# RAYLEIGH WAVES IN A VISCOELASTIC HALF-SPACE UNDER INITIAL HYDROSTATIC STRESS IN PRESENCE OF THE TEMPERATURE FIELD

SUSHIL KUMAR ADDY AND NIL RATAN CHAKRABORTY

The effect of the temperature and initial hydrostatic stress has been shown on the propagation of Rayleigh waves in a viscoelastic half-space. It has been explained how the velocity of Rayleigh waves depends not only on the parameters pertaining to the viscoelastic properties of the half-space, but on the temperature and the initial hydrostatic stress of the half-space also. The variations of the phase velocity of Rayleigh waves in dimensionless form with respect to the magnitude of the initial hydrostatic stress under certain practical assumptions have been depicted in graphs after numerical computations. If the temperature and the initial hydrostatic stress of the half-space are neglected, the results obtained are in perfect agreement with the classical case as obtained by Caloi for the propagation of Rayleigh waves in a viscoelastic medium.

## 1. Introduction

The propagation of thermoelastic waves has been discussed long ago by Lockett [5] and Nowacki and Sokołowski [7] in different media. Recently, this has been explained in a different manner by some authors such as Chandrasekharaiah [3]. The effect of viscosity on the propagation of these waves has also been shown by a few authors such as Das and Sengupta [4]. But none of them considered the initial stress that might be present in the media. But the earth is an initially stressed medium. Hence it should be of geophysical interest to see how the initial stress influences the propagation of waves in elastic or a viscoelastic medium when the medium is heated.

This paper has discussed the effect of the temperature as well as the initial hydrostatic stress on the propagation of Rayleigh waves in a viscoelastic half-space. Here, a new frequency equation of viscoelastic Rayleigh waves has been derived, which involves the parameters connected with the temperature and the initial hydrostatic stress besides the viscoelastic properties of the half-space. The values of the phase velocity of Rayleigh waves have been computed for different values of the initial hydrostatic stress of the half-space in dimensionless form for certain values of the coupling coefficient of temperature and strain fields. These graphs show that the phase velocity of Rayleigh waves changes
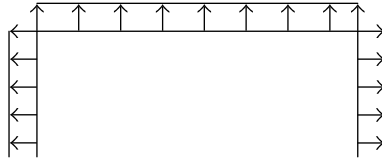
Figure 2.1. Viscoelastic half-space under initial hydrostatic stress in presence of temperature field.

remarkably with respect to the initial hydrostatic stress of the half-space as well as the coupling coefficient of temperature and strain fields of the medium.

## 2. Formulation and solution of the problem

Let us consider a Voigt-type viscoelastic half-space $y \geq 0$, the boundary of which $y = 0$ is free from tractions, but does permit heat exchange with the surroundings. Let the half-space be under an initial hydrostatic stress $H$ at an initial temperature $T_o$ (see Figure 2.1). When the temperature of the half-space is changed, incremental thermal stresses $s_{ij}$ together with incremental strains $e_{ij}$ are produced in it, which are measured with reference to the rotated axes as explained by [1].

The dynamical equations of equilibrium under initial hydrostatic stress are given by [1]

$$\frac{\partial s_{11}}{\partial x} + \frac{\partial s_{12}}{\partial y} = \rho \frac{\partial^2 u}{\partial t^2}, \qquad \frac{\partial s_{12}}{\partial x} + \frac{\partial s_{22}}{\partial y} = \rho \frac{\partial^2 v}{\partial t^2}. \tag{2.1}$$

Here, $s_{11}$, $s_{22}$ are the incremental normal thermal stresses along $x$- and $y$-axes, respectively. $s_{12}$ is the incremental shear thermal stress in the $xy$ plane. $u$ and $v$ are the displacement components along $x$- and $y$-axes, respectively.

The stress-strain relations in the Voigt-type viscoelastic half-space under thermal condition are given by

$$s_{11} = \left[ (\lambda + 2\mu) + (\lambda' + 2\mu') \frac{\partial}{\partial t} \right] e_{xx} + \left[ \lambda + \lambda' \frac{\partial}{\partial t} \right] e_{yy} - \gamma T,$$

$$s_{22} = \left[ \lambda + \lambda' \frac{\partial}{\partial t} \right] e_{xx} + \left[ (\lambda + 2\mu) + (\lambda' + 2\mu') \frac{\partial}{\partial t} \right] e_{yy} - \gamma T, \tag{2.2}$$

$$s_{12} = 2 \left[ \mu + \mu' \frac{\partial}{\partial t} \right] e_{xy},$$

where $\gamma = (3\lambda + 2\mu)\alpha_t$, $\alpha_t$ is the coefficient of linear expansion, and $T$ is the incremental change of temperature from the initial state. The incremental strain components are given by [6]

$$e_{xx} = \frac{\partial u}{\partial x}, \qquad e_{yy} = \frac{\partial v}{\partial y}, \qquad e_{xy} = \frac{1}{2} \left[ \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right]. \tag{2.3}$$
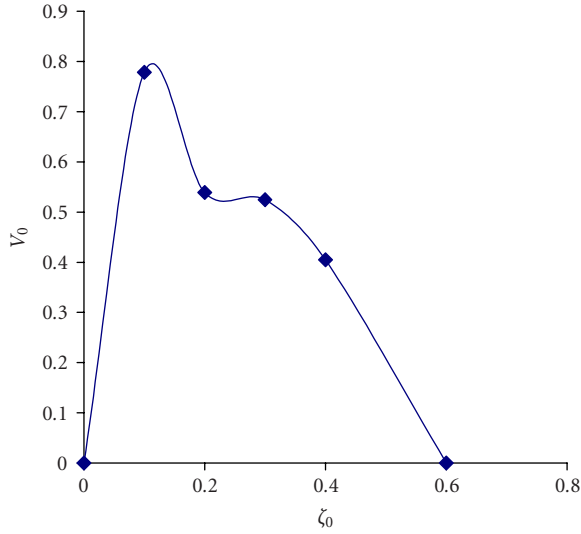
Figure 2.2. $\in = -0.1$.

Equation (2.1) with the help of (2.2) and (2.3) changes to

$$
\begin{aligned}
&\left[\lambda + 2\mu + (\lambda' + 2\mu')\frac{\partial}{\partial t}\right]\frac{\partial^2 u}{\partial x^2} + \left[(\lambda + \mu) + (\lambda' + \mu')\frac{\partial}{\partial t}\right]\frac{\partial^2 v}{\partial x \partial y} \\
&+ \left[\mu + \mu'\frac{\partial}{\partial t}\right]\frac{\partial^2 u}{\partial y^2} = \rho\frac{\partial^2 u}{\partial t^2} + \gamma\frac{\partial T}{\partial x}, \\
&\left[\lambda + 2\mu + (\lambda' + 2\mu')\frac{\partial}{\partial t}\right]\frac{\partial^2 v}{\partial y^2} + \left[\lambda + \mu + (\lambda' + \mu')\frac{\partial}{\partial t}\right]\frac{\partial^2 u}{\partial x \partial y} \\
&+ \left(\mu + \mu'\frac{\partial}{\partial t}\right)\frac{\partial^2 v}{\partial x^2} = \rho\frac{\partial^2 v}{\partial t^2} + \gamma\frac{\partial T}{\partial y}.
\end{aligned}
\tag{2.4}
$$

The displacement components $u$ and $v$ may be expressed in terms of the potential functions $\phi$ and $\psi$ as follows:

$$
u = \frac{\partial \phi}{\partial x} - \frac{\partial \psi}{\partial y}, \qquad v = \frac{\partial \phi}{\partial y} + \frac{\partial \psi}{\partial x}.
\tag{2.5}
$$

Equations (2.4) and (2.5) show that potential functions $\phi$ and $\psi$ satisfy the wave equations

$$
(\lambda + 2\mu)\nabla^2\phi + (\lambda' + 2\mu')\frac{\partial}{\partial t}(\nabla^2\phi) = \rho\frac{\partial^2\phi}{\partial t^2} + \gamma T,
\tag{2.6a}
$$

$$
\mu\nabla^2\psi + \mu'\frac{\partial}{\partial t}(\nabla^2\psi) = \rho\frac{\partial^2\psi}{\partial t^2},
\tag{2.6b}
$$

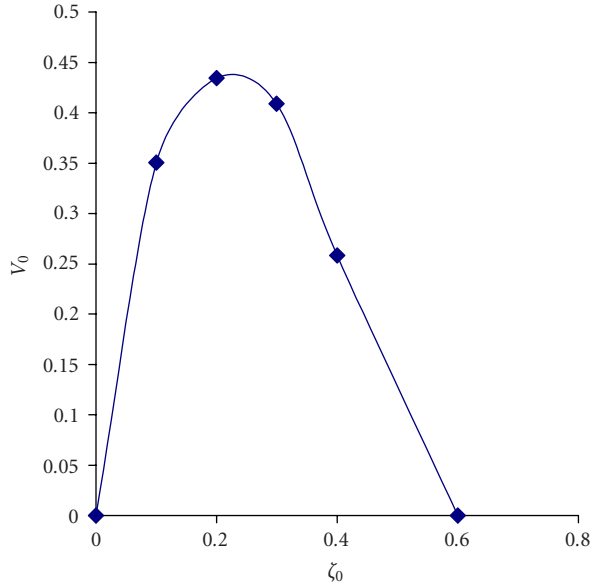where $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$.

Figure 2.3. $\in = 0$.

The heat conduction equation is given by [6]

$$\nabla^2 T - \frac{S\rho}{\delta}\frac{\partial T}{\partial t} - \frac{\gamma T_o}{\delta}\nabla^2\frac{\partial \phi}{\partial t} = 0, \qquad (2.7)$$

where $S$ is the specific heat capacity and $\delta$ is the thermal conductivity of the medium.

To find the solution of (2.6a), (2.6b), and (2.7), it is assumed that

$$\phi(x, y, t) = \phi_1(y)\exp\left[i(\kappa x - \omega t)\right], \qquad (2.8a)$$

$$\psi(x, y, t) = \psi_1(y)\exp\left[i(\kappa x - \omega t)\right], \qquad (2.8b)$$

$$T(x, y, t) = T_1(y)\exp\left[i(\kappa x - \omega t)\right], \qquad (2.8c)$$

which are plane harmonic waves moving along the $x$-axis.

Using (2.8a) and (2.8c), $T$ is eliminated from (2.6a) and (2.7), and we get

$$\left[\nabla^2 - \frac{S\rho}{\delta}\frac{\partial}{\partial t}\right]\left[\nabla^2 - \frac{1}{c_1^2 - i\omega c_1'^2}\frac{\partial^2}{\partial t^2}\right]\phi - \frac{\gamma^2 T_o}{[(\lambda + 2\mu) - i\omega(\lambda' + 2\mu')]\delta}\nabla^2\left(\frac{\partial\phi}{\partial t}\right) = 0.$$
$$(2.9a)$$

Equation (2.6b) with the help of (2.8b) can be written as

$$\left[\nabla^2 - \frac{1}{c_2^2 - i\omega c_2'^2}\frac{\partial^2}{\partial t^2}\right]\psi = 0, \qquad (2.9b)$$

where

$$c_1^2 = \frac{(\lambda + 2\mu)}{\rho}, \qquad c_1'^2 = \frac{(\lambda' + 2\mu')}{\rho}, \qquad c_2^2 = \frac{\mu}{\rho}, \qquad c_2'^2 = \frac{\mu'}{\rho}. \qquad (2.10)$$

Introducing (2.8a) into (2.9a) and (2.8b) into (2.9b), we obtain the following differential equations:

$$\left(\frac{\partial^2}{\partial y^2} - \lambda_1^2\right)\left(\frac{\partial^2}{\partial y^2} - \lambda_2^2\right)\phi_1(y) = 0,$$

$$\left(\frac{\partial^2}{\partial y^2} - \nu^2\right)\psi_1(y) = 0, \qquad (2.11)$$

where

$$\lambda_1^2 = \kappa^2 - k_1^2, \qquad \lambda_2^2 = \kappa^2 - k_2^2, \qquad \nu^2 = \kappa^2 - \tau^2, \qquad \tau = \frac{\omega^2}{(c_2^2 - i\omega c_2'^2)}. \qquad (2.12)$$

Here, $k_1^2$ and $k_2^2$ are the roots of the biquadratic equation

$$k^4 - k^2[\sigma^2 + q(1 + \in)] + q\sigma^2 = 0, \qquad (2.13)$$

where $k^2 = -\nabla^2$ and the roots $k_1^2$ and $k_2^2$ are given by

$$k_1^2 = q\left[1 + \frac{q \in}{q - \sigma^2}\right], \qquad k_2^2 = \sigma^2\left[1 - \frac{q \in}{q - \sigma^2}\right], \qquad (2.14)$$

where

$$\sigma^2 = \frac{\omega^2}{c_1^2 - i\omega c_1'^2}, \qquad q = \frac{i\omega S\rho}{\delta}, \qquad \in = \frac{\gamma^2 T_o}{S\rho[(\lambda + 2\mu) - i\omega(\lambda' + 2\mu')]}. \qquad (2.15)$$

The requirement that the stresses and hence the potential functions $\phi$ and $\psi$ vanish as $x^2 + y^2$ tends to infinity leads to the following solution of (2.11):

$$\phi_1 = Ae^{-\lambda_1 y} + Be^{-\lambda_2 y}, \qquad (2.16a)$$

$$\psi_1 = Ce^{-\nu y}. \qquad (2.16b)$$

Combining (2.8a), (2.8b), and (2.16a), (2.16b) respectively, we get

$$\phi(x, y, t) = \left\lfloor Ae^{-\lambda_1 y} + Be^{-\lambda_2 y}\right\rfloor \exp\left[i(\kappa x - \omega t)\right], \qquad (2.17a)$$

$$\psi(x, y, t) = Ce^{-\nu y} \exp\left[i(\kappa x - \omega t)\right]. \qquad (2.17b)$$

Using (2.6a), (2.8c), and (2.17a), we get

$$T = \frac{\rho m^2}{\gamma}\left[A\eta_1 e^{-\lambda_1 y} + B\eta_2 e^{-\lambda_2 y}\right] \exp\left[i(\kappa x - \omega t)\right], \qquad (2.18)$$
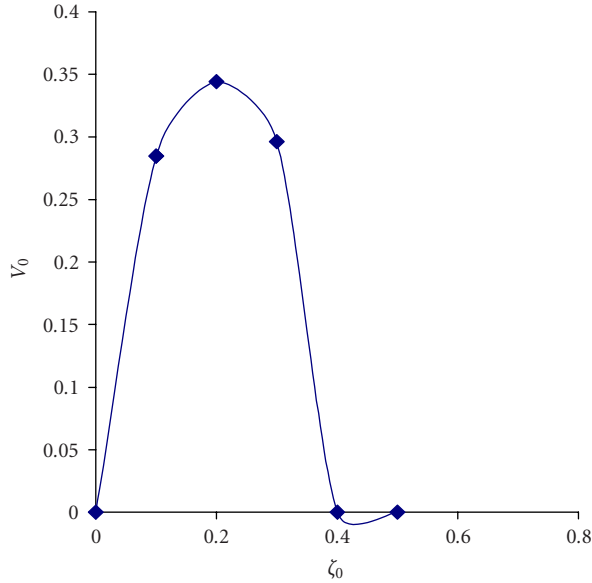
Figure 2.4. $\epsilon = +0.1$. Variation of velocity of Rayleigh waves with initial stress in viscoelastic medium when $\lambda = 0$ and $\mu = 0$ (Figures 2.2, 2.3, 2.4).

where

$$m^2 = c_1^2 - i\omega c_1'^2, \qquad \eta_1 = \sigma^2 - k_1^2, \qquad \eta_2 = \sigma^2 - k_2^2. \tag{2.19}$$

The boundary conditions on the plane $y = 0$ are given by [1]

$$\Delta f_x = s_{12} - H\frac{\partial v}{\partial x} = 0, \qquad \Delta f_y = s_{22} + H\frac{\partial u}{\partial x} = 0, \qquad \frac{\partial T}{\partial y} + hT = 0. \tag{2.20}$$

Here, $\Delta f_x$ and $\Delta f_y$ are incremental boundary forces per unit initial area along $x$- and $y$-axes, respectively, and $h$ is the ratio between the coefficient of heat transfer and thermal conductivity.

By using (2.2), (2.3), (2.5), (2.17a), (2.17b), and (2.20), we change these boundary conditions to

$$A\left[\kappa\lambda_1\{i(2\mu - H) + 2\omega\mu'\}\right] + B\left[\kappa\lambda_2\{i(2\mu - H) + 2\omega\mu'\}\right]$$
$$+ C\left[\kappa^2\{(2\mu - H) - 2i\omega\mu'\} - \tau^2(\mu - i\omega\mu')\right] = 0,$$
$$A\left[\rho m^2(\kappa^2 - \sigma^2) - \kappa^2\{(\lambda + H) - i\omega\lambda'\}\right] + B\left[\rho m^2(\kappa^2 - \sigma^2) - \kappa^2\{(\lambda + H) - i\omega\lambda'\}\right]$$
$$+ C\left[-\kappa\nu\{2\mu'\omega + i(2\mu - H)\}\right] = 0,$$
$$A\left[\eta_1(h - \lambda_1)\right] + B\left[\eta_2(h - \lambda_2)\right] = 0. \tag{2.21}$$

Eliminating $A$, $B$, $C$ from (2.21), we get the following frequency equation of Rayleigh waves:

$$
\begin{bmatrix}
[\kappa\lambda_1\{i(2\mu - H) + 2\mu'\omega\}] & [\kappa\lambda_2\{i(2\mu - H) + 2\mu'\omega\}] & \begin{matrix}[\kappa^2\{(2\mu - H) - 2i\omega\mu'\} \\ -\tau^2(\mu - i\omega\mu')]\end{matrix} \\
\begin{matrix}[\rho m^2(\kappa^2 - \sigma^2) \\ -\kappa^2(\lambda + H - i\omega\lambda')]\end{matrix} & \begin{matrix}[\rho m^2(\kappa^2 - \sigma^2) \\ -\kappa^2(\lambda + H - i\omega\lambda')]\end{matrix} & [-\kappa\nu\{2\mu'\omega + i(2\mu - H)\}] \\
\eta_1(h - \lambda_1) & \eta_2(h - \lambda_2) & 0
\end{bmatrix} = 0.
$$

$$(2.22)$$

If we ignore the presence of the initial stress and the temperature field in the half-space, then $H$ and $q$ both vanish, $h = 0$ and $\lambda_1 = \kappa$ and (2.22) reduces to

$$
\begin{bmatrix}
\left[\kappa^2 \dfrac{2i\rho\omega^2}{k_\beta^2}\right] & \left[\kappa\sqrt{\kappa^2 - k_\alpha^2}\dfrac{2i\rho\omega^2}{k_\beta^2}\right] & \left[\dfrac{2\kappa^2\rho\omega^2}{k_\beta^2} - \rho\omega^2\right] \\
\left[\dfrac{\rho\omega^2}{k_\alpha^2}(\kappa^2 - k_\alpha^2) - \kappa^2\rho\omega^2\left(\dfrac{1}{k_\alpha^2} - \dfrac{2}{k_\beta^2}\right)\right] & \left[\dfrac{\rho\omega^2}{k_\alpha^2}(\kappa^2 - k_\alpha^2) - \kappa^2\rho\omega^2\left(\dfrac{1}{k_\alpha^2} - \dfrac{2}{k_\beta^2}\right)\right] & \left[-\dfrac{2i\rho\omega^2}{k_\beta^2}\kappa\sqrt{\kappa^2 - k_\beta^2}\right] \\
-\kappa k_\alpha^2 & 0 & 0
\end{bmatrix} = 0,
$$

$$(2.23)$$

where we have used

$$
k_\alpha^2 = \frac{\rho\omega^2}{(\lambda + 2\mu) - i\omega(\lambda' + 2\mu')}, \qquad k_\beta^2 = \frac{\rho\omega^2}{\mu - i\omega\mu'}.
$$

$$(2.24)$$

Expanding the determinant (2.23) and simplifying, we get

$$
1 - 8\frac{\kappa^2}{k_\beta^2} + \left[24 - 16\frac{k_\alpha^2}{k_\beta^2}\right]\frac{\kappa^4}{k_\beta^4} - 16\left(1 - \frac{k_\alpha^2}{k_\beta^2}\right)\frac{\kappa^6}{k_\beta^6} = 0,
$$

$$(2.25)$$

which is the same equation as obtained by Caloi [2] for the propagation of Rayleigh waves in a viscoelastic half-space without initial stress and temperature.

But considering the initial hydrostatic stress and the temperature field to be present in the half-space and assuming that $\lambda = \mu$ and $\lambda' = (2/3)\mu'$; $\sigma$, $k_1$, $k_2$, and $\in$ all change, respectively, to $\sigma'$, $k_1'$, $k_2'$, and $\in'$ are given by

$$
\sigma'^2 = \frac{\omega^2}{c_{01}^2 - i\omega c_{01}'^2}, \quad c_{01}^2 = \frac{2\mu}{\rho}, \quad c_{01}'^2 = \frac{8\mu'}{3\rho},
$$

$$
k_1'^2 = q\left[1 + \frac{q\in'}{q - \sigma'^2}\right], \qquad k_2'^2 = \sigma'^2\left[1 - \frac{q\in'}{q - \sigma'^2}\right], \qquad \in' = \frac{3\gamma^2 T_o}{S\rho[9\mu - i\omega 8\mu']}.
$$

$$(2.26)$$

Then, (2.22) reduces to

$$
\begin{bmatrix}
2i\beta_1\dfrac{(x\mu - i\omega\mu')}{(\mu - i\omega\mu')} & 2i\beta_2\dfrac{(x\mu - i\omega\mu')}{(\mu - i\omega\mu')} & \dfrac{\tau^2}{\kappa^2} - \dfrac{2(x\mu - i\omega\mu')}{(\mu - i\omega\mu')} \\
\dfrac{2(x\mu - i\omega\mu')}{(\mu - i\omega\mu')} - \dfrac{\tau^2}{\kappa^2} & \dfrac{2(x\mu - i\omega\mu')}{(\mu - i\omega\mu')} - \dfrac{\tau^2}{\kappa^2} & \dfrac{2i\beta_3(x\mu - i\omega\mu')}{(\mu - i\omega\mu')} \\
\eta_1(h - \kappa\beta_1) & \eta_2(h - \kappa\beta_2) & 0
\end{bmatrix} = 0,
$$

$$(2.27)$$

where

$$\beta_1 = \sqrt{1 - \frac{k_1'^2}{\kappa^2}}, \qquad \beta_2 = \sqrt{1 - \frac{k_2'^2}{\kappa^2}}, \qquad \beta_3 = \sqrt{1 - \frac{\tau^2}{\kappa^2}}, \qquad x = 1 - \frac{H}{2\mu}. \tag{2.28}$$

Expanding the determinant in (2.27), we get

$$\left[ \frac{2(x\mu - i\omega\mu')}{(\mu - i\omega\mu')} - \frac{\tau^2}{\kappa^2} \right]^2 = 4\beta_3 \left[ \frac{x\mu - i\omega\mu'}{\mu - i\omega\mu'} \right]^2 \left[ \frac{\kappa\beta_1\beta_2(\eta_1 - \eta_2) + h(\beta_1\eta_2 - \beta_2\eta_1)}{h(\eta_2 - \eta_1) + \kappa(\beta_1\eta_1 - \beta_2\eta_2)} \right]. \tag{2.29}$$

In deducing this equation, we assumed a convection condition for the temperature on the boundary of the half-space. For thermal insulation, $h = 0$ and (2.29) reduces to

$$\left[ \frac{2(x\mu - i\omega\mu')}{(\mu - i\omega\mu')} - \frac{\tau^2}{\kappa^2} \right]^2 = 4\beta_3 \left[ \frac{x\mu - i\omega\mu'}{\mu - i\omega\mu'} \right]^2 \frac{\beta_1\beta_2(\eta_1 - \eta_2)}{(\beta_1\eta_1 - \beta_2\eta_2)}. \tag{2.30}$$

Assuming heat transfer to be infinitely large, $h = \infty$ and (2.29) reduces to

$$\left[ \frac{2(x\mu - i\omega\mu')}{(\mu - i\omega\mu')} - \frac{\tau^2}{\kappa^2} \right]^2 = 4\beta_3 \left[ \frac{x\mu - i\omega\mu'}{\mu - i\omega\mu'} \right]^2 \frac{(\beta_1\eta_2 - \beta_2\eta_1)}{(\eta_2 - \eta_1)}. \tag{2.31}$$

Expressing the quantities $\lambda_1$, $\lambda_2$, $\nu$, $\eta_1$, and $\eta_2$ in terms of the quantities $\beta_1$, $\beta_2$, and $\beta_3$, we find that (2.29) reduces to

$$\begin{aligned}
&\left[ \frac{2(x\mu - i\omega\mu')}{(\mu - i\omega\mu')} - \frac{c^2}{c_2^2 - i\omega c_2'^2} \right]^2 \left[ \beta_1^2 + \beta_2^2 + \beta_1\beta_2 - 1 + \frac{c^2}{(c_{01}^2 - i\omega c_{01}'^2)} \right] \\
&\quad - 4\left[ \frac{x\mu - i\omega\mu'}{\mu - i\omega\mu'} \right]^2 \beta_1\beta_2\beta_3(\beta_1 + \beta_2) \\
&= \frac{h}{\kappa} \left[ \left\{ \frac{2(x\mu - i\omega\mu')}{(\mu - i\omega\mu')} - \frac{c^2}{c_2^2 - i\omega c_2'^2} \right\} (\beta_1 + \beta_2) \right. \\
&\quad \left. - 4\left( \frac{x\mu - i\omega\mu'}{\mu - i\omega\mu'} \right)^2 \beta_3 \left( \beta_1\beta_2 + 1 - \frac{c^2}{c_{01}^2 - i\omega c_{01}'^2} \right) \right],
\end{aligned} \tag{2.32}$$

where $c^2 = \omega^2/\kappa^2$ is the phase velocity of Rayleigh waves.

The quantity $1/\mathrm{Re}(1/c)$ is a measure of the phase velocity and $\omega I_m(1/c)$ is a measure of the damping of Rayleigh waves propagating along the positive $x$-axis.

Under the assumptions $\lambda = \mu$ and $\lambda' = (2/3)\mu'$, (2.13) changes to

$$k_1'^2 + k_2'^2 = \sigma'^2 + q(1+ \in'), \qquad k_1'^2 k_2'^2 = \sigma'^2 q. \tag{2.33}$$

Using (2.28) and (2.33), we get

$$\beta_1^2 + \beta_2^2 = 2 - \frac{c^2}{c_{01}^2 - i\omega c_{01}'^2} - \frac{ic^2(1+\in')}{f(c_{01}^2 - i\omega c_{01}'^2)},$$

$$\beta_1^2\beta_2^2 = 1 - \frac{c^2}{c_{01}^2 - i\omega c_{01}'^2} - \frac{ic^2}{f(c_{01}^2 - i\omega c_{01}'^2)}\left[1 + \in' - \frac{c^2}{c_{01}^2 - i\omega c_{01}'^2}\right],$$

$$(2.34)$$

where $f = \delta\omega/S\rho(c_{01}^2 - i\omega c_{01}'^2)$ is the reduced frequency.

Substituting (2.34) into (2.32), expanding the quantities $\beta_1, \beta_2$ in a series of "$f$," and neglecting the terms of the order $f^{1/2}$, we get

$$\left[\left(x\mu - \frac{\rho c^2}{2}\right) - i\omega\mu'\right]^2 = (x\mu - i\omega\mu')^2 \left(1 - \frac{\rho c^2}{\mu - i\omega\mu'}\right)^{1/2}\left(1 - \frac{3\rho c^2}{(1+\in')(9\mu - 8i\omega\mu')}\right)^{1/2}.$$

$$(2.35)$$

Considering a liquid medium, we write $\lambda = \mu = 0$ and (2.35) simplifies to

$$\left[\frac{H}{2} + \frac{\rho c^2}{2} + i\omega\mu'\right]^2 = \left(\frac{H}{2} + i\omega\mu'\right)^2\left(1 - \frac{i\rho c^2}{\omega\mu'}\right)^{1/2}\left(1 - \frac{3i\rho c^2}{8(1+i\in_0)\omega\mu'}\right)^{1/2},$$

$$(2.36)$$

where $\in_0 = 3\gamma^2 T_0/8\rho S\omega\mu'$, which is deduced from (2.26).

Also, from (2.28), $x\mu = (1 - H/2\mu)\mu = -H/2$.

Rationalizing and squaring (2.36), we get

$$\left(\frac{H}{2\omega\mu'} + \frac{\rho c^2}{2\omega\mu'} + i\right)^4 = \left(\frac{H}{2\omega\mu'} + i\right)^4\left(1 - \frac{i\rho c^2}{\omega\mu'}\right)\left(1 - \frac{3\rho c^2 \in_0}{8(1+\in_0^2)\omega\mu'} - \frac{3i\rho c^2}{8(1+\in_0^2)\omega\mu'}\right).$$

$$(2.37)$$

Expanding (2.37) and equating the real parts only, we get

$$\left(\varsigma_0 + \frac{V_0^2}{2}\right)^4 - 6\left(\varsigma_0 + \frac{V_0^2}{2}\right)^2 + 1 = (\varsigma_0^4 - 6\varsigma_0^2 + 1)\left[1 - \frac{3V_0^2 \in_0}{8(1+\in_0^2)} - \frac{3V_0^4}{8(1+\in_0^2)}\right]$$

$$+ 4V_0^2\varsigma_0(\varsigma_0^2 - 1)\left[1 - \frac{3V_0^2 \in_0}{8(1+\in_0^2)} + \frac{3}{8(1+\in_0^2)}\right],$$

$$(2.38)$$

where $V_0^2 = c^2/\omega\mu'/\rho$ and $\varsigma_0 = H/2\omega\mu'$.

For different values of $\varsigma_0 = H/2\omega\mu'$, the values of $V_0$ are calculated for some specific value of $\in = \gamma^2 T_o/S\rho[(\lambda + 2\mu) - i\omega(\lambda' + 2\mu')]$ and the results are plotted in graphs. From these graphs, we find that the maximum value of the phase velocity $V_0$ of Rayleigh waves in viscoelastic liquid decreases as $\in$ changes from $-0.1$ to $0.1$.

When $\in = -0.1$, $V_0$ vanishes at $\varsigma_0 = H/2\omega\mu' = 0$. It attains its maximum value 0.77850 at $\varsigma_0 = 0.1$, then decreases gradually as $\varsigma_0$ increases, and finally vanishes again at $\varsigma_0 = 0.5$ (Figure 2.2).

When $\in = 0$, $V_0$ vanishes at $\varsigma_0 = H/2\omega\mu' = 0$, attains a maximum value 0.43444 at $\varsigma_0 = 0.2$, then gradually decreases as $\varsigma_0$ increases and vanishes again at $\varsigma_0 = 0.5$ (Figure 2.3).
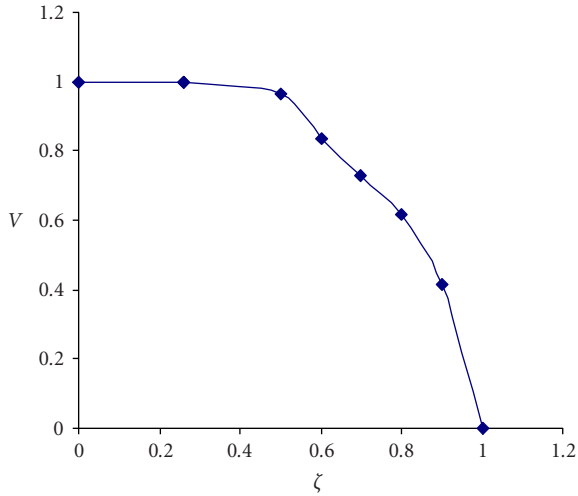
Figure 2.5. $\theta = 1/2$.



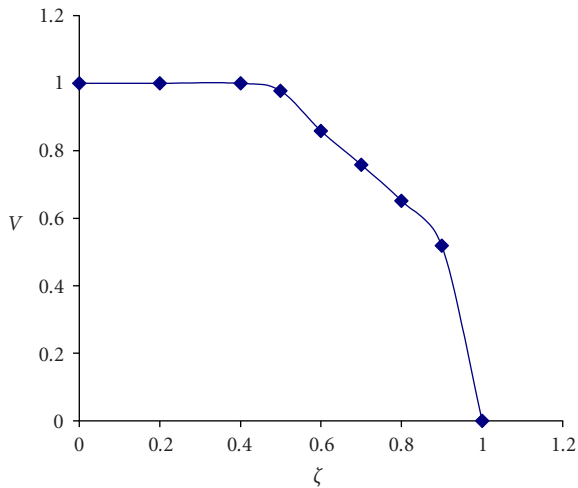Figure 2.6. $\theta = 1/3$.

When $\in = +0.1$, $V_0$ vanishes at $\varsigma_0 = H/2\omega\mu' = 0$, attains a maximum value $0.34443$ at $\varsigma_0 = 0.2$, then decreases gradually as $\varsigma_0$ increases, and vanishes again at $\varsigma_0 = 0.5$ (**Figure 2.4**).

In all the above three cases, the phase velocity $V_0$ ceases to exist at two particular values of $\varsigma_0$, which are $\varsigma_0 = 0$ and $\varsigma_0 = 0.5$, that is, when there is no initial stress and when the initial stress is equal to the product of the angular frequency $\omega$ of Rayleigh waves and the rigidity $\mu'$ of the fluid. But the maximum values of $V_0$ are different for different values of $\in$.
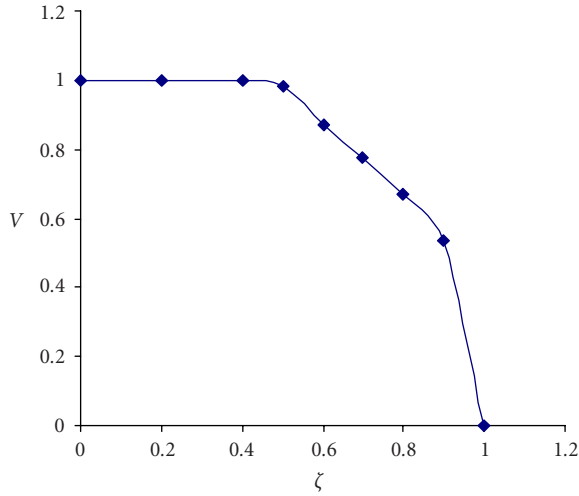
Figure 2.7. $\theta = 1/4$. Variation of velocity of Rayleigh waves with initial stress when viscous effect is neglected (Figures 2.5, 2.6, 2.7).

If we neglect the viscous properties of the half-space, then $\mu' = 0$ and (2.35) reduces to

$$[2(1-\varsigma) - V^2]^2 = 4(1-\varsigma)^2[1 - V^2]^{1/2}[1 - V^2\theta]^{1/2}, \tag{2.39}$$

where $\varsigma = H/2\mu$, $V^2 = c^2/c_2^2$, and $\theta = c_2^2/(1+\in)c_1^2$.

Ignoring the initial stress, that is, with $H = 0$, (2.39) reduces to

$$[2 - V^2]^2 = 4[1 - V^2]^{1/2}[1 - V^2\theta]^{1/2}. \tag{2.40}$$

This frequency equation for Rayleigh waves in an elastic solid medium is in perfect agreement with that obtained by Lockett [5]. Comparing (2.39) and (2.40), we find that in the result obtained by Lockett, the phase velocity of Rayleigh waves depends on the coupling coefficient $\theta = c_2^2/(1+\in)c_1^2$ between temperature and strain fields, but the results of the present authors show that the phase velocity depends on the initial stress also, besides the factor $\theta$ in an elastic solid medium.

If we vary $\varsigma$ keeping $\theta$ fixed, then the trend of phase velocity of Rayleigh waves changes remarkably, which is quite a new result in this paper. This fact may be used to understand the nature of Rayleigh waves accurately in seismology.

For different values of $\varsigma = H/2\mu$, the values of $V$ are calculated for some specific values of $\theta = c_2^2/(1+\in)c_1^2$ and these results are plotted in graphs. From these graphs, we find that the maximum and minimum values of phase velocity of Rayleigh waves are the same which are 1 and 0, respectively, for $\zeta = 0$ and 1 if we take $\theta = 1/2$, 1/3, or 1/4 (Figures 2.5, 2.6, 2.7). Here, $\zeta = 0$ implies that there is no initial stress and $\zeta = 1$ implies that the initial hydrostatic stress is twice the rigidity of the solid elastic medium. We also note from the graphs that due to different values of $\theta$, there are slight variations in the overall

nature of the graphs. This signifies that in an elastic solid medium, the phase velocity of Rayleigh waves depends much more on the initial stress of the medium than the coupling coefficient $\theta$ between temperature and strain fields. This finding is also of paramount importance in seismology.

## References

[1]    M. A. Biot, *Mechanics of Incremental Deformations. Theory of Elasticity and Viscoelasticity of Initially Stressed Solids and Fluids, Including Thermodynamic Foundations and Applications to Finite Strain*, John Wiley & Sons, New York, 1965.

[2]    P. Caloi, *Comportement des ondes de Rayleigh dans un milieu firmo-élastique indéfini*, Publ. Bureau Central Seismol. Internat. Sér. A. Trav. Sci. **17** (1950), 89–108.

[3]    D. S. Chandrasekharaiah, *One-dimensional wave propagation in the linear theory of thermoelasticity without energy dissipation*, J. Thermal Stresses **19** (1996), 695–710.

[4]    T. K. Das and P. R. Sengupta, *Effect of gravity on visco-elastic surface waves in solids involving time rate of strain and stress of first order*, Sadhana **17** (1992), part 2, 315–323.

[5]    F. J. Lockett, *Effect of thermal properties of a solid on the velocity of Rayleigh waves*, J. Mech. Phys. Solids **7** (1958), no. 1, 71–75.

[6]    W. Nowacki, *Thermoelasticity*, Addison-Wesley, London, 1962.

[7]    W. Nowacki and M. Sokołowski, *Propagation of thermoelastic waves in plates*, Arch. Mech. (Arch. Mech. Stos.) **11** (1959), no. 6, 715–727.

Sushil Kumar Addy: Department of Mathematics, Jamshedpur Co-operative College, Jamshedpur Ranchi University, 831001 Jharkhand, India
    *E-mail address*: skaddy@sify.com

Nil Ratan Chakraborty: Department of Physics, Jamshedpur Co-operative College, Jamshedpur Ranchi University, 831001 Jharkhand, India
    *E-mail address*: nil_c53@yahoo.co.in

# Special Issue on Time-Dependent Billiards

## Call for Papers

This subject has been extensively studied in the past years for one-, two-, and three-dimensional space. Additionally, such dynamical systems can exhibit a very important and still unexplained phenomenon, called as the Fermi acceleration phenomenon. Basically, the phenomenon of Fermi acceleration (FA) is a process in which a classical particle can acquire unbounded energy from collisions with a heavy moving wall. This phenomenon was originally proposed by Enrico Fermi in 1949 as a possible explanation of the origin of the large energies of the cosmic particles. His original model was then modified and considered under different approaches and using many versions. Moreover, applications of FA have been of a large broad interest in many different fields of science including plasma physics, astrophysics, atomic physics, optics, and time-dependent billiard problems and they are useful for controlling chaos in Engineering and dynamical systems exhibiting chaos (both conservative and dissipative chaos).

We intend to publish in this special issue papers reporting research on time-dependent billiards. The topic includes both conservative and dissipative dynamics. Papers discussing dynamical properties, statistical and mathematical results, stability investigation of the phase space structure, the phenomenon of Fermi acceleration, conditions for having suppression of Fermi acceleration, and computational and numerical methods for exploring these structures and applications are welcome.

To be acceptable for publication in the special issue of Mathematical Problems in Engineering, papers must make significant, original, and correct contributions to one or more of the topics above mentioned. Mathematical papers regarding the topics above are also welcome.

Authors should follow the Mathematical Problems in Engineering manuscript format described at http://www.hindawi.com/journals/mpe/. Prospective authors should submit an electronic copy of their complete manuscript through the journal Manuscript Tracking System at http://mts.hindawi.com/ according to the following timetable:

| Manuscript Due | December 1, 2008 |
| --- | --- |
| First Round of Reviews | March 1, 2009 |
| Publication Date | June 1, 2009 |

### Guest Editors

**Edson Denis Leonel,** Departamento de Estatística, Matemática Aplicada e Computação, Instituto de Geociências e Ciências Exatas, Universidade Estadual Paulista, Avenida 24A, 1515 Bela Vista, 13506-700 Rio Claro, SP, Brazil ; edleonel@rc.unesp.br

**Alexander Loskutov,** Physics Faculty, Moscow State University, Vorob'evy Gory, Moscow 119992, Russia; loskutov@chaos.phys.msu.ru