TU/e EINDHOVEN UNIVERSITY OF TECHNOLOGY

Re-sourcing teacher work and interaction : new perspectives on resource design, use and teacher collaboration

Citation for published version (APA):
Pepin, B., Gueudet, G., & Trouche, L. (2013). Re-sourcing teacher work and interaction : new perspectives on resource design, use and teacher collaboration. *ZDM : The International Journal on Mathematics Education*, *45*(7), 929-943. https://doi.org/10.1007/s11858-013-0534-2

DOI:
10.1007/s11858-013-0534-2

Document status and date:
Published: 01/01/2013

Document Version:
Publisher's PDF, also known as Version of Record (includes final page, issue and volume numbers)

Please check the document version of this publication:

• A submitted manuscript is the version of the article upon submission and before peer-review. There can be important differences between the submitted version and the official published version of record. People interested in the research are advised to contact the author for the final version of the publication, or visit the DOI to the publisher's website.
• The final author version and the galley proof are versions of the publication after peer review.
• The final published version features the final layout of the paper including the volume, issue and page numbers.

Link to publication

General rights
Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

• Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
• You may not further distribute the material or use it for any profit-making activity or commercial gain
• You may freely distribute the URL identifying the publication in the public portal.

If the publication is distributed under the terms of Article 25fa of the Dutch Copyright Act, indicated by the "Taverne" license above, please follow below link for the End User Agreement:

www.tue.nl/taverne

Take down policy
If you believe that this document breaches copyright please contact us at:

openaccess@tue.nl

providing details and we will investigate your claim.

Download date: 22. Aug. 2022

ZDM Mathematics Education (2013) 45:929–943
DOI 10.1007/s11858-013-0534-2

ORIGINAL ARTICLE

Re-sourcing teachers' work and interactions: a collective perspective on resources, their use and transformation

B. Pepin · G. Gueudet · L. Trouche

Accepted: 15 August 2013 / Published online: 24 August 2013
© FIZ Karlsruhe 2013

Abstract This paper reviews the literature on the theme of mathematics teachers' work and interactions with resources, taking a particular perspective, the so-called 'collective perspective' on resources, their use and transformation. The review is presented under three headings: (1) theoretical frameworks commonly used in this area of research; (2) teachers' interactions with resources in terms of their design and use; and (3) teachers' interactions with resources in terms of teacher learning and professional development. From the literature, and the collection of papers in this issue, we argue that the collective dimensions play an important role in mathematics teachers' work with resources and in their professional learning/development. Further empirical investigations are likely to be needed on: how teachers may work in collectives and with resources, and in which ways 'productive' collectives may form and work together; which roles particular resources can play in these delicate constellations and how particular resources may support teachers in their work and learning; and which kinds of resources offer opportunities for community building.

B. Pepin (✉)
Sør-Trøndelag University College, Trondheim, Norway
e-mail: birgit.pepin@hist.no

G. Gueudet
University of Brest, Brest, France
e-mail: ghislaine.gueudet.1@univ-rennes1.fr

L. Trouche
French Institute of Education, Ecole Normale Supérieure de Lyon, Lyon, France
e-mail: luc.trouche@ens-lyon.fr

1 Introduction

Over the past decade the concept of 'mathematics teacher resources' has been changing (Adler 2000; Gueudet et al. 2012). It has widened from conventional resources, such as textbooks (Haggarty and Pepin 2002) and curriculum materials (Remillard 1999) to include ICT for communicating and 'doing' mathematics, and a variety of devices for implementing and combining resources. Even textbooks are neither conceived nor used in the same way as before—whereas before the textbook meant literally the book, nowadays teachers refer to textbooks in connection with various resources linked to the book (e.g. computer programmes).

We define *mathematics teaching resources* as all the resources which are developed and used by teachers (and pupils) in their interaction with mathematics in/for teaching and learning, inside and outside the classroom. Resources are essential for mathematics teachers, and teachers use different kinds of resources which shape the mathematical content presented to, and used by, pupils in their mathematics learning. Moreover, when appropriating resources, teachers adapt them to their needs and customs. This process of 'design' and interpretation of resources then continues 'in use'—hence transformation is seen here as 'design-in-use'.

As the nature of resources changes, and their availability increases, so too do the possibilities of teacher collaboration with and via these resources: e-mail and forums, for example, permit distant collaboration. We consider that these developments show the need for research on teachers' interactions with resources, and in particular the collective dimensions of these interactions. Furthermore, we consider that collective processes are very important for the design of and interaction with resources: they may take

place in *design-based-research* collectives; in Communities of Practice (Wenger 1998); and also in other kinds of groups (e.g. informal teacher collectives in schools).

In this introductory paper we propose a particular perspective, the *collective perspective* of teachers' interactions with resources. Looking at the 'world' of teachers' work with resources, we regard as 'collective work' teachers' work with colleagues in-school and out-of-school, with teacher educators in professional development, and also with pupils and parents in school—in fact teachers working with 'other participants', that is, teachers working with and in teams, communities and networks (Krainer 2008). We provide a review of the state-of-the-art research literature of this relatively new field. Our guiding questions for reviewing the literature are:

(1) What are the main theoretical frameworks that are relevant in this field?
(2) What do we know about teachers' interaction with resources in terms of their use and transformation?
(3) What do we know about teachers' interaction with resources in terms of teacher learning and professional development?

According to these questions, we start by presenting theoretical frames and associated methodologies commonly used in this area of research and subsequently focus on two main dimensions: teachers' interactions with resources in terms of their design and quality; and teachers' interactions with resources in terms of teacher learning and professional development.

2 Theoretical perspectives

A variety of theoretical perspectives have been used to study teachers' interactions with resources, in particular teachers' collaborative work with resources, and the implications for professional learning. From the literature (e.g. Stylianides and Stylianides 2013), it is clear that many contemporary resources are the results of processes of *design research* (e.g. Swan and Dorman 2013). At the same time it is now recognized that design continues in use, and two theories that attempt to frame this process are those of (a) interpretation of, and participation with, a resource (Remillard 2005); and (b) documentational genesis (Gueudet and Trouche 2009). These processes of design and interpretation take place in Communities of Practice, but also, we argue, in other collectives (those that are not strictly Communities of Practice as defined by Wenger 1998). In line with this, we present four perspectives in this section: (1) Design-Based Research; (2) Remillard's framework of interpretation of/participation with resources; (3) Gueudet and Trouche's framework of documentational genesis; and (4) Communities of Practice (CoP). For each framework surveyed we identify key studies which have employed it.

2.1 Design-Based Research

"If you want to change something, you have to understand it, and if you want to understand something, you have to change it" (adage in Gravemeijer and Cobb 2006).

Design research can be considered as a specific methodology (e.g. Confrey 2006), as well as a particular theoretical stance (Gravemeijer and Cobb 2006). Methodologically, the Design-Based Research (DBR) approach involves processes such as iteration and feedback loops in such ways that development and research take place through cycles of design, enactment, analysis and redesign (Cobb et al. 2003), and hence the insights and interventions evolve over time through multiple iterations (of investigation, development, testing and refinement): this may involve a small number of participants at first, and through iterative cycles the number of participants grows. Recent conceptions of design research (e.g. Design-Based Research Collective 2003) define it as a methodology which is 'collaborative' by nature, hence particularly suitable for research on collective work with resources: it requires collaboration among a range of actors (e.g. teachers, teacher educators, policy makers) for the problem at hand (McKenney and Reeves 2012). Tabak (2004, p. 226) argues that "one of the hallmarks of Design-Based Research methods is collaboration with participants from the settings in which the research is set."

In the research literature it is also argued that design research "blends empirical educational research with the theory-driven design of learning environments" (Design-Based Research Collective 2003, p. 8). In DBR "local theories" concerning specific domains are produced through design experiments (Cobb et al. 2003). A first design is implemented and experimented in class (field testing), which, after reflection and feedback, informs the re-design of a 'new resource': this can be a particular pedagogic practice, a professional development module, or a particular mathematical task or 'tool', to name but a few. Whilst the basic principles of design research stay the same, the literature names a number of related research approaches which have these characteristics (e.g. *design experiments*, *development research*, *formative evaluation*, *engineering research*; van den Akker et al. 2006).

The underlying philosophy of design research is said to develop a better understanding of "the innovative forms of education that [one] might want to bring about in order to be able to produce them" (Gravemeijer and Cobb 2006, p. 17). Hence, and linking to the adage quoted above, the one side is the socio-constructivist approach which is

inspired by the desire to understand; the other side by the desire for educational change.

There have been a number of proposals to define design research in mathematics education, and Brown's (1992) and Wittman's (2001) articles may be the most notable. In the Netherlands Freudenthal et al. (1976) were probably the first to propose an approach of this type with the concept of 'developmental research', an idea which was further developed by Streefland (1990) and Gravemeijer (1993). On the basis of Freudenthal's ideas, and the work of the Freudenthal Institute in terms of design research, the so-called domain-specific instruction theory of Realistic Mathematics Education (RME) was developed, which, according to Gravemeijer, can be reconstructed as a 'generalization over numerous local instruction theories' (Gravemeijer and Cobb 2006)—hence educational change. The other predecessor of design research is the constructivist "teaching experiment methodology" (Cobb and Steffe 1983), and these studies (rather 'experiments') aimed predominantly at understanding how students learn: first one-to-one (e.g. Cobb and Steffe 1983); later expanded into classroom teaching experiments (e.g. Cobb 2000); and further to school and district restructuring experiments (e.g. Confrey et al. 2001). Hence, in these studies the focus on understanding remained a salient characteristic of design research (Gravemeijer and Cobb 2006).

Drawing on European traditions of didactical research, which is said to be concerned about the bridging between theoretical principles and design processes, Ruthven et al. (2009) point to the European notion of didactical design, which encompasses several European theories of mathematics didactics, for example the theory of *didactical situations* (Brousseau 1997), in particular *didactical engineering* (Artigue 1994). In their words didactical design is "the design of learning environments and teaching sequences informed by close analysis of the specific topic of concern and its framing within a particular subject area" (p. 329). It is argued that "the promise of design research is in providing more powerful and direct means of developing such teaching sequences and associated theorizations that can assist local adaptation of such sequences to take account of crucial contextual features" (p. 329), hence assumed to (overcome the concerns mentioned above and) mediate between grand theory and the practice of mathematics teaching. Recent studies that employ didactical design are various: for example, Ruthven and Hofmann (2013) report on the design of a probability module intended for implementation at scale in English early secondary education; as another example, Sensevy (2012) develops the notion of *collaborative didactical engineering* where primary school teachers, teacher educators and researchers work together in terms of didactical designs.

Likely to be inspired by poor outcomes of the US in recent large-scale cross-national achievement studies, (e.g. TIMSS, PISA) and claims that mathematics education research does not influence practice (Wiliam and Lester 2008), several recent American mathematics education design research studies (e.g. Cobb and Jackson 2011) have drawn attention to the need to investigate and support "improvements in the quality of mathematics teaching at scale" (p.1; Cobb et al. 2013). They point to the importance of "establishing research-practitioner partnerships that involve co-designing, testing, and refining current school and district design conjectures" and emphasize the necessity of "conducting systematic inquiry to develop theory related to improving quality of classroom instruction and student learning at the system level" (p. 33). For this, they propagate *design-based-implementation research* (DBIR—see Penuel et al. 2011), an emerging methodology akin to DBR. The important issue, whether in small-scale or large-scale reforms, appears to be the important role the interventions play, whether these are curriculum materials (e.g. Stein and Kim 2009), or whole modules (see Swan and Dorman 2013). Interestingly, a recent *ZDM* Special Issue (Stylianides and Stylianides 2013) addresses similar concerns in terms of 'answering' problems of practice with research-based solutions/theories.

2.2 Interpretation of/participation with resources

With the increase in availability of reform-inspired curriculum materials, in particular in the United States and in China, studies of teachers' use of mathematics curriculum materials become increasingly relevant. In her seminal review article (covering 25 years of research on mathematics curriculum use), Remillard (2005) examined 'key concepts in research on teachers' use of mathematics curricula'. She termed curriculum materials, curriculum and textbooks to refer to printed, and often published, resources designed for use by teachers and students in their work together in lessons. Using the commonly utilized terms of 'intended' and 'enacted' curriculum, it can be argued that 'intended curriculum' refers to published resources, such as programs or textbooks, and the 'enacted curriculum' refers to what teachers do using these resources. Remillard indeed sees teachers as 'active' designers and users of the curriculum materials (and not as simple transmitters), and it leads her to analyse teachers' usages of resources as *interpretation of* and *participation with* the resources (Fig. 1).

In her conception of teachers' work with and use of resources, the teacher/curriculum resource relationship emerges as a significant construct. This view emphasizes the curriculum use as *transaction with* resources, and such studies (e.g. Brown 2002; Sherin and Drake 2009;

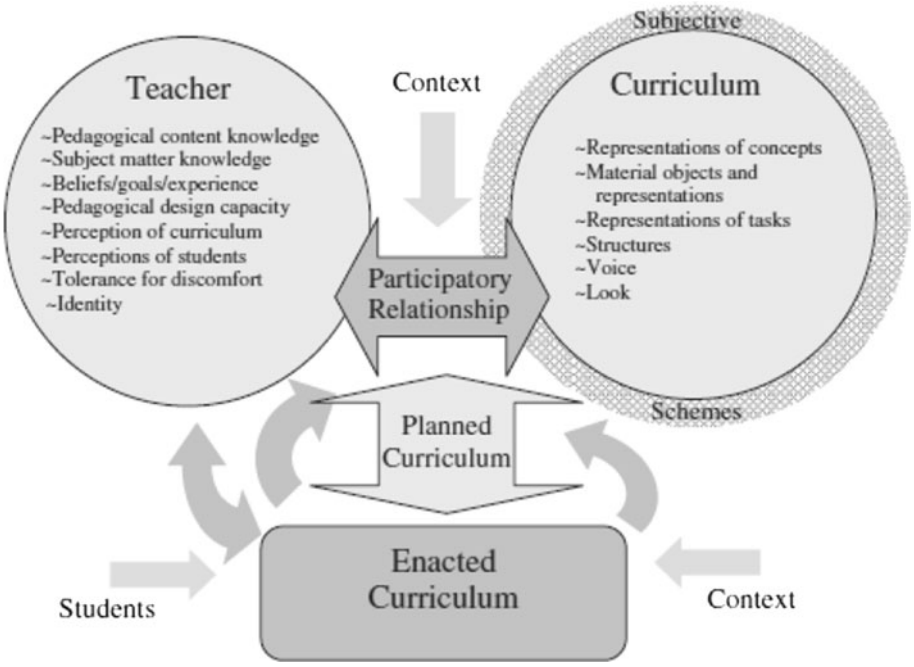

Fig. 1 Remillard's (2005) framework of components of the teacher/curriculum relationship

Remillard 1999) typically examine how teachers actively engage and work with resources, and how in turn the resources are shaped, and how they shape teachers—hence the emphasis is on the active and interactive nature of teachers' work with resources. What comes into play are the various ways teachers may draw on their own knowledge, how they evaluate resources, and subsequently select (or not) them, how they make sense of them in terms of their own teaching and perhaps adapt them. Brown (2002) asks for an 'integrated analysis' of the teacher's resources and how they interact—this view proposes that the features of the resource matter as much to their use as the teachers' characteristics (e.g. knowledge). Hence, resource analysis (e.g. structures, features, quality) becomes an integral part of developing a deeper understanding of their use, and teachers' participation with resources can expose important differences in how they interact with these different features and characteristics (Remillard 2005). The above framework highlights dimensions seen as essential by Remillard for understanding the teacher/curriculum relationship: the teacher; the curriculum; the participatory relationship between them; and the resulting planned and enacted curriculum. This view is underpinned by the assumption that 'curriculum use involves a participatory relationship between teacher and the curriculum' (p. 236). This is supported, and taken further, by Brown's (2002) conception of resources as *artefacts* (or cultural tools), framing both resources and teachers as 'active players in an interactive relationship'. Teachers select resources; interpret them; reconcile their (teachers') perceptions of intended goals with their own; accommodate potential interests of their students; add, modify or omit parts; and 'offload, adapt, improvise' in their use of resources (Brown 2002). In the same way as Brown's (2002) *Design Capacity for Enactment Framework*, Remillard (2005) emphasizes the 'participatory relationship, the interaction between the teacher and the curricular resources' (p. 236). The theoretical perspective we introduce in the following section takes a very similar stance, and at the same time proposes a different model for the understanding of these phenomena.

2.3 Documentational genesis

Gueudet and Trouche (2009) coined the terms *document* and *documentational* genesis: teachers' (or pupils') documents incorporate both resources and knowledge 'piloting' their usages, that is, a document = resource/s + utilization *scheme* (Fig. 2), a scheme (Vergnaud 1998) being an invariant organization of activity for performing a given task, encapsulating knowledge and beliefs both piloting this activity and emerging from this activity. The documentational genesis is the process of the teacher's interaction with the resource (or sets of resources). It comprises two intertwined processes: (1) the features of the resources influence the teacher's practice, the knowledge s/he develops—this process is called *instrumentation*; (2) the teacher's knowledge guides the choice made between different resources, the way they are modified—this process is called *instrumentalization*. This perspective is close to Remillard' view (Sect. 2.2) emphasizing how the resources are shaped, and how they shape teachers' work.

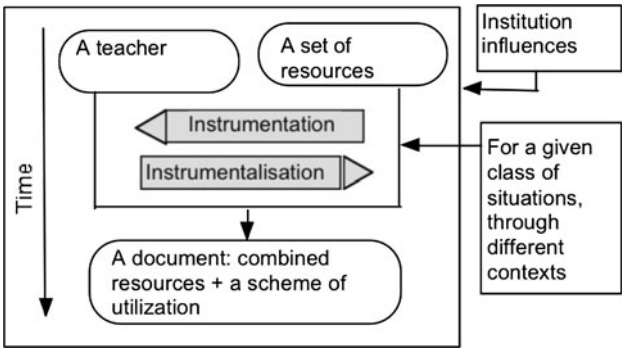

Fig. 2 Schematic representation of the documentational genesis approach

The documentational approach proposes a model of the interactions between teachers and resources, and of the implications for teachers' professional development.

This theorization links to Rabardel's (1995) *instrumental approach*, which introduced the terms instrument and instrumental genesis: an artefact, appropriated by a user to perform a given task, gives birth to an instrument through a complex process named instrumental genesis. The documentational approach enlarges this approach, taking into account a great variety of 'things' intervening when a teacher creates and implements a given lesson, similar to Brown's (2002) notion of *cultural artefact* (Sect. 2.2).

Following the documentational work of a teacher is a complex matter. Teachers typically work in different places, in school and out-of-school, at different moments. In order to analyse teachers' documentation work, a specific methodology, named *reflective investigation*, has been developed (Gueudet and Trouche 2012a). The researcher collects, as far as possible, all the teacher's resources: files; papers; references of books; etc. The teacher him/herself fills in a logbook, describing his/her documentation work. Classroom lessons are observed and videotaped, and interviews are conducted with the teacher. All these data are analysed and compared, in order to access as 'precisely' as possible the teacher's documentation work.

Documentation work (searching for resources, transforming them, etc.) also takes place in collectives, thus documentational geneses also happen within collectives, development of documents and communities being regarded as interrelated 'stories' (Gueudet and Trouche 2012b).

2.4 Communities of Practice

The collective work of teachers is often organized in a variety of ways: formal and school-organized teams (e.g. teachers of a particular grade working together); self-organized and informal groups (e.g. a group of teachers working on a particular theme or mathematical topic); loosely or tightly coupled networks of teachers (e.g. working on a net-based textbook).

In terms of teacher learning/professional development and 'collectives', we refer to theory which goes beyond seeing the individual's learning as participation in social practice or activity (Vygotsky 1978). Learning in collectives (and interacting with resources), we assume, involves notions of *co-learning* (Wagner 1997), and the notions of *community of practice* (Lave and Wenger 1991).

According to Wenger (1998) a *community of practice* is a group of persons sharing the same practice. It has three central features: the members of a community of practice have *a mutual enterprise*; *a shared commitment*; and *a common repertoire*. This repertoire can contain material objects, but also stories or signs that are shared by the members of the community. We consider the repertoire as a *repertoire of resources*, and thus studies referring to teachers' Communities of Practice involving interactions of teachers and resources are relevant for our review. The notion of Communities of Practice has also been widely used in research on *lesson studies* (Fernandez and Yoshida 2004; see Sect. 4), more recently for analysing online Communities of Practice (Borba and Llinares 2012a), where discussions in forums enrich the shared repertoire.

Rooted in the theory of community of practice, Jaworski (2008) introduced the notion of 'inquiry community': a group of teachers and researchers inquire together about teaching/learning issues, where inquiry becomes a tool for learning (see Huang and Jaworski, *ZDM* Special Issue 2014). Most of these studies involve communities of teachers (or teachers and researchers) preparing, observing and analysing a lesson (taught by a member/s of the community). We consider again this aspect of the literature in Sect. 4, in connection with teacher education. In the next section, we focus on teachers' use of resources.

3 Teachers' interactions with resources: use and transformation

In this section we investigate teachers' interactions with resources focusing on the 'design' and use of resources encompassing the transformation of resources by the teacher. Here we consider resources as: text resources (e.g. curriculum materials); ICT resources; and also other materials, such as manipulatives. We pay particular attention to research on textbooks, and compare this with research on technology resources. These two fields of mathematics education research have previously been separate: with some exceptions (e.g. Rezat 2012), textbook research has mainly focused on interactions between teachers and textbooks (e.g. Remillard 2005; Remillard et al. 2008; Pepin and Haggarty 2001), whereas research on

technology resources has principally been concerned with pupil interactions with resources (e.g. Hoyles and Lagrange 2010). As Healy and Lagrange (2010, p. 287) state: "Despite the fact that teachers have a central role in the mathematics classroom, they have been somewhat neglected players in research considering the relations between digital technologies and mathematics education." It could be argued that mathematics education research has initially considered textbooks as tools for teachers, whilst technologies were seen as tools for students. Nevertheless, over the last 10 years this situation has changed: digital textbooks are now available, and this technology is clearly a tool for both teachers and pupils; the same holds for many internet resources. In this section we focus on issues which have been studied in both textbook/text and technology research: the adoption of resources; their appropriation/transformation by the teacher; and the emergence of a new perspective on resource quality. Further, we identify the phenomena which are similar, and which are different, in both fields, and why; moreover, we always keep our focus on collective aspects.

3.1 Adoption/integration

We consider here research questions such as "Why does a teacher use, or not, a given textbook or software?" When addressing this kind of question, the textbook research literature predominantly uses the term "adoption" (e.g. Ball and Cohen 1996; Lloyd et al. 2008), whilst research on ICT employs the notion of "integration" (e.g. Ruthven and Hennessy 2002; Hoyles et al. 2004; Monaghan 2004; Assude 2007; Haspekian and Artigue 2007). This difference appears significant; in fact the questions addressed are not the same. For textbooks, the question of adoption is raised for specific textbooks: typically, a new textbook in the context of reform. Teachers are likely to use a textbook; but they could reject a new textbook, for example if the book does not correspond to their views of mathematics, or they disagree with the learning progressions suggested in the textbook. For technology resources, questioning integration generally starts by acknowledging the difference between the institutional expectations and the actual classroom use (e.g. of software). Each study generally focuses on specific software; nevertheless the question/challenge of integration does not seem to be attached to the features of this particular software, but concerns technology as a whole, the obstacles for adoption being sometimes (but not always) linked to the difficulty for the teacher to develop technical skills (e.g. Monaghan 2001; Lagrange and Erdogan 2009; Kieran et al. 2012).

Nevertheless, there are also similarities in the two areas of research. An essential condition for adoption of a textbook or software is its potential integration/inclusion into the teacher's 'normal' practice. The teacher has one, or several, usual "activity formats" in the classroom (Monaghan 2004; Behm and Lloyd 2009; Ruthven 2012), and is more likely to use a given textbook or software if it does not require a completely new activity format. Another important factor, for adoption or rejection, is related to teachers' views of mathematics (including its teaching and learning). The literature proposes that it can be expected that a given text resource 'shapes' (and portrays) the mathematics in a particular way, which in turn can be rejected by a teacher if this does not correspond to his/her views (e.g. Silver et al. 2009). Similar phenomena have been identified for selected technology resources: for example, Assude (2007) shows that primary school teachers in France rejected calculators because teaching calculation techniques was a central objective, and of value for them, as primary school teachers.

Research studies on 'teacher information behaviour' (e.g. Diekema and Olsen Whitney 2012) claim that the main reason for the choice of a particular teaching resource is its recommendation by colleagues, that is, drawing on shared experience and information from colleagues. This very important collective dimension holds true for both text resources (e.g. textbooks, internet resources) and software.

3.2 Genesis/appropriation

Most of the studies investigating teacher–resource interactions analyse a two-way process (Remillard et al. 2008): the teacher's beliefs and practices shape his/her use of the curriculum resources, whilst at the same time the features of the curriculum resources can contribute to teacher development, and thus 'produce' an evolution in the teacher's beliefs and practices. The documentational approach (Gueudet et al. 2012) captures this two-fold process in the concept of documentational genesis. We consider here separately the two aspects of this process, in order to focus more precisely on research results: on the one hand, what can be considered as appropriation, that is, transformation of the resources when used by the teachers; and on the other hand, on teacher learning. Arguably, this separation is somehow artificial, and perhaps more importantly, appropriation is also strongly connected with adoption, both for text and technology resources.

In terms of textbook research, many studies (e.g. Lloyd 1999; Sherin and Drake 2009) have investigated the ways teachers' characteristics (e.g. knowledge in particular) shape their use of curriculum materials. These studies consider that teachers interpret the materials according to personal characteristics. Sherin and Drake (2009), for example, introduce the concept of *curriculum strategy framework*, focusing on three key interpretive activities: reading; evaluating; and adapting. Remillard (2012)

considers that teachers have a *mode of engagement* with textbooks, which have developed along their use of various textbooks, and guides their use of a new textbook, independently of the features of this textbook.

Concerning ICT, appropriation has been conceptualized by some authors by referring to an *instrumental genesis* perspective (Haspekian 2005); this genesis includes in particular an *instrumentalization* aspect, which describes how the teacher appropriates a given technology, according to his/her agency, or knowledge. This means that two different teachers are likely to develop two different instruments from the same artefact. Beyond this first conceptualization, appropriation from an instrumental perspective is captured in the notion of *instrumental orchestration*. The concept of instrumental orchestration has been introduced by Trouche (2004), analysing how a teacher guides the instrumental geneses of the students with a given software. It has been progressively generalized, to describe didactical configurations (arrangements of artefacts in the environment) and exploitation modes (the way the teacher decides to exploit this didactical configuration). Drijvers (2012) has refined the concept of orchestration, introducing a distinction between what has been planned, and what actually happens in class. In terms of textbook use by teachers, the notion of orchestration seems less commonly used (it is used by several authors, e.g. Grant et al. 2008, but not formally presented as a concept), whereas in 'digital environments' it appears a more common concept. Interestingly, the authors who use the notion of orchestration in textbook research consider the use of textbooks in class, for example the use of curriculum material to pursue student thinking (Grant et al. 2008; Choppin 2011).

We retain that, for text resources and for technology resources, the teacher–resource interaction is not isolated. It takes place in a given environment and context, typically in the classroom; and hence students, and the way teachers orchestrate their work with resources, are important elements in the appropriation processes.

3.3 Use, transformation and (evaluation of the) quality of resources

In the field of textbooks research, evaluation is one of the central fields of research (e.g. Fan et al. 2013). For example, the evaluation may concern the *adequacy of textbooks* with respect to a given curriculum (e.g. Pepin, et al., 2013; Haggarty and Pepin 2002; Shield and Dole 2008). However, typically the evaluation of a textbook is linked to a consideration of its use, or at least its potential use: for example, to evaluate the *adequacy of situations* to a didactical goal (Brousseau 1997).

In recent years Davis and Krajcik (2005) coined the term "educative curriculum materials" (based on Ball and Cohen's (1996) call for teacher learning materials) denoting those materials which are intended to promote teacher learning, that is, curriculum materials (including teacher guides) which support teachers in the process of enacting the curriculum. In their study they define particular design heuristics for educative curriculum materials, amongst them that educative curriculum materials should promote teachers' *pedagogical design capacity* (Brown 2009). Remillard (2013) took up this challenge and analysed five mathematics curricula in terms of whether these materials are helpful for teachers to design quality instruction. Leaning on the work of Ball and Cohen (1996) and Davis and Krajcik (2005) she devised a conceptual framework for the assessment of educative features in curriculum materials, all in her quest for "examining the capacity required for teachers to make productive use of the resources available in curriculum materials to design quality instruction" (Remillard 2013, p. 1). This is clearly based on an understanding of teachers' curriculum use as an interactive process between the teacher and the curriculum, and it implies that a teacher's pedagogical design capacity must be influenced by the particular curriculum resources (and their educative features).

At the same time the notion of "resource evaluation" appears to have been replaced by the notion of "resource quality" (albeit with the same objective). Studies on "resource quality" also seem to have been further developed (e.g. Trouche et al. 2013), arguably because of the emergence of more (and more sophisticated) digital resources, apparently *easily accessible* and *easily shareable* (Pedauque 2006). These developments have led to a questioning of the relevance of such resources for teachers' work. The quality can be evaluated in different ways; nevertheless, it is always linked to teachers' use.

A possible way of evaluating quality consists of deciding on *criteria* for evaluating the quality of a resource for a particular teaching/learning purpose, in particular in mathematics education (Caprotti and Seppälä 2007). In the field of ergonomics, Tricot et al. (2003) identified three main criteria for a given resource: *utility*—it allows the learner, or the teacher, to reach his/her goal; (b) *usability*—it is easy to learn how to use it; (c) *acceptance*—it is compatible with the context and with the learner's motivation. Bueno-Ravel et al. (2010) also consider *didactical* and *epistemological* criteria, for example whether resources are aligned with particular learning objectives (e.g. of the national curriculum). These criteria are useful for institutional and research purposes, but are not likely to constitute useful tools for practitioners in terms of helping them to integrate their own resources into a new resource system.

Considering both Remillard's notion of *transaction* with resources (Sect. 2.2) as well as the notion of documentational genesis (Sect. 2.3) allows researchers to view the notion of quality from a different perspective. Quality can be considered in a dynamic way, taking into account the productive process of appropriation. The notion of *design-in-use* involves the users, not only after the resource has been designed but also 'upstream' of the process of design, that is, whilst the resource is being designed. This process has first been highlighted in the field of ergonomics (Rabardel and Béguin 2005) and seems all the more relevant for digital resources, where several potential users can intervene from the beginning of the design process. Viewed from this perspective, the development of resources stems from the engagement of a user collective, and the *quality process* becomes an important element (Gueudet et al. 2013b). Each community sharing resources shares indeed ideas, sometimes implicitly, in terms of the ways the resources fit participants' needs. Hence, a resource is not 'of good quality' per se, but it may be a 'good resource' for a given context, for a given community, at a given stage of its development. The notion of quality thus appears *collective* and dynamic. This view has been further developed for online resources (Trgalová et al. 2011), which may be linked with more varied potential developments of online resources, whilst a traditional textbook (not digital) may be more "frozen" and hence does not suggest a dynamic notion of quality. We argue below that, even for textbooks, important processes of appropriation and design-in-use can take place.

4 Teachers' interactions with resources and professional learning

In this section we focus on teachers' interactions with resources in collectives and for mathematics teacher professional learning. We address different interrelated aspects of this theme: (1) teacher professional learning/development and resources; (2) online teacher education; and (3) collectives of teachers as instructional designers (of resources) and for organizational aspects of change.

4.1 Genesis/teacher learning

The appropriation of resources, their modification in use and the development of teacher agency are closely connected to a particular process: teacher learning through the use of resources. As mentioned above, Remillard (2012) considers that teachers have a *mode of engagement* with textbooks; simultaneously, she also considers that textbooks have a *mode of address*-ing the reader, and that this mode of address shapes the use of textbooks by the teacher.

It is acknowledged that physical resources, and in particular manipulatives, are important resources for teacher learning (O'Shea 1993), and they serve a dual purpose in mathematics teacher learning: they are learning objects for developing a deeper understanding of mathematics as well as practical knowledge. The resource's value is linked to its nature and the relationships with the corresponding mathematical concepts (Fischbein 1977), thus viewing manipulatives not as mechanical but as symbolic devices (Nührenbörger and Steinbring 2008). Radford (2012) considers that the 'possible roles' that one may attribute to resources (and machines demonstrate this perhaps better than other resources) depend on one's theoretical view of cognition: he outlines the cognitive, epistemic and ontological role a resource can play.

Concerning technologies, several authors referring to the instrumental approach (Haspekian 2005) consider that teachers are engaged, along their use of technologies, in instrumental geneses. We mentioned above the instrumentalization aspect of the geneses, which is associated with an *instrumentation* process: development of schemes of use (Vergnaud 1998), which encompasses in particular knowledge. Interestingly, the works of Bartolini Bussi and Maschietto (2008) view machines, and these include digital/ICT 'machine' resources, in terms of at least three analytical components: epistemological, didactical and cognitive. They use *semiotic mediation* to investigate machines in teacher learning, thus examining the elaboration and evolution of instruments in terms of instrumental genesis.

The collective dimensions of teacher learning with text resources, or with technologies, appear mainly in studies of teacher education/professional development. For example, the concept of *lesson study* (originating in Japan, see Fernandez and Yoshida 2004) brings together collectives of mathematics teachers to discuss lessons, which they previously jointly planned and then observed as they unfolded in actual classrooms. Lesson studies have a clearly defined goal (which needs the agreement of all teachers involved in the lesson study), and this can be the work with a particular resource (or resources). The *learning study*, which originates in Sweden, is a hybrid of the Japanese lesson study model and design experiment (Runesson 2008). In learning studies a particular theoretical framework 'variation theory' is used as a guiding principle (Marton and Tsui 2004). Interestingly, the Chinese form of lesson study typically also uses 'variation' to develop pupil learning trajectories. The Chinese lesson study brings together groups of teachers, master teachers and researchers to work on learning trajectories and exemplary lesson development in iterative cycles (Yang 2009; Huang and Li 2009). In all these studies the mathematics is at the centre of the activities and discussions, and the resources are those that

are perceived to be beneficial for studying particular mathematical topic areas and learning sequences. Winsløw (2012) focuses on the collective use of resources by teachers in a lesson study concerning the topic of proportion in grade 6. He identifies rich resources, including a short and a detailed version of the lesson plan, but also very precise objectives, different possible ways for introducing the ratio, elements about students' skills and potential difficulties. All these resources are connected with the mathematical content, from a didactic point of view, and this is an important aspect of the lesson study. Collective interactions with such resources seem especially appropriate for professional development.

In terms of helping teachers to learn with and from curriculum resources, in particular in the context of reforms, many studies have investigated the influence of curriculum material on teacher knowledge and practice (e.g. Ball and Cohen 1996; Davis and Krajcik 2005). Can resources support a change in the curriculum? Can a textbook contribute to teacher learning? These questions highlight a complex situation, with often no evidence of learning, where teachers individually use these resources without specific training. Pepin (2012) analyses, by contrast, how a 'task analysis tool' can act as a catalyst for teacher learning, in the context of teacher professional development.

More recently, video has received particular attention as a resource for teacher learning. The literature claims that video in professional development can strengthen teachers' mathematical understanding in the process of trying to make sense of students' work or analysing instances of classroom practice (Borko et al. 2008). A number of studies claim that the use of video in professional learning situations can help teachers to 'learn to notice', value and analyse more appropriately students' mathematical thinking (van Es and Sherin 2008). As teachers (and teacher educators) work with video to notice and analyse students' thinking, they are more likely to better evaluate the strengths and weaknesses in students' mathematical thinking, and how to respond to those, and thus develop and refine their instruction and response to students' needs (Kazemi and Franke 2004).

In a forthcoming issue Huang and Jaworski (2014) provide an international perspective on 'Practices and strategies of promoting professional development of didacticians and teachers of mathematics', focusing squarely on the collective of teacher educators and mathematics teachers working and learning together.

4.2 Resources and online teacher education

In terms of ICT resources and teacher education, technology resources can play a significant role as means for the collective work of teachers: platforms, wikis or other tools open networking possibilities (Borba and Gadanidis 2008). The researchers investigating these issues typically adopt a Vygotskian socio-cultural perspective, where the activity and the interactions between teachers are mediated by technology.

In particular, a recent *ZDM* Special Issue (Borba and Llinares 2012b) drew together studies on online mathematics education. They identified several challenges and key topics for research, amongst them the sustainability of communities of teachers and the kinds of organizational structures; and the knowledge-building practices in such group interactions. Most of these issues are linked to resources, and it can be argued that a study of resources is intrinsically linked to online teacher interaction and education. The aforementioned work of Gueudet and Trouche in terms of documentational genesis (2009; 2012a, 2012b) is particularly pertinent here, as they include new developments in ICT which in turn have led to developments of new forms of teacher education programs. In terms of collaborative work, ICT provides various communication tools which can serve as opportunities for teacher collaborative learning—hence new resources offer new pathways for teacher collective work (see Gueudet et al. 2012).

Other studies (e.g. Llinares and Olivera 2008) have addressed the issue of 'virtual communities' in/for mathematics teacher learning. They use a range of computer-based communication tools/resources (e.g. video paper) which are now available, for teacher educators to adapt and transform them into pedagogical tools. Interestingly, Borba and Gadanidis (2008) investigate the role of virtual technology resources, that is, how these mediate and interact in the way teachers collaborate and learn/construct knowledge together.

4.3 Teacher learning with resources and at scale

We now turn to teacher learning in large numbers and to organizational aspects of change (developments of schools, districts, etc.)—the issue of teacher learning with resources and change at scale. In this scenario new potential participants and environments come into play, which make the interactional situation and definition of collective very complex: different resources, different places, different participants/actors, to name but a few. How can schools and mathematics departments employ and organize their financial support so that mathematics teacher collective learning of mathematics with resources can be supported? Which resources should be used/bought/provided? What does *organizational learning* mean (Kazemi 2008) in this context? How can teacher knowledge be generated that links to resources and which contributes to the ongoing improvement of mathematics teaching and learning at scale?

In several countries, national policies have retained a "cascading model" (Wedell 2005): providing training to a relatively small number of teachers, who are expected afterwards to train other teachers, and so on. For example, in several European Union 7th framework projects (e.g. PRIMAS—Promoting Inquiry-based learning in Mathematics and Science education across Europe), multipliers are trained to teach colleagues in their or other schools, mainly using prepared modules as their resources, which are expected to help them run the professional development sessions. This is a very specific, "top-down" model of collaboration aimed at reaching large numbers of teachers. However, several authors claim that the successive steps of training are likely to lead to the development of misunderstanding and transmission of wrong interpretations (e.g. Ono and Ferreira 2010).

Up-scaling successful experiments requires appropriate support structures (for learning at scale and with resources) and includes networks, teams and collectives at different levels. Studies of that kind (Cobb and Smith 2008) are typically design-based studies which engage different collectives in work with particular resources/interventions, and in repeated cycles of analysis and design (Cobb et al. 2003) at the level of the school and district in order to bring about organizational learning.

5 'Re-sourcing' teacher work and interaction: towards a holistic perspective of collaborative design, (ICT) resources and professional development

This section introduces the Special Issue and its contributions, and draws together selected ideas on and issues concerning mathematics teachers' interaction and work with resources. In the reaction (at the end of the Special Issue) Kenneth Ruthven rounds off the discussion by outlining significant results and implications for practice and future research.

The Special Issue is presented under three themes: the first predominantly concerned with the design of resources; the second with technology resources; and the third with teacher learning/professional development linked to resources. Under the first theme, the three groups of authors study the design and use of resources using different theoretical frames: in the case of Swan and Dorman a traditional design-based approach is used, whereas Sensevy et al. have developed a specific Design-Based Research approach, *cooperative engineering*. Interestingly, both research groups use text and video methods for teacher collective learning, but the latter group insists on the cooperative aspect, whereas the former is clear about their stance as providers of 'goal-directed and tool-mediated activity' for teacher professional development. Kieran et al. lean on the documentational genesis approach to outline the various phases of documentation work done by the design researchers and the production of three documents in the process.

The second group of papers is concerned with the quality and use of digital/ICT resources in and for mathematics teacher learning. Maschietto and Soury-Lavergne study the design of a duo of artefacts: a material artefact being associated with its virtual version. They considered the material, didactical and digital affordances and constraints of working with a physical machine (pascaline), as compared to its digital counterpart (e-pascaline). The use of such resources also raises questions, as the other two papers in this section evidence. Trgalová and Jahn and Drijvers et al. claim that whilst there are a large number of digital resources, also on the internet, which offer opportunities for the development of mathematics teaching and learning, these are rarely taken up by teachers; hence teachers need to be guided and supported to be able to exploit the technological affordances. Drijvers et al. contend that face-to-face meetings are 'still' central to teacher professional development and for building inquiry and learning communities. Interestingly, Trgalová and Jahn assert that whilst teachers in their study have been supported in the resource quality evaluation of particular dynamic geometry resources, they have seldom taken up the opportunities offered in terms of reviewing and modifying the resources. However, when examining teachers' practices (case study of seven teachers), their involvement in the quality resource considerations had considerable and positive impact on their practices: these teachers shifted from focusing on technical aspects (related to the mastery of the resource system) to didactical and pedagogical considerations for the resource's integration into their practice—the 'double instrumental genesis'. Hoyles et al. also study the question of resources supporting the integration of ICT by teachers, and the authors identify conditions for what we interpret as a specific form of interactions between teachers and the resources provided: the adaptation of these resources. The teachers do not align with the recommended use, but actively participate in the design of their own resources.

The third group of papers focuses squarely on the collective aspect of teachers working with resources. Gueudet et al. examine the resource systems of two teachers working in very different environments. They claim that whilst the collective dimensions are always present in teachers' ordinary work, and these are linked to the design and use of common resources, 'true collaborative work' appears to be rare and dependent on particular conditions. Visnovska and Cobb take a different stance on 'collectives': they show how classroom video can be utilized for the creation of a shared repertoire of reasoning within a

group of teachers—a community of practice develops during the professional development program.

From the collection of papers we suggest that there is evidence, albeit limited, to claim the following:

(1) Particular resources (together with the support) have the potential to draw 'participants' (e.g. mathematics teachers; teacher educators; pupils) together as a working collective (see Sensevy et al. 2013; Gueudet et al. 2013a; Hoyles et al. 2013; Visnovska and Cobb 2013; Kieran et al. 2013);
(2) The design and quality of resources are by no means a trivial matter: they need careful consideration, including in terms of newly developing perceptions and concepts of 'quality' of resources, in particular in the light of 'new' and 'different' IT resources (see Swan and Dorman 2013; Maschietto and Soury-Lavergne 2013);
(3) Particular support is needed, either in terms of systemic support or individuals taking the lead, in order to develop well-functioning collectives (see Gueudet et al. 2013a; Drijvers et al. 2013; Trgalova and Jahn 2013).

In terms of theoretical and methodological considerations, we argue that the existing theoretical and methodological frameworks capture only parts of the phenomena concerning teachers working in collectives with resources. For example, affective issues when working together with particular resources (e.g. 'the lack of communication and collaboration tools in the repository may create a feeling of isolation', see Trgalová and Jahn 2013) are likely to play a crucial role, but are not captured within the present frames.

In conclusion, drawing on the literature and the collection of papers in this issue, we claim that, although not the focus of each paper, the collective dimensions play an important role in mathematics teachers' work with resources and teacher professional learning/development. How teachers may work in collectives, in which ways 'productive' collectives may form and work together, the roles particular resources can play in these delicate constellations, and which kinds of resources offer opportunities for community building, are likely to need further empirical investigation.

References

Adler, J. (2000). Conceptualising resources as a theme for teacher education. *Journal of Mathematics Teacher Education, 3*, 205–224.

Artigue, M. (1994). Didactical engineering as a framework for the conception of teaching products. In R. Biehler, R. W. Scholz, R. Strasser, & B. Winkelman (Eds.), *Didactics of mathematics as a scientific discipline* (pp. 27–39). Dordrecht: Kluwer.

Assude, T. (2007). Changements et résistances à propos de l'intégration des nouvelles technologies dans l'enseignement des mathématiques au primaire. *Informations, Savoirs, Décisions et Médiations (ISDM), 29*. http://isdm.univ-tln.fr/articles/num_archives.htm#isdm29. Accessed 6 August 2013.

Ball, D. L., & Cohen, D. (1996). Reform by the book: What is—or might be—the role of curriculum materials in teacher learning and instructional reform? *Educational Researcher*, *25*(9), 6–8, 14.

Bartolini Bussi, M. G., & Maschietto, M. (2008). Machines as tools in teacher education. In D. Tirosh & T. Wood (Eds.), *International handbook of mathematics teacher education: Tools and processes in mathematics teacher education* (Vol. 2, pp. 183–208). Rotterdam: Sense Publishers.

Behm, S. L., & Lloyd, G. M. (2009). Factors influencing student teachers' use of mathematics curriculum materials. In J. T. Remillard, B. A. Herbel-Eisenmann, & G. M. Lloyd (Eds.), *Mathematics teachers at work—Connecting curriculum materials and classroom instruction* (pp. 205–222). New York: Routledge.

Borba, M. C., & Gadanidis, G. (2008). Virtual communities and networks of practising mathematics teachers. In K. Krainer & T. Wood (Eds.), *International handbook of mathematics teacher education: Participants in mathematics teacher education* (Vol. 3, pp. 181–206). Rotterdam: Sense Publishers.

Borba, M. C., & Llinares, S. (2012a). Online mathematics teacher education: overview of an emergent field of research. *ZDM—The International Journal on Mathematics Education, 44*(6), 697–704.

Borba, M.C., & Llinares, S. (2012b). Special issue: Online mathematics education. *ZDM—The International Journal on Mathematics Education*, *44*(6).

Borko, H., Jacobs, J. K., Eiteljorg, E., & Pittman, M. E. (2008). Video as a tool for fostering productive discussions in mathematics professional development. *Teaching and Teacher Education, 24*(2), 417–436.

Brousseau, G. (1997). *Theory of didactical situations in mathematics*. Dordrecht: Kluwer.

Brown, A. L. (1992). Design experiments: theoretical and methodological challenges in creating complex interventions in classroom settings. *The Journal of the Learning Sciences, 2*(2), 141–178.

Brown, M. W. (2002). *Teaching by design: Understanding the intersection between teacher practice and the design of curricular innovations*. Unpublished doctoral dissertation, Northwestern University, Evanston.

Brown, M. W. (2009). The teacher-tool relationship: Theorizing the design and use of curriculum materials. In J. T. Remillard, B. A. Herbel-Eisenmann, & G. M. Lloyd (Eds.), *Mathematics teachers at work—Connecting curriculum materials and classroom instruction* (pp. 205–222). New York: Routledge.

Bueno-Ravel, L., Forest, D., Gueudet, G., & Sensevy, G. (2010). Guidelines for design of on line resources for IBST. *Mind the Gap FP7 Project 217725*. http://uv-net.uio.no/mind-the-gap/Finished%20Deliverables/5.2_Guidelines%20for%20ICT%20design.doc.pdf. Accessed 6 August 2013.

Caprotti, O., & Seppälä, M. (2007). Evaluation criteria for eContent quality. *Joining Educational Mathematics, ECP-2005-EDU-038208*. http://www.jem-thematic.net/files_private/D%202.1.pdf. Accessed 6 August 2013.

Choppin, J. (2011). Learned adaptations: Teachers' understanding and use of curriculum resources. *Journal of Mathematics Teacher Education, 14*(5), 331–353.

Cobb, P. (2000). Conducting teaching experiments in collaboration with teachers. In A. Kelly & R. Lesh (Eds.), *Handbook of research design in mathematics and science education* (pp. 307–333). Mahwah: Lawrence Erlbaum.

Cobb, P., Confrey, J., diSessa, A., Lehrer, R., & Schauble, L. (2003). Design experiments in educational research. *Educational Researcher, 31*(1), 9–13.

Cobb, P., & Jackson, K. (2011). Towards an empirically grounded theory of action for improving the quality of mathematics teaching at scale. *Mathematics Teacher Education and Development, 13*(1), 6–33.

Cobb, P., Jackson, K., Smith, T., Sorum, M., & Henrick, E.C. (2013). Design research with educational systems: investigating and supporting improvements in the quality of mathematics teaching at scale. In B. J. Fishman, W. R. Penuel, A.-R. Allen, & B. H. Cheng (Eds.), *Design-based implementation research: Theories, methods, and exemplars. National Society for the Study of Education Yearbook*. New York: Teachers College Record. http://www.isls.org/icls2014/Resources.htm. Accessed 6 August 2013 (in press).

Cobb, P., & Smith, T. (2008). District development as a means of improving mathematics teaching and learning at scale. In K. Krainer, & T. Wood (Eds.), *International handbook of mathematics teacher education: Participants in mathematics teacher education* (Vol. 3, pp. 231–254). Rotterdam: Sense Publishers.

Cobb, P., & Steffe, L. P. (1983). The constructivist researcher as teacher and model builder. *Journal for Research in Mathematics Education, 14*(2), 83–95.

Confrey, J. (2006). The evolution of design studies as methodology. In R. K. Sawyer (Ed.), *The Cambridge handbook of the learning sciences* (pp. 135–151). Cambridge: Cambridge University Press.

Confrey, J., Bell, K., & Carrejo, D. (2001). *Systemic crossfire: What implementation research reveals about urban reform in mathematics*. Austin: University of Texas.

Davis, E. A., & Krajcik, J. S. (2005). Designing educative curriculum materials to promote teacher learning. *Educational Researcher, 34*(3), 3–14.

Design-Based Research Collective. (2003). Design-based research: An emerging paradigm for educational inquiry. *Educational Researcher, 31*(1), 5–8.

Diekema, A. R. & Olsen Whitney, M. (2012). The notion of relevance in teacher information behavior. In *Proceedings of the American Society for Information Science and Technology*, Baltimore.

Drijvers, P. (2012). Teachers transforming resources into orchestrations. In G. Gueudet, B. Pepin, & L. Trouche, (Eds.) (2012), *From text to 'lived' resources: Mathematics curriculum materials and teacher development* (pp. 265–281). New York: Springer.

Drijvers, P., Tacoma, S., Besamusca, A., Doorman, M., & Boon, P. (2013). Digital resources inviting mathematics teachers' professional development: the case of the DPICT project. *ZDM—The International Journal on Mathematics Education, 45*(7) (this issue). doi:10.1007/s11858-013-0535-1

Fan, L., Jones, K., Wang, J., & Xu, B. (Eds.) (2013). Textbook research in mathematics education. *ZDM—The International Journal on Mathematics Education*, *45*(5).

Fernandez, C., & Yoshida, M. (2004). *Lesson studies: A Japanese approach to improving mathematics teaching and learning*. London: Lawrence Erlbaum.

Fischbein, E. (1977). Image and concept in learning mathematics. *Educational Studies in Mathematics, 8*, 153–165.

Freudenthal, H., Janssen, G. M., & Sweers, W. J. (1976). *Five years IOWO on H. Freudenthal's retirement from the Directorship of IOWO: IOWO snapshots*. Dordrecht: Reidel.

Grant, T. J., Kline, K., Crumbaugh, C., Kim, O. K., & Cengiz, N. (2008). How can curriculum material support teachers in pursuing student thinking during whole-group discussion? In J. T. Remillard, B. A. Herbel-Eisenmann, & G. M. Lloyd (Eds.), *Mathematics teachers at work: Connecting curriculum materials and classroom instruction* (pp. 103–117). New York: Routledge.

Gravemeijer, K. (1993). The empty number line as an alternative means of representation for addition and subtraction. In J. de Lange, I. Huntley, C. Keitel, & M. Niss (Eds.), *Innovation in mathematics education by modeling and applications* (pp. 141–159). Chichester: Ellis Horwood.

Gravemeijer, K., & Cobb, P. (2006). Design research from a learning design perspective. In J. van den Akker, K. Gravemeijer, S. McKenney, & N. Nieveen (Eds.), *Educational design research* (pp. 17–51). London: Routledge.

Gueudet, G., Pepin, B., & Trouche, L. (Eds.). (2012). *From text to 'lived' resources: Mathematics curriculum materials and teacher development*. New York: Springer.

Gueudet, G., Pepin, B., & Trouche, L. (2013a). Collective work with resources: an essential dimension for teacher documentation. *ZDM—The International Journal on Mathematics Education*, *45*(7) (this issue). doi:10.1007/s11858-013-0527-1.

Gueudet, G., Pepin, B., & Trouche, L. (2013b). Textbooks design and digital resources. In C. Margolinas (Ed.), *Designing and using tasks for learning mathematics. 22nd ICMI study*. http://hal.archives-ouvertes.fr/docs/00/83/74/88/PDF/ICMI_STudy_22_proceedings_2013-FINAL_V2.pdf. Accessed 6 August 2013.

Gueudet, G., & Trouche, L. (2009). Towards new documentation systems for mathematics teachers? *Educational Studies in Mathematics, 71*(3), 199–218.

Gueudet, G., & Trouche, L. (2012a). Teachers' work with resources: Documentation geneses and professional geneses. In G. Gueudet, B. Pepin, & L. Trouche (Eds.), *From text to 'lived' resources: Mathematics curriculum materials and teacher development*. New York: Springer.

Gueudet, G., & Trouche, L. (2012b). Communities, documents and professional geneses: Interrelated stories. In G. Gueudet, B. Pepin, & L. Trouche (Eds.), *From text to 'lived' resources: Mathematics curriculum materials and teacher development* (pp. 305–322). New York: Springer.

Haggarty, L., & Pepin, B. (2002). An investigation of mathematics textbooks and their use in English, French and German classrooms: Who gets an opportunity to learn what? *British Educational Research Journal, 28*(4), 567–590.

Haspekian, M. (2005). An instrumental approach to study the integration of a computer tool into mathematics teaching: The case of spreadsheets. *International Journal of Computers for Mathematical Learning, 10*(2), 109–141.

Haspekian, M., & Artigue, M. (2007). L'intégration d'artefacts informatiques professionnels à l'enseignement dans une perspective instrumentale: le cas des tableurs. In M. Baron, D. Guin, & L. Trouche (Eds.), *Environnements informatisés et ressources numériques pour l'apprentissage* (pp. 37–63). Paris: Hermès.

Healy, L., & Lagrange, J.-B. (2010). Introduction to section 3. In C. Hoyles & J. B. Lagrange (Eds.), *Mathematical education and digital technologies: Rethinking the terrain* (pp. 287–292). New York: Springer.

Hoyles, C., & Lagrange, J. B. (Eds.). (2010). *Mathematical education and digital technologies: Rethinking the terrain*. New York: Springer.

Hoyles, C., Noss, R., & Kent, P. (2004). On the integration of digital technologies into mathematics classrooms. *International Journal of Computers for Mathematical Learning, 9*(3), 309–326.

Hoyles, C., Noss, R., Vahey, P., &, Roschelle, J. (2013). Cornerstone mathematics: designing digital technology for teacher adaptation

and scaling. *ZDM—The International Journal on Mathematics Education*, *45*(7) (this issue).

Huang, R., & Jaworski, B. (Eds.) (2014). Practices and strategies of promoting professional development of didacticians and teachers of mathematics: an international perspective. *ZDM—The International Journal on Mathematics Education*, *46*(2) (forthcoming).

Huang, R., & Li, Y. (2009). Pursuing excellence in mathematics classroom instruction through exemplary lesson development in China: A case study. *ZDM—The International Journal on Mathematics Education, 41*, 297–309.

Jaworski, B. (2008). Building and sustaining inquiry communities in mathematics teaching development: Teachers and didactitians in collaboration. In K. Krainer & T. Wood (Eds.), *International handbook of mathematics teacher education: Participants in mathematics teacher education* (Vol. 3, pp. 309–330). Rotterdam: Sense Publishers.

Kazemi, E. (2008). School development as a means of improving mathematics teaching and learning: Towards multidirectional analyses of learning across contexts. In K. Krainer & T. Wood (Eds.), *International handbook of mathematics teacher education: Participants in mathematics teacher education* (Vol. 3, pp. 209–230). Rotterdam: Sense Publishers.

Kazemi, E., & Franke, M. L. (2004). Teacher learning in mathematics: Using student work to promote collective inquiry. *Journal of Mathematics Teacher Education, 7*(3), 203–235.

Kieran, C., Boileau, A., Tanguay, D., & Drijvers, P. (2013) Design researchers' documentational genesis in a study on equivalence of algebraic expressions. *ZDM—The International Journal on Mathematics Education*, *45*(7). (this issue). doi:10.1007/s11858-013-0516-4

Kieran, C., Tanguay, D., & Solares, A. (2012). Researcher-designed resources and their adaptation within classroom teaching practice: shaping both the implicit and the explicit. In G. Gueudet, B. Pepin, & L. Trouche (Eds.), *From text to 'lived' resources: Mathematics curriculum materials and teacher development* (pp. 189–214). New York: Springer.

Krainer, K. (2008). Individuals, teams, communities and networks; participants and ways of participation in mathematics teacher education. In K. Krainer & T. Wood (eds.) *International handbook of mathematics teacher education: Participants in mathematics teacher education* (Vol. 3, pp. 1–10), Rotterdam: Sense Publishers.

Lagrange, J.-B., & Erdogan, E. O. (2009). Teachers' emergent goals in spreadsheet-based lessons: Analysing the complexity of technology integration. *Educational Studies in Mathematics, 71*(1), 65–84.

Lave, J., & Wenger, E. (1991). *Situated learning: Legitimate peripheral participation*. Cambridge: Cambridge University Press.

Llinares, S., & Olivera, F. (2008). Virtual communities of prospective mathematics teachers: technologies, interactions and new forms of discourse. In K. Krainer & T. Wood (Eds.), *International handbook of mathematics teacher education: Participants in mathematics teacher education* (Vol. 3, pp. 155–180). Rotterdam: Sense Publishers.

Lloyd, G. M. (1999). Two teachers' conceptions of a reform-oriented curriculum: implications for mathematics teacher development. *Journal of Mathematics Teacher Education, 2*(3), 227–252.

Lloyd, G. M., Remillard, J. T., & Herbel-Eisenmann, B. A. (2008). Teachers' use of curriculum material: an emerging field. In J. T. Remillard, B. A. Herbel-Eisenmann, & G. M. Lloyd (Eds.) (2008). *Mathematics teachers at work: Connecting curriculum materials and classroom instruction* (pp. 3–14). New York: Routledge.

Marton, F., & Tsui, A. B. M. (Eds.). (2004). *Classroom discourse and the space of learning*. Mahwah: Lawrence Erlbaum.

Maschietto, M., & Soury-Lavergne S. (2013). Merging material, didactical and digital resources: an example for primary school mathematics with the pascaline and Cabri Elem. *ZDM—The International Journal on Mathematics Education*, *45*(7) (this issue). doi:10.1007/s11858-013-0533-3

McKenney, S., & Reeves, T. C. (2012). *Conducting educational design research*. London: Routledge.

Monaghan, J. (2001). Teachers' classroom interactions in ICT-based mathematics lessons. In M. van den Heuvel-Panhuizen (Ed.), *Proceedings of the 25th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 3, pp. 383–390). Nottingham: University of Nottingham.

Monaghan, J. (2004). Teachers' activities in technology-based mathematics lessons. *International Journal of Computers for Mathematical Learning, 9*(3), 327–357.

Nührenbörger, M., & Steinbring, H. (2008). Manipulatives as tools in mathematics teacher education. In D. Tirosh & T. Wood (Eds.), *International handbook of mathematics teacher education, Vol. 2: Tools and processes in mathematics teacher education* (pp. 157–181). Rotterdam: Sense Publishers.

O'Shea, T. (1993). The role of manipulatives in mathematics education. *Contemporary Education, 65*(1), 6–9.

Ono, Y., & Ferreira, J. (2010). A case study of continuing teacher professional development through lesson study in South Africa. *South African Journal of Education, 30*, 59–74.

Pédauque, R. T. (coll.) (2006). *Le document à la lumière du numérique*. Caen: C & F éditions.

Penuel, W. R., Fishman, B. J., Cheng, B. H., & Sabelli, N. (2011). Organising research and development at the intersection of learning, implementation, and design. *Educational Researcher, 40*(7), 331–337.

Pepin, B. (2012). Working with teachers on curriculum materials to develop mathematical knowledge in/for teaching: Task analysis as 'catalytic tool' for feedback and teacher learning. In G. Gueudet, B. Pepin, & L. Trouche (Eds.), *From text to 'lived' resources: Mathematics curriculum materials and teacher development* (pp. 123–142). New York: Springer.

Pepin, B., Gueudet, G., & Trouche, L. (2013). Investigating textbooks as crucial interfaces between culture, policy and teacher curricular practice: two contrasted case studies in France and Norway. *ZDM—The International Journal on Mathematics Education*, *45*(5). doi:10.1007/s11858-013-0526-2

Pepin, B., & Haggarty, L. (2001). Mathematics textbooks and their use in English, French and German classrooms: a way to understand teaching and learning cultures. *ZDM—The International Journal on Mathematics Education, 33*(5), 158–175.

Rabardel, P. (1995). *Les hommes et les technologies: Approche cognitive des instruments contemporains*. Paris: Armand Colin.

Rabardel, P., & Béguin, P. (2005). Instrument mediated activity: from subject development to anthropocentric design. Theoretical issues. *Ergonomics Science, 6*(5), 429–461.

Radford, L. (2012). On the cognitive, epistemic, and ontological roles of artefacts. In G. Gueudet, B. Pepin, & L. Trouche (Eds.), *From text to 'lived' resources: Mathematics curriculum materials and teacher development* (pp. 283–288). New York: Springer.

Remillard, J. (1999). Curriculum materials in mathematics education reform: A framework for examining teachers' curriculum development. *Curriculum Inquiry, 29*(3), 315–342.

Remillard, J. (2005). Examining key concepts in research on teachers' use of mathematics curricula. *Review of Educational Research, 75*(2), 211–246.

Remillard, J. (2012). Modes of engagement: Understanding teachers' transactions with mathematics curriculum resources. In G. Gueudet, B. Pepin, & L. Trouche (Eds.), *From text to 'lived' resources: Mathematics curriculum materials and teacher development* (pp. 105–122). New York: Springer.

Remillard, J. (2013) Beyond the script: Educative features of five mathematics curricula and how teachers use them. In *Introductory symposium presentation at the annual meeting of American Educational Research Association in San Francisco*, May 2013.

Remillard, J., Herbel-Eisenmann, B. A., & Lloyd, G. M. (Eds.). (2008). *Mathematics teachers at work: Connecting curriculum materials and classroom instruction*. New York: Routledge.

Rezat, S. (2012). Interactions of teachers' and students' use of mathematics textbooks. In G. Gueudet, B. Pepin, & L. Trouche (Eds.), *From text to 'lived' resources: Mathematics curriculum materials and teacher development* (pp. 231–246). New York: Springer.

Runesson, U. (2008). Learning to design for learning. In P. Sullivan & T. Wood (Eds.), *International handbook of mathematics teacher education* (Vol. 1, pp. 153–172)., Knowledge and beliefs in mathematics teaching and teaching development Rotterdam: Sense Publishers.

Ruthven, K. (2012). Constituting digital tools and material as classroom resources: An example in dynamic geometry. In G. Gueudet, B. Pepin, & L. Trouche (Eds.), *From text to 'lived' resources: Mathematics curriculum materials and teacher development* (pp. 83–103). New York: Springer.

Ruthven, K., & Hennessy, S. (2002). A practitioner model of the use of computer-based tools and resources to support mathematics teaching and learning. *Educational Studies in Mathematics, 49*(1), 47–88.

Ruthven, K., & Hofmann, R. (2013). Chance by design: Devising an introductory probability module for implementation at scale in English early-secondary education. *ZDM—The International Journal on Mathematics Education, 45*(3), 409–423.

Ruthven, K., Laborde, C., Leach, J., & Tiberghien, A. (2009). Design tools in didactical research: Instrumenting the epistemological and cognitive aspects of the design of teaching sequences. *Educational Researcher, 38*(5), 329–342.

Sensevy, G. (2012). Patterns of didactic intentions. Thought collectives and documentation work. In G. Gueudet, B. Pepin, & L. Trouche (Eds.), *From text to 'lived' resources: Mathematics curriculum materials and teacher development* (pp. 43–57). New York: Springer.

Sensevy, G., Forest, D., Quilio, S., & Morales, G. (2013). Cooperative engineering as a specific design-based research in joint action theory in didactics. *ZDM—The International Journal on Mathematics Education*, *45*(7) (this issue). doi:10.1007/s11858-013-0532-4

Sherin, M. G., & Drake, C. (2009). Curriculum strategy framework: Investigating patterns in teachers' use of a reform-based elementary mathematics curriculum. *Journal of Curriculum Studies, 41*(4), 467–500.

Shield, M., & Dole, S. (2008). *A methodology for evaluating middle-years mathematics textbooks.* Paper presented at ICME 11, Monterrey, Mexico.

Silver, E. A., Ghousseini, H., Charamlambous, C. Y., & Mills, V. (2009). Exploring curriculum implementation plateau: An instructional perspective. In J. T. Remillard, B. A. Herbel-Eisenmann, & G. M. Lloyd (Eds.), *Mathematics teachers at work—Connecting curriculum materials and classroom instruction* (pp. 245–265). New York: Routledge.

Stein, M. K., & Kim, G. (2009). The role of mathematics curriculum materials in large scale urban reform: An analysis of demands and opportunities for teacher learning. In J. Remillard, G. Lloyd, & B. Herbel-Eisenmann (Eds.), *Teachers' use of mathematics curriculum materials: Research perspectives on relationships between teachers and curriculum* (pp. 37–55). New York: Routledge.

Streefland, L. (1990). *Fractions in realistic mathematics education, a paradigm of developmental research*. Dordrecht: Kluwer.

Stylianides, A. J. & Stylianides, G. J. (Eds.) (2013) Special Issue: Classroom-based interventions in mathematics education. *ZDM—The International Journal on Mathematics Education, 45,* 333–341.

Swan, M., & Dorman, M. (2013). Designing and using professional development resources for inquiry-based learning. *ZDM—The International Journal on Mathematics Education*, *45*(7) (this issue). doi:10.1007/s11858-013-0520-8.

Tabak, I. (2004). Reconstructing context: Negotiating the tension between exogenous and endogenous educational design. *Educational Psychologist, 39*(4), 225–233.

Trgalová, J., & Jahn, A.-P. (2013). Quality issue in the design and use of resources by mathematics teachers. *ZDM—The International Journal on Mathematics Education*, *45*(7) (this issue). doi:10.1007/s11858-013-0525-3.

Trgalová, J., Soury-Lavergne, S., & Jahn, A. P. (2011). Quality assessment process for dynamic geometry resources in Intergeo project: rationale and experiments. *ZDM—The International Journal on Mathematics Education, 43*(3), 337–351.

Tricot, A., Plégat-Soutjis, F., Camps, J.-F., Amiel, A., Lutz, G., & Morcillo, A. (2003). Utilité, utilisabilité, acceptabilité: Interpréter les relations entre trois dimensions de l'évaluation des EIAH. In C. Desmoulins, P. Marquet, & D. Bouhineau (Eds.), *Environnements informatiques pour l'apprentissage humain* (pp. 391–402). Paris: ATIEF/INRP.

Trouche, L. (2004). Managing the complexity of human/machine interactions in computerized learning environments: Guiding students' command process through instrumental orchestrations. *International Journal of Computers for Mathematical Learning, 9*, 281–307.

Trouche, L., Drijvers, P., Gueudet, G., & Sacristan, A. I. (2013). Technology-driven developments and policy implications for mathematics education. In A. J. Bishop, M. A. Clements, C. Keitel, J. Kilpatrick, & F. K. S. Leung (Eds.), *Third international handbook of mathematics education* (pp. 753–790). New York: Springer.

Van den Akker, J., Gravemeijer, K., KcKenney, S., & Nieveen, N. (2006). Introducing educational design research. In J. van den Akker, K. Gravemeijer, S. KcKenney, & N. Nieveen (Eds.), *Educational design research* (pp. 3–7). London: Routledge.

Van Es, E. A., & Sherin, M. G. (2008). Mathematics teachers' 'learning to notice' in the context of a video club. *Teaching and Teacher Education, 24*(2), 244–276.

Vergnaud, G. (1998). Toward a cognitive theory of practice. In A. Sierpinska & J. Kilpatrick (Eds.), *Mathematics education as a research domain: A search for identity* (pp. 227–241). Dordrecht: Kluwer.

Visnovska, J., & Cobb, P. (2013). Classroom video in teacher professional development program: community documentational genesis perspective. *ZDM—The International Journal on Mathematics Education, 45*(7) (this issue). doi:10.1007/s11858-013-0523-5.

Vygotsky, L. (1978). *Mind in society*. Cambridge: Harvard University Press.

Wagner, J. (1997). The unavoidable intervention of educational research: A framework for reconsidering research–practitioner cooperation. *Educational Researcher, 26*(7), 13–22.

Wedell, M. (2005). Cascading training down into the classroom: the need for parallel planning. *International Journal of Educational Development, 25*(6), 637–651.

Wenger, E. (1998). *Communities of practice: Learning, meaning, and identity*. Cambridge: Cambridge University Press.

Wiliam, D., & Lester, F. K. (2008). On the purpose of mathematics education research: Making productive contributions to policy and practice. In L. D. English (Ed.), *Handbook of international*

research in mathematics education (2nd ed., pp. 32–48). New York: Routledge.

Winsløw, C. (2012). A comparative perspective on teacher collaboration: The cases of lesson study in Japan and of multidisciplinary teaching in Denmark. In G. Gueudet, B. Pepin, & L. Trouche (Eds.), *From text to 'lived' resources: Mathematics curriculum materials and teacher development* (pp. 291–304). New York: Springer.

Wittman, E. C. (2001). Developing mathematics education in a systemic process. *Educational Studies in Mathematics, 48*(1), 1–20.

Yang, Y. (2009). How a Chinese teacher improved classroom teaching in Teaching Research Group: A case study on Pythagoras theorem teaching in Shanghai. *ZDM—The International Journal on Mathematics Education, 41*, 279–296.