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Research Article

Reachable Set Bounding for Homogeneous Nonlinear Systems with Delay and Disturbance

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Reachable set bounding for homogeneous nonlinear systems with delay and disturbance is studied. By the usage of a new method for stability analysis of positive systems, an explicit necessary and sufficient condition is first derived to guarantee that all the states of positive homogeneous time-delay systems with degree $p > 1$ converge asymptotically within a specific ball. Furthermore, the main result is extended to a class of nonlinear time variant systems. A numerical example is given to demonstrate the effectiveness of the obtained results.

1. Introduction

Recent years have witnessed a rapid development of reachable set bounding for linear systems in [1–11], to name a few. In most of existing references, the traditional Lyapunov-Krasovskii function method is most commonly used. However, such a method is usually difficult to derive explicit conditions for reachable set estimation of nonlinear systems with delay and disturbance.

Due to the ubiquitous existence of time delay in practical engineering and its adverse effect on stability [12–15] and oscillation [16–19], it has attracted wide attention in recent years. So far, less attention has been paid to reachable set bounding for nonlinear time-delay systems. Such a problem was discussed in [20, 21] for certain nonlinear perturbed systems with delay, where the involved nonlinear terms satisfy a linear growth condition. Reachable set bounding for continuous-time and discrete-time homogeneous time-delay positive systems of degree one was studied in [22]. The decay rates of homogeneous positive systems of any degree with time-varying delays were given in [23]. Recently, the same problem was considered in [24] for homogeneous positive systems of degree $p > 1$, while time delay was not taken into consideration. The problem of reachable set estimation of switched positive systems with discrete and distributed delays subject to bounded disturbances was investigated in [25].

Positive systems are dynamical systems whose states remain nonnegative whenever the initial states are nonnegative ([26, 27]). In view of the special structure of positive systems, a special method was commonly used for stability analysis of positive systems in [28–33], which is different from the traditional Lyapunov-Krasovskii function method.

Motivated by the work in [23, 24], we study in this paper reachable set bounding for homogeneous nonlinear time-delay systems with bounded disturbance. By developing the methods used in [23, 24], we first establish a necessary and sufficient condition such that all the solutions of positive homogeneous time-delay systems with degree $p > 1$ converge asymptotically within a specific ball, which contains those results in [23, 24] in special cases. The main result is also applied to certain nonlinear time variant systems with delay and disturbance.

Throughout this paper, $\mathbb{R}^n$ is the set of n-dimensional real vectors. Denote by x_i the ith coordinate of $x \in \mathbb{R}^n$ for $i \in \langle n \rangle = \{1, 2, \ldots, n\}$. Given $x, y \in \mathbb{R}^n$, say $x \succ y$ (or $y \prec x$) if $x_i > y_i$, $x \succeq y$ (or $y \preceq x$) if $x_i \geq y_i$, $i \in \langle n \rangle$. Denote $\mathbb{R}^n_+ = \{x \in \mathbb{R}^n : x \succeq 0\}$. For $x = (x_i) \in \mathbb{R}^n$, denote $|x| = (|x_i|) \in \mathbb{R}^n_+$ and $\|x\|_\infty = \max_{i\in\langle n\rangle}|x_i|$. Let $\mathscr{B}(\varepsilon)=\{x \in \mathbb{R}^n | \|x\|_\infty \le \varepsilon\}$, where $\varepsilon > 0$ is a constant. For given $r > 0$, denote $\mathbb{BF}_r([0,\infty],\mathbb{R}^n) = \{\omega : [0,\infty] \longrightarrow \mathbb{R}^n | \|\omega(t)\|_\infty \le r, \forall t \ge 0\}$. An $n\times n$-dimensional matrix A is called Metzler if all its off-diagonal entries are nonnegative.

2. Preliminaries

In this paper, nonlinear time-delay systems of the form

$$\begin{aligned} &\dot{x}(t) = f(x(t)) + g((x(t-\tau(t)))) + \omega(t), \quad t \ge 0 \\ &x(t) = \varphi(t), \quad t \in [-h,0], \end{aligned} \tag{1}$$

are investigated, where $x(t) \in \mathbb{R}^n$ is the state vector, $f, g : \mathbb{R}^n \longrightarrow \mathbb{R}^n$ are continuous vector functions satisfying $f(0) = g(0) = 0$, $\tau(t)$ is a time delay satisfying $0 \le \tau(t) \le h$, $h > 0$ is a constant, $\omega(t) \in \mathbb{BF}_r([0,\infty],\mathbb{R}^n)$ is the disturbance, and the initial state $\varphi(t) : [-h,0] \longrightarrow \mathbb{R}^n$ is continuous. Note that when $\tau(t) \equiv 0$, system (1) takes the form of the system considered in [24].

The following definitions and lemma in [34] will be required.

Definition 1. Assume that $f : \mathbb{R}^n \longrightarrow \mathbb{R}^n$ is continuous on $\mathbb{R}^n$ and continuously differentiable on $\mathbb{R}^n \setminus \{0\}$. The vector function f is called cooperative if the Jacobian matrix $(\partial f/\partial x)(x)$, $x \in \mathbb{R}^n \setminus \{0\}$, is Metzler.

Definition 2. A vector function $f : \mathbb{R}^n \longrightarrow \mathbb{R}^n$ is called homogeneous of degree $p > 0$ if $f(\lambda x) = \lambda^p f(x)$, $x \in \mathbb{R}^n$, $\lambda > 0$.

Definition 3. A vector function $g : \mathbb{R}^n \longrightarrow \mathbb{R}^n$ is called order-preserving on $\mathbb{R}^n_+$ provided that $g(x) \ge g(y)$, where $x, y \in \mathbb{R}^n_+$, $x \ge y$.

Lemma 4. *A cooperative vector function f satisfies $f_i(u) \ge f_i(v)$, where $u, v \in \mathbb{R}^n \setminus \{0\}$, $u \ge v$, $u_i = v_i$, $i \in \langle n\rangle$.*

In this paper, we need the following assumptions:

(H1) f and g are continuously differentiable on $\mathbb{R}^n\setminus\{0\}$ and homogeneous of degree $p > 1$;

(H2) f is cooperative and g is order-preserving on $\mathbb{R}^n_+$;

(H3) $\omega(t) \ge 0$ for $t \ge 0$.

Following the proof given in [22], we can easily obtain the following lemma.

Lemma 5. *System (1) is positive under assumptions (H2) and (H3).*

3. Main Results

Theorem 6. *Suppose that (H1)-(H3) are valid. Then, we have the following equivalent statements:*

(i) *There is an n-dimensional vector $v \succ 0$ satisfying $f(v) + g(v) \prec 0$.*

(ii) *The solution $x(t)$ of system (1) satisfies*

$$\|x(t)\|_\infty \le \alpha + (\beta + \gamma t)^{-1/(p-1)} \tag{2}$$

for any $t \ge 0$, any initial state $\varphi(t) \in \mathscr{C}([-h,0],\mathbb{R}^n_+)$, any disturbance $\omega(t) \in \mathbb{BF}_r([0,\infty],\mathbb{R}^n_+)$, and any bounded delay $\tau(t)$, where α, β, and γ are appropriate nonnegative constants dependent on r, h, and the initial state φ, and $\alpha = 0$ if $r = 0$.

In addition, if condition (i) holds, α, β, and γ can be chosen as follows:

$$\begin{aligned} \alpha &= \theta\rho, \\ \beta &= (K\rho)^{1-p}, \\ \gamma &= (p-1)\eta\rho^{1-p}, \end{aligned} \tag{3}$$

where $\rho = \max_{i\in\langle n\rangle} v_i$,

$$\begin{aligned} \theta &= \left(\frac{r}{-\max_{i\in\langle n\rangle}[f_i(v) + g_i(v)]}\right)^{1/p}, \\ K &= \begin{cases} 0, & \|\varphi\|_v \le \theta, \\ \left[(\|\varphi\|_v)^p - \theta^p\right]^{1/p}, & \|\varphi\|_v > \theta, \end{cases} \end{aligned} \tag{4}$$

$\|\varphi\|_v = \max_{i\in\langle n\rangle, t\in[-h,0]}(|\varphi_i(t)|/v_i)$, η satisfies $0 < \eta < \min_{i\in\langle n\rangle}\eta_i$, and η_i satisfies the following equation:

$$\frac{f_i(v)}{v_i} + \frac{g_i(v)}{v_i}\left[1 + (p-1)K^{p-1}\eta_i h\right]^{p/(p-1)} + \eta_i = 0, \quad i \in \langle n\rangle. \tag{5}$$

Proof. (i)$\Longrightarrow$(ii) Given the initial state $\varphi \in \mathscr{C}([-h,0],\mathbb{R}^n_+)$, from Lemma 5 we have $x(t) \ge 0$, $t \ge 0$. Based on definitions of K and $\|\varphi\|_v$, we have

$$\frac{x_i(t)}{v_i} \le (\theta^p + K^p)^{1/p}, \quad t \in [-h,0],\ i \in \langle n\rangle. \tag{6}$$

Set

$$z_i(t) = \begin{cases} \dfrac{x_i(t)}{v_i} - \left\{\theta^p + \left[K^{1-p} + (p-1)\eta t\right]^{-p/(p-1)}\right\}^{1/p}, & t \ge 0,\ i \in \langle n\rangle, \\ \dfrac{x_i(t)}{v_i} - (\theta^p + K^p)^{1/p}, & t \in [-h,0],\ i \in \langle n\rangle. \end{cases} \tag{7}$$

Then (6) and (7) yield $z_i(t) \le 0$, $t \in [-h, 0]$, $i \in \langle n \rangle$. Next, we show that $z_i(t) \le 0$ for $i \in \langle n \rangle$ and $t \ge 0$. If it is not true, there is a constant $t_* \ge 0$ and an index $k \in \langle n \rangle$ guaranteeing $z_i(t) \le 0$ for $i \in \langle n \rangle$, $t \in [0, t_*]$, and $z_k(t_*) = 0$. Therefore,

$$\dot{z}_k(t_*) \ge 0, \tag{8}$$

$$\frac{x_i(t)}{v_i} \le \left\{\theta^p + \left[K^{1-p} + (p-1)\eta t\right]^{-p/(p-1)}\right\}^{1/p}, \quad t \in [0, t_*], \; i \in \langle n \rangle. \tag{9}$$

$$\frac{x_k(t_*)}{v_k} = \left\{\theta^p + \left[K^{1-p} + (p-1)\eta t_*\right]^{-p/(p-1)}\right\}^{1/p}. \tag{10}$$

Using Lemma 4 and the homogeneity of f, we get from (9) and (10) that

$$\begin{aligned} f_k(x(t_*)) &\le f_k\left(\left\{\theta^p + \left[K^{1-p} + (p-1)\eta t_*\right]^{-p/(p-1)}\right\}^{1/p} v\right) \\ &= \left\{\theta^p + \left[K^{1-p} + (p-1)\eta t_*\right]^{-p/(p-1)}\right\} f_k(v) \\ &= \theta^p f_k(v) + \left[K^{1-p} + (p-1)\eta t_*\right]^{-p/(p-1)} f_k(v). \end{aligned} \tag{11}$$

For the case when $\tau(t_*) \le t_*$, it holds that

$$\frac{x_i(t_* - \tau(t_*))}{v_i} \le \left\{\theta^p + \left[K^{1-p} + (p-1)\eta(t_* - \tau(t_*))\right]^{-p/(p-1)}\right\}^{1/p}, \quad i \in \langle n \rangle. \tag{12}$$

Considering g is homogeneous and order-preserving, we conclude

$$\begin{aligned} g_k(x(t_* - \tau(t_*))) &\le g_k\left(\left\{\theta^p + \left[K^{1-p} + (p-1)\eta(t_* - \tau(t_*))\right]^{-p/(p-1)}\right\}^{1/p} v\right) \\ &= \left\{\theta^p + \left[K^{1-p} + (p-1)\eta(t_* - \tau(t_*))\right]^{-p/(p-1)}\right\} \cdot g_k(v) = \theta^p g_k(v) + \left[K^{1-p} + (p-1)\eta(t_* - \tau(t_*))\right]^{-p/(p-1)} g_k(v). \end{aligned} \tag{13}$$

Note that

$$\begin{aligned} &\left[K^{1-p} + (p-1)\eta(t_* - \tau(t_*))\right]^{-p/(p-1)} \\ &= \left[K^{1-p} + (p-1)\eta t_*\right]^{-p/(p-1)} \times \left[\frac{\eta(p-1)\tau(t_*)}{K^{1-p} + \eta(p-1)(t_* - \tau(t_*))} + 1\right]^{p/(p-1)} \\ &\le \left[K^{1-p} + (p-1)\eta t_*\right]^{-p/(p-1)} \times \left[1 + (p-1)K^{p-1}\eta h\right]^{p/(p-1)}. \end{aligned} \tag{14}$$

We further get from (13) and (14) that

$$g_k(x(t_* - \tau(t_*))) \le \theta^p g_k(v) + \left[K^{1-p} + \eta(p-1)t_*\right]^{-p/(p-1)} \cdot \left[1 + (p-1)K^{p-1}\eta h\right]^{p/(p-1)} g_k(v). \tag{15}$$

For the case when $\tau(t_*) > t_*$, it holds that $z_i(t_* - \tau(t_*)) \le 0$; i.e.,

$$\frac{x_i(t_* - \tau(t_*))}{v_i} \le (\theta^p + K^p)^{1/p}, \quad i \in \langle n \rangle. \tag{16}$$

It thus follows that

$$\begin{aligned} g_k(x(t_* - \tau(t_*))) &\le (\theta^p + K^p) g_k(v) = \theta^p g_k(v) + \left[K^{1-p} + (p-1)\eta t_*\right]^{-p/(p-1)} \cdot K^p \left[K^{1-p} + (p-1)\eta t_*\right]^{p/(p-1)} g_k(v) \\ &\le \theta^p g_k(v) + \left[K^{1-p} + (p-1)\eta t_*\right]^{-p/(p-1)} \cdot \left[1 + (p-1)K^{p-1}\eta h\right]^{p/(p-1)} g_k(v). \end{aligned} \tag{17}$$

Next, we can conclude from (1) and (7) that

$$\begin{aligned} \dot{z}_k(t_*) &= \frac{f_k(x(t_*)) + g_k(x(t_* - \tau(t_*))) + w_k(t_*)}{v_k} + \eta\left\{\theta^p + \left[K^{1-p} + (p-1)\eta t_*\right]^{-p/(p-1)}\right\}^{(1-p)/p} \cdot \left[K^{1-p} + (p-1)\eta t_*\right]^{(1-2p)/(p-1)} \\ &\le \frac{f_k(x(t_*)) + g_k(x(t_* - \tau(t_*))) + w_k(t_*)}{v_k} + \eta\left[K^{1-p} + (p-1)\eta t_*\right]^{-p/(p-1)}. \end{aligned} \tag{18}$$

Consequently, (11), (15), (17), and (18) imply that

$$\dot{z}_k(t_*) \le \frac{\theta^p[f_k(v)+g_k(v)]+r}{v_k} + \left[K^{1-p} + (p-1)\eta t_*\right]^{-p/(p-1)} \times \left\{\frac{f_k(v)}{v_k} + \frac{g_k(v)}{v_k}\left[1+(p-1)K^{p-1}\eta h\right]^{p/(p-1)} + \eta\right\}. \tag{19}$$

On the other hand, the definitions of θ and η yield that

$$\theta^p[f_k(v)+g_k(v)]+r \le \theta^p \max_{i\in\langle n\rangle}[f_i(v)+g_i(v)]+r = 0, \tag{20}$$

and

$$\frac{f_k(v)}{v_k} + \frac{g_k(v)}{v_k}\left[1+(p-1)K^{p-1}\eta h\right]^{p/(p-1)} + \eta < 0. \tag{21}$$

Combining this with (19), we have $\dot{z}_k(t_*) < 0$, which contradicts (8). Therefore, $z_i(t) \le 0, t \ge 0, i \in \langle n\rangle$; i.e.,

$$\frac{x_i(t)}{v_i} \le \left\{\theta^p + \left[K^{1-p} + (p-1)\eta t\right]^{-p/(p-1)}\right\}^{1/p}, \quad t \ge 0,\ i \in \langle n\rangle. \tag{22}$$

From the well-known inequality $(a+b)^q \le a^q + b^q$ for $a, b \ge 0$ and $0 < q < 1$, we further get

$$\frac{x_i(t)}{v_i} \le \theta + \left[K^{1-p} + (p-1)\eta t\right]^{-1/(p-1)}, \quad t \ge 0,\ i \in \langle n\rangle. \tag{23}$$

It implies (2).

(ii)⟹(i) For the particular case when $r = 0$ and $h = 0$, system (1) reduces to

$$\dot{x}(t) = f(x(t)) + g(x(t)), \quad t \ge 0. \tag{24}$$

Given the initial condition $x(0) \succeq 0$, each solution of system (24) satisfies

$$\|x(t)\|_\infty \le (\beta + \gamma t)^{-1/(p-1)}. \tag{25}$$

That is, system (24) is asymptotically stable. Based on Proposition 4.1 in [35], there is a vector $v \succ 0$ such that $f(v)+g(v) \prec 0$. The proof is complete. □

Remark 7. It can be seen from Theorem 6 that the bound of the reachable set is determined by the bound of disturbances, the choice of v, and the value of p. When the bound of disturbances and the value of p are given, an appropriate vector v can be chosen to guarantee a minimal bound of the reachable set by solving the following nonlinear optimization problem: $\min_{v\succ 0}\theta$ subject to $f(v) + g(v) \prec 0$, where θ is defined as in Theorem 6.

Remark 8. If $\omega(t) = 0$ for $t \ge 0$, then Theorem 6 reduces to the main result given in [23]. If $g(x) = 0$ for $x \in \mathbb{R}^n$, then Theorem 6 reduces to the main result given in [24].

Finally, consider the following nonlinear time-varying system

$$\dot{x}(t) = \tilde{f}(t, x(t)) + \tilde{g}(t, (x(t-\tau(t)))) + \omega(t), \quad t \ge 0$$
$$x(t) = \varphi(t), \quad t \in [-h, 0], \tag{26}$$

where $x(t), \tau(t), \omega(t)$, and $\varphi(t)$ are the same as in (1), and $\tilde{f}, \tilde{g} : [0,\infty) \times \mathbb{R}^n \longrightarrow \mathbb{R}^n$ are vector functions satisfying $\tilde{f}(t,0) = \tilde{g}(t,0) = 0$.

Suppose that $\tilde{f}$ and $\tilde{g}$ satisfy the following assumption:

(H4) $\tilde{f}$ and $\tilde{g}$ are continuous on $[0,\infty)\times\mathbb{R}^n$, continuously differentiable with respect to x on $\mathbb{R}^n\backslash\{0\}$, and there are vector functions f and g satisfying (H1) and (H2), and for $x_i \ne 0$,

$$\tilde{f}_i(t,x)\,\text{sign}\, x_i \le f_i(|x|),$$
$$|\tilde{g}_i(t,x)| \le g_i(|x|), \quad t \ge 0,\ i \in \langle n\rangle. \tag{27}$$

Without the restriction on the disturbance that $\omega(t) \succeq 0$ for $t \ge 0$, we can get the following reachable set bounding criterion for system (26).

Theorem 9. *Suppose that (H4) is valid. If there is an n-dimensional vector $v \succ 0$ such that $f(v) + g(v) \prec 0$, the solution of system (26) satisfies (2), where constants α, β, and γ are defined by (3).*

Proof. Set

$$y_i(t) = \begin{cases} \dfrac{|x_i(t)|}{v_i} - \left\{\theta^p + \left[K^{1-p} + (p-1)\eta t\right]^{-p/(p-1)}\right\}^{1/p}, & t \ge 0,\ i \in \langle n\rangle, \\ \dfrac{|x_i(t)|}{v_i} - (\theta^p + K^p)^{1/p}, & t \in [-h, 0],\ i \in \langle n\rangle. \end{cases} \tag{28}$$

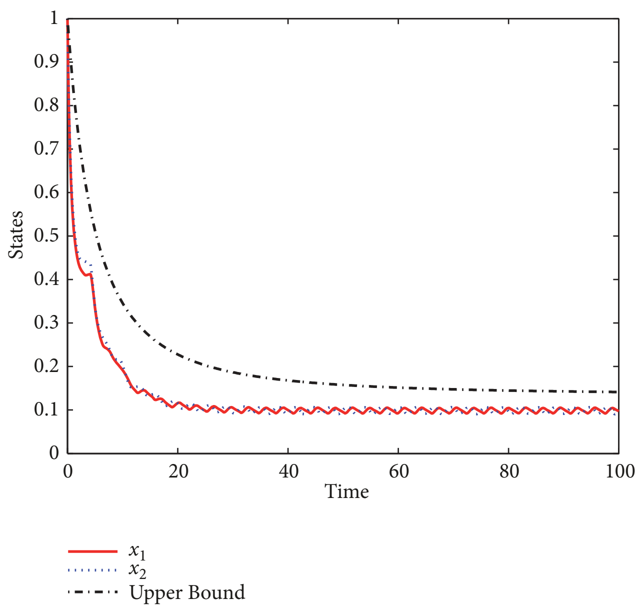

Figure 1: The states of system (1).

Based on definitions of K and $\|\varphi\|_v$, it holds that $y_i(t) \leq 0$, $t \in [-h, 0]$, $i \in \langle n \rangle$. For the case when $x_i(t) \neq 0$, $t \geq 0$, notice that

$$\begin{aligned} \dot{y}_i(t) &= D_- |x_i(t)| = \dot{x}_i(t) \operatorname{sign} x_i(t) \\ &\leq f_i(|x(t)|) + g_i(|x(t-\tau(t))|) + |w_i(t)|, \\ & \qquad i \in \langle n \rangle. \end{aligned} \tag{29}$$

Here D_- denotes the left derivative. Similar to the analysis in Theorem 6, it is not difficult to conclude that $y_i(t) \leq 0$, $t \geq 0$, $i \in \langle n \rangle$. Consequently, (2) holds. The proof is complete. □

4. Numerical Example

Consider system (1) with

$$\begin{aligned} f(x_1, x_2) &= \begin{pmatrix} -3 & 6 \\ 2 & -2 \end{pmatrix} \begin{pmatrix} x_1^{3/2} \\ x_2^{3/2} \end{pmatrix} - \sqrt{x_1^3 + x_2^3} \begin{pmatrix} 3 \\ 1 \end{pmatrix}, \\ g(x_1, x_2) &= \begin{pmatrix} 0.2x_2^{3/2} \\ 0.4x_1^{3/2} \end{pmatrix}, \\ \omega(t) &= \begin{pmatrix} 0.05|\sin t| \\ 0.04|\cos t| \end{pmatrix}, \\ \tau(t) &= 5 + \sin t, \quad t \geq 0. \end{aligned} \tag{30}$$

It is easy to verify that assumptions (H1)-(H2) hold. Let $v = (1,1)^T$. Then $f(v) + g(v) \prec 0$. By a direct calculation, it yields $h = 6$, $r = 0.05$, $\alpha \approx 0.1345$, and $\gamma \approx 0.11$.

We conclude from Theorem 6 that there is a ball $\mathscr{B}(0.1345)$ such that all the states of system (1) converge asymptotically within it. Given the initial state $\varphi(t) = (1,1)^T$, $t \in [-6, 0]$, noting that $\|\varphi\|_v = 1$ and $\beta \approx 1.0749$, solution (1) satisfies

$$\|x(t)\|_\infty \leq 0.1345 + (1.0749 + 0.11t)^{-2}, \quad t \geq 0. \tag{31}$$

Figure 1 presents the simulation.

5. Conclusion

This paper has been concerned with reachable set bounding for homogeneous nonlinear time-delay systems with disturbance. We not only derive explicit reachable set bounding criterion independent of delay, but also estimate the decay rate. It will be interesting to extend our work to the case of unbounded delays and discrete-time systems.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

We declare that there are no conflicts of interest regarding this paper.

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