

Reaction Matrix Theory for Cluster States in Light Nuclei. II

—Clustering Dependence of the Effective Interaction—

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The states with triangular and linear chain configurations of three α -clusters in C^{12} are treated by the reaction matrix theory on the basis of a realistic nuclear force. Clustering dependence of G-matrix is found to be essential for clusterization. It also brings on a change of effective interaction between the two configurations, which has a role in reducing the excitation energy of the linear chain state. Its origin is mainly in the triplet even-state tensor force.

§ 1. Introduction

Clustering phenomena in nuclei are closely related to the saturation property of nuclei. The saturation property makes two phases comparable in energy: the one where all nucleons are combined in one nucleus and the other where nucleons are separated in several smaller nuclei. Consequently, we find at rather low excitation energy the states which seem to have cluster structure, and we also find in some cases the clustering feature in the ground state. Thus, the mechanism which produces the saturation property is considered to influence the clustering phenomena too.

We have some knowledge of nuclear saturation, though it has not yet been thoroughly clarified even for infinite nuclear matter. We must investigate how physical effects of nuclear saturation, which we have clarified to date, appear in clustering phenomena. Here we discuss the effective nucleon-nucleon interaction within nuclei. It is known that the effective interaction changes, depending on the mass number of nuclei due to many-body effects. For example, we cannot obtain proper saturation for nuclear matter if we use as the effective interaction the phase shifts of free two-nucleon scattering.¹⁰⁾ This means that the nuclear force should act less attractively within nuclei than in free space. Such reduction of the effective attraction appears also from α -particle to heavier nuclei.¹¹⁾ This effect is very important for the overall saturation of binding energy from light to heavy nuclei. This effect originates mainly in the shielding of a strong tensor force due to many-body effects.

Such a change of the effective interaction itself will also occur when a nu-

cleus changes from shell phase to cluster phase. It will be especially distinct when α -clustering occurs, since the shielding of the tensor force is weak because of only four constituent nucleons. This can be said as some additional attractive interaction is induced by α -clustering, and this induced interaction enhances the α -clustering itself. Thus, a physical situation which assures the saturation property will produce here some clustering-induced attraction. This is the subject in this paper and our previous paper¹⁾ (referred to as I).

For this purpose, we proposed in I to use the Brueckner theory on the basis of the polycentre-shell model and made calculations for Be^8 nucleus. Clustering phenomena have many aspects which should themselves be investigated. Thus, we adopt model wave functions which are simple but include the clustering features essential for our aim, that is, the poly-centre-shell model wave functions. This was discussed in I regarding the applicability of the reaction matrix theory.

We showed in I that in the ground state band of Be^8 the effective interaction becomes more attractive as the distance between two α -clusters becomes larger. Such a change of the effective interaction itself should appear more generally, not only in one state as treated in I, but also between states with different configurations in a nucleus, for instance, the states with shell structure and with cluster structure. This feature of interdependence between the effective interaction and the structure of a state is characteristic of the nuclear system. In this paper we treat the triangular and the linear chain configurations of three α -clusters in C^{12} (abbreviated as T - and L -configurations hereafter). T -configuration corresponds to the ground state of C^{12} from an α -cluster point of view. L -configuration was proposed by Morinaga³⁾ as a possible interpretation of the excited 0^+ state at 7.66 MeV of C^{12} . Though this interpretation is not confirmed now,⁴⁾ we take L -configuration as a distinct contrast to the ground T -configuration. The T - and L -configurations should have small and large clusterization, respectively. So the change of effective interaction between them will play an important role in lowering the excitation energy of the L -configuration. This effect can then be considered an important element of "clustering correlation" or "four-body correlation".

As the model wave functions of three α -clusters, we use the s -orbits around three points in space corresponding to the configurations.⁵⁾ From these, we construct single particle wave functions by using geometrical symmetry or the Hartree-Fock condition. The parameters are the size of the α -clusters and the distance between them. Since these model wave functions include harmonic oscillator shell model wave functions in a limit of parameters, we can describe the degree of clustering by these parameters. For each value of parameters we calculate the reaction matrices and single particle energies self-consistently. Thus, we can obtain the energy surface for parameters, which includes the effect of clustering dependence of the effective interaction (reaction matrix). Our method is formulated in § 2 and the calculated results are discussed in § 3.

§ 2. Model wave functions and reaction matrix

2-1 Triangular configuration

We take the triangular configuration of three α -clusters (shown in Fig. 1) as the intrinsic wave function of the ground state of C^{12} .

The s -orbit around the centre of each α -cluster is occupied by 2 protons and 2 neutrons. They are written as

$$\varphi_i(\mathbf{r}) = (\sqrt{\pi}b)^{-3/2} e^{-(r-c_i)^2/2b^2}, \quad i=1, 2, 3, \quad (2.1)$$

where \mathbf{c}_i (shown in Fig. 1) satisfies $\mathbf{c}_1 + \mathbf{c}_2 + \mathbf{c}_3 = 0$ and $|\mathbf{c}_i| = d/\sqrt{3}$ (d : distance between α -clusters). Our model wave function is the Slater determinant of $\varphi_i(\mathbf{r})$'s. We can easily construct ortho-normal single particle wave functions from $\varphi_i(\mathbf{r})$, considering the symmetry for $2\pi/3$ rotation within xy -plane. They are expressed as

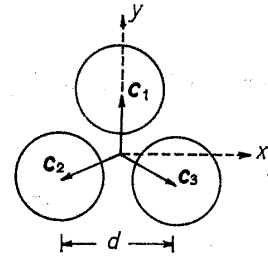
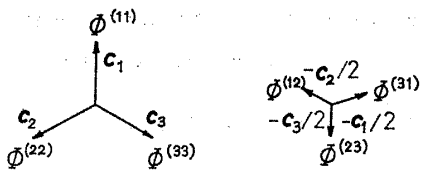


Fig. 1. Triangular configuration of three α -clusters.

$$\begin{aligned} \chi_1(\mathbf{r}) &= \{3(1 + 2\Delta^{1/4})\}^{-1/2} (\varphi_1 + \varphi_2 + \varphi_3), \\ \chi_2(\mathbf{r}) &= \{\frac{3}{2}(1 - \Delta^{1/4})\}^{-1/2} (\varphi_1 - \frac{1}{2}\varphi_2 - \frac{1}{2}\varphi_3), \\ \chi_3(\mathbf{r}) &= \{2(1 - \Delta^{1/4})\}^{-1/2} (\varphi_2 - \varphi_3), \end{aligned} \quad (2.2)$$

where $\Delta = e^{-\delta}$ and $\delta = d^2/b^2$. χ_2 and χ_3 are degenerate. In the limit of $d \rightarrow 0$, χ_1 , χ_2 and χ_3 tend to $(0s)$ and $(0p)$ states with a quantum in y -axis and in x -axis, respectively, of the usual h.o. shell model.

Now, let us transform the pair wave functions $\chi_i(\mathbf{r}_1)\chi_j(\mathbf{r}_2)$ into the relative (\mathbf{r}) and centre of mass (\mathbf{R}) coordinates. There appear six independent basis functions for the centre of mass and seven for the relative coordinates, which are written as

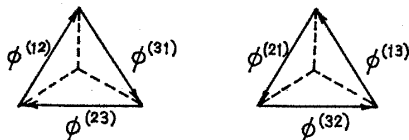


(a)

$$\Phi^{(ij)}(\mathbf{R}) = \left(\sqrt{\frac{\pi}{2}}b\right)^{-3/2} \exp\left[-\left(\mathbf{R} - \frac{\mathbf{c}_i + \mathbf{c}_j}{2}\right)^2/b^2\right] \quad (2.3)$$

and

$$\begin{aligned} \phi^{(0)}(\mathbf{r}) &= (\sqrt{2\pi}b)^{-3/2} e^{-r^2/4b^2}, \\ \phi^{(ij)}(\mathbf{r}) &= (\sqrt{2\pi}b)^{-3/2} e^{-\{r - (\mathbf{c}_i - \mathbf{c}_j)\}^2/4b^2}. \quad (i \neq j) \end{aligned} \quad (2.4)$$



(b)

Fig. 2. Geometrical illustration of the independent basis functions for c.m. (a) and relative (b) coordinates.

Equations (2.3) and (2.4) are illustrated in Fig. 2. Again we can construct orthonormal sets with certain symmetry from $\Phi^{(ij)}$ and $\phi^{(0)}$, $\phi^{(ij)}$, respectively.

As for the c.m. coordinates, they are expressed by

$$\begin{aligned}
\Phi_1(\mathbf{R}) &= \{2(1+N_0)\}^{-1/2} \{A_0(\mathbf{R}) + B_0(\mathbf{R})\}, \\
\Phi_2(\mathbf{R}) &= \{2(1-N_1)\}^{-1/2} \{A_y(\mathbf{R}) - B_y(\mathbf{R})\}, \\
\Phi_3(\mathbf{R}) &= \{2(1-N_1)\}^{-1/2} \{A_x(\mathbf{R}) - B_x(\mathbf{R})\}, \\
\Phi_4(\mathbf{R}) &= \{2(1-N_0)\}^{-1/2} \{A_0(\mathbf{R}) - B_0(\mathbf{R})\}, \\
\Phi_5(\mathbf{R}) &= \{2(1+N_1)\}^{-1/2} \{A_x(\mathbf{R}) + B_x(\mathbf{R})\}, \\
\Phi_6(\mathbf{R}) &= \{2(1+N_1)\}^{-1/2} \{A_y(\mathbf{R}) + B_y(\mathbf{R})\},
\end{aligned} \tag{2.5}$$

where $A(\mathbf{R})$ and $B(\mathbf{R})$ are defined by

$$\begin{aligned}
A_0(\mathbf{R}) &= \{3(1+2\mathcal{A}^{1/2})\}^{-1/2} \{\Phi^{(11)} + \Phi^{(22)} + \Phi^{(33)}\}, \\
A_y(\mathbf{R}) &= \{\frac{3}{2}(1-\mathcal{A}^{1/2})\}^{-1/2} \{\Phi^{(11)} - \frac{1}{2}\Phi^{(22)} - \frac{1}{2}\Phi^{(33)}\}, \\
A_x(\mathbf{R}) &= \{2(1-\mathcal{A}^{1/2})\}^{-1/2} \{\Phi^{(22)} - \Phi^{(33)}\}, \\
B_0(\mathbf{R}) &= \{3(1+2\mathcal{A}^{1/8})\}^{-1/2} \{\Phi^{(33)} + \Phi^{(31)} + \Phi^{(12)}\}, \\
B_y(\mathbf{R}) &= \{\frac{3}{2}(1-\mathcal{A}^{1/8})\}^{-1/2} \{\Phi^{(23)} - \frac{1}{2}\Phi^{(31)} - \frac{1}{2}\Phi^{(12)}\}, \\
B_x(\mathbf{R}) &= \{2(1-\mathcal{A}^{1/8})\}^{-1/2} \{\Phi^{(31)} - \Phi^{(12)}\}
\end{aligned} \tag{2.6}$$

and

$$\begin{aligned}
N_0 &= \langle A_0(\mathbf{R}) | B_0(\mathbf{R}) \rangle = (\mathcal{A}^{3/8} + 2\mathcal{A}^{1/8}) (1 + 2\mathcal{A}^{1/2})^{-1/2} (1 + 2\mathcal{A}^{1/8})^{-1/2}, \\
N_1 &= \langle A_y(\mathbf{R}) | B_y(\mathbf{R}) \rangle = \langle A_x(\mathbf{R}) | B_x(\mathbf{R}) \rangle \\
&= (\mathcal{A}^{3/8} - \mathcal{A}^{1/8}) (1 - \mathcal{A}^{1/2})^{-1/2} (1 - \mathcal{A}^{1/8})^{-1/2}.
\end{aligned} \tag{2.7}$$

In the limit $d \rightarrow 0$, $\Phi_N(\mathbf{R})$ tends to $(0s)$ for $N=1$, $(0p)$ for $N=2, 3$ and $(1s, 0d)$ for $N=4 \sim 6$, in h.o. wave functions.

Similarly we can construct orthonormal relative wave functions with definite symmetry and parity, taking account of additional $\phi^{(0)}(\mathbf{r})$. The results are

$$\begin{aligned}
\phi_1(\mathbf{r}) &= \{2(1+\nu_0)\}^{-1/2} [\phi^{(0)}(\mathbf{r}) + \{2(1+\nu_0)\}^{-1/2} \{\alpha_0(\mathbf{r}) + \beta_0(\mathbf{r})\}], \\
\phi_2(\mathbf{r}) &= \{2(1-\nu_1)\}^{-1/2} \{\alpha_y(\mathbf{r}) - \beta_y(\mathbf{r})\}, \\
\phi_3(\mathbf{r}) &= \{2(1-\nu_1)\}^{-1/2} \{\alpha_x(\mathbf{r}) - \beta_x(\mathbf{r})\}, \\
\phi_4(\mathbf{r}) &= \{2(1-\nu_0)\}^{-1/2} [\phi^{(0)}(\mathbf{r}) - \{2(1+\nu_0)\}^{-1/2} \{\alpha_0(\mathbf{r}) + \beta_0(\mathbf{r})\}], \\
\phi_5(\mathbf{r}) &= \{2(1+\nu_1)\}^{-1/2} \{\alpha_x(\mathbf{r}) + \beta_x(\mathbf{r})\}, \\
\phi_6(\mathbf{r}) &= \{2(1+\nu_1)\}^{-1/2} \{\alpha_y(\mathbf{r}) + \beta_y(\mathbf{r})\}, \\
\phi_7(\mathbf{r}) &= \{2(1-\nu_0)\}^{-1/2} \{\alpha_0(\mathbf{r}) - \beta_0(\mathbf{r})\},
\end{aligned} \tag{2.8}$$

where $\alpha(\mathbf{r})$ and $\beta(\mathbf{r})$ are defined by

$$\begin{aligned}
\alpha_0(\mathbf{r}) &= \{3(1+2\mathcal{A}^{3/8})\}^{-1/2} \{\phi^{(33)} + \phi^{(31)} + \phi^{(12)}\}, \\
\alpha_x(\mathbf{r}) &= \{\frac{3}{2}(1-\mathcal{A}^{3/8})\}^{-1/2} \{\phi^{(23)} - \frac{1}{2}\phi^{(31)} - \frac{1}{2}\phi^{(12)}\},
\end{aligned}$$

$$\begin{aligned}
 \alpha_y(\mathbf{r}) &= \{2(1 - \Delta^{3/8})\}^{-1/2} \{\phi^{(31)} - \phi^{(12)}\}, \\
 \beta_0(\mathbf{r}) &= \{3(1 + 2\Delta^{3/8})\}^{-1/2} \{\phi^{(32)} + \phi^{(13)} + \phi^{(21)}\}, \\
 \beta_x(\mathbf{r}) &= \{\frac{3}{2}(1 - \Delta^{3/8})\}^{-1/2} \{\phi^{(32)} - \frac{1}{2}\phi^{(13)} - \frac{1}{2}\phi^{(21)}\}, \\
 \beta_y(\mathbf{r}) &= \{2(1 - \Delta^{3/8})\}^{-1/2} \{\phi^{(13)} - \phi^{(21)}\}
 \end{aligned} \tag{2.9}$$

and

$$\begin{aligned}
 \nu_0 &= \langle \alpha_0(\mathbf{r}) | \beta_0(\mathbf{r}) \rangle = \Delta^{1/8} (1 + 2\Delta^{3/8})^{-1} (2 + \Delta^{3/8}), \\
 \nu_1 &= \langle \alpha_x(\mathbf{r}) | \beta_x(\mathbf{r}) \rangle = \langle \alpha_y(\mathbf{r}) | \beta_y(\mathbf{r}) \rangle = -\Delta^{1/8}, \\
 \nu_{00} &= \sqrt{6} \Delta^{1/8} (1 + 2\Delta^{3/8})^{-1/2} (1 + \nu_0)^{-1/2}.
 \end{aligned} \tag{2.10}$$

In the limit $d \rightarrow 0$, $\phi_n(\mathbf{r})$ tends to $(0s)$ for $n=1$, $(0p)$ for $n=2, 3$, $(1s, 0d)$ for $n=4 \sim 6$ and $(1p, 0f)$ for $n=7$, in h.o. wave functions. In order to solve the reaction matrix equation, we expand the relative wave functions $\phi_n(\mathbf{r})$ in partial waves, using the expansion

$$\phi^{(ij)}(\mathbf{r}) = 4\pi(\sqrt{2\pi}b)^{-3/2} \Delta^{1/4} e^{-r^2/4b^2} \sum_{lm} \mathcal{G}_l\left(\frac{dr}{2b^2}\right) Y_{lm}^*(\widehat{\mathbf{c}_i - \mathbf{c}_j}) Y_{lm}(\widehat{\mathbf{r}}), \tag{2.11}$$

where $\mathcal{G}(z) = \sqrt{\pi/2z} I_{l+1/2}(z)$. $\phi_1, \phi_4, \phi_5, \phi_6$ and ϕ_2, ϕ_3, ϕ_7 include even and odd l waves, respectively. ϕ_5 and ϕ_6 are found to have higher than d waves only and ϕ_7 higher than f waves only.

Using the inverse relations of Eqs. (2.5), (2.6), (2.8) and (2.9) we express $\chi_i(\mathbf{r}_1)\chi_j(\mathbf{r}_2)$ in terms of $\Phi_N(\mathbf{R})$ and $\phi_n(\mathbf{r})$, and then expand the latter in partial waves. The resultant expressions are shown in the Appendix. There appear eight kinds of s -wave functions, $\xi_0^{(n)}(r)$. The reaction matrix equation is solved for each unperturbed wave function $\xi(r)$.

2-2 Linear chain configuration

The model wave function in this case is the Slater determinant of $\varphi_i(\mathbf{r})$ given by

$$\begin{aligned}
 \varphi_1(\mathbf{r}) &= (\sqrt{\pi}b)^{-3/2} e^{-r^2/2b^2}, \\
 \varphi_2(\mathbf{r}) &= (\sqrt{\pi}b)^{-3/2} e^{-(r-d)^2/2b^2}, \\
 \varphi_3(\mathbf{r}) &= (\sqrt{\pi}b)^{-3/2} e^{-(r+d)^2/2b^2}.
 \end{aligned} \tag{2.12}$$

The orthonormal single particle wave functions $\chi_i(\mathbf{r})$ are not determined

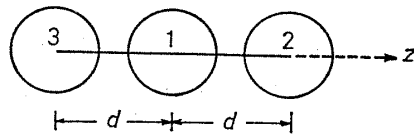


Fig. 3. Linear chain configuration of three α -clusters.

uniquely by geometrical symmetry in this case, since $(\varphi_2 - \varphi_3)$ is the unique negative parity state, while φ_1 and $(\varphi_2 + \varphi_3)$ with positive parity can admit in an arbitrary way. We determine them by the Hartree-Fock condition. Thus $\chi_i(\mathbf{r})$ is written as follows:

$$\begin{aligned}\chi_1(\mathbf{r}) &= \{1 + \sin(2\theta_1) \cosh^{-1/2}(\delta/2)\}^{-1/2} [\cos \theta_1 \varphi_1 + \sin \theta_1 \{2(1 + \mathcal{A})\}^{-1/2} (\varphi_2 + \varphi_3)], \\ \chi_2(\mathbf{r}) &= \{2(1 - \mathcal{A})\}^{-1/2} (\varphi_2 - \varphi_3), \\ \chi_3(\mathbf{r}) &= \{1 - \sin(2\theta_3) \cosh^{-1/2}(\delta/2)\}^{-1/2} [-\sin \theta_3 \varphi_1 + \cos \theta_3 \{2(1 + \mathcal{A})\}^{-1/2} (\varphi_2 + \varphi_3)].\end{aligned}\quad (2.13)$$

Orthogonality of χ_1 and χ_3 imposes the condition

$$\sin(\theta_1 - \theta_3) + \cos(\theta_1 + \theta_3) \cosh^{-1/2}(\delta/2) = 0. \quad (2.14)$$

χ_1 , χ_2 and χ_3 tend to the $(0s)$, $(0p)$ and $(0s, 1d)$ states, respectively, of the h.o. shell model in the limit $d \rightarrow 0$, so far as $0 \leq \theta_1 \leq \pi/2$. The parameter θ_1 (θ_3) is determined later by the H-F condition

$$\langle \chi_1 | T + U | \chi_3 \rangle = 0. \quad (2.15)$$

Now we transform the pair wave functions $\chi_i(\mathbf{r}_1)\chi_j(\mathbf{r}_2)$ into the c.m. and relative coordinates. From $\varphi_i(\mathbf{r}_1)\varphi_j(\mathbf{r}_2)$ there appear five independent basis functions for each \mathbf{R} and \mathbf{r} , which are

$$\Phi^{(\pm N)}(\mathbf{R}) = \left(\sqrt{\frac{\pi}{2}}b\right)^{-3/2} e^{-(\mathbf{R} \mp N\mathbf{d}/2)^2/b^2}, \quad N=0, 1, 2, \quad (2.16)$$

$$\phi^{(\pm n)}(\mathbf{r}) = (\sqrt{2\pi}b)^{-3/2} e^{-(\mathbf{r} \mp n\mathbf{d})^2/4b^2}, \quad n=0, 1, 2. \quad (2.17)$$

There is arbitrariness in constructing orthonormal sets from $\Phi^{(\pm N)}$ and $\phi^{(\pm n)}$, so we adopt a simple way of conserving parity. The results are

$$\begin{aligned}\Phi_1(\mathbf{R}) &= \Phi^{(0)}(\mathbf{R}), \\ \Phi_2(\mathbf{R}) &= \{4\mathcal{A}^{1/4} \sinh(\delta/4)\}^{-1/2} \{\Phi^{(+1)}(\mathbf{R}) - \Phi^{(-1)}(\mathbf{R})\}, \\ \Phi_3(\mathbf{R}) &= \{\sqrt{8}\mathcal{A}^{1/8} \sinh(\delta/8)\}^{-1} \{\Phi^{(+1)}(\mathbf{R}) + \Phi^{(-1)}(\mathbf{R}) - 2\mathcal{A}^{1/8}\Phi^{(0)}(\mathbf{R})\}, \\ \Phi_4(\mathbf{R}) &= [4\mathcal{A} \sinh^{-1}(\delta/4) \{\sinh(\delta) \sinh(\delta/4) - \sinh^2(\delta/2)\}]^{-1/2} \\ &\quad \times [\{\Phi^{(+2)}(\mathbf{R}) - \Phi^{(-2)}(\mathbf{R})\} - \mathcal{A}^{3/8} \sinh(\delta/2) \sinh^{-1}(\delta/4) \\ &\quad \times \{\Phi^{(+1)}(\mathbf{R}) - \Phi^{(-1)}(\mathbf{R})\}], \\ \Phi_5(\mathbf{R}) &= [8\mathcal{A} \sinh^{-2}(\delta/8) \{\sinh^2(\delta/2) \sinh^2(\delta/8) - \sinh^4(\delta/4)\}]^{-1/2} \\ &\quad \times [\{\Phi^{(+2)}(\mathbf{R}) + \Phi^{(-2)}(\mathbf{R})\} - \mathcal{A}^{3/8} \sinh^2(\delta/4) \sinh^{-2}(\delta/8) \\ &\quad \times \{\Phi^{(+1)}(\mathbf{R}) + \Phi^{(-1)}(\mathbf{R})\} + 2\mathcal{A}^{1/2} \{\sinh^2(\delta/4) \sinh^{-2}(\delta/8) - 1\} \Phi^{(0)}(\mathbf{R})].\end{aligned}\quad (2.18)$$

In the limit $d \rightarrow 0$, $\Phi_N(\mathbf{R})$ tends to h.o. wave functions with $(N-1)$ quanta in z -direction. Φ_1 , Φ_3 , Φ_5 and Φ_2 , Φ_4 have even and odd parity, respectively. The definition of the orthonormal relative wave functions $\phi_n(\mathbf{r})$ is the same as Eq.

(2.18), if Φ 's are replaced by ϕ 's. Partial wave expansion of $\phi_n(\mathbf{r})$ is also straightforward. The expressions of $\chi_i(\mathbf{r}_1)\chi_j(\mathbf{r}_2)$ are more complicated than in the case of T -configuration and are not shown explicitly here.

2-3 Reaction matrix and single particle energy

The reaction matrix G is solved for each $\xi(r)$ defined in the Appendix by the equation

$$G = v + v \frac{Q}{(\varepsilon_1 + \varepsilon_2) - (T_1 + T_2)} G, \tag{2.19}$$

where Q is the Pauli projection operator, ε_i is the single particle energy of the state χ_i which is determined self-consistently and T_i is the kinetic energy operator. It has been shown¹²⁾ in the case of nuclear matter that single particle potential energy in off-energy-shell propagation is negligibly small. This fact is related to three-body correlation which decreases by taking into account all order diagrams for the singular two-body force. Though numerical calculations of three-body correlation have not yet been published for finite nuclei, the situation will be the same as in nuclear matter because the above-mentioned result is concerned mainly with high momentum correlations. Therefore, we take single particle potential energy at virtual state to be zero.¹³⁾

We solve Eq. (2.19) in two steps, first omitting Pauli operator Q and then taking it into account. The method of calculation is the same as explained in I. Approximation in our calculation is only in the over-estimate of the contribution of one particle jump. The G matrix depends on single particle states i and j and c.m. quantum state N through the starting energy. It will be found that the starting energy plays an important role in inducing some changes in the reaction matrices depending on clusterization.

Single particle potential energies are given by G as follows:

$$U_i = \sum_j \sum_{T S m_s} \frac{2T+1}{2} \langle \chi_i \chi_j | G_{T S m_s} | \chi_i \chi_j \rangle. \tag{2.20}$$

Single particle energies are expressed by

$$\varepsilon_i = \langle \chi_i | T | \chi_i \rangle + U_i. \tag{2.21}$$

ε_i thus obtained should equal the values put in Eq. (2.19). Computations are performed until this self-consistency requirement is satisfied. In the case of L -configuration, an additional condition Eq. (2.15) is necessary in order to determine the parameter $\theta_1(\theta_2)$. Four quantities, ε_i and θ_1 , are, therefore, to be determined self-consistently in this case, while two, ε_1 and $\varepsilon_2(=\varepsilon_3)$, are to be determined self-consistently in the case of T -configuration.

Thus, total energy is given by

$$E(b, d) = 4 \sum_{i=1}^3 \left(\varepsilon_i - \frac{1}{2} U_i \right) - \frac{3}{4} \frac{\hbar^2}{M b^2}, \tag{2.22}$$

in which the zero point oscillation energy of the total system is subtracted in the last term.

§ 3. Results and discussion

We adopt OPEG potential⁶⁾ (with one-pion-exchange tail and gaussian soft core) as a realistic nuclear force. This force gives about -22 MeV at $b=1.4$ fm for the binding energy B_α of an α -particle¹⁾. B_α is related to $E(b, d)$ in Eq. (2.22) by

$$B_\alpha = \frac{1}{3} \left\{ E(b, d \rightarrow \infty) - 2 \cdot \frac{3}{4} \frac{\hbar^2}{Mb^2} \right\}, \quad (3.1)$$

in which the subtracted term corresponds to the zero point oscillation energies of the α -clusters.

We take only relative s -waves in the computations here, because the odd state contributions are rather small, and our aim is to show the importance of clustering dependence of effective interaction in an even state of relative angular momentum.

In Fig. 4, calculated single particle energies for T - and L - configurations are shown with respect to the distance between α -clusters. For the L -configuration, values of θ_1 are also shown. Corresponding total energies are shown in Fig. 5 in terms of $\Delta E(b, d) = E(b, d) - E(b, d \rightarrow \infty)$.

Single particle energies tend to coincide for large values of d and split gradually for smaller values of d . In the limit of small d , they correspond to the values in the usual h.o. shell model. The value of θ_1 goes up to $\pi/2$ at $d \approx 1$ fm, and at smaller d no solution of θ_1 is found which satisfies the H-F condition (2.15). This means that h.o. wave functions can not satisfy the H-F

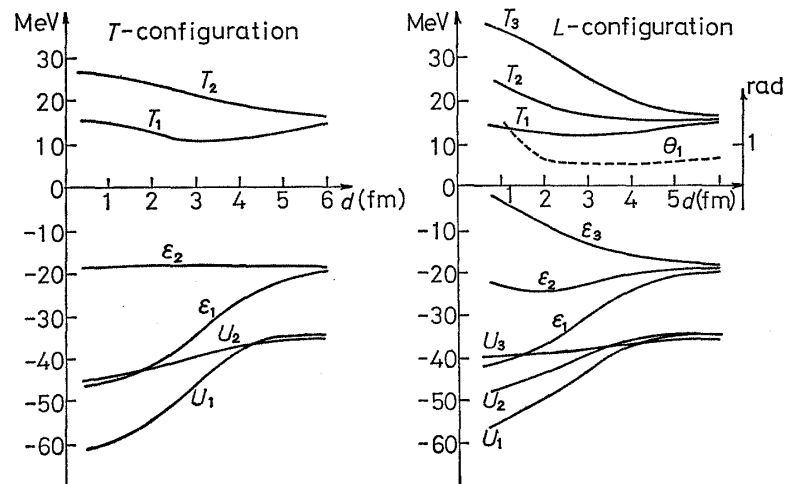


Fig. 4. Single particle energies for $b=1.4$ fm (T_i : kinetic energy, U_i : potential energy and $E_i = T_i + U_i$) and the mixing parameter θ_1 for L -configuration.

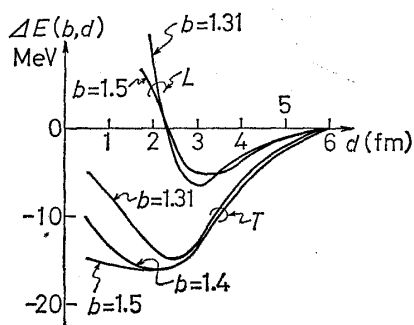


Fig. 5. Total energy $\Delta E(b, d)$ for two configurations.

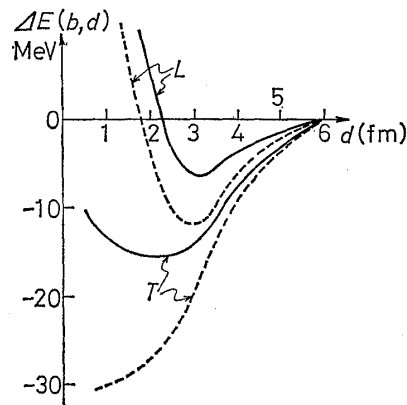


Fig. 6. $\Delta E(b, d)$ for $b=1.4$ fm. Dashed curves represent the values obtained by using $G(d = \infty)$ for all d -values (no clustering dependence of G -matrix).

condition by themselves.

The total energy $\Delta E(b, d)$ has a shallow minimum around $d=2$ fm for T -configuration and a steep minimum around $d=3$ fm for L -configuration. This means that the latter has more distinct and more stable clusterization than the former. It is emphasized that these energy minima at non-zero values of d are obtained by using no odd-state repulsive force. Clustering dependence of effective interaction, that is, d -dependence of G -matrix is responsible for bringing about these clusterizations. In order to see the effect explicitly, let us compare $\Delta E(b, d)$ in Fig. 5 with those calculated by using the G -matrix obtained at $d = \infty$ for all values of d . At $d = \infty$ the G -matrix is same for both configurations. They are shown in Fig. 6. In T -configuration, the energy minimum is found at $d=0$, if we use artificially the same G -matrix for all values of d . This means that without the d -dependent G -matrix T -configuration would have no clusterization. This situation occurs usually when we use simple phenomenological interactions without strong odd-state repulsion. Our results show that the effective interaction is generally more attractive as the clustering becomes more noticeable with larger values of d . In other words, clusterization makes the effective interaction more favourable to clusterization. In the case of L -configuration, we see in Fig. 6. that there remains stable clustering even if the d -dependence of the G -matrix is ignored. This is due to the strong effect of the Pauli principle.⁷⁾

In order to find the difference of the effective interaction between T - and L -configurations, we measure in Fig. 6 the total energies from the dashed curves which were obtained by use of the same G -matrix. Energy differences from the dashed lines are about 11 MeV at the energy minimum for T -configuration, but only about 5 MeV at the energy minimum for L -configuration. This means that the effective interaction is more attractive in L - than in T - configuration, and this configuration dependence makes the excitation energy of

the L -state from the ground state, i.e. the T -state, about 6 MeV lower than that obtained without this effect.

The above discussion is a generalization. Actually, G -matrices; even for a fixed value of d , depend in a complicated way on the states of the two interacting nucleons. In Fig. 7 are shown the ratios of the matrix elements $\langle \chi_i \chi_j | G_{TS} | \chi_i \chi_j \rangle$ to those calculated by use of the G -matrix at $d = \infty$, which are used to obtain the dashed curves in Fig. 6. We find that $T=0, S=1$ matrix elements change markedly with d , while $T=1, S=0$ change only slightly.

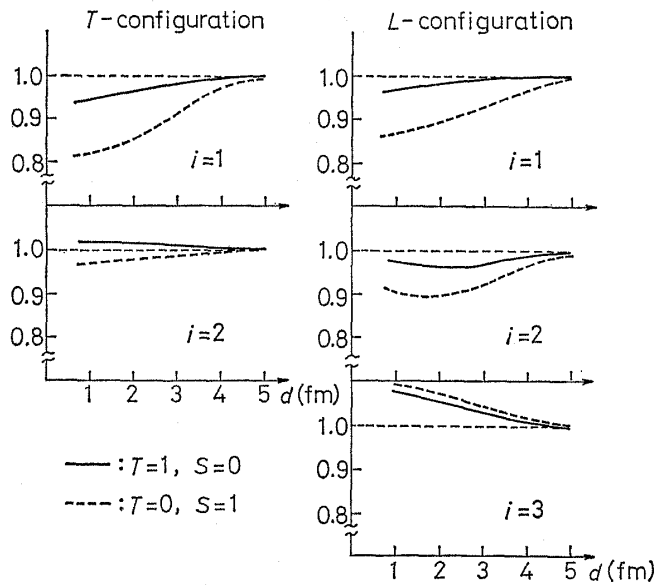


Fig. 7. The ratios $\langle \chi_i(d) \chi_i(d) | G_{TS}(d) | \chi_i(d) \chi_i(d) \rangle / \langle \chi_i(d) \chi_i(d) | G_{TS}(d=\infty) | \chi_i(d) \chi_i(d) \rangle$ for $b=1.4$ fm.

potential energy in our calculation.

§ 4. Conclusions

We have shown in this paper that

- i) there exists a "clustering dependence of effective interaction" which enhances clustering itself (this may be regarded as clustering-induced attraction),
- ii) it has a decisive role in bringing about some clusterization for the ground T -configuration of C^{12} ,
- iii) it makes the effective interaction more attractive in the excited L -configuration than in the ground T -configuration and, as the result, reduces the excitation energy of the L -state,
- iv) it appears most markedly in the triplet even state and is understood as originating from a strong tensor force.

These results may give a theoretical foundation to cluster structure in light nuclei

force in the triplet even state. It is seen from Figs. 4 and 7 that renormalizations of the tensor force to the central part depend on the changes of single particle energies ϵ_i 's and on the existence of levels to be rejected by the Pauli principle. Thus we can say that the 3E -tensor force is mainly responsible for the clustering dependence of the effective interactions, which enhances clustering itself as a whole.

The above results should be thought in a qualitative and not a quantitative sense, since the binding energy of an α -particle is deficient by about 10% of the

from the view-point of the effective interaction. We have adopted simple model wave functions here. Many effects such as polarization of α -clusters,⁸⁾ $l \cdot s$ splitting,⁹⁾ and so on, must be taken into consideration in order to give realistic results. Still then, clustering dependence of the effective interaction will play an important role.

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The numerical computations are carried out on the Computer System at the Data Processing Centre of Kyoto University.

Appendix

In T -configuration, $\chi_i(\mathbf{r}_1)\chi_j(\mathbf{r}_2)$ are transformed into c.m. and relative coordinates as follows:

$$\chi_1(\mathbf{r}_1)\chi_1(\mathbf{r}_2) = \frac{1}{\sqrt{6}}(1+2A^{1/4})^{-1} \{ (1+N_0)^{1/2}\Phi_1(\mathbf{R})\xi_0^{(1)}(r) + (1-N_0)^{1/2}\Phi_4(\mathbf{R})\xi_0^{(2)}(r) \},$$

$$\begin{aligned} \chi_2(\mathbf{r}_1)\chi_2(\mathbf{r}_2) = & \frac{1}{\sqrt{6}}(1-A^{1/4})^{-1} \left\{ (1+N_0)^{1/2}\Phi_1(\mathbf{R})\xi_0^{(4)}(r) + \frac{1}{\sqrt{2}}(1-N_1)^{1/2}\Phi_2(\mathbf{R})\xi_0^{(6)}(r) \right. \\ & \left. + (1-N_0)^{1/2}\Phi_4(\mathbf{R})\xi_0^{(3)}(r) + \frac{1}{\sqrt{2}}(1+N_1)^{1/2}\Phi_5(\mathbf{R})\xi_0^{(5)}(r) \right\}, \end{aligned}$$

$$\chi_3(\mathbf{r}_1)\chi_3(\mathbf{r}_2) = \chi_2(\mathbf{r}_1)\chi_2(\mathbf{r}_2),$$

$$\begin{aligned} \chi_1(\mathbf{r}_1)\chi_2(\mathbf{r}_2) = & \frac{1}{\sqrt{6}}(1+2A^{1/4})^{-1/2}(1-A^{1/4})^{-1/2} \{ (1-N_1)^{1/2}\Phi_2(\mathbf{R})\xi_0^{(7)}(r) \\ & + (1+N_1)^{1/2}\Phi_5(\mathbf{R})\xi_0^{(8)}(r) \}, \end{aligned}$$

$$\begin{aligned} \chi_2(\mathbf{r}_1)\chi_3(\mathbf{r}_2) = & -\frac{1}{\sqrt{12}}(1-A^{1/4})^{-1} \{ (1-N_1)^{1/2}\Phi_3(\mathbf{R})\xi_0^{(6)}(r) \\ & + (1+N_1)^{1/2}\Phi_6(\mathbf{R})\xi_0^{(5)}(r) \}, \end{aligned}$$

$$\begin{aligned} \chi_3(\mathbf{r}_1)\chi_1(\mathbf{r}_2) = & \frac{1}{\sqrt{6}}(1+2A^{1/4})^{-1/2}(1-A^{1/4})^{-1/2} \{ (1-N_1)^{1/2}\Phi_3(\mathbf{R})\xi_0^{(7)}(r) \\ & + (1+N_1)^{1/2}\Phi_6(\mathbf{R})\xi_0^{(8)}(r) \}. \end{aligned}$$

Here the s -state wave functions $\xi_0^{(n)}(r)$ are defined by

$$\xi_0^{(n)}(r) = (\sqrt{2\pi}b)^{-3/2}e^{-r^2/4b^2} \{ (1+2A^{1/2})^{1/2} - (-)^n p(n) \}$$

$$\times (1 + 2A^{1/8})^{1/2} A^{1/4} g_0(dr/2b^2)\}, \quad n=1\sim 4$$

and

$$\begin{aligned} \xi_0^{(n)}(r) &= (\sqrt{2\pi}b)^{-3/2} e^{-r^2/4b^2} \{ (1 - A^{1/2})^{1/2} - (-)^n p(n) \\ &\quad \times (1 - A^{1/8})^{1/2} A^{1/4} g_0(dr/2b^2)\}, \quad n=5\sim 8, \end{aligned}$$

where $p(n)=2$ for $n=1, 2, 5, 6$ and 1 for $n=3, 4, 7, 8$. Other than s-waves are omitted in the above expressions.

The expressions in the case of L -configuration are more complicated and are not written here explicitly.

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