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REACHABILITY OF CONE FRACTIONAL CONTINUOUS–TIME LINEAR SYSTEMS

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A new class of cone fractional continuous-time linear systems is introduced. Necessary and sufficient conditions for a fractional linear system to be a cone fractional one are established. Sufficient conditions for the reachability of cone fractional systems are given. The discussion is illustrated with an example of linear cone fractional systems.

Keywords: cone fractional system, linear systems, reachability.

1. Introduction

In positive systems inputs, state variables and outputs take only non-negative values. Examples of positive systems are industrial processes involving chemical reactors, heat exchangers and distillation columns, storage systems, compartmental systems, water and atmospheric pollution models. A variety of models having positive linear systems behaviour can be found in engineering, management science, economics, social sciences, biology and medicine, etc.

Positive linear systems are defined on cones and not on linear spaces. Therefore, the theory of positive systems is more complicated and less advanced. An overview of state of the art in positive systems is given in the monographs (Farina and Rinaldi, 2000; Kaczorek, 2002). An extension of positive systems are cone systems. The notion of cone systems was introduced in (Kaczorek, 2006). Roughly speaking, a cone system is a system obtained from a positive one by the substitution of the positive orthants of states, inputs and outputs by suitable arbitrary cones. The realization problem for cone systems was addressed in (Kaczorek, 2006; Kaczorek, 2007c).

The first definition of the fractional derivative was introduced by Liouville and Rieman at the end of the 19th century (Miller and Ross, 1993; Nishimoto, 1984; Podlubny, 1999). This idea was used by engineers for modelling various processes in the late 1960s (Vinagre *et al.*, 1960; Vinagre and Feliu, 2002; Zaborowsky and Meylaov, 2001). Mathematical fundamentals of fractional calculus are given in the monographs (Miller and Ross, 1993; Nishimoto, 1984; Oldham and Spanier, 1974; Oustaloup, 1995; Podlubny, 1999). Fractional order controllers were developed in (Oustaloup, 1993; Podlubny *et al.*, 1997). A generalization of the Kalman filter for fractional order systems was proposed in (Sierociuk and Dzieliński, 2007). Some other applications of fractional order systems can be found in (Engheta, 1997; Ferreira and Machado, 2003; Ostalczyk, 2000; Ostalczyk, 2004; Ostalczyk, 2004; Reyes-Melo *et al.*, 2004; Riu *et al.*, 2001; Samko *et al.*, 1993; Sjöberg and Kari, 2002; Vinagre and Feliu, 2002).

In (Ortigueira, 1997), a method was set forth for the computation of the impulse responses from the frequency responses for fractional standard (non-positive) discrete-time linear systems. Fractional polynomials and nD systems were investigated in (Gałkowski, 2005). Necessary and sufficient conditions for the reachability and controllability to zero of cone fractional discrete-time linear systems were established in (Kaczorek, 2007a; Kaczorek, 2007b; Kaczorek, 2007d).

In this paper, the notion of cone fractional linear continuous-time systems will be introduced. Sufficient conditions for the reachability of cone fractional linear systems will be established. To the best of the author's knowledge, cone fractional continuous-time linear systems have not been considered yet.

2. Positive fractional linear systems

Let $\mathbb{R}^{n\times m}$ be the set of $n \times m$ real matrices and $\mathbb{R}^n := \mathbb{R}^{n\times 1}$. The set of $m \times n$ real matrices with nonnegative entries will be denoted by $\mathbb{R}_+^{m\times n}$ and $\mathbb{R}_+^n := \mathbb{R}_+^{n\times 1}$. The set of nonnegative integers will be denoted by $\mathbb{Z}_+$ and the $n \times n$ identity matrix by I_n.

The following Caputo definition of the fractional derivative will be used (Miller and Ross, 1993; Podlubny, 1999):

$$\begin{aligned} D^\alpha f(t) &= \frac{\mathrm{d}^\alpha}{\mathrm{d}t^\alpha} f(t) \\ &= \frac{1}{\Gamma(k-\alpha)} \int_0^t \frac{f^{(k)}(\tau)}{(t-\tau)^{\alpha+1-k}}\,\mathrm{d}\tau, \\ &\qquad k-1 < \alpha \le k \in \mathbb{N} = \{1, 2, \ldots\}, \end{aligned} \tag{1}$$

where $\alpha \in \mathbb{R}$ is the order of the fractional derivative and $f^{(k)}(\tau) = \mathrm{d}^k f(\tau)/\mathrm{d}\tau^k$. Consider the continuous-time fractional linear system described by the state equation

$$\frac{\mathrm{d}^\alpha}{\mathrm{d}t^\alpha} x(t) = Ax(t) + Bu(t), \quad 0 < \alpha \le 1, \tag{2a}$$

$$y(t) = Cx(t) + Du(t), \tag{2b}$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$, $y(t) \in \mathbb{R}^p$ are respectively the state, input and output vectors, and $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $C \in \mathbb{R}^{p\times n}$, $D \in \mathbb{R}^{p\times m}$.

Theorem 1. *The solution of Eqn. (2a) is given by*

$$x(t) = \Phi_0(t)x_0 + \int_0^t \Phi(t-\tau)Bu(t)\,\mathrm{d}\tau, \quad x(0) = x_0, \tag{3}$$

where

$$\Phi_0(t) = E_\alpha(At^\alpha) = \sum_{k=0}^{\infty} \frac{A^k t^{k\alpha}}{\Gamma(k\alpha+1)}, \tag{4}$$

$$\Phi(t) = \sum_{k=0}^{\infty} \frac{A^k t^{(k+1)\alpha-1}}{\Gamma[(k+1)\alpha]}, \tag{5}$$

and $E_\alpha(At^\alpha)$ is the Mittag-Leffler matrix function. What is more,

$$\Gamma(x) = \int_0^\infty e^{-t} t^{x-1}\,\mathrm{d}t$$

is the Gamma function.

Definition 1. The system (2) is called the *internally positive fractional system* if and only if $x(t) \in \mathbb{R}_+^n$ and $y(t) \in \mathbb{R}_+^p$, $t \ge 0$ for any initial conditions $x_0 \in \mathbb{R}_+^n$ and all inputs $u(t) \in \mathbb{R}_+^m$, $t \ge 0$.

A square rational matrix $A = [a_{ij}]$ is called a *Metzler matrix* if its off-diagonal entries are nonnegative, i.e., $a_{ij} \ge 0$ for $i \ne j$ (Farina and Rinaldi, 2000; Kaczorek, 2002). The set of $n \times n$ all Metzler matrices will be denoted by M_n.

Theorem 2. *The continuous-time fractional system (2) is internally positive if and only if*

$$A \in M_n, \quad B \in \mathbb{R}_+^{n\times m}, \quad C \in \mathbb{R}_+^{p\times n}, \quad D \in \mathbb{R}_+^{p\times m}. \tag{6}$$

3. Cone fractional systems

Based on (Kaczorek, 2008), the following definitions are recalled.

Definition 2. Let

$$P = \begin{bmatrix} p_1 \\ \vdots \\ p_n \end{bmatrix} \in \mathbb{R}^{n\times n}$$

be nonsingular and p_k be its k-th row ($k = 1, \ldots, n$). The set

$$\mathcal{P} := \left\{ x \in \mathbb{R}^n : \bigcap_{k=1}^{n} p_k x \ge 0 \right\} \tag{7}$$

is called a *linear cone generated by the matrix P*. In a similar way, for the inputs u, we may define the *linear cone*

$$\mathcal{Q} := \left\{ u \in \mathbb{R}^m : \bigcap_{k=1}^{m} q_k u \ge 0 \right\} \tag{8}$$

generated by the nonsingular matrix

$$Q = \begin{bmatrix} q_1 \\ \vdots \\ q_m \end{bmatrix} \in \mathbb{R}^{m\times m},$$

and for the outputs y, the *linear cone*

$$\mathcal{V} := \left\{ y \in \mathbb{R}^p : \bigcap_{k=1}^{p} v_k y \ge 0 \right\} \tag{9}$$

generated by the nonsingular matrix

$$V = \begin{bmatrix} v_1 \\ \vdots \\ v_p \end{bmatrix} \in \mathbb{R}^{p\times p}.$$

Definition 3. The fractional system (2) is called a $(\mathcal{P},\mathcal{Q},\mathcal{V})$ *cone fractional system* if $x(t) \in \mathcal{P}$ and $y(t) \in \mathcal{V}$, $t \ge 0$ for every $x_0 \in \mathcal{P}$, $u(t) \in \mathcal{Q}$, $t \ge 0$.

The $(\mathcal{P},\mathcal{Q},\mathcal{V})$ cone fractional system (2) will be briefly called the cone fractional system. If $\mathcal{P} = \mathbb{R}_+^n$, $\mathcal{Q} = \mathbb{R}_+^m$, $\mathcal{V} = \mathbb{R}_+^n$, then the $(\mathbb{R}_+^n, \_\ \mathbb{R}_+^m,\ \mathbb{R}_+^p)$ cone system is equivalent to the classical positive system (Farina and Rinaldi, 2000; Kaczorek, 2002).

Theorem 3. *The fractional system (2) is a $(\mathcal{P},\mathcal{Q},\mathcal{V})$ cone fractional one if and only if*

$$\begin{aligned} \bar{A} &= PAP^{-1} \in M_n, & \bar{B} &= PBQ^{-1} \in \mathbb{R}_+^{n\times m}, \\ \bar{C} &= VCP^{-1} \in \mathbb{R}_+^{p\times n}, & \bar{D} &= VDQ^{-1} \in \mathbb{R}_+^{p\times m}. \end{aligned} \tag{10}$$

Proof. Let

$$\begin{aligned}\bar{x}(t) &= Px(t), \quad \bar{u}(t) = Qu(t),\\ \bar{y}(t) &= Vy(t), \quad t \geq 0.\end{aligned} \tag{11}$$

From Definition 3 it follows that if $x(t) \in \mathcal{P}$, then $\bar{x}(t) \in \mathbb{R}_+^n$, if $u(t) \in \mathcal{Q}$, then $\bar{u}(t) \in \mathbb{R}_+^m$, and if $y(t) \in \mathcal{V}$, then $\bar{y}(t) \in \mathbb{R}_+^p$. From (2) and (11) we have

$$\begin{aligned}\frac{\mathrm{d}^\alpha \bar{x}(t)}{\mathrm{d}t^\alpha} &= \frac{\mathrm{d}^\alpha Px(t)}{\mathrm{d}t^\alpha} = PAx(t) + PBu(t)\\ &= PAP^{-1}\bar{x}(t) + PBQ^{-1}\bar{u}(t)\\ &= \bar{A}\bar{x}(t) + \bar{B}\bar{u}(t), \quad t \geq 0\end{aligned} \tag{12a}$$

and

$$\begin{aligned}\bar{y}(t) &= Vy(t) = VCx(t) + VDu(t)\\ &= VCP^{-1}\bar{x}(t) + VDQ^{-1}\bar{u}(t)\\ &= \bar{C}\bar{x}(t) + \bar{D}\bar{u}(t), \quad t \geq 0.\end{aligned} \tag{12b}$$

It is well known (Kaczorek, 2002; Kaczorek, 2007b) that the system (12) is positive if and only if the conditions (10) are satisfied. ■

4. Reachability

4.1. Positive fractional systems.

Definition 4. The state $x_f \in \mathbb{R}_+^n$ of the positive fractional system (2) is called reachable in time t_f if there exists an input $u(t) \in \mathbb{R}_+^m$, $t \in [0, t_f]$ which steers the state of the system (2) from the zero initial state $x_0 = 0$ to the state x_f. If each state $x_f \in \mathbb{R}_+^n$ is reachable in time t_f, then the system is called reachable in time t_f. If for each state $x_f \in \mathbb{R}_+^n$ there exists a time t_f such that the state is reachable in time t_f, then the system (2) is called *reachable*.

A real square matrix is called *monomial* if and only if each of its rows and columns contains only one positive entry and the remaining entries are zero.

Theorem 4. (Kaczorek, 2008) *The positive fractional system (2) is reachable in time t_f if the matrix*

$$R(t_f) = \int_0^{t_f} \Phi(\tau)BB^T\Phi^T(\tau)\,\mathrm{d}\tau \tag{13}$$

is monomial. The input which steers the state of (2) from $x_0 = 0$ to x_f is given by

$$u(t) = B^T\Phi^T(t_f - t)R^{-1}(t_f)x_f, \tag{14}$$

where 'T' denotes the transpose.

4.2. Cone fractional systems.

Definition 5. A state $x_f \in \mathcal{P}$ of the cone fractional system (2) is called reachable in time t_f if there exists an input sequence $u(t) \in \mathcal{Q}$, $t \in [0, t_f]$ which steers the state of the system from the zero initial state $x_0 = 0$ to the desired state x_f, i.e., $x(t_f) = x_f$. If each state $x_f \in \mathcal{P}$ is reachable in time t_f, then the cone fractional system is called reachable in time t_f. If for every state $x_f \in \mathcal{P}$ there exists a time t_f such that the state is reachable in time t_f, then the cone fractional system is called *reachable*.

Theorem 5. *The cone fractional system (2) is reachable in time t_f if the matrix*

$$\bar{R}(t_f) = P\int_0^{t_f} \Phi(\tau)BQ^{-1}Q^{-T}B^T\Phi^T(\tau)\,\mathrm{d}\tau P^T \tag{15}$$

here ($Q^{-T} = (Q^{-1})^T$) is a monomial matrix.

Proof. From the relations (11) it follows that if $x(t) \in \mathcal{P}$, then $\bar{x}(t) = Px(t) \in \mathbb{R}_+^n$, $t \geq 0$, and if $u(t) \in \mathcal{Q}$, then $\bar{u}(t) = Qu(t) \in \mathbb{R}_+^m$, $t \geq 0$. Hence, by Definitions 4 and 5, the cone fractional system (2) is reachable in time t_f if the positive fractional system (12) is reachable in time t_f.

From (10) and (5) we have

$$\begin{aligned}\bar{\Phi}(t) &= \sum_{k=0}^{\infty} \frac{\bar{A}^k t^{(k+1)\alpha-1}}{\Gamma[(k+1)\alpha]}\\ &= \sum_{k=0}^{\infty} \frac{(PAP^{-1})^k t^{(k+1)\alpha-1}}{\Gamma[(k+1)\alpha]} = P\Phi(t)P^{-1}\end{aligned} \tag{16}$$

since $\bar{A}^k = (PAP^{-1})^k = PA^kP^{-1}$ for $k = 1, 2, \ldots$ and

$$\bar{\Phi}(t)\bar{B} = P\Phi(t)P^{-1}PBQ^{-1} = P\Phi(t)BQ^{-1}. \tag{17}$$

Using (13) and (17), we obtain

$$\begin{aligned}\bar{R}(t_f) &= \int_0^{t_f} \bar{\Phi}(\tau)\bar{B}\bar{B}^T\bar{\Phi}^T(\tau)\,\mathrm{d}\tau\\ &= \int_0^{t_f} (P\Phi(\tau)BQ^{-1})(P\Phi(\tau)BQ^{-1})^T\,\mathrm{d}\tau\\ &= P\int_0^{t_f} \Phi(\tau)BQ^{-1}Q^{-T}B^T\Phi^T(\tau)\,\mathrm{d}\tau P^T.\end{aligned} \tag{18}$$

Therefore, by Theorem 4, the positive fractional system (12) and the cone fractional system (2) are reachable in time t_f if the matrix (15) is monomial. ■

By Theorem 5 we have the following result.

Corollary 1. *If $Q = I_m$, then $\bar{R}(t_f) = PR(t_f)P^T$, and the cone fractional system (2) is reachable in time t_f if the positive fractional system is reachable and P is a monomial matrix.*

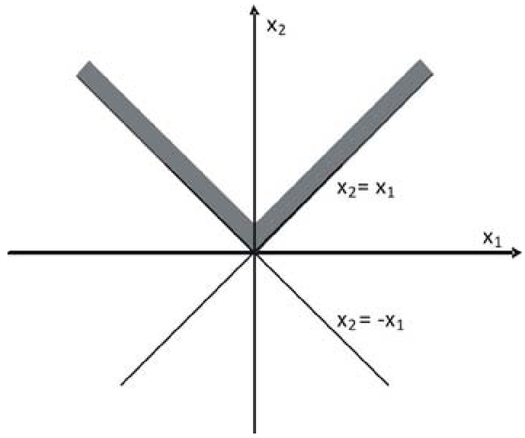

Fig. 1. $\mathcal{P}$-cone generated by the matrix P.

Example 1. Consider the cone fractional system (2) with

$$P = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}, \quad Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$
$$A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}. \tag{19}$$

The $\mathcal{P}$-cone generated by the matrix P is shown in Fig. 1.

In (Kaczorek, 2007d), it was shown that

$$\Phi(t)B = \begin{bmatrix} 0 & \Phi_1(t) \\ \Phi_2(t) & 0 \end{bmatrix} \tag{20}$$

and

$$\begin{aligned} R(t_f) &= \int_0^{t_f} \Phi(\tau)BB^T\Phi^T(\tau)\,\mathrm{d}\tau \\ &= \int_0^{t_f} \begin{bmatrix} \Phi_1^2(\tau) & 0 \\ 0 & \Phi_2^2(\tau) \end{bmatrix} \mathrm{d}\tau, \end{aligned} \tag{21}$$

where

$$\Phi_1(t) = \sum_{k=0}^{\infty} \frac{t^{(k+1)\alpha-1}}{\Gamma[(k+1)\alpha]}, \quad \Phi_2(t) = \frac{t^{\alpha-1}}{\Gamma(\alpha)} \tag{22}$$

for $0 < \alpha < 1$. The matrix (21) is monomial and by Theorem 4 the positive fractional system is reachable in time t_f.

In case $Q = I_2$, the matrix

$$\begin{aligned} &\bar{R}\ (t_f) \\ &= PR(t_f)P^T \\ &= \int_0^{t_f} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} \Phi_1^2(\tau) & 0 \\ 0 & \Phi_2^2(\tau) \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \mathrm{d}\tau \\ &= \int_0^{t_f} \begin{bmatrix} \Phi_1^2(\tau)+\Phi_2^2(\tau) & \Phi_2^2(\tau)-\Phi_1^2(\tau) \\ \Phi_2^2(\tau)-\Phi_1^2(\tau) & \Phi_1^2(\tau)+\Phi_2^2(\tau) \end{bmatrix} \mathrm{d}\tau \end{aligned} \tag{23}$$

is not monomial since $\Phi_1^2(t) \neq \Phi_2^2(t)$. Therefore, the sufficient condition of Theorem 5 for the reachability in time t_f is not satisfied. ♦

From this example and the comparison of (13) and (15), it follows that the sufficient conditions for the reachability of cone fractional systems is much stronger than for positive fractional systems.

5. Concluding remarks

The concept of cone fractional linear systems has been introduced. Necessary and sufficient conditions for fractional systems to be cone fractional ones were established. Sufficient conditions for the reachability of cone fractional linear systems were also given. The conditions were illustrated with an example of a linear cone fractional system. Following (Kaczorek, 2007d), the results can be extended to the controllability to zero for cone fractional linear systems.

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