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Re-determination of the decade fluctuations in the rotation of the Earth in the period 1861–1978

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Summary. The 'decade' fluctuations in the Earth's rate of rotation are derived from a comparison of the universal time-scale, based on the Earth's diurnal rotation, with: (a) the dynamical time-scale of the lunar ephemeris from 1861 to 1955 (using the collected times of occultations of stars), and (b) the international atomic time-scale from 1955 to 1978. It is concluded from these fluctuations that the torques which operated on the Earth's mantle varied in magnitude in a characteristic period of about 30 yr and reached a maximum of 10^{18} Nm around 1900.

1 Introduction

Brouwer (1952) deduced from a study of the motion of the Moon that there are fluctuations in the rate of rotation of the Earth with a characteristic period of 10–30 yr. It is now generally accepted that these so-called decade fluctuations are caused by the transfer of angular momentum between the core and mantle. Various mechanisms have been invoked to explain how the angular momentum is transferred and these have met with varying degrees of success (Rochester 1974). In order to try to discriminate between these hypotheses we need a better determination of the characteristics of the fluctuations and hence of the magnitude and duration of the torques acting between the core and mantle. The purpose of this paper is to re-determine these decade fluctuations in the period 1861 to 1978 with improved accuracy.

The fluctuations in the Earth's rate of rotation introduce irregularities in the universal time-scale which is derived from the diurnal rotation of the Earth. On the other hand, the orbital motions of the Sun (the Earth's reflected motion), Moon and planets are free from fluctuations, apart, of course, from known gravitational perturbations, and observations of their positions can be used to derive a uniform time-scale that is independent of the Earth's rotational time-scale, universal time. The Moon's comparatively rapid angular motion, and the existence of a continuous series of accurate observations of its position relative to the stars, make it the most suitable object for measuring the uniform time-scale. The difference between the uniform time-scale from the Moon's motion and the universal time-scale gives a measure of the fluctuations in the Earth's rotation. In particular, the difference in the time-scales can be determined directly from a comparison of the observed times on the universal time-scale at which the Moon occults stars in its path and the predicted times on the uniform time-scale defined by the time-argument of the lunar ephemeris.

We have collected about 50 000 times of occultations in the period 1861 to 1955 and in this paper we analyse them for the difference between the universal time-scale and the uniform reference time-scale. Our results are expected to be an improvement upon those of Brouwer because we have more data available and we can apply corrections for the irregularities of the Moon's profile (Watts 1963). At the time that Brouwer carried out his analysis in 1952 he did not have the benefit of the high-precision data on the Earth's rotation which followed from the replacement in 1955 of the Moon's motion by atomic clocks as the long-term standard of time-keeping. The well-determined behaviour of the Earth's rotation in the period 1955 to 1978 is used in this analysis as a guide to eliminating fluctuations in the lunar data which are not due to variations in the Earth's rotation.

The essential features of the results of this analysis are presented in three figures (Figs 3, 4 and 5), the third of which shows the magnitude and temporal behaviour of the torques operating on the mantle in the period 1861–1978. The torques have a characteristic period of about 30 yr and reach a maximum of 10^{18} Nm around 1900.

2 Observations

Although Brouwer had analysed most of the available occultation observations, they were nowhere collected or published in one place prior to 1943. Beginning with the year 1943, HM Nautical Almanac Office took over the international responsibility for collecting and processing the occultation observations and these are published in a uniform layout by Morrison (1978a) for the years 1943 to 1971. Prior to 1943 the work of locating and copying the observations was divided between the Nautical Almanac Offices of the Royal Greenwich Observatory and US Naval Observatory. From 1899 onwards the *Astronomische Jahresbericht* was used as the reference source: for the years 1861 to 1898 the subject indices of the astronomical journals and the annals of observatories were searched. The present analysis is confined to the years 1861 to 1978 because the period before 1861 has already been treated by Martin (1969) and it is already known from Brouwer's work that particularly large and interesting fluctuations in the rate of rotation occurred around 1900.

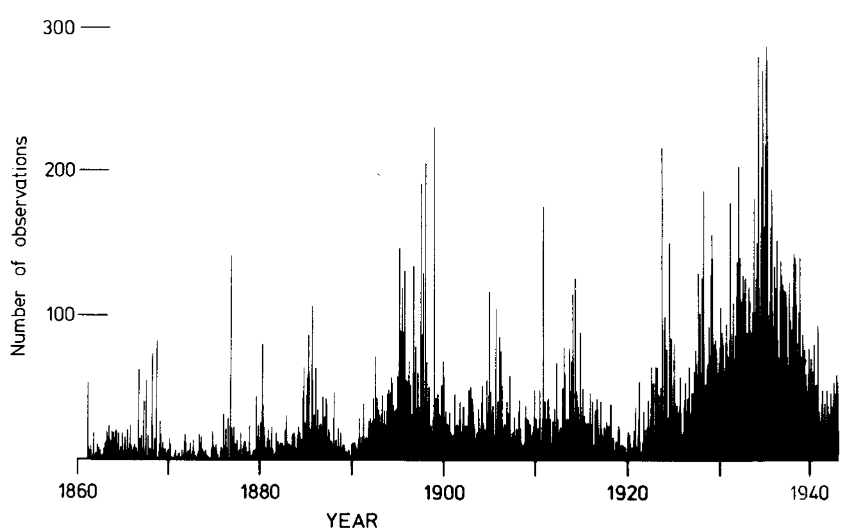

Figure 1. Distribution by lunation of occultation observations in the years 1861 to 1942.

About 35 000 times of occultations made in the years 1861 to 1942 recorded to a precision of 0.1 s were transcribed from the literature and reduced to universal time. Their distribution by lunation is shown in Fig. 1. The distribution of the data from 1943 onwards can be found in the analysis of these data by Morrison (1979). The annual cycle in the number of observations, having their modes near the beginning of each year, is due to the fact that most of the observations were made in the northern hemisphere during the longer nights of the winter months. This periodicity in the distribution of the data lends itself to grouping the observations into yearly batches when solving for the differences between the time-scales.

The occultation observations for the years 1861 to 1942 will be published in a *Royal Greenwich Observatory Bulletin* in a similar format to the data for the years 1943 onwards.

3 Reduction

The precise method of reducing occultation observations used in this analysis is given by Morrison (1979), and only the essential outline will be given here. The lunar ephemeris used in this analysis is that used in the current *Astronomical Ephemeris* and is designated by $j = 2$. It is based on Brown's theory with modifications for recent determinations of some of the empirical parameters. The expression for the mean longitude in $j = 2$ was amended by the addition of the correction

$$\Delta L = -1''.54 + 2''.33T - 1''.78T^2,$$

where T is measured in Julian centuries from the epoch 1900.0. This correction was derived by Morrison (1979) in order that the argument of time in the ephemeris $j = 2$ should be dynamical time (TD), where TD is defined relative to the international atomic time-scale (TAI) by the relation

$$\text{TD} = \text{TAI} + 32^{\text{s}}.184$$

at the arbitrary epoch 1977 January 1.0. The unit of measure on the TD scale at this epoch is defined to be one day of exactly 86 400 SI seconds (see IAU 1977).

The lunar ephemeris, amended by the expression for ΔL above, was entered for the time of observation using a preliminary approximation, $\Delta T'$, to the difference between the dynamical and universal time-scales. The values of $\Delta T'$ were taken from a smooth curve fitted through the values in table VIIIa of Brouwer (1952) and amended for the change in mean longitude given above, by the addition of $-1.821\,\Delta L$ (s), where 1.821 is the reciprocal of the Moon's mean motion in s/arcsec. The residual angular separation between the position of the star and the outline of the Moon, conventionally denoted by $\Delta\sigma$, was calculated for the time of each observation as described by Morrison (1979) for the data for the years 1943 onwards. The present discussion of these residuals is confined to the years 1861 to 1942.

No limb-profile corrections were available from Watts' charts for about 5 per cent of the 35 000 observations made in the years 1861 to 1942, so these were excluded from further analysis. The distribution of the remaining residuals lying in the range $-3''$ to $+3''$ is shown as a dotted line in Fig. 2. There is an evident bias towards the positive residuals in the figure. This bias is due to errors in timing occultations at the illuminated limb where the star is lost in the glare of the limb, thus increasing the residual positively. Almost 3000 observations were rejected for this reason, leaving about 30 000 for further analysis. The distribution of these is shown by the solid line in Fig. 2. The mean and standard deviation of the distribution were found to be $-0''.16$ and $\pm 0''.63$ respectively. The normal frequency curve

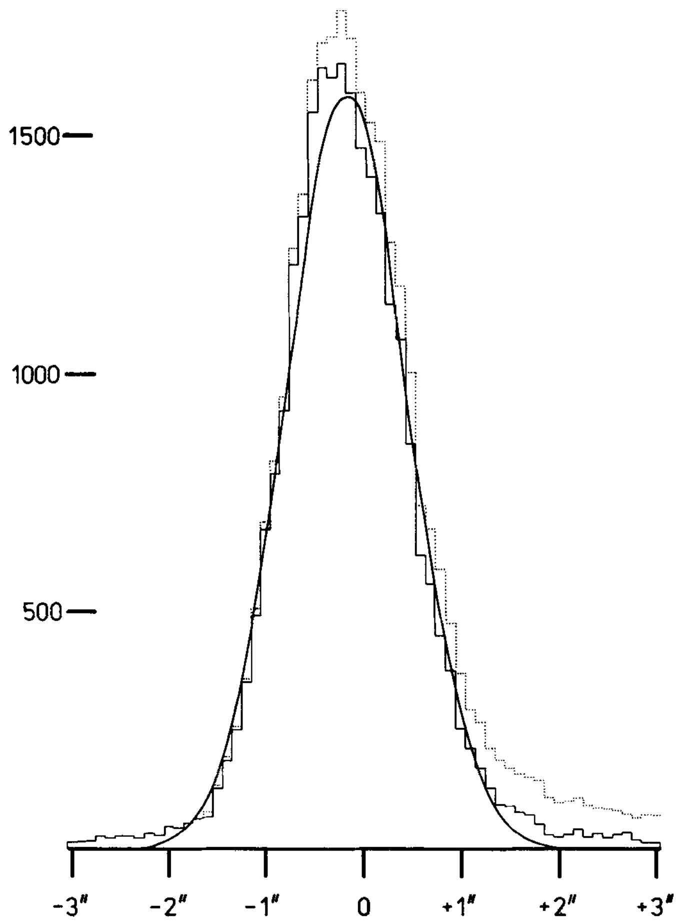

Figure 2. Histogram of the computed angular residuals ($\Delta\sigma$) with fitted normal frequency curve having a mean of $-0''.16$ and a standard deviation of $0''.63$.

corresponding to these values is shown in Fig. 2. The off-set of the mean from zero is due to a bias in the approximation, $\Delta T'$, to the difference between the dynamical and universal time-scales used in reducing the observations.

It was decided to retain for analysis only the 25 000 residuals lying within the range $-2''.0$ to $+2''.0$, which is slightly greater than three times the standard deviation. This leaves about 5000 observations outside the range of three standard deviations. The excessive number of residuals in this range arises from mistakes made in recording the date and time of occultation or in reporting the number of the star.

The 25 000 angular residuals ($\Delta\sigma$) in the range $-2''.0$ to $+2''.0$ were analysed for the orthogonal components in orbital longitude and latitude. The component in longitude was considered to arise wholly from a correction $\delta(\Delta T)$ to the preliminary values, $\Delta T'$. Morrison (1979) found that the greatest correction to the longitude in the ephemeris $j = 2$ has a semi-amplitude of about $0''.15$ with the period of anomalistic month (27.55 day). Therefore the maximum systematic error in $\delta(\Delta T)$ due to the lunar ephemeris will have a semi-amplitude of about 0.3 s and a period which will be some alias of the anomalistic month, depending on the interval of time between solutions for $\delta(\Delta T)$. Systematic errors in $\delta(\Delta T)$ may also arise from the positions of the stars mostly taken from the *Catalog of 3539 Zodiacal Stars,* Robertson (1940), which were corrected to the system of the *FK4* by the application of systematic corrections as described in Morrison (1979, section 3.5). Over the period 1860 to 1978, the main systematic error in right ascension in the *FK4,* which has a mean epoch around 1915, is an approximately sinusoidal term with a semi-amplitude of no more than 0″.05. This would contribute an error of less than 0.1 s to $\delta(\Delta T)$ with a period which is some alias of the sidereal month (27.32 day), depending on the interval between solutions for $\delta(\Delta T)$. These systematic errors will be considered further in the discussion of the results.

The component in latitude, δB, was introduced to absorb the effects of periodic corrections in latitude in the lunar ephemeris, the displacement between the centre of mass and the centre of figure, and the displacement between the position of the dynamical equinox and that of the *FK4.* The solutions for δB are not of immediate interest to this paper and they will be ignored henceforth.

The equation of condition, which was weighted by the method given in Morrison (1979), took the form

$$\frac{\partial \sigma}{\partial(\Delta T)}\,\delta(\Delta T)+\frac{\partial \sigma}{\partial B}\,\delta B=-\Delta\sigma.$$

The partial derivatives were calculated by numerical differentiation.

In order to obtain a compromise between the reliability and frequency of solutions for $\delta(\Delta T)$, it was decided to solve the observational equations in annual groups. As shown in Section 2 the mode of the distribution of the observations falls near the beginning of the year. For this reason the annual groupings were taken from July 1 of one year to the next and the annual solutions were regarded as applying to January 1 of each year. In fact, the modes of the distribution of the observations always fell within two months of January 1, and 80 per cent were within one month.

The annual solutions for $\delta(\Delta T)$ for the years 1861 to 1942, obtained by the method of least squares, were added to the preliminary values, $\Delta T'$, which were computed for January 1 of each year. This gave the final values, ΔT, listed in Table 1 and plotted in Fig. 3. The standard errors of the solutions are also given in Table 1. Estimates of the real uncertainties of the results must include the effect of the possible systematic errors of 0.3 and 0.1 s due

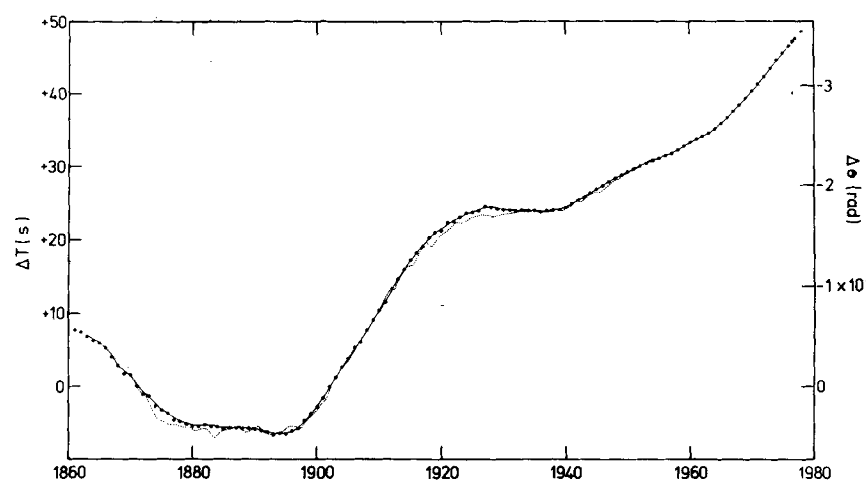

Figure 3. Annual solutions for ΔT (shown as dots) with fitted smooth curve. The standard errors of the solutions are given in Table 1. Brouwer's (1952) modified values (see text) are joined by dotted lines.

Table 1. Annual solutions for ΔT.

Date	Solution	s.e.	Smoothed (5-pt)	Date	Solution	s.e.	Smoothed (5-pt)	Date	Solution	s.e.	Smoothed (5-pt)
	s	s	s								
1860.0				1900.0	- 2^{s}.72	±0^{s}.07	- 2^{s}.73	1940.0	+24^{s}.33	±0^{s}.05	+24^{s}.34
1.0	+ 7.82	±0.27		1.0	1.54	0.07	1.47	1.0	24.83	0.06	24.81
2.0	7.54	0.26		2.0	- 0.02	0.07	- 0.11	2.0	25.30	0.05	25.28
3.0	6.97	0.20	+ 6.98	3.0	+ 1.24	0.06	+ 1.30	3.0	25.70		25.73
4.0	6.40	0.13	6.45	4.0	2.64	0.09	2.57	4.0	26.24		26.23
5.0	+ 6.02	±0.12	+ 6.02	5.0	+ 3.86	±0.07	+ 3.99	5.0	+26.77		+26.77
6.0	5.41	0.19	5.30	6.0	5.37	0.08	5.14	6.0	27.28		27.29
7.0	4.10	0.10	4.18	7.0	6.14	0.07	6.37	7.0	27.78		27.78
8.0	2.92	0.11	2.85	8.0	7.75	0.09	7.64	8.0	28.25		28.26
9.0	1.82	0.15	2.08	9.0	9.13	0.10	9.16	9.0	28.71		28.71
1870.0	+ 1.61	±0.32	+ 1.28	1910.0	+10.46	±0.08	+10.36	1950.0	+29.15		+29.15
1.0	+ 0.10	0.22	+ 0.21	1.0	11.53	0.06	11.73	1.0	29.57		29.57
2.0	- 1.02	0.18	- 0.81	2.0	13.36	0.08	13.20	2.0	29.97		29.98
3.0	1.28	0.26	1.62	3.0	14.65	0.09	14.72	3.0	30.36		30.36
4.0	2.69	0.18	2.45	4.0	16.01	0.05	15.99	4.0	30.72		30.73
5.0	- 3.24	±0.24	- 3.24	5.0	+17.20	±0.06	+17.21	5.0	+31.07		+31.06
6.0	3.64	0.22	3.80	6.0	18.24	0.08	18.18	6.0	31.35		31.35
7.0	4.54	0.11	4.35	7.0	19.06	0.07	19.18	7.0	31.68		31.71
8.0	4.71	0.18	4.82	8.0	20.25	0.09	20.18	8.0	32.18		32.17
9.0	5.11	0.21	5.10	9.0	20.95	0.08	20.83	9.0	32.68		32.68
1880.0	- 5.40	±0.09	- 5.38	1920.0	+21.16	±0.10	+21.43	1960.0	+33.15		+33.15
1.0	5.42	0.15	5.36	1.0	22.25	0.08	21.98	1.0	33.59		33.59
2.0	5.20	0.14	5.32	2.0	22.41	0.07	22.58	2.0	34.00		34.01
3.0	5.46	0.12	5.34	3.0	23.03	0.06	23.00	3.0	34.47		34.47
4.0	5.46	0.12	5.58	4.0	23.49	0.05	23.44	4.0	35.03		35.04
5.0	- 5.79	±0.06	- 5.66	5.0	+23.62	±0.05	+23.64	5.0	+35.73		+35.73
6.0	5.63	0.09	5.69	6.0	23.86	0.07	23.98	6.0	36.54		36.55
7.0	5.64	0.09	5.68	7.0	24.49	0.06	24.33	7.0	37.43		37.42
8.0	5.80	0.11	5.71	8.0	24.34	0.03	24.37	8.0	38.29		38.30
9.0	5.66	0.15	5.76	9.0	24.08	0.04	24.12	9.0	39.20		39.21
1890.0	- 5.87	±0.21	- 5.82	1930.0	+24.02	±0.04	+24.02	1970.0	+40.18		+40.17
1.0	6.01	0.13	6.00	1.1	24.00	0.04	23.95	1.0	41.17		41.17
2.0	6.19	0.11	6.28	2.0	23.87	0.03	23.93	2.0	42.22		42.24
3.0	6.64	0.08	6.48	3.0	23.95	0.03	23.89	3.0	43.37		43.37
4.0	6.44	0.06	6.57	4.0	23.86	0.03	23.92	4.0	44.48		44.47
5.0	- 6.47	±0.06	- 6.37	5.0	+23.93	±0.03	+23.83	5.0	+45.47		+45.47
6.0	6.09	0.05	6.20	6.0	23.73	0.03	23.83	6.0	46.46		46.48
7.0	5.76	0.07	5.61	7.0	23.92	0.03	23.86	7.0	47.52		
8.0	4.66	0.04	4.77	8.0	23.96	0.03	23.95	8.0	48.52		
9.0	3.74	0.06	3.72	9.0	24.02	0.05	24.04				

to the lunar ephemeris and the star positions. These systematic errors will behave in a periodic manner and this will be considered further in Section 4.1.

The values for the years 1943 to 1955 were evaluated from the following quadratic relation found by Morrison (1979) from an analysis of occultations in that period:

$$\Delta T = 31^{s}.24 + 0^{s}.331\,(Y - 1955.5) - 0^{s}.009\,(Y - 1955.5)^2,$$

where Y is the calendar date expressed in years. The values for January 1 in the years 1956 to 1978 were taken with respect to the international atomic time-scale, TAI, from the relation

$$\Delta T = 32^{s}.184 + (\mathrm{TAI}-\mathrm{UT1}),$$

where UT1 is a precise measure of UT, freed from the effects of polar motion. The values of TAI–UT1 are published by the Bureau International de l'Heure. The errors in these values are negligible compared with those derived with respect to dynamical time for the years 1861 to 1955. Brouwer's values for ΔT, amended for the change in mean longitude given at the beginning of this section, are joined by dotted lines in Fig. 3.

4 Analysis and discussion

4.1 DIFFERENCE IN TIME-SCALES, ΔT, AND THE ROTATIONAL DISPLACEMENT ANGLE, $\Delta\theta$

The sidereal displacement angle of the Greenwich meridian, $\Delta\theta$, measured in radians, is related to ΔT, measured in seconds of mean solar time, by

$$\Delta\theta = -1.002737\,\Delta T(15/206265)\text{ rad}.$$

This is shown as the right ordinate of Fig. 3. If $\omega = \omega(t)$ is the actual sidereal rate of rotation and ω' is the constant reference rate, then the value of $\Delta\theta$ at some epoch t is given by

$$\Delta\theta = \int_{t_0}^{t} (\omega - \omega')\,dt + \Delta\theta(t_0).$$

The constant reference rate is given by (see *Explanatory Supplement* 1961, p. 76)

$$\omega' = 1.00273\ 78119\ 06\ (2\pi/86400)\text{ rad/s},$$

which is fairly close to the average rate of rotation during the nineteenth century.

As a consequence of a resolution of the International Astronomical Union at Grenoble in 1976 (IAU 1977), ΔT is defined to have the value given by the following relation at the epoch $t_0 = 1977$ January 1 0^{h} TAI,

$$\Delta T(t_0) = \text{TD} - \text{UT1} = (\text{TAI} - \text{UT1}) + 32^{\text{s}}.184.$$

The value of TAI–UT1 for this epoch, published by the Bureau International de l'Heure, is $+15^{\text{s}}.34$; therefore

$$\Delta T(t_0) = +47^{\text{s}}.52,$$

and hence $\Delta\theta(t_0) = -0.003465$ rad. Thus,

$$\Delta\theta = -0.003465\text{ rad} + \int_{t_0}^{t} (\omega - \omega')\,dt.$$

In Fig. 3 we note the considerable improvement in the consistency of the revised values of ΔT over Brouwer's values (joined by dotted lines). The erratic departures have been almost completely removed. This was to be expected, because the recent high-precision values of ΔT for the years 1956 to 1978, obtained from TAI–UT1, show no erratic fluctuations on the time-scale of Brouwer's values. The improvements in the results are due to the use of more observations and the application of corrections for the limb-profile of the Moon. The second reason almost certainly accounts for the systematic departures of Brouwer's values from ours. The scatter in our results is due to accidental errors in timing and star positions. As explained in Section 3, the greatest periodic error in ΔT that can arise from the lunar ephemeris has a semi-amplitude of 0.3 s and since values of ΔT have been derived at yearly intervals the original period of the anomalistic month will appear as an aliasing period of approximately 4 yr. A possible periodic error of semi-amplitude 0.1 s arising from systematic errors in the star positions will appear with an aliasing period of approximately 3 yr.

The cause of what may be the greatest systematic error in ΔT is the uncertainty of the value of the tidal deceleration of the Moon in longitude. The value adopted in this analysis is $-13''T^2$ (where T is measured in centuries from the epoch 1900.0) and this has an uncertainty of $\pm 1''T^2$ (Morrison & Ward 1975). If the correct value were $-14''T^2$ (Muller 1975) then the values of ΔT obtained in this analysis would have to be increased by approximately $+1^s.8T^2$.

In order to reduce the effect of a possible 4-yr periodic term in ΔT arising from the lunar ephemeris and in order to smooth the annual values, the following 5-point quadratic convolute was applied to the tabular values of ΔT in Table 1

$$35s(0) = 17f(0) + 12[f(+1) + f(-1)] - 3[f(+2) + f(-2)],$$

where $f(n)$ is the value at point n and $s(0)$ is the smoothed result for point 0. Essentially, this method of smoothing is equivalent to fitting a quadratic polynomial by least-squares to five successive (equally spaced) data points and evaluating it at the central point. The smoothed values obtained by this procedure are joined by a continuous line in Fig. 3 and tabulated in Table 1.

4.2 ANGULAR VELOCITY OF ROTATION

The first derivatives of ΔT with respect to dynamical time are plotted in Fig. 4. The values of the derivatives were found at yearly intervals by applying the following 5-point quadratic convolute to the solutions for ΔT listed in Table 1,

$$10\dot{s}(0) = [f(+1) - f(-1)] + [f(+2) - f(-2)].$$

The notation of the left ordinate is derived as follows: we have

$$d(\Delta T)/dt = d(\text{TD}-\text{UT})/dt$$

$$= 86\,400 \text{ SI seconds per SI day} - \text{number of mean solar seconds per SI day.}$$

If the Earth's rate of rotation decreases with respect to TD, the mean solar second increases in length with respect to the SI second, and hence there will be fewer than 86 400 mean solar seconds in one SI day. So, $d(\Delta T)/dt$ is positive when the Earth's rate of rotation is decreasing and negative when it is increasing. The first derivative of ΔT can be regarded as expressing the excess length of the mean solar day over the SI day, and is conveniently measured in units of milliseconds.

The right ordinate gives the rate of change of $\Delta\theta$ with respect to dynamical time:

$$d(\Delta\theta)/dt = \omega - \omega' = \Delta\omega \text{ rad/s},$$

where ω' is the reference angular velocity whose value is given in Section 4.1.

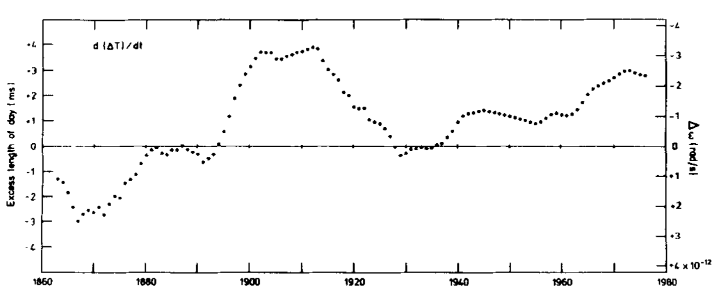

Figure 4. First time-derivative of ΔT obtained by fitting a moving quadratic polynomial to five successive values in Table 1 and evaluating the first derivative at the central point.

4.3 ANGULAR ACCELERATION OF ROTATION

The second derivatives of ΔT with respect to dynamical time are plotted in Fig. 5. The values of the derivatives were found at yearly intervals by applying the following 11-point quadratic convolute to the solutions for ΔT in Table 1:

$$429\ddot{s}(0) = -10f(0) - 9[f(+1) + f(-1)] - 6[f(+2) + f(-2)] - 1[f(+3) + f(-3)]$$
$$+ 6[f(+4) + f(-4)] + 15[f(+5) + f(-5)].$$

The 11-point smoothing was required to eliminate erratic short-term fluctuations in the second derivatives due to noise in the data. This level of smoothing is also supported later by the results of a power spectrum analysis of the data.

By an extension of the interpretation of the first derivative in Section 4.2, $d^2(\Delta T)/dt^2$ can be regarded as the rate of change per day in the length of the mean solar day and is conveniently measured in units of microseconds per day. The right ordinate gives the acceleration in the angular rotation in units of rad s^{-2}. Multiplying this by the principal moment of inertia of the mantle, $C = 7.2 \times 10^{37}$ kg m^2, gives the magnitude of the torque acting on the mantle (in units of Newton metres). Morrison (1978b) finds the combined tidal torque due to the Moon, Sun and atmosphere of the Earth to be -0.49×10^{17} Nm: this is marked on the inside of the right ordinate of the figure. By contrast, the maximum value of the torque displayed in Fig. 5 is about 10^{18} Nm.

The similarity in the behaviour of the second derivative in the periods 1930–45 and 1955–70 is noteworthy because the values of ΔT in these periods were derived by completely independent methods; the first being obtained by using the time-argument of the lunar ephemeris as the standard reference, and the second by using atomic time.

The apparently smooth transitions in acceleration may result simply from the 11-yr smoothing process. These data do not preclude the possibility that the accelerations change on a time-scale shorter than 11 yr. In order that this possibility might be tested and that the

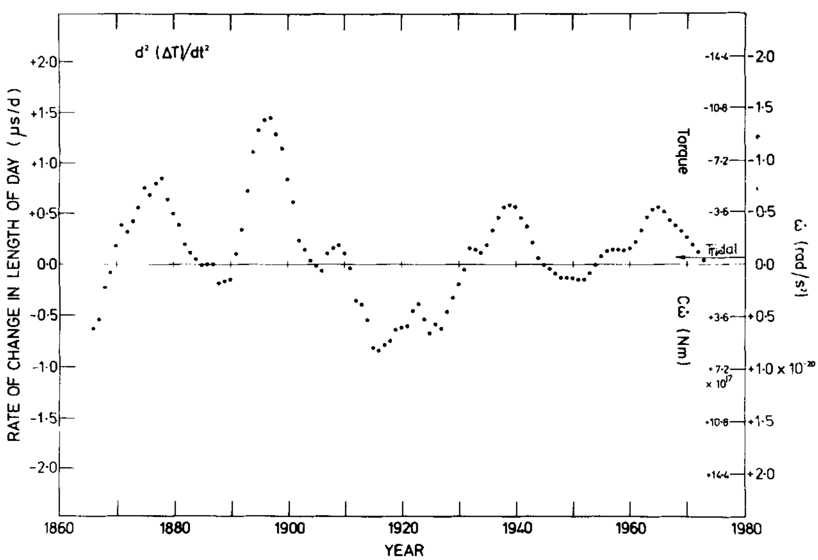

Figure 5. Second time-derivative of ΔT obtained by fitting a moving quadratic polynomial to 11 successive values in Table 1 and evaluating the second derivative at the central point. The inside right ordinate measures the torque operating on the mantle in units of 10^{17} Newton metres.

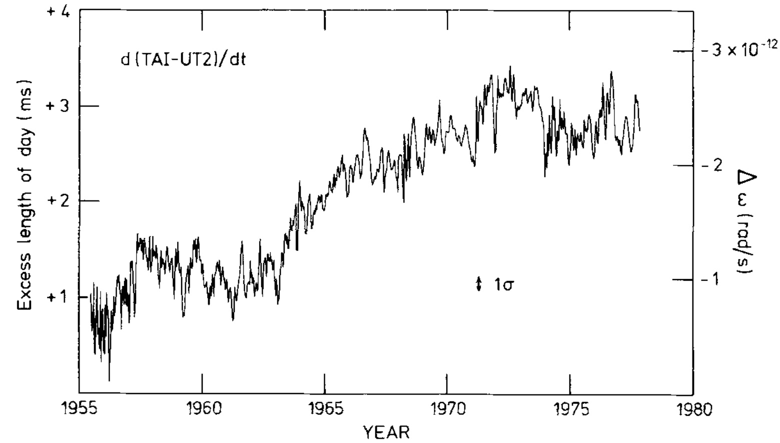

Figure 6. First time-derivative of TAI–UT2 obtained by differencing the 10-day values of TAI–UT2 published by the Bureau International de l'Heure.

11-yr period of smoothing might be justified, a closer inspection was undertaken of the high-precision data which are available since the introduction of the atomic time-scale in 1955 July 1. Values of TAI–UT2 were taken at 10-day intervals from the publications of the Bureau International de l'Heure and differenced in order to form the estimate of the first derivative which is displayed in Fig. 6. The universal time-scale, UT2, is derived from UT1 by removing annual and 6-monthly terms of constant amplitude and phase. These so-called 'seasonal' terms are attributed to the action of the atmosphere on the mantle and bodily tides.

The comparatively rapid changes in angular velocity in Fig. 6 clearly show that the accelerations, and hence torques, fluctuate in periods of a year or less, but, apart from the general trend in the slope, it is not clear whether the accelerations fluctuate with characteristic periods in the range 2–20 yr. In order that this might be investigated further, 50-day averages of TAI–UT2 were calculated and the second differences of these averages were formed. A power spectrum analysis was made of the differences and the result is shown

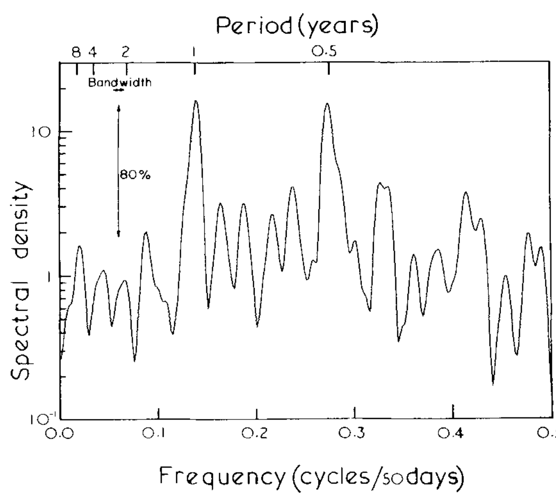

Figure 7. Power spectrum of δ^2 (TAI–UT2) using 50-day averages of TAI–UT2. The vertical bar shows the 80 per cent confidence limits.

in Fig. 7. The power in the spectrum at periods of 6 month and 1 yr is caused by the randomness in the amplitude and phase of the actual seasonal fluctuations. There are no significant peaks in the spectrum in the range 1–10 yr. However, this result for the 23-yr period from 1955 to 1978 may not necessarily be characteristic of the whole period since 1860; therefore a power spectrum analysis of the second differences of the annual solutions for ΔT listed in Table 1, which are independent (apart from the years 1943 to 1955) was also carried out and the result is shown in Fig. 8. The dashed line is the spectrum that would result if the second differences were purely random.

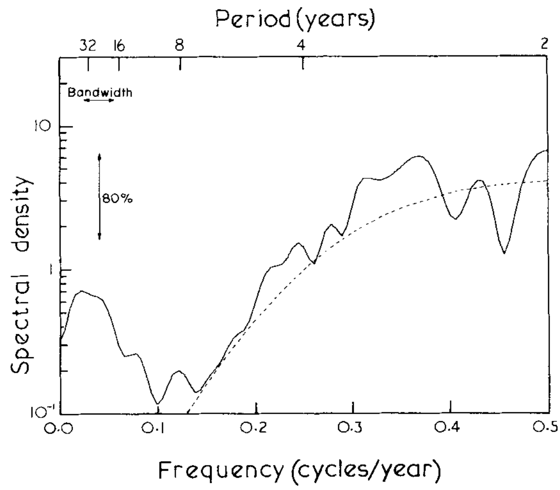

Figure 8. Power spectrum of $\delta^2(\Delta T)$ using the values of ΔT in Table 1. The vertical bar shows the 80 per cent confidence limits. The dashed curve is the spectrum for random values of $\delta^2(\Delta T)$.

The most significant feature in the figure is the broad peak around the period of 30 yr. I interpret the rise in the spectrum near the period of 2 yr as an indication of the presence of a peak in the spectrum whose period is shorter than the limiting resolution of the data. It is recalled that the annual solutions for ΔT in the period 1861 to 1942 apply to epochs scattered around January 1 in each year and therefore they may contain some of the signal from the annual term. Fig. 8 should be compared with Fig. 11.12(f) on page 196 of *The Rotation of the Earth* (Munk & MacDonald 1960). Their figure was derived from Brouwer's data and it shows no rise in the spectrum at low frequencies, which they took to be evidence that the accelerations changed on a time-scale shorter than the sampling interval of 1 yr. The improved results for ΔT in this analysis reveal that there is power in the spectrum around a period of 30 yr and there is much less power in the range 2–10 yr, which is consistent with the power spectrum for the TAI–UT2 data shown in Fig. 7.

5 Conclusions

This re-analysis of the collected times of lunar occultations of stars in the period 1861 to 1955 leads to the results shown in Fig. 5 for the magnitude and temporal behaviour of the torques operating on the mantle during that period. The results for the years after 1955 were obtained from atomic clock data. A power spectrum analysis of these results shows that the intervals between changes in direction of torque have a characteristic length of about 30 yr.

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