partial T}{\partial x}-\frac{\partial C}{\partial x} = (1+\delta_5)\frac{\partial^2 u}{\partial t^2}, \tag{22}$$

$$(\delta_1+\delta_4)\frac{\partial^2 u}{\partial x\partial z}+\delta_2\frac{\partial^2 w}{\partial x^2}+(\delta_3+\delta_4)\frac{\partial^2 w}{\partial z^2} -\delta_7\frac{\partial T}{\partial z}-\delta_8\frac{\partial C}{\partial z}=(1+\delta_5)\frac{\partial^2 w}{\partial t^2} \tag{23}$$

$$\left(1+\tau_T\frac{\partial}{\partial t}\right)\left(\delta_9\frac{\partial^2 \dot{T}}{\partial x^2}+\delta_{12}\frac{\partial^2 \dot{T}}{\partial z^2}\right)+\left(1+\tau_v\frac{\partial}{\partial t}\right)\left(\delta_{10}\frac{\partial^2 T}{\partial x^2}+\delta_{11}\frac{\partial^2 T}{\partial z^2}\right) = \left(1+\tau_q\frac{\partial^\alpha}{\partial t^\alpha}+{\tau_q}^2\frac{\partial^2}{\partial t^2}\right) \left[\delta_9\ddot{T}+\delta_{13}\frac{\partial \ddot{u}}{\partial x}+\delta_{14}\frac{\partial \ddot{w}}{\partial z}+\delta_{15}\ddot{C}\right]. \tag{24}$$

$$q_1\frac{\partial^3 u}{\partial x^3}+q_2\frac{\partial^3 w}{\partial x^2\partial z}+q_3\frac{\partial^3 u}{\partial x\partial z^2}+q_4\frac{\partial^3 w^*}{\partial z^3{}_3}+q_5\frac{\partial^2 C}{\partial x^2}+q_6\frac{\partial^2 C}{\partial z^2} +q_7\frac{\partial^2 T}{\partial x^2}+q_8\frac{\partial^2 T}{\partial z^2}+q_9\frac{\partial C}{\partial t}=0 \tag{25}$$

where

$$\delta_1=\frac{c_{13}+c_{44}}{c_{11}},\delta_2=\frac{c_{44}}{c_{11}},\delta_3=\frac{c_{33}}{c_{11}},\delta_4=\frac{\mu_0H_0^2}{\rho C_1^2},\delta_5=\frac{\varepsilon_0\mu_0^2H_0^2}{\rho},$$
$$\delta_7=\frac{a_3}{a_1},\delta_8=\frac{b_3}{b_1},\delta_9=\frac{\rho\omega_1^{*3}}{a_1},\delta_{10}=\frac{\rho\omega_1^{*2}K_1^*}{a_1K_1},$$
$$\delta_{11}=\frac{\rho\omega_1^{*2}K_3^*}{a_1K_1},\delta_{12}=\frac{\rho\omega_1^{*3}K_3}{a_1K_1},\delta_{13}=\frac{T_0\omega_1^{*2}a_1}{K_1},\delta_{14}=\frac{T_0\omega_1^{*2}a_3}{K_1},$$
$$\delta_{15}=\frac{a\rho C_1^2T_0\omega_1^{*2}}{K_1b_1}.$$

Rayleigh wave propagation

We pursue Rayleigh wave solution of the equations of the form

$$\begin{pmatrix}u\\w\\T\\C\end{pmatrix}=\begin{pmatrix}1\\W\\S\\R\end{pmatrix}Ue^{i\xi(x+mz-ct)} \tag{26}$$

where $c=\frac{\omega}{\xi}$ is the non-dimensional phase velocity and m is an unknown parameter. 1, W, S, and R are the amplitude ratios of displacements u, w, temperature change T, and concentration C, respectively.

Upon using Eq. (26) in Eqs. (22–25), we get

$$U\left[l_1+l_6+l_2m^2\right]+W[l_3m]+S[l_5]+R[l_5]=0, \tag{27}$$

$$U[l_3m]+W\left[l_2+l_6+l_7m^2\right]+S[l_8m]+R[l_9m]=0, \tag{28}$$

$$U[l_{12}]+W[l_{13}m]+S\left[l_{10}+l_{11}m^2\right]+R[l_{14}]=0, \tag{29}$$

$$U\left[l_{15}+l_{16}m^2\right]+W\left[l_{17}m+l_{18}m^3\right]+S\left[l_{21}+l_{22}m^2\right] +R\left[l_{19}+l_{20}m^2\right]=0, \tag{30}$$

where

$$l_1=-\xi^2(1+\delta_4),l_2=-\delta_2\xi^2,l_3=-\xi^2(\delta_1+\delta_4),l_5=-i\xi,$$
$$l_6=(1+\delta_5)\xi^2c^2,l_7=-\xi^2(\delta_3+\delta_4),l_8=-i\xi\delta_7,l_9=-i\xi\delta_8,$$
$$l_{10}=-\delta_{10}(1-i\xi c\tau_v)\xi^2+\delta_9(1-i\xi c\tau_T)i\xi^3c-\delta_9\xi^2c^2\left(1-i\xi c\tau_q-\frac{{\tau_q}^2\xi^2c^2}{2}\right),$$
$$l_{11}=-\delta_{11}(1-i\xi c\tau_v)\xi^2+\delta_{12}(1-i\xi c\tau_T)i\xi^3c,$$
$$l_{12}=-\delta_{13}i\xi^3c^2\left(1-i\xi c\tau_q-\frac{{\tau_q}^2\xi^2c^2}{2}\right),$$
$$l_{13}=-\delta_{14}i\xi^3c^2\left(1-i\xi c\tau_q-\frac{{\tau_q}^2\xi^2c^2}{2}\right),$$
$$l_{14}=-\delta_{15}\xi^2c^2\left(1-i\xi c\tau_q-\frac{{\tau_q}^2\xi^2c^2}{2}\right),$$
$$l_{15}=-q_1i\xi^3,l_{16}=-q_3i\xi^3,l_{17}=-q_2i\xi^3,l_{18}=-q_4i\xi^3,$$
$$l_{19}=-q_5\xi^2-q_9i\xi c,l_{20}=-q_6\xi^2,l_{21}=-q_7\xi^2,$$
$$l_{22}=-q_8\xi^2,q_1=\frac{\alpha_1^*b_1\omega_1^{*2}}{c_1^2},q_2=\frac{\alpha_1^*b_3\omega_1^{*2}}{c_1^2},q_3=\frac{\alpha_3^*b_1\omega_1^{*2}}{c_1^2},$$
$$q_4=\frac{\alpha_3^*b_3\omega_1^{*2}}{c_1^2},q_5=-\frac{\alpha_1^*b\omega_1^{*2}\rho}{b_1}q_6=-\frac{\alpha_3^*b\omega_1^{*2}\rho}{b_1},$$
$$q_7=-\frac{\alpha_1^*a\omega_1^{*2}\rho}{a_1},q_8=\frac{\alpha_3^*a\omega_1^{*2}\rho}{a_1},q_9=-\frac{\omega_1^*c_1^2\rho}{b_1}.$$

and from (27–30), the characteristic equation is a biquadratic equation in m^2 given by

$$m^8+\frac{B}{A}m^6+\frac{C}{A}m^4+\frac{D}{A}m^2+\frac{E}{A}=0, \tag{31}$$

where

$$A=l_2l_7l_{11}l_{20}-l_2l_9l_{18}l_{11},$$

$$B=l_1l_7l_{11}l_{20}-l_1l_9l_{18}l_{11}+l_2l_6l_{11}l_{20}+l_2l_7l_{10}l_{20}-l_{14}l_2l_{22}l_7 +l_{14}l_2l_8l_{18}+l_2l_9l_{13}l_{22}-l_2l_9l_{13}l_{22}-l_{10}l_9l_2l_{18}+l_2l_{11}l_{17}l_9 -l_3l_3l_{11}l_{20}+l_3l_9l_{16}l_{11}+l_5l_3l_{11}l_{18}+l_5l_7l_{11}l_{15}+l_2l_8l_{13}l_{20},$$

$$C=l_1l_6l_{11}l_{20}+l_1l_7l_{19}l_{11}+l_1l_7l_{10}l_{20}-l_1l_7l_{14}l_{22}-l_{14}l_1l_8l_{18} -l_{10}l_1l_9l_{18}+l_1l_9l_{13}l_{22}-l_1l_9l_{11}l_{17}+l_2l_6l_{11}l_{19}+l_2l_6l_{10}l_{20} -l_2l_6l_{14}l_{22}+l_1l_7l_{10}l_{19}-l_2l_7l_{14}l_{21}-l_2l_8l_{13}l_{19}+l_1l_8l_{13}l_{20} -l_2l_8l_{14}l_{17}-l_2l_9l_{13}l_{21}-l_2l_9l_{10}l_{17}-l_3^2l_{11}l_{19}-l_3^2l_{10}l_{20}+l_3^2l_{14}l_{22} +l_3l_8l_{12}l_{20}-l_3l_8l_{14}l_{16}-l_3l_9l_{12}l_{22}-l_5l_3l_{13}l_{22}+l_5l_3l_{10}l_{18} +l_5l_3l_{11}l_{17}-l_5l_6l_{16}l_{11}-l_5l_7l_{12}l_{22}+l_5l_7l_{10}l_{16}+l_5l_7l_{11}l_{16} -l_5l_8l_{12}l_{18}+l_5l_8l_{16}l_{13}+l_5l_3l_{13}l_{20}-l_5l_3l_{14}l_{18}-l_5l_7l_{12}l_{20} +l_5l_7l_{14}l_{16}+l_5l_9l_{12}l_{18}-l_5l_9l_{13}l_{16},$$

$$\begin{aligned}D = {} & l_1l_6l_{11}l_{19} + l_1l_6l_{10}l_{20} - l_1l_6l_{14}l_{22} + l_1l_7l_{10}l_{19} + l_5l_7l_{10}l_{15}\\ & - l_1l_7l_{14}l_{21} - l_1l_8l_{13}l_{19} - l_1l_8l_{14}l_{17} + l_1l_9l_{13}l_{21} - l_5l_8l_{12}l_{17}\\ & - l_1l_9l_{10}l_{17} + l_2l_6l_{10}l_{19} - l_2l_6l_{14}l_{21} - l_3^2l_{10}l_{19} + l_5l_8l_{13}l_{15}\\ & + l_3^2l_{14}l_{21} + l_3l_8l_{12}l_{19} - l_3l_8l_{14}l_{15} - l_3l_9l_{12}l_{21} + l_5l_3l_{13}l_{19}\\ & + l_3l_9l_{10}l_{15} - l_3l_5l_{13}l_{21} + l_3l_5l_{10}l_{17} + l_5l_6l_{12}l_{22} - l_5l_6l_{12}l_{20}\\ & - l_{10}l_5l_6l_{16} - l_5l_3l_{14}l_{17} - l_5l_6l_{10}l_{16} - l_5l_6l_{15}l_{11} - l_5l_7l_{12}l_{21}\\ & + l_5l_6l_{14}l_{16} - l_5l_7l_{12}l_{19} + l_5l_7l_{14}l_{15} + l_5l_9l_{12}l_{17} - l_5l_9l_{13}l_{15}\\ & + l_5l_6l_{14}l_{15},\end{aligned}$$

$$E = l_1l_6l_{10}l_{19} - l_1l_6l_{14}l_{21} - l_5l_6l_{12}l_{21} - l_5l_6l_{10}l_{15} - l_5l_6l_{12}l_{19}.$$

The characteristic in Eq. (27) gives four roots $m_p^2$where $p = 1, 2, 3, 4$. Since we consider only the surface waves, therefore, motion is restricted to the free surface $z = 0$ of the half-space, hence, satisfy the radiation conditions $\mathrm{Re}(m_p) \geq 0$.

The displacements, temperature change, and concentration can be written as

$$\begin{pmatrix} u \\ w \\ T \\ C \end{pmatrix} = \sum_{p=1}^{4} \begin{pmatrix} 1 \\ n_{1p} \\ n_{2p} \\ n_{3p} \end{pmatrix} A_p e^{i\xi\left(x + i m_p z - ct\right)} \tag{32}$$

where A_p ($p = 1, 2, 3, 4$) are arbitrary constants and coupling constants are

Boundary conditions

The boundary conditions at $z = 0$ are given by

$$t_{zz} = 0, t_{zx} = 0, \frac{\partial T}{\partial z} + hT = 0, P = 0. \tag{33}$$

After applying dimensionless quantities from Eq. (21), the above boundary conditions reduces to

$$\begin{gathered}(\delta_1 - \delta_2)\frac{\partial u}{\partial x} + \delta_3\frac{\partial w}{\partial z} - \delta_7 T - \delta_8 C = 0,\\ \delta_2\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right) = 0,\\ \frac{\partial T}{\partial z} + hT = 0,\\ \frac{\partial u}{\partial x} + \epsilon_2\frac{\partial w}{\partial z} - \eta_2 C + \eta_1 T = 0,\end{gathered}$$

where

$$\eta_1 = \frac{aC_{11}}{a_1b_1}, \eta_2 = \frac{bC_{11}}{b_1^2},$$

Derivations of the secular equations

By using the values of u, w, T, and C from (28) in (29), we get four linear equations as

$$n_{1p} = \frac{(-l_9l_{16}l_{11} + l_3l_{11}l_{20})m_p^5 + \begin{pmatrix} l_3l_{11}l_{19} + l_3l_{10}l_{20} - l_3l_{14}l_{22} \\ -l_8l_{12}l_{20} + l_8l_{14}l_{16} + l_9l_{12}l_{22} \end{pmatrix} m_p^3 + \begin{pmatrix} l_3l_{10}l_{19} + l_3l_{14}l_{21} - l_8l_{12}l_{19} \\ +l_8l_{14}l_{15} + l_9l_{12}l_{21} - l_9l_{15}l_{10} \end{pmatrix} m_p}{\begin{pmatrix} l_7l_{11}l_{20} \\ -l_9l_{18}l_{11} \end{pmatrix} m_p^6 + \begin{pmatrix} l_6l_{11}l_{20} + l_7l_{11}l_{19} + l_7l_{10}l_{20} - l_7l_{14}l_{22} \\ +l_8l_{13}l_{20} - l_8l_{14}l_{18} + l_9l_{13}l_{22} - l_9l_{10}l_{18} \end{pmatrix} m_p^4 + \begin{pmatrix} l_6l_{11}l_{19} + l_6l_{10}l_{20} - l_6l_{14}l_{22} + l_7l_{10}l_{19} - l_7l_{14}l_{21} \\ -l_8l_{13}l_{19} - l_8l_{14}l_{17} + l_9l_{13}l_{21} - l_9l_{10}l_{17} \end{pmatrix} m_p^2 + \begin{pmatrix} l_6l_{10}l_{19} \\ -l_6l_{14}l_{21} \end{pmatrix}},$$

$$n_{2p} = \frac{\begin{pmatrix} l_3l_{13}l_{20} - l_3l_{14}l_{18} - l_7l_{12}l_{20} \\ +l_7l_{14}l_{16} + l_9l_{12}l_{18} - l_9l_{13}l_{16} \end{pmatrix} m_p^4 + \begin{pmatrix} l_3l_{13}l_{19} - l_3l_{14}l_{17} - l_6l_{12}l_{20} + l_6l_{11}l_{16} - l_7l_{12}l_{19} \\ +l_7l_{14}l_{15} + l_9l_{12}l_{17} - l_9l_{13}l_{15} \end{pmatrix} m_p^3 + (-l_6l_{12}l_{19} + l_6l_{14}l_{15})}{\begin{pmatrix} l_7l_{11}l_{20} \\ -l_9l_{18}l_{11} \end{pmatrix} m_p^6 + \begin{pmatrix} l_6l_{11}l_{20} + l_7l_{11}l_{19} + l_7l_{10}l_{20} - l_7l_{14}l_{22} \\ +l_8l_{13}l_{20} - l_8l_{14}l_{18} + l_9l_{13}l_{22} - l_9l_{10}l_{18} \end{pmatrix} m_p^4 + \begin{pmatrix} l_6l_{11}l_{19} + l_6l_{10}l_{20} - l_6l_{14}l_{22} + l_7l_{10}l_{19} - l_7l_{14}l_{21} \\ -l_8l_{13}l_{19} - l_8l_{14}l_{17} + l_9l_{13}l_{21} - l_9l_{10}l_{17} \end{pmatrix} m_p^2 + \begin{pmatrix} l_6l_{10}l_{19} \\ -l_6l_{14}l_{21} \end{pmatrix}},$$

$$n_{3p} = \frac{\begin{pmatrix} -l_3l_{11}l_{18} \\ -l_7l_{11}l_{15} \end{pmatrix} m_p^6 + \begin{pmatrix} l_3l_{13}l_{22} - l_3l_{10}l_{18} - l_3l_{11}l_{17} + l_6l_{11}l_{16} - l_7l_{12}l_{22} \\ -l_7l_{10}l_{16} - l_7l_{11}l_{16} + l_8l_{12}l_{18} - l_8l_{16}l_{13} \end{pmatrix} m_p^4 + \begin{pmatrix} l_3l_{13}l_{21} - l_3l_{10}l_{17} - l_6l_{12}l_{22} + l_6l_{10}l_{16} + l_6l_{11}l_{15} \\ +l_7l_{12}l_{21} - l_7l_{15}l_{10} + l_8l_{12}l_{17} - l_8l_{15}l_{13} \end{pmatrix} m_p^2 + \begin{pmatrix} l_6l_{10}l_{15} \\ -l_6l_{12}l_{21} \end{pmatrix}}{\begin{pmatrix} l_7l_{11}l_{20} \\ -l_9l_{18}l_{11} \end{pmatrix} m_p^6 + \begin{pmatrix} l_6l_{11}l_{20} + l_7l_{11}l_{19} + l_7l_{10}l_{20} - l_7l_{14}l_{22} \\ +l_8l_{13}l_{20} - l_8l_{14}l_{18} + l_9l_{13}l_{22} - l_9l_{10}l_{18} \end{pmatrix} m_p^4 + \begin{pmatrix} l_6l_{11}l_{19} + l_6l_{10}l_{20} - l_6l_{14}l_{22} + l_7l_{10}l_{19} - l_7l_{14}l_{21} \\ -l_8l_{13}l_{19} - l_8l_{14}l_{17} + l_9l_{13}l_{21} - l_9l_{10}l_{17} \end{pmatrix} m_p^2 + \begin{pmatrix} l_6l_{10}l_{19} \\ -l_6l_{14}l_{21} \end{pmatrix}}.$$

$$\sum_{p=1}^{4} Q_{jp}A_p = 0, j = 1, 2, 3, 4. \tag{34}$$

where

$$Q_{1p} = (\delta_1 - \delta_2) + \delta_3 i m_p n_{1p} + \frac{i\delta_7 n_{2p}}{\xi} + \frac{i\delta_8 n_{3p}}{\xi},$$
$$Q_{2p} = i m_p + n_{1p},$$
$$Q_{3p} = \left(-\xi m_p + h\right) n_{2p},$$
$$Q_{4p} = 1 + i\epsilon_2 m_p n_{1p} - \frac{i\eta_1 n_{2p}}{\xi} + \frac{i\eta_2 n_{3p}}{\xi}.$$

Secular equations are

$$\begin{bmatrix} Q_{11} & Q_{12} & Q_{13} & Q_{14} \\ Q_{21} & Q_{22} & Q_{23} & Q_{24} \\ Q_{31} & Q_{32} & Q_{33} & Q_{34} \\ Q_{41} & Q_{42} & Q_{43} & Q_{44} \end{bmatrix} = 0, \quad \text{or} \tag{35}$$
$$-Q_{31}D_1 + Q_{32}D_2 - Q_{33}D_3 + Q_{34}D_4 = 0,$$

where

$$D_1 = \begin{bmatrix} Q_{12} & Q_{13} & Q_{14} \\ Q_{22} & Q_{23} & Q_{24} \\ Q_{42} & Q_{43} & Q_{44} \end{bmatrix},$$
$$D_1 = Q_{12}(Q_{23}Q_{44} - Q_{24}Q_{43}) - Q_{13}(Q_{22}Q_{44} - Q_{24}Q_{42}) + Q_{14}(Q_{22}Q_{43} - Q_{23}Q_{42}),$$
$$D_2 = \begin{bmatrix} Q_{11} & Q_{13} & Q_{14} \\ Q_{21} & Q_{23} & Q_{24} \\ Q_{41} & Q_{43} & Q_{44} \end{bmatrix},$$
$$D_2 = Q_{11}(Q_{23}Q_{44} - Q_{24}Q_{43}) - Q_{13}(Q_{21}Q_{44} - Q_{24}Q_{42}) + Q_{14}(Q_{21}Q_{43} - Q_{23}Q_{41}),$$
$$D_3 = \begin{bmatrix} Q_{11} & Q_{12} & Q_{14} \\ Q_{21} & Q_{22} & Q_{24} \\ Q_{41} & Q_{42} & Q_{44} \end{bmatrix},$$
$$D_3 = Q_{11}(Q_{22}Q_{44} - Q_{24}Q_{42}) - Q_{12}(Q_{21}Q_{44} - Q_{24}Q_{41}) + Q_{14}(Q_{21}Q_{42} - Q_{22}Q_{41}),$$
$$D_4 = \begin{bmatrix} Q_{11} & Q_{12} & Q_{13} \\ Q_{21} & Q_{22} & Q_{23} \\ Q_{31} & Q_{32} & Q_{33} \end{bmatrix},$$
$$D_4 = Q_{11}(Q_{22}Q_{33} - Q_{23}Q_{32}) - Q_{12}(Q_{21}Q_{33} - Q_2Q_{31}) + Q_{13}(Q_{21}Q_{32} - Q_{22}Q_{31}).$$

These secular equations have entire information regarding the wavenumber, phase velocity, and attenuation coefficient of Rayleigh waves in the transversely isotropic magneto-thermoelastic medium. Moreover, If we write

$$c^{-1} = \nu^{-1} + Fi\omega^{-1}, \tag{36}$$

then $\xi = E + iF$, where $E = \frac{\omega}{\nu}, \nu$ (velocity), and F (attenuation coefficient) are real.

The roots of the characteristic in Eq. (27) are complex and therefore, we assume that $m_p = Q_p + ip_q$, so that the exponent in Rayleigh wave solutions (28) becomes

$$iE\left(x - im_p^i z - vt\right) - E\left(\frac{F}{E}x + m_p^r z\right), \tag{37}$$

where

$$m_p^r = Q_p - p_q\frac{F}{E}, m_p^i = p_q + Q_p\frac{F}{E}.$$

Equation (28) can be written as

$$\begin{pmatrix} u \\ w \\ T \\ C \end{pmatrix} = \sum_{p=1}^{4} \begin{pmatrix} 1 \\ n_{1p} \\ n_{2p} \\ n_{3p} \end{pmatrix} A_p e^{\left(-Fx - \chi_p^r\right)} \times e^{i\left[E(x - vt) - \chi_p^i\right]}, \tag{38}$$

where

$$\left|\chi_p^r\right|^2 - \left|\chi_p^i\right|^2 = E^2\left\{\left(m_p^r\right)^2 - \left(m_p^i\right)^2\right\},$$
$$\left|\chi_p^r\right|\left|\chi_p^i\right| cos\theta = \frac{1}{2}E^2 m_p^r m_p^i,$$

θ is the angle between the real and imaginary part of the vector χ_p.

Phase velocity

Phase velocity defines the speed at which waves oscillating at a particular frequency propagate and it depends on the real component of the wave number. The phase velocities are given by

$$V = \frac{\omega}{Re(\xi)} \tag{39}$$

Attenuation coefficient

The attenuation coefficient is the gradual loss of flux intensity through a medium, and it depends on the imaginary component of the wavenumber. The attenuation coefficient is defined as

$$Q = Img(\xi), \tag{40}$$

Specific loss

The specific loss is the most direct way of defining internal resistance for a material. The specific loss W is given by

$$W = \left(\frac{\Delta W}{W}\right) = 4\pi\left|\frac{Img(\xi)}{Re(\xi)}\right|, \tag{41}$$

Penetration depth

Penetration depth describes how deep a wave can penetrate into a material and describes the decay of waves inside of a material. The penetration depth S is defined by

$$S = \frac{1}{Img(\xi)} \tag{42}$$

Particular cases

1. If $\tau_T \neq 0$, $\tau_\nu \neq 0$, $\tau_q \neq 0$, we obtain results for Rayleigh wave propagation in transversely isotropic magneto-thermoelastic solid with diffusion and with and without energy dissipation and TPL (three-phase-lag) effects.
2. If $\tau_T = 0$, $\tau_\nu = 0$, $\tau_q = 0$, and $K^* \neq 0$, we obtain results for Rayleigh wave propagation in magneto-thermoelastic transversely isotropic solid with diffusion and GN-III theory (thermoelasticity with energy dissipation).
3. If $\tau_T = 0$, $\tau_\nu = 0$, $\tau_q = 0$, and $K^* = 0$, we obtain results for Rayleigh wave propagation in magneto-thermoelastic transversely isotropic solid with diffusion and GN-II theory (generalized thermoelasticity without energy dissipation).
4. If $\tau_T \neq 0$, $\tau_\nu \neq 0$, $\tau_q \neq 0$, and $K^* = 0$, we obtain results for Rayleigh wave propagation in magneto-thermoelastic transversely isotropic solid with diffusion and GN-II theory with TPL effect
5. If $\tau_T = 0$, $\tau_\nu = 0$, $\tau_q = \tau_0 > 0$, and $K^* = 0$, and ignoring τ_q^2, we obtain results for Rayleigh wave propagation in magneto-thermoelastic transversely isotropic solid with diffusion and Lord-Shulman (L-S) model.
6. If $\tau_T = 0$, $\tau_\nu = 0$, and $\tau_q = 0$ and if the medium is not permitted with the magnetic field, i.e., $\mu_0 = H_0 = 0$, then we obtain results for Rayleigh wave propagation in transversely isotropic thermoelastic solid with diffusion and without TPL effect
7. If $C_{11} = C_{33} = \lambda + 2\mu, C_{12} = C_{13} = \lambda, C_{44} = \mu, \alpha_1 = \alpha_3 = \alpha', a_1 = a_3 = a, b_1 = b_3 = b, K_1 = K_3 = K, K_1^* = K_3^* = K^*$, we obtain expressions for Rayleigh wave propagation in magneto-thermoelastic isotropic materials with diffusion and with and without energy dissipation with TPL effect.

Numerical results and discussion

In order to illustrate our theoretical results in the proceeding section and to show the effect of Hall current and fractional order parameter, we now present some numerical results. Following Dhaliwal and Sherief (1980), cobalt material has been taken for thermoelastic material as

$$c_{11} = 3.07 \times 10^{11} Nm^{-2}, c_{33} = 3.581 \times 10^{11} Nm^{-2},$$

$$c_{13} = 1.027 \times 10^{10} Nm^{-2}, c_{44} = 1.510 \times 10^{11} Nm^{-2},$$

$$\beta_1 = 7.04 \times 10^6 Nm^{-2}\ deg^{-1},$$

$$\beta_3 = 6.90 \times 10^6 Nm^{-2}\ deg^{-1}, \rho = 8.836 \times 10^3 Kgm^{-3},$$

$$C_E = 4.27 \times 10^2 jKg^{-1}\ deg^{-1},$$

$$K_1 = 0.690 \times 10^2 Wm^{-1} Kdeg^{-1},$$

$$K_3 = 0.690 \times 10^2 Wm^{-1} K^{-1}, T_0 = 298\ \mathrm{K},$$

$$H_0 = 1\,\mathrm{Jm^{-1}nb^{-1}}, \varepsilon_0 = 8.838 \times 10^{-12}\,\mathrm{Fm^{-1}}, L = 1.$$

Using the above values, the graphical representations of stress components, temperature change, and concentration, Rayleigh wave velocity, attenuation coefficient, specific loss, and penetration depth of Raleigh wave in the transversely isotropic thermoelastic medium have been investigated with three-phase-lag, GN-III, and LS theory of thermoelasticity and demonstrated graphically as

1. The solid line relates to the three-phase lag theory $\tau_T \neq 0$, $\tau_\nu \neq 0$, $\tau_q \neq 0$,
2. The dashed line relates to GN-III theory $\tau_T = 0$, $\tau_\nu = 0$, $\tau_q = 0$, and $K^* \neq 0$,
3. The dotted line relates to LS theory $\tau_T = 0$, $\tau_\nu = 0$, $\tau_q = \tau_0 > 0$, and $K^* = 0$.

Figure 1 illustrates the deviations of tangential stress t_{zx} with wave number. From the graph, we observe that tangential stress t_{zx} decreases with wave number in all the three theories with a little difference in magnitude. Figure 2 shows the deviations of normal stress t_{zz} with

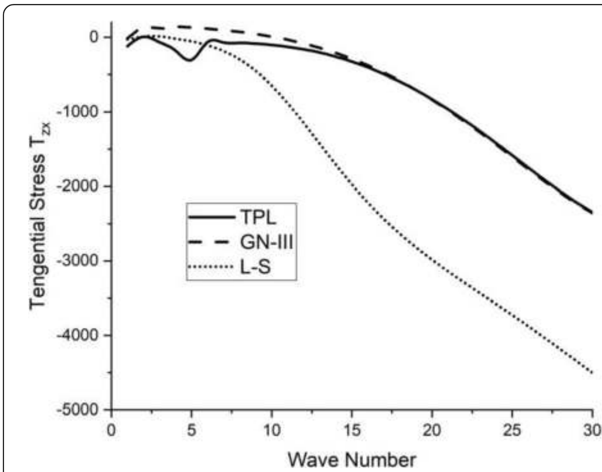

Fig. 1 Variations of tangential stress t_{zx} with wavenumber

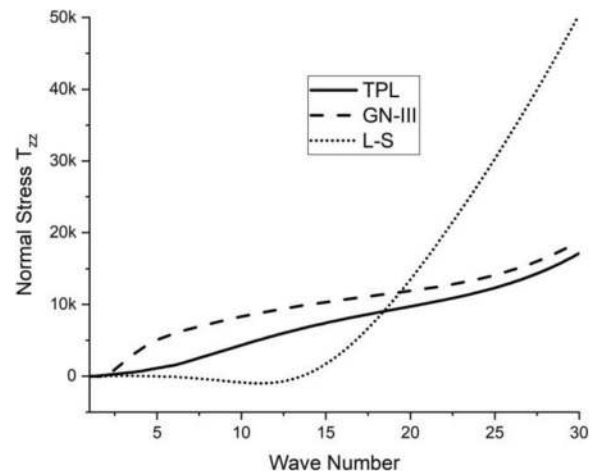

Fig. 2 Variations of normal stress t_{zz} with wavenumber

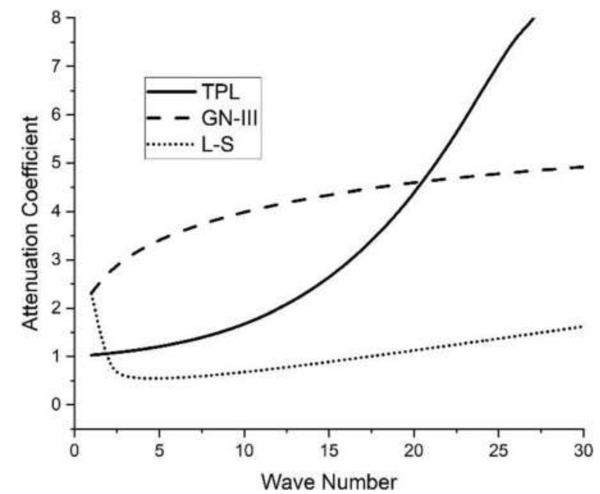

Fig. 3 Variations of attenuation coefficient with wavenumber

wavenumber. Here, we observe that the normal stress t_{zz} increases with increase in wavenumber with a small magnitude difference in all the three theories. Figure 3 illustrates the deviations of the attenuation coefficient with wavenumber. For the TPL theory, we observe that increase in attenuation coefficient is a gradually increasing which shows that for TPL theory attenuation coefficient is directly proportional to wavenumber. For GN-III theory, the attenuation coefficient increases in the form of a curve with an increase in wavenumber, while for L-S theory, the value of the attenuation coefficient decreases with increase in wavenumber. Figure 4 shows the deviations of penetration depth with wavenumber. From the graphs, we observe that the penetration depth decreases for TPL and GN-II theories, while for L-S theory, it first increases and then starts decreasing with increase in wavenumber and hence shows the influence of three different theories on penetration depth. Figure 5 illustrates the variations of specific loss with wavenumber. From the graphs, we observe that the value of specific loss first decreases and then becomes stationary with an increase in wavenumber for TPL theory. In GN-III theory, specific loss increases with increase in wavenumber, while for L-S theory, the value of specific loss first increases and then starts decreasing after attaining a maximum value at wavenumber = 2.5. Figure 6 shows variations of concentration C with wavenumber. From the graph, we observe that the concentration C increases with increase in wavenumber for all the three theories with a little magnitude difference. Figure 7 shows variations of Rayleigh wave velocity with wavenumber. The Rayleigh wave velocity increases for the GN-III theory case and no change for TPL case, while for L-S theory, it first decreases and then remains the same with an increase in wavenumber. Figure 8 shows variations of temperature T with wavenumber. From the graph, we observe that the temperature T increases with increase in wavenumber for all the three theories with a little magnitude difference. Thus, we conclude that there is a significant influence of three-phase-lag GN-III and LS on the deformation wave parameter attenuation coefficients, specific loss, wave velocity, penetration depth, temperature, concentration, tangential stress, normal stress components, and of the transversely isotropic magneto-thermoelastic medium.

Conclusion

From the above study, we conclude the following:

- A mathematical model to study the Rayleigh wave propagation in the homogeneous transversely

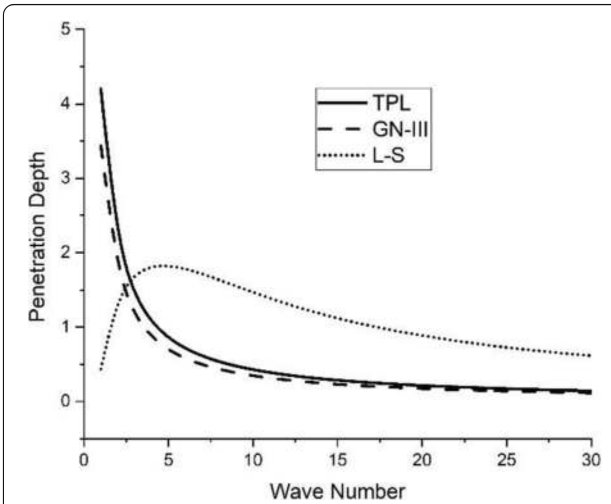

Fig. 4 Variations of penetration depth with wavenumber

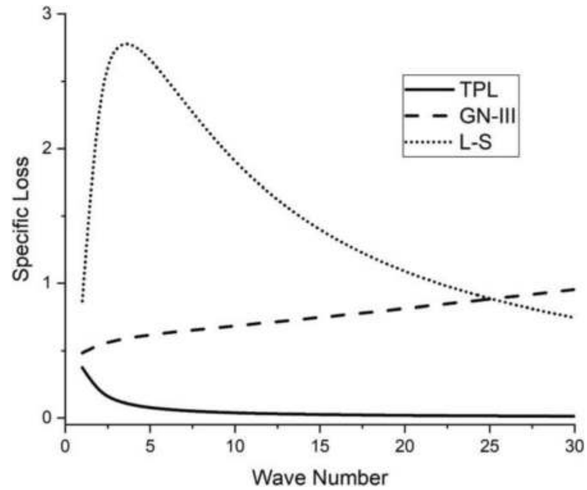

Fig. 5 Variations of specific loss with wavenumber

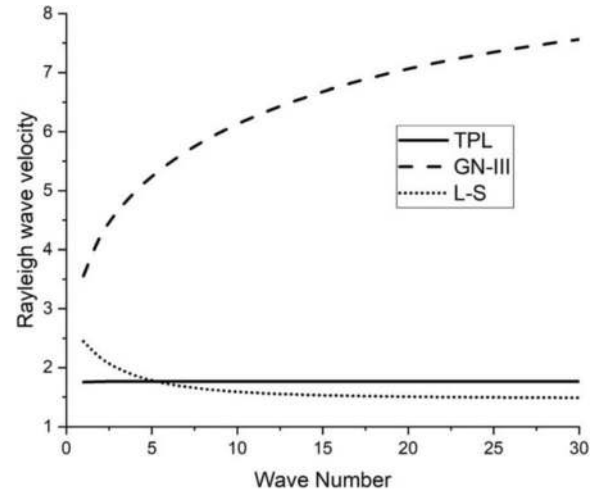

Fig. 7 Variations of Rayleigh wave velocity with wave number

isotropic magneto-thermoelastic medium in the presence of mass diffusion and the three-phase-lag heat transfer has been developed, and various wave characteristics, i.e., attenuation coefficients, specific loss, wave velocity, penetration depth, temperature, concentration, tangential stress, and normal stress components have been derived and represented graphically. The secular equation of Rayleigh waves in the presence of the effect of diffusion in a transversely isotropic magneto-thermoelastic medium has been derived. The comparison of different theories of thermoelasticity, i.e., TPL, GN-III, and L-S theories are carried out.

- From the graphs, we observe a significant influence of three-phase-lag, GN-III and LS theories on the various wave characteristics, i.e., attenuation coefficients, specific loss, wave velocity, penetration depth, temperature, concentration, tangential stress, and normal stress components in transversely isotropic magneto-thermoelastic medium. Attenuation of waves increases, whereas the penetration depth decreases with the increase in wavenumber.
- The study of elastic wave attenuation particularly in transversely isotropic magneto-thermoelastic medium carries information about transversely isotropic magneto-thermoelastic medium properties and is important for the design of geophysics and seismic investigations.
- Significant resemblance and non-resemblance among the results under TPL, GN-III, and L-S theory of thermoelasticity have been identified.
- However, the problem is theoretical, but it can deliver useful information for experimental

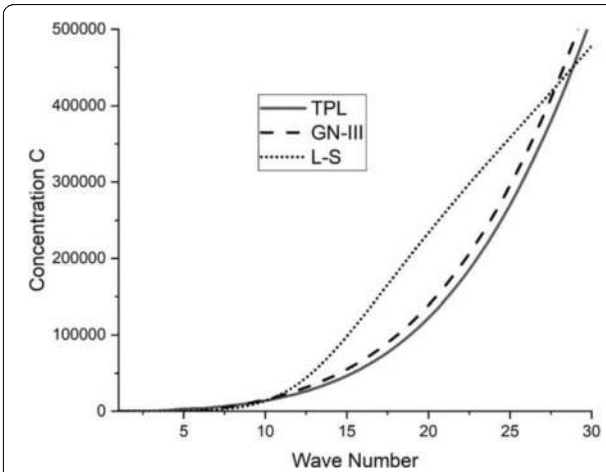

Fig. 6 Variations of concentration *C* with wavenumber

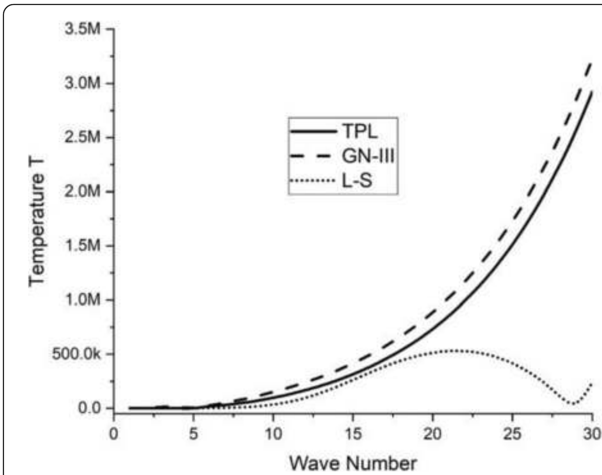

Fig. 8 Variations of temperature *T* with wave number

researchers working in the field of geophysics and earthquake engineering and seismologist working in the field of mining tremors and drilling into the Earth crust.

Nomenclature

δ_{ij} Kronecker delta
C_{ijkl} Elastic parameters
β_{ij} Thermal elastic coupling tensor
T Absolute temperature
T_0 Reference temperature
ϕ Conductive temperature
t_{ij} Stress tensors
e_{ij} Strain tensors
u_i Components of displacement
ρ Medium density
C_E Specific heat
a_{ij} Tensor of thermal moduli
α_{ij} Linear thermal expansion coefficient
K_{ij} Materialistic constant
K_{ij}^* Thermal conductivity
ω Angular frequency
μ_0 Magnetic permeability
$\mathbf{\Omega}$ Angular velocity of the solid and equal to $\Omega\boldsymbol{n}$, where $\mathbf{n}$ is a unit vector
$\overrightarrow{u}$ Displacement vector
$\overrightarrow{H}_0$ Magnetic field intensity vector
$\overrightarrow{j}$ Current density vector
F_i Components of the Lorentz force
τ_0 Relaxation time
ε_0 Electric permeability
$\delta(t)$ Dirac's delta function
τ_t Phase lag of heat flux
τ_v Phase lag of temperature gradient
τ_q Phase lag of thermal displacement
α Fractional-order derivative
ξ Wavenumber
b_{ij} Tensor of diffusion moduli
C The concentration of the diffusion material
α_{ij}^* Diffusion parameters
η_i The flow of diffusion mass vector
q_i Components of heat flux vector
P Chemical potential per unit mass
S Entropy per unit mass
k Material constant
ω_1^* Characteristics frequency of the medium
C_1 Longitudinal wave velocity

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Authors' contributions
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Availability of data and materials
For the numerical results, cobalt material has been taken for thermoelastic material from Dhaliwal and Sherief (1980).

Competing interests
The authors declare that they have no competing interests.

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