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Real and Nominal Equilibrium Yield Curves with Endogenous Inflation: A Quantitative Assessment*

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Abstract

The links between real and nominal bond risk premia and macroeconomic dynamics are explored analytically and quantitatively in a model with nominal rigidities and monetary policy. The interest-rate policy rule becomes a restriction linking real and nominal risk premia through endogenous inflation. The estimated model captures macroeconomic and yield curve properties of the U.S. economy, implying significantly positive real term and inflation risk bond premia. Both premia are induced by wage rigidities as a compensation for permanent productivity shocks. Stronger policy-rule responses to inflation (output) increase (decrease) both premia. Policy surprises generate significant yield volatility but negligible risk premia.

JEL Classification: D51, E43, E44, E52, G12.

Keywords: Term structure of interest rates, bond risk premia, monetary policy, nominal rigidities.

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1 Introduction

Understanding the economic drivers of long-term real bond yields and risk premia is a fundamental concern in financial economics. This understanding is critical, for instance, to explain the dynamics of the real pricing kernel, the diversification benefits of real bonds, or the transmission of monetary policy. Its importance, however, radically contrasts with our knowledge of the subject. Insights from the empirical analysis of inflation-linked government bonds are, at best, incomplete. These bonds have been traded only recently in developed economies, are imperfect substitutes of real bonds, and are potentially affected by illiquidity and mispricing.¹ In addition, the available evidence from the United Kingdom and United States inflation-linked bonds does not present a clear picture of their risk properties and link to the economy. Common risk measures such as the average slope of the yield curve, realized excess returns, or the bond return correlations with macroeconomic variables and stock returns provide mixed results across countries and sub-periods. For instance, the average inflation-linked U.K. and U.S. yield curves during 1999-2008 are sharply downward and upward sloping, respectively. The same curve in the U.K. is slightly upward sloping for 1985-2008. Thus, the limited evidence can be highly benefited from the theoretical analysis of real yields and risk premia. This paper provides such an analysis by developing and estimating a New Keynesian model that delivers equilibrium real and nominal yield curves. It focuses on understanding (i) the effect of nominal rigidities, several fundamental shocks, and monetary policy on real term and inflation risk premia in real and nominal bonds, respectively, and (ii) the link imposed by endogenous inflation on these two types of premia.

The theoretical framework is motivated by two reasons. First, New Keynesian models have become the workhorse model for understanding economic dynamics, and are widely used for policy analysis. Second, the framework generates endogenous inflation dynamics that depend on economic fundamentals, linking the properties of real term and inflation risk premia. This link

¹See Garcia and van Rixtel (2007) for recent history on inflation-linked bond markets, D'Amico, Kim and Wei (2014) for evidence on significant time-varying liquidity premia in U.S. and U.K. inflation-linked bonds, and Fleckenstein, Longstaff and Lustig (2014) for evidence on mispricing in the TIPS market.

acts as an additional restriction relative to term structure models with exogenous inflation. Rudebusch and Swanson (2012), and Andreasen (2012), among others, study bond risk premia in this framework with relative quantitative success. It is not well known, however, what frictions and shocks are essential or quantitatively important in explaining yield dynamics, the decomposition and link between real term and inflation risk premia, and the economic mechanisms driving the results. This paper contributes to fill this gap.

The model contains the standard elements of the New Keynesian framework, and recursive preferences on consumption and labor for the representative household. These preferences have the ability to simultaneously capture macroeconomic and term structure dynamics.² The analysis is focused on understanding the contribution of the following key elements to real and nominal bond risk premia. First, nominal price and wage rigidities. Both frictions generate real effects in monetary policy, but have different implications for economic dynamics, as highlighted in Christiano, Eichenbaum and Evans (2005). Second, productivity, monetary policy, and inflation-target shocks. Productivity is modeled to contain permanent (difference-stationary) and transitory (trend-stationary) components. As shown by Campbell (1986) and Labadie (1994), these two components have different implications for bond risk premia. They also have different effects on the permanent component of the marginal utility of wealth, and then in their ability to price other financial assets such as stocks, as demonstrated by Alvarez and Jermann (2005). Monetary policy and inflation-target shocks are standard in this framework but their implications for bond risk premia have been less studied. Third, a nominal interest-rate policy rule. The response to economic conditions in this rule has important implications for the joint dynamics of real variables and inflation, and then the link between real term and inflation risk premia.³

The model is estimated via the Generalized Method of Moments (GMM) to match a series

²See Rudebusch and Swanson (2008, 2012) for differences in this ability between habit formation and recursive preferences, respectively.

³Alternatively, a more structural approach to monetary policy is to consider the monetary authority as a social planner that maximizes welfare, as in Palomino (2012). This approach may have different implications and is not explored in this paper.

of key U.S. macroeconomic moments. The baseline estimation captures the nominal price and wage rigidities in the data, and is affected by the four shocks described above. It simultaneously captures macroeconomic and nominal bond return dynamics better than all the tested estimations with alternative rigidity and shock specifications. It matches the Sharpe ratio of the 5-year nominal bond with a risk aversion in the range of values reported in the term structure literature. It implies upward sloping real and nominal yield curves, with average 5-year real and nominal bond spreads of 82 and 112 bps., respectively. This is the result of positive real term and inflation risk premia.

Explaining the economic drivers of positive real term and inflation risk premia in the baseline model is central to the analysis. It relies on comparisons with alternative model specifications, and complemented with approximate closed-form solutions for these premia. In particular, two properties of the pricing kernel are useful to describe the main findings: Real term and inflation risk premia are positive when the real pricing kernel is negatively autocorrelated and positively correlated with inflation, respectively. Consumption growth is the main driver of the pricing kernel in the baseline model, and these two conditions become a negative autocorrelation of consumption growth, and a negative correlation of it with inflation, respectively.⁴

There are three main findings. First, permanent productivity shocks, in combination with wage rigidities, are crucial to generate large and positive real term and inflation risk premia. Permanent productivity shocks contribute with almost all the variability in the pricing kernel, and thus bond risk premia are mainly a compensation for this risk. Wage rigidities, in the presence of these shocks, generate a consumption growth process that is both negatively autocorrelated and negatively correlated with inflation. This is explained by the impact of labor dynamics on consumption with and without rigidities. In the absence of rigidities, prices and wages freely adjust after permanent shocks to preserve product and labor markups, keeping labor constant. Since consumption depends on productivity and labor, consumption growth inherits the positive

⁴As discussed below, Section 5 presents a model extension that breaks the strong link between consumption growth and the pricing kernel in the baseline model. It allows us to capture a positive autocorrelation of consumption growth without affecting the main term structure implications of the model.

autocorrelation of the permanent shocks. After a negative shock, expected consumption growth and inflation decline, as well as real and nominal yields. Bond returns are thus high, implying negative real and nominal term premia. On the contrary, nominal rigidities generate procyclical and mean-reverting labor that affects consumption growth. After a negative shock, prices and wages are higher than in the frictionless case, depressing labor and reducing consumption further. However, expected future labor and consumption increase since prices and wages gradually adjust downwards. In the baseline model, the effect of wage rigidities on labor is strong enough to generate a negative autocorrelation in consumption growth, and a negative correlation of it with inflation. Therefore, real term and inflation risk premia become positive under wage rigidities.

Second, responses to economic conditions in the interest-rate policy rule, and surprises in monetary policy affect bond risk premia and yield volatility, respectively. A stronger response to inflation in the policy rule increases real term and inflation risk premia by increasing the sensitivity of the real rate (and pricing kernel) to permanent shocks. A stronger response to the output gap, or an increase in the interest-rate smoothing coefficient, have the opposite effect. Differently, monetary policy and inflation-target shocks have a negligible effect on bond risk premia, but significantly affect real and nominal bond yield volatility. In the absence of nominal rigidities, monetary policy does not have any real effects, and these shocks do not affect real yields. In the presence of rigidities, however, monetary policy affects the real economy and thus real yields for all maturities. As a result, monetary policy shocks become the main driver of real yields. Inflation-target shocks do not have a significant impact on real yields, but its persistence considerably affect long-term nominal bond yield volatility.

Third, the interest-rate policy rule restricts the joint behavior of real term and inflation risk premia. This restriction is reflected in the baseline model in real term and inflation risk premia that share the same sign. It is explained by the dynamics of the real pricing kernel and inflation implied by the policy rule. A rule with a sufficiently strong response to inflation imposes a positive relation between inflation and the short-term nominal interest rate. This rate, in turn,

is negatively related to the future pricing kernel, as implied by optimality conditions and bond market clearing. That is, in equilibrium, inflation is negatively correlated with the real pricing kernel. Inflation risk premia are thus positive (negative) when the pricing kernel is negatively (positively) autocorrelated. This condition is the same as for positive (negative) real term premia. This restriction is not existent in models with exogenous inflation, where parameters are freely set to simultaneously generate negative real term and positive inflation risk premia.

Finally, the paper addresses three limitations of the baseline model. First, the baseline estimation does not capture the positive autocorrelation of consumption growth in the data. A negative autocorrelation in consumption growth induced by permanent productivity shocks is essential to generate positive bond risk premia. Introducing habit persistence in preferences breaks the tight link between the pricing kernel and consumption growth. It generates negative autocorrelation in the former and positive in the latter, without compromising the main macroeconomic and term structure implications of the model. Second, the baseline model generates negligible variation in expected excess bond returns, at odds with the well documented evidence on bond return predictability. This shortcoming can be overcome by introducing countercyclical volatility in permanent or transitory productivity shocks. Third, the baseline model abstracts from capital accumulation. An estimation of a model with capital shows that the main implications of the baseline model survive, but the link between real term and inflation risk premia becomes weaker.

This paper joins the literature that analyzes the term structure of interest rates using New Keynesian models (see Woodford (2003) for the standard framework). It complements the current literature by providing an estimation of the real yield curve, and a detailed analysis of real term and inflation risk premia, their economic link, and the quantitative contribution of different model elements to the results. It is closely related to the studies in Rudebusch and Swanson (2012), Andreasen (2012), Dew-Becker (2014) and Kung (2014), which add recursive preferences to the standard framework with different model elements. Rudebusch and Swanson (2012) rely on transitory productivity shocks and price rigidities to capture nominal yield curve properties, and do

not study real yield curve implications. These elements are present in our model, but their effects are not as quantitatively important as those of wage rigidities and permanent shocks. Andreasen (2012) incorporates both permanent and transitory components in productivity, and Dew-Becker (2014) adds wage rigidities to the analysis. These studies focus on the time-variation in bond risk premia by fitting macroeconomic and yield dynamics. Our model has these elements but with a different focus. The GMM approach allows us to target unconditional moments, provide quantitative comparisons across model specifications, and focus on explaining the economic mechanisms behind the results. Kung (2014) presents a model with an endogenous growth channel that is complementary to our model structure. The effect of this channel on our results is beyond the scope of the paper. The paper is also related to term structure models with exogenous inflation such as the endowment economies in Wachter (2006), Piazzesi and Schneider (2007), and Bansal and Shaliastovich (2013), or the real business cycle model in Van Binsbergen et al. (2012). These models require a negative correlation between these variables to match an average upward sloping nominal yield curve, and have different implications for the real yield curve.⁵ Our model generates the negative correlation of consumption growth and inflation endogenously, and links real and nominal bond risk premia from first principles. An advantage of our framework is that it allows us to predict changes in yield curve dynamics related to structural economic and policy changes. Finally, Buraschi and Jiltsov (2005) studies real and nominal bond risk premia in a monetary real business cycle model. This model also generates endogenous consumption growth and inflation. Their monetary policy and friction specifications are substantially different.

The paper is organized as follows. Section 2 describes the data and reports descriptive statistics for nominal and inflation-linked yield curves in the United States and the United Kingdom. Section 3 describes the model. Section 4 provides details of the model estimation and its quantitative performance, presents its main implications, and explores the economic mechanism behind the results. Section 5 presents some model limitations and extensions, and Section 6 concludes.

⁵The models with recursive preferences in Piazzesi and Schneider (2007) and Bansal and Shaliastovich (2013) imply an average downward sloping real yield curve, while the habit model in Wachter (2006) implies the opposite.

2 Empirical Evidence

This section presents descriptive statistics of inflation-linked and nominal government bonds in the United Kingdom and the United States. While the empirical properties of nominal bond yields are well known, the study of the real yield curve, its risks, and links to the nominal one is limited by data availability. Inflation-linked government bonds are, at best, imperfect substitutes of real bonds, and have only been traded in the United Kingdom and the United States since 1981 and 1997, respectively. Their inflation protection is limited by a lagged indexation to price levels and the embedded deflation optionality they provide. In addition, pricing in these markets has been affected by liquidity concerns and potential unexploited arbitrage opportunities.⁶ The evidence illustrates several difficulties in exploring properties of real bonds from the available data, motivating the joint theoretical analysis of real and nominal yields that follows.⁷

We use quarterly data from January 1985 to September 2008 for the U.S. and the U.K., and report statistics for the periods 1985-2008 and 1999-2008. The data sample periods are motivated by two reasons. First, TIPS data in the U.S. and inflation-linked gilts data in the U.K. are only available since 1999 and 1985, respectively.⁸ Second, the period September-December 2008 coincides with the collapse of Lehman Brothers that drove short-term interest rates close to zero, and triggered a switch to unconventional monetary policies. The period after September 2008 is then not covered to focus on the effects on bond yields of a (conventional) monetary policy conducted using an interest-rate rule.

⁶See D’Amico, Kim and Wei (2014) for evidence on significant time-varying liquidity premia in U.S. and U.K. inflation linked bonds. See Fleckenstein, Longstaff and Lustig (2014) for evidence of mispricing in the TIPS and inflation swaps markets.

⁷There is an extensive empirical literature studying the real term and inflation risk premia with and without inflation-linked bonds with no-arbitrage term structure models. This literature shows a wide range for the sign and size of inflation risk premia in the U.K and the U.S. An incomplete list includes Barr and Campbell (1997) Evans (1998), and Joyce, Lildholdt and Sorensen (2010) for the U.K., and Ang, Bekaert and Wei (2008), D’Amico, Kim and Wei (2014), Chen, Liu and Cheng (2010), Christensen, Lopez and Rudebusch (2010), Chernov and Mueller (2012), Grishchenko and Huang (2013), and Abrahams et al. (2013) for the U.S. Hördahl and Tristani (2012) provide a similar study for the Euro zone.

⁸Results using comparable monthly data are very similar. We present results for quarterly data to be consistent with the model estimation. The same macroeconomic and term structure data for the United States are used to estimate the model, for the longer period January 1982 to September 2008.

The consumption growth and inflation series for the U.S. are constructed using quarterly data from the Bureau of Economic Analysis, following the methodology in Piazzesi and Schneider (2007). These series capture only consumption of non-durables and services and its related inflation, and then consistent with the model variables. Wages are real wages per hour of non-farm business from the Federal Reserve Economic Data (FRED) database from the Federal Reserve Bank of St. Louis. The data on U.S. zero-coupon nominal bond and TIPS yields are constructed following the procedure in Gurkaynak, Sack and Wright (2006, 2008), respectively. These data are obtained from the Federal Reserve website. The short-term nominal interest rate is the 3-month T-bill rate from the Fama risk-free rates database. The three-month real rate is estimated using the methodology described in Pflueger and Viceira (2011).⁹ Dividends and stock market returns correspond to the market portfolio obtained from the Center for Research in Security Prices (CRSP). For the U.K., consumption growth and inflation are obtained directly from the FRED database. The historical yields for U.K. real and nominal bonds are taken from the Bank of England website. The three-month real rate in the U.K. is estimated using the same methodology used to estimate the U.S. real rate. Stock returns are for the UK FTSE All-Shares Index. The bond yields under study correspond to maturities from 2 to 10 years. The long end of the curves has been excluded for comparison purposes across countries. Greenwood and Vayanos (2010) document a significant effect on long-term inflation-linked bond yields in the U.K, resulting from the increased demand from pension funds to meet the Minimum Funding Requirements. Table 1 summarizes the empirical evidence.

The properties of bond risk premia are frequently characterized by the average slope of a yield curve, the average excess bond returns relative to a risk-free rate, or the correlation of excess bond and stock returns. Panel A reports, a slightly and a significantly upward-sloping average nominal

⁹Specifically, the computation is based on the regression

$$i_t - \pi_{t+1} = \text{constant} + \beta_i i_t + \beta_r (i_{t-1} - \pi_t) + \varepsilon_t,$$

where i_t is the three-month nominal rate and π_t is the three-month inflation rate. The real rate is then computed as $r_t = i_t - \mathbb{E}_t[\pi_{t+1}]$ under the assumption that the inflation risk premium in three-month nominal bonds is negligible.

yield curves in the U.K and the U.S, respectively, suggesting positive risk premia in nominal bonds. The picture is less clear for inflation-linked bonds. The average yield curve for these bonds is slightly upward sloping for the U.K. for the sample 1985-2008, but becomes drastically downward sloping for the sample 1999-2008. During the same period, the comparable average yield curve in the U.S. is significantly upward sloping. Figure 1 shows the average U.K. yield curves for both samples. These findings suggest negative and positive risk premia in inflation-linked bonds in the U.K. and the U.S. respectively. The average excess returns in Panel B support these claims.¹⁰ Nominal bonds exhibit positive average excess returns for both countries, and inflation-linked bonds in the U.K. and the U.S. have negative and positive average excess returns, respectively. However, the correlations between excess bond and stock returns in Panel C suggest a different story. While inflation-linked bond excess returns in the U.K. have shown positive correlations with stock excess returns in both samples, U.K. nominal bonds switch from a positive correlation for 1985-2008 to a negative one for 1999-2008. The opposite occurs for U.S. nominal bonds, while the correlation between TIPS and stock excess returns is negative for 1999-2008. According to the CAPM, the evidence for the recent sample implies negative risk premia for U.K. and U.S. nominal and inflation-linked bonds, at odds with evidence from panels A and B.¹¹

The link between macroeconomic variables and the yield curve also is of interest to understand bond risk premia. Panel C reports correlations of consumption growth and inflation with bond yields. The correlations of U.K. and U.S. inflation-linked and nominal bond yields with consumption growth are significantly positive during both samples. On the other hand, the correlations of inflation with these yields change from positive for 1985-2008 to negative for 1999-2008. These changes are accompanied by a reduced autocorrelation of inflation in both the U.K. and the U.S., higher and lower autocorrelations of consumption growth in the U.S. and the U.K respectively, and a correlation between consumption growth and inflation that is negative in the U.S. and

¹⁰Excess bond returns are computed as the difference of realized nominal returns on inflation-linked and nominal bonds with the respective 3-month nominal rate for each country.

¹¹The time-varying nature of the correlation between nominal bond and stock returns is highlighted and studied by Viceira (2012) and Campbell, Sunderam and Viceira (2013).

switching from positive to negative in recent years in the U.K. These evidence can be linked to bond risk premia through economic theory. According to standard equilibrium models, a positive autocorrelation of consumption growth implies negative premia for real bonds, and a negative correlation between consumption growth and inflation implies positive inflation risk premia.¹²

Interestingly, Panel A also shows that the standard deviations of inflation-linked and nominal bonds are similar for 1999-2008 in both U.K. and the U.S. This is intriguing given the additional exposure of nominal yields to inflation risk. In summary, the descriptive statistics presented here do not provide a clear pattern to describe salient properties of real bond risk premia and its link to macroeconomic variables. This is not surprising given the short-sample for inflation-linked bonds, the limitations of these bonds presented above, and potential structural changes in the economy. The theoretical model in Section 3 allows us to analyze the link between real and nominal bond risk premia and macroeconomic variables. This analysis can provide testable implications to understand better the dynamics of real bond yields.

3 Model

We model a production economy with a representative household, a production sector for differentiated goods, and monetary policy. The representative household derives utility from the consumption of a basket of goods and disutility from supplying differentiated labor to the production sector. Labor and product markets are characterized by monopolistic competition and nominal wage and price rigidities, respectively. Monetary policy is modeled as an interest-rate policy rule that reacts to economic conditions. All markets are complete. Default-free real and nominal bonds are in zero net supply. The model can be seen as an extension of the standard New-

¹²Campbell (1986) shows the link between the autocorrelation of consumption growth and the real yield curve under constant relative risk aversion preferences. The same intuition applies under recursive preferences on consumption, as shown in Bansal and Shaliastovich (2013). The Campbell and Cochrane (1999) habits model can imply the opposite, as shown by Wachter (2006). Piazzesi and Schneider (2007) highlight the link between positive inflation risk premia and the negative correlation between (expected) consumption growth and inflation under recursive preferences.

Keynesian framework (see Woodford (2003), for instance) to capture bond pricing dynamics. It incorporates recursive preferences for the representative household, as in Rudebusch and Swanson (2012) and Li and Palomino (2014), to disentangle risk aversion from the elasticity of intertemporal substitution of consumption. This separation allows us to match observed macroeconomic dynamics by choosing an appropriate level for the elasticity of intertemporal substitution, while increasing the degree of risk aversion to capture large expected excess returns. Nominal prices and/or wages that are not adjusted optimally generate relative price and wage distortions that affect production decisions. In this setting, different monetary policy rules have different implications on inflation and real activity. As a result, the dynamics and riskiness of real and nominal bond yields are affected by both nominal rigidities and monetary policy. This section describes the characteristics of the model economy.

3.1 Household

A representative agent chooses consumption C_t and labor supply N_t^s to maximize the Epstein and Zin (1989) recursive utility function

$$V_t = (1 - \beta)U(C_t, N_t^s)^{1-\varphi} + \beta\mathbf{E}_t \left[V_{t+1}^{\frac{1-\gamma}{1-\varphi}} \right]^{\frac{1-\varphi}{1-\gamma}}, \quad (1)$$

where $\beta > 0$ is the subjective discount factor, and φ and γ determine the elasticity of intertemporal substitution (EIS) and the coefficient of relative risk aversion, respectively.¹³ The recursive utility formulation relaxes the strong assumption of $\gamma = \varphi$ implied by constant relative risk aversion. The intra-temporal utility is defined over consumption and labor supply as

$$U(C_t, N_t^s) = \left(\frac{C_t^{1-\varphi}}{1-\varphi} - \kappa_t \frac{(N_t^s)^{1+\omega}}{1+\omega} \right)^{\frac{1}{1-\varphi}}, \quad (2)$$

¹³The elasticity of intertemporal substitution of the utility bundle of consumption and labor is φ^{-1} . The coefficient of relative risk aversion is defined in Section 4.

where $\omega^{-1} > 0$ captures the Frisch elasticity of labor supply, and the process κ_t is chosen to ensure balanced growth (it is specified in the production sector section below). The consumption good is a basket of differentiated goods produced by a continuum of firms. Specifically, the consumption basket is

$$C_t = \left[\int_0^1 C_t(j)^{\frac{\theta_p-1}{\theta_p}} dj \right]^{\frac{\theta_p}{\theta_p-1}}, \quad (3)$$

where $\theta_p > 1$ is the elasticity of substitution across differentiated goods, and $C_t(j)$ is the consumption of the differentiated good j . Labor supply is the aggregate of a continuum of different labor types supplied to the production sector, such that

$$N_t^s = \int_0^1 N_t^s(k) dk, \quad (4)$$

where $N_t^s(k)$ is the supply of labor type k .

The representative consumer is subject to the intertemporal budget constraint

$$\mathbb{E}_t \left[\sum_{s=0}^{\infty} M_{t,t+s}^{\$} P_{t+s} C_{t+s} \right] \leq \mathbb{E}_t \left[\sum_{s=0}^{\infty} M_{t,t+s}^{\$} P_{t+s} (LI_{t+s} + D_{t+s}) \right], \quad (5)$$

where $M_{t,t+s}^{\$}$ is the nominal discount factor for cashflows at time $t+s$, P_t is the nominal price of a unit of the basket of goods, LI_t is the real labor income from supplying labor to the production sector, and D_t is the real dividend from owning the production sector.

Appendix A shows that the household's optimality conditions imply that the one-period real and nominal discount factors are

$$M_{t,t+1} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\varphi} \left(\frac{V_{t+1}^{1/(1-\varphi)}}{\mathbb{E}_t \left[V_{t+1}^{(1-\gamma)/(1-\varphi)} \right]^{1/(1-\gamma)}} \right)^{\varphi-\gamma}, \quad \text{and} \quad M_{t,t+1}^{\$} = M_{t,t+1} \left(\frac{P_{t+1}}{P_t} \right)^{-1}, \quad (6)$$

respectively. The one-period (continuously compounded) real and nominal interest rates are obtained from

$$r_t = -\log \mathbb{E}_t [M_{t,t+1}], \quad \text{and} \quad i_t = -\log \mathbb{E}_t [M_{t,t+1}^s], \quad (7)$$

respectively. The nominal interest rate i_t is the instrument of monetary policy.

3.1.1 Wage Setting

Following Schmitt-Grohe and Uribe (2007), an imperfectly competitive labor market is modeled where the representative household monopolistically provides a continuum of labor types indexed by $k \in [0, 1]$.¹⁴ The supply of labor type k satisfies the demand equation

$$N_t^s(k) = \left(\frac{W_t(k)}{W_t} \right)^{-\theta_w} N_t^d, \quad (8)$$

where N_t^d is the aggregate labor demand of the production sector, $W_t(k)$ is the wage for labor type k , and W_t is the aggregate wage index given by

$$W_t = \left[\int_0^1 W_t^{1-\theta_w}(k) dk \right]^{\frac{1}{1-\theta_w}}. \quad (9)$$

The labor demand equation (8) is obtained from the production sector problem presented in the section below. The household chooses wages $W_t(k)$ for all labor types k under Calvo (1983) staggered wage setting. Specifically, at each time t , the household is only able to adjust wages optimally for a fraction $1 - \alpha_w$ of labor types. The remaining fraction α_w of labor types adjust their previous period wages by the wage indexation factor $\Lambda_{w,t-1,t}$. The specific functional form

¹⁴This approach is different from the standard heterogeneous households approach to model wage rigidities in Erceg, Henderson and Levin (2000), where each household supplies a differentiated type of labor. In the presence of recursive preferences, this approach introduces heterogeneity in the marginal rate of substitution of consumption across households since it depends on labor. We avoid this difficulty and obtain a unique marginal rate of substitution by modeling a representative agent who provides all different types of labor.

of this factor is presented in Section 4. The optimal wage maximizes (1), subject to demand functions (8) for all labor types k , and the budget constraint (5). Notice that real labor income is given by

$$LI_t = \int_0^1 \frac{W_t(k)}{P_t} N_t^s(k) dk. \quad (10)$$

Since the demand curve and the cost of labor supply are identical across different labor types, the household chooses the same wage W_t^* for all labor types subject to an optimal wage change at time t . Appendix A shows that the optimal wage satisfies

$$\frac{W_t^*}{P_t} = \mu_w \kappa_t (N_t^s)^\omega C_t^{\varphi} \frac{G_{w,t}}{H_{w,t}}, \quad (11)$$

where $\mu_w \equiv \frac{\theta_w}{\theta_w - 1}$. The recursive equations describing $G_{w,t}$ and $H_{w,t}$ are presented in the appendix. Equation (11) can be interpreted as follows: In the absence of wage rigidities ($\alpha_w = 0$), the marginal rate of substitution between labor and consumption is $\kappa_t (N_t^s)^\omega C_t^{\varphi}$, and the optimal wage is this rate adjusted by the optimal markup μ_w . Wage rigidities generate the time-varying markup $\mu_w \frac{G_{w,t}}{H_{w,t}}$, since the wage of some labor types is not adjusted optimally.

3.2 Production Sector

The production of differentiated goods is characterized by monopolistic competition and price rigidities in a continuum of firms. Firms set the price of their differentiated goods in a Calvo (1983) staggered price setting: At each time t , with probability α_p , a firm sets the price of the good as the previous period price adjusted by the price indexation factor $\Lambda_{p,t-1,t}$. The specific functional form of this factor is presented in Section 4. With probability $1 - \alpha_p$, the firm sets the product price to maximize the present value of profits. The maximization problem for firm j can

be written as

$$\max_{\{P_t(j)\}} \mathbb{E}_t \left\{ \sum_{s=0}^{\infty} \alpha_p^s M_{t,t+s}^s [\Lambda_{p,t,t+s} P_t(j) Y_{t+s|t}(j) - W_{t+s|t}(j) N_{t+s|t}^d(j)] \right\}, \quad (12)$$

subject to the production function

$$Y_{t+s|t}(j) = A_{t+s} N_{t+s|t}^d(j), \quad (13)$$

and the demand function

$$Y_{t+s|t}(j) = \left(\frac{P_t(j) \Lambda_{p,t,t+s}}{P_{t+s}} \right)^{-\theta} Y_{t+s}. \quad (14)$$

The output $Y_{t+s|t}(j)$ is the production of firm j at time $t+s$ given that the last optimal price change was at time t . The wage $W_{t+s|t}(j)$ and the labor demand $N_{t+s|t}^d(j)$ have a similar interpretation. The production problem takes into account the probability of not being able to adjust the price optimally in the future, and the corresponding indexation $\Lambda_{p,t,t+s}$.

The production function depends on labor productivity A_t and labor. We assume that labor productivity contains difference- and trend-stationary components.¹⁵ Specifically, $A_t = A_t^p Z_t$, where $a_t \equiv \log A_t^p$ and $z_t \equiv \log Z_t$, are the difference- and trend-stationary components of productivity, respectively. These components follow the processes

$$\Delta a_{t+1} = (1 - \phi_a) g_a + \phi_a \Delta a_t + \sigma_a \varepsilon_{a,t+1}, \quad \text{and} \quad z_{t+1} = \phi_z z_t + \sigma_z \varepsilon_{z,t+1}, \quad (15)$$

where Δ is the difference operator, g_a is the average growth rate in the economy, and innovations $\varepsilon_{a,t}$ and $\varepsilon_{z,t} \sim \text{IID}\mathcal{N}(0,1)$. For simplicity, throughout the paper we refer to the difference- and

¹⁵The two components are incorporated given the different effects on bond risk premia of these two processes for consumption in endowment economies. A difference-stationary process for consumption with positive autocorrelation coefficient generates negative term premia. A trend-stationary process for consumption with positive autocorrelation coefficient generates positive term premia.

trend-stationary components as the permanent and transitory shocks to productivity, respectively.

Labor demand is a composite of a continuum of differentiated labor types indexed by $k \in [0, 1]$ via the aggregator

$$N_t^d(j) = \left[\int_0^1 N_t^d(j, k)^{\frac{\theta_w - 1}{\theta_w}} dj \right]^{\frac{\theta_w}{\theta_w - 1}}, \quad (16)$$

where $\theta_w > 1$ is the elasticity of substitution across differentiated labor types.

All firms that set prices optimally are identical and set the same optimal price P_t^* . Appendix B shows that the optimal price satisfies

$$\left(\frac{P_t^*}{P_t} \right) H_{p,t} = \frac{\mu_p}{A_t} \frac{W_t}{P_t} G_{p,t}, \quad (17)$$

where $\mu_p = \frac{\theta_p}{\theta_p - 1}$. The recursive equations for $H_{p,t}$ and $G_{p,t}$ are presented in the appendix. Equation (17) can be interpreted as follows: In the absence of price rigidities, the product price is the markup-adjusted marginal cost of production, with optimal markup μ_p . Price rigidities generate the time-varying markup $\mu_p \frac{G_{p,t}}{H_{p,t}}$, since some firms do not adjust their prices optimally.

We define $\kappa_t \equiv (A_t^p)^{1-\varphi}$ to preserve balanced growth. It can be shown from equation (11) that wages and consumption share the same average trend as long as $\kappa_t \propto (A_t^p)^{1-\varphi}$, and implies stationary labor.

3.3 Monetary Policy

Monetary policy is described by the interest-rate policy rule

$$i_t = \rho i_{t-1} + (1 - \rho) \left[\bar{i} + \iota_\pi (\pi_t - \pi_{t-1}^*) + \iota_x (x_t - x_{ss}) \right] + u_t. \quad (18)$$

The policy rule has an interest-rate smoothing component captured by the sensitivity ρ to the lagged term, i_{t-1} , and responds to aggregate inflation $\pi_t \equiv \log \frac{P_t}{P_{t-1}}$, the output gap x_t , and a policy

shock u_t . The output gap is defined as the log deviation of total output, Y_t , from the output in an economy under flexible prices and wages, Y_t^f . That is, $X_t \equiv \frac{Y_t}{Y_t^f}$, and $x_t \equiv \log X_t$. The coefficients ι_π and ι_x capture the response of the monetary authority to the deviations of inflation and the output gap from their targets, respectively. The constant \bar{r} is defined as the nominal rate when the inflation rate and the output gap are at their targets, i.e., $\bar{r} \equiv -\log \beta + \varphi g_a + g_\pi$. The process π_t^* denotes the time-varying inflation target. The inflation target is time-varying as in Ireland (2007) and Rudebusch and Swanson (2012).¹⁶ Its process is

$$\pi_t^* = (1 - \phi_{\pi^*})g_\pi + \phi_{\pi^*}\pi_{t-1}^* + \sigma_{\pi^*}\varepsilon_{\pi^*,t}, \quad (19)$$

where $\varepsilon_{\pi^*,t} \sim \text{IID}\mathcal{N}(0, 1)$. The output gap target x_{ss} corresponds to the output gap in steady state. The policy shocks u_t follow the process

$$u_{t+1} = \phi_u u_t + \sigma_u \varepsilon_{u,t+1}, \quad (20)$$

where $\varepsilon_{u,t} \sim \text{IID}\mathcal{N}(0, 1)$.

3.4 Bond Prices and Yields

Real and nominal default-free zero-coupon bonds with maturity at $t + n$ pay a unit of real and nominal consumption, respectively, at maturity. Their prices are

$$B_t^{c,(n)} = \exp\left(-nr_t^{(n)}\right) = \mathbb{E}_t[M_{t,t+n}], \quad \text{and} \quad B_t^{s,(n)} = \exp\left(-ni_t^{(n)}\right) = \mathbb{E}_t[M_{t,t+n}^s], \quad (21)$$

for real and nominal bonds, respectively, where $r_t^{(n)}$ and $i_t^{(n)}$ are the associated real and nominal bond yields, and $M_{t,t+n}$ and $M_{t,t+n}^s$ are the real and nominal discount factors for payoffs at $t + n$.¹⁷

¹⁶The inflation target has also been used in the macro finance literature by Bekaert, Cho and Moreno (2010), Campbell, Pflueger and Viceira (2014) and Dew-Becker (2014).

¹⁷Notice that $B_t^{c,(n)}$ is the real price of the real bond, while $B_t^{s,(n)}$ is the nominal price of the nominal bond.

3.5 Equilibrium

Equilibrium requires product, labor, and financial market clearing. Product market clearing is characterized by $C_t(j) = Y_t(j)$ for all $j \in [0, 1]$, and then $C_t = Y_t$. Labor market clearing requires that supply and demand of labor type k employed by firm j are equal, $N_t^s(j, k) = N_t^d(j, k)$. As shown in appendix C, it implies the aggregate labor market clearing condition $N_t^s = N_t^d F_{w,t}$ where $N_t^d = \frac{Y_t}{A_t} F_{p,t}$. The distortions $F_{w,t}$ and $F_{p,t}$ measure wage and price dispersion caused by wage and price rigidities, respectively, and are defined in the appendix. Equilibrium in the financial market implies that the nominal interest rate from household maximization in equation (7) is equal to the interest rate set by the monetary policy rule in equation (18). Equilibrium implies the absence of arbitrage opportunities in real and nominal bond markets. Appendix D provides a summary of the equilibrium conditions.

3.6 Expected Excess Bond Returns and Risk Premia

Risk differences between short- and long-term bonds, and between real and nominal bonds are analyzed in terms of differences in their expected returns, risk premia, or implied yields. The link between these measures is presented in this section. It allows us to decompose and quantify the compensations for real and nominal risks in real and nominal bond yields. In particular, real term and inflation risk premia are useful to decompose bond yields into compensations for real and nominal risks, respectively. The model determinants of these premia are analyzed in Section 4.

One-period gross bond returns are $R_{t,t+1}^{\ell,(n)} \equiv \frac{B_{t+1}^{\ell,(n-1)}}{B_t^{\ell,(n)}}$, for $\ell = \{c, \$\}$. Real and nominal gross risk-free rates are $R_{f,t}^c \equiv \exp(r_t)$ and $R_{f,t}^\$ \equiv \exp(i_t)$, respectively. One-period expected excess returns relative to the risk-free rate are $\mathbb{E}_t \left[X R_{t,t+1}^{\ell,(n)} \right] = \mathbb{E}_t \left[R_{t,t+1}^{\ell,(n)} \right] - R_{f,t}^\ell$, and Sharpe ratios are $S R_t^{\ell,(n)} \equiv \frac{\mathbb{E}_t \left[X R_{t,t+1}^{\ell,(n)} \right]}{\sigma_t \left(X R_{t,t+1}^{\ell,(n)} \right)}$, for $\ell = \{c, \$\}$. In equilibrium, $\mathbb{E}_t \left[X R_{t,t+1}^{\ell,(n)} \right] = -R_{f,t}^\ell \text{cov}_t \left(M_{t,t+1}^\ell, X R_{t,t+1}^{\ell,(n)} \right)$, where $M_{t,t+1}^c \equiv M_{t,t+1}$. Expected excess bond returns capture the compensation for macroeconomic risk in long-term bonds. This compensation depends on the correlation between bond returns and the

marginal utility of consumption.

The one-period real term premium of an n -period (real) bond is defined as

$$rTP_t^{(n)} \equiv \log \mathbb{E}_t \left[R_{t,t+1}^{c,(n)} \right] - \log R_{f,t}^c. \quad (22)$$

Appendix E shows that this premium and the average spread $r_t^{(n)} - r_t$ can be approximated as¹⁸

$$rTP_t^{(n)} = \text{cov}_t \left(m_{t,t+1}, (n-1)r_{t+1}^{(n-1)} \right), \quad \text{and} \quad \mathbb{E} \left[r_t^{(n)} - r_t \right] = \text{J.I.}_r^{(n)} + \frac{1}{n} \sum_{s=0}^{n-2} \mathbb{E} \left[rTP_{t+s}^{(n-s)} \right], \quad (23)$$

respectively, where $m_{t,t+1} \equiv \log M_{t,t+1}$, and J.I. denotes Jensen's inequality terms not important for the analysis. The real term premium captures the correlation between the marginal utility of consumption and the bond one-period return. This return depends on the bond yield at the end of the period. A positive correlation between marginal utility and the bond yield implies low bond real returns during periods of high marginal utility and, therefore, positive expected excess bond returns. The unconditional yield spread can be seen as an average of one-period real term premia during the life of the bond.

The one-period inflation risk premium $\pi TP_t^{(n)}$ is the difference in (log) real return for investing in an n -period nominal bond over an n -period real bond for one-period. That is,

$$\pi TP_t^{(n)} \equiv \log \mathbb{E}_t \left[R_{t,t+1}^{\$, (n)} P_t / P_{t+1} \right] - \log \mathbb{E}_t \left[R_{t,t+1}^{c,(n)} \right], \quad (24)$$

Appendix E shows that this premium and the average spread $i_t^{(n)} - r_t^{(n)}$ can be approximated as

$$\pi TP_t^{(n)} = \text{cov}_t \left(m_{t,t+1}, \sum_{s=1}^n \pi_{t+s} \right), \quad \text{and} \quad \mathbb{E} \left[i_t^{(n)} - r_t^{(n)} \right] = \mathbb{E}[\pi_t] + \text{J.I.}_\pi^{(n)} + \frac{1}{n} \sum_{s=0}^{n-1} \mathbb{E} \left[\pi TP_{t+s}^{(n-s)} \right], \quad (25)$$

¹⁸As shown in the appendix, this derivation relies on the the assumption of joint normality for the log-pricing kernel and bond yields. This is used only for illustration purposes, since the economic model is solved using a second-order perturbation method, which does not imply log-normality. Similar approximations are used throughout the paper for illustration purposes only. Equation (22) is used for the computation of real term premia in the quantitative analysis.

The inflation risk premium is then an expected return compensation in nominal bonds for the correlation between the marginal utility of consumption and inflation. If this correlation is positive, the expected real returns of nominal bonds are higher than for real bonds: during periods of high marginal utility, high inflation has a negative impact on nominal bond returns. The unconditional spread between nominal and real rates captures average inflation and inflation risk premia.

4 Model Implications and Analysis

This section reports and analyzes the main model implications for bond yields and risk premia. It describes first the model estimation and its quantitative performance capturing macroeconomic and yield curve dynamics simultaneously. The main findings are highlighted by comparing the baseline model's performance with alternative model specifications for nominal price and wage rigidities, model shocks, and monetary policy. The economic mechanisms behind the quantitative findings are explained based on approximate analytical solutions for real term and inflation risk premia, and their link.

4.1 Estimation Strategy

The purpose of the model estimation is (i) to examine the model's quantitative ability to simultaneously capture observed macroeconomic and nominal yield curve dynamics, and (ii) to provide a quantitative framework for the economic analysis of the real yield curve and bond risk premia. Model parameters are chosen to capture key quarterly properties of U.S. data for the period 1982:Q1 to 2008:Q3 using the Generalized Method of Moments (GMM). The sample period is chosen to focus on a monetary policy with a stable response to economic conditions, which can be described by an interest-rate policy rule. Clarida, Galí and Gertler (2000) provide empirical evidence of a change in monetary policy after 1979. The monetary experiment period 1979 – 1981 is excluded since the short-term rate was replaced by monetary aggregates as the policy instru-

ment during this period. Data after the third quarter of 2008 are not included since the ability to conduct policy using the Federal Funds rate was limited by the zero bound after December 2008.

Table 2 reports the parameter values for the baseline model. The model estimation involves three sets of parameters.¹⁹ For the first set, parameters values are assigned to match a direct empirical counterpart or to be consistent with the literature. The average productivity growth rate g_a is chosen to match the average consumption growth during the period. Non-optimal changes in prices and wages are assumed to be perfectly indexed to the inflation target, such that $\log \Lambda_{p,t,t+1} = \pi_t^*$, and $\log \Lambda_{w,t,t+1} = g_a + \pi_t^*$. The wage indexation implies no deviations from real wages on average. The price duration of $-1/\log(\alpha_p) \approx 2.4$ quarters is consistent with the empirical evidence in Bils and Klenow (2004). The wage duration of $-1/\log(\alpha_w) \approx 4$ quarters is consistent with the evidence in Barattieri, Basu and Gottschalk (2014). The elasticity parameters θ_p and θ_w imply price and wage markups of 20%. The value chosen for ω implies a Frisch labor elasticity of $1/\omega = 2$, in the lower range of the values used in the macro literature to capture labor and wage dynamics. The policy responses to inflation $\iota_\pi = 1.5$ and the output gap $\iota_x = 0.125$ are standard in the literature. The persistence $\phi_{\pi^*} = 0.9999$ and volatility $\sigma_{\pi^*} = 0.001\%$ of the inflation target process are chosen to maximize the model’s ability to capture the high volatility of long-term yields, and are in line with the ones used by Rudebusch and Swanson (2012), and the unit root process in Campbell, Pflueger and Viceira (2014).

For a second set of parameters, values are estimated using the Generalized Method of Moments. This procedure focuses on maximizing the model’s ability to capture macroeconomic dynamics. Eight parameter values are chosen to minimize percentage deviations of nine model moments from their data counterparts.²⁰ The moments are the volatility and autocorrelation of consumption growth, inflation, wage growth, and the short-term nominal interest rate, and the correlation of

¹⁹The parametrization has elements of both estimation and calibration. For simplicity, we refer to it as “estimation” throughout the paper. The method is similar to that in Andreasen, Fernández-Villaverde and Rubio-Ramírez (2014). The model is solved using the Dynare package, available from www.dynare.org.

²⁰The estimation is restricted within a range of parameter values that are economically sensible.

consumption growth and inflation. The data series are described in Section 2.²¹ The estimated parameters are φ , ρ , and the persistence and volatility parameters of productivity and monetary policy shocks. The estimated elasticity of intertemporal substitution $1/\varphi = 0.05$, is in the lower range of values in the macroeconomic literature, and contrasts dramatically with the values used in the asset-pricing long-run risk literature. The interest-rate smoothing coefficient $\rho \approx 0.62$ in the policy rule is slightly lower than the one estimated by Clarida, Galí and Gertler (2000) for the period, but in line with values used in the literature. The persistence of policy shocks $\phi_u \approx 0.4$ is in the upper range of values estimated in the literature. The persistence parameters for both permanent and transitory productivity components are lower than those in Andreasen (2012).

Finally, values for the subjective discount factor β , the average inflation target g_π , and the risk aversion parameter γ are chosen to match the average (annualized) inflation rate of 3.26%, the short-term nominal (annualized) interest rate of 5.20%, and the Sharpe ratio of 0.32 implied by excess returns of the 5-year bond simultaneously.²² The policy rule constant $\bar{i} \equiv -\log \beta + \psi g_a + g_\pi$ is the nominal rate when both inflation and the output gap are at their respective targets. The average coefficient of risk aversion in the presence of leisure preferences, as shown by Swanson (2012), is given by²³

$$\frac{\varphi}{1 + \frac{\varphi}{\omega} \frac{\mu_w}{\mu_p}} + \frac{\gamma - \varphi}{1 - \frac{1-\varphi}{1+\omega} \frac{\mu_w}{\mu_p}} \approx 52.$$

This value is comparable to those used in models of the term structure with recursive preferences. For instance, Piazzesi and Schneider (2007) estimate a value of 59 in an endowment economy, and Rudebusch and Swanson (2012) and Andreasen (2012) use values between 75 and 110 in models

²¹Allowing ω , ϕ_{π^*} , and σ_{π^*} to be estimated implies a very similar performance matching these moments.

²²The model is solved using a second-order approximation around the non-stochastic steady state. The high value for γ generates large precautionary savings terms that affect the means of inflation and the short-term interest rate. The precautionary savings terms are offset by a large values for g_π , reducing its interpretation as a long-term inflation target. The approach does not generate distortions in expected excess bond returns.

²³In the presence of recursive preferences on consumption and labor, the coefficient of relative risk aversion is not solely determined by γ , since the ability to smooth consumption using labor changes the representative agent's attitudes towards risk. The coefficient is computed relative to intertemporal gambles on consumption-related wealth, since the coefficient related to total wealth (including the value of leisure) is not well defined.

with price rigidities.²⁴

4.2 Quantitative Performance of the Baseline Model

This section describes the model’s ability to simultaneously match macroeconomic and yield curve properties of the economy. The estimation centers almost entirely on matching macroeconomic moments, and uses only yield curve information to match the Sharpe ratio of the 5-year nominal bond. It is then important to verify that other properties of the nominal yield curve are captured by the estimation and provide a reasonable baseline for the analysis of the implied real yield curve.

Table 3 reports moments for the baseline model and their empirical counterparts. Simulated 90% confidence intervals are added from samples with the size of the data sample (107 quarters). Panel A reports the macroeconomic moments. The model captures well the volatilities of consumption growth and inflation, the autocorrelations of inflation and wage growth, and the negative correlation between consumption growth and inflation. This correlation is important for explaining a positive inflation risk premium. The model, however, generates lower volatility in wage growth than in the data, and fails to capture the reported empirical positive autocorrelation of consumption growth.²⁵ As explained below, a negative autocorrelation of consumption growth is crucial in the baseline model to generate negative autocorrelation in the pricing kernel and then a positive real term premium. Section 5 shows that an external habit in consumption preferences can simultaneously generate negative and positive autocorrelations in the pricing kernel and consumption growth, respectively, while preserving the main implications of the baseline model.

Panels B and C of Table 3 report yield curve and bond excess return statistics, respectively.

²⁴This value could be reduced by incorporating persistent sources of long-run risk as in Bansal and Shaliastovich (2013), or Kung (2014). Bansal and Shaliastovich (2013) achieve this in an endowment economy with exogenous inflation. Kung (2014) introduces endogenous growth to a New Keynesian model and generates an endogenous persistent source of long-run risk. We do not follow this approach to highlight the different effects of price and wage rigidities and different shocks in a standard New Keynesian framework.

²⁵It is well known however, that the autocorrelation of consumption growth is poorly measured and different empirical studies show mixed evidence. While Campbell and Mankiw (1989) and Cochrane (1994), for instance, find that U.S. consumption growth is almost unforecastable, Kandel and Stambaugh (1991) and Mehra and Prescott (1985) imply a small persistent predictability component in this variable.

The baseline model implies an average 5-year nominal bond spread of 112 bps. vs. 138 bps. in the data, and a positive 5-year real bond spread of 82 bps. The model does a reasonable job capturing the volatility of the short-term nominal interest rate but fails to reproduce the high volatility of long-term nominal yields. This is a well-known shortcoming of most equilibrium models, given the lack of enough persistence in the explanatory macroeconomic variables. The model implications for the volatility of real and nominal yields are explored in the analysis below. By construction, the model reproduces the Sharpe ratio of the 5-year nominal bond, higher than the implied Sharpe ratio for the comparable real bond. However, the one-quarter expected bond excess return in the model is small relative to the average realized excess return in the data. It reflects the model limitation to capture the high volatility of bond returns. The positive 5-year real bond spread implies a real term premium and expected excess return of around 1%. The higher expected excess return for the comparable nominal bond reflects a positive inflation risk premium of 86 bps.

It is worth mentioning additional model implications not targeted in the estimation. Panel E of Table 3 shows a volatility of dividend growth in the model, $\sigma(\Delta d_t)$, similar to the one in the data. It implies an equity premium $\mathbb{E}[XR_d]$ slightly below the data counterpart. The R^2 's of regressions of yields on consumption growth and inflation also are presented in the table to highlight that the model can reasonably capture the joint dynamics of yields and macroeconomic variables.

In summary, the baseline model provides a reasonable description of U.S. macroeconomic and yield dynamics, and then a good framework for the quantitative analysis of the real term structure. The model, however, has some limitations that will be addressed in Section 5.

4.2.1 Alternative model specifications and comparison to similar models

This section presents results of alternative estimations for models with only one rigidity (prices or wages) or none, and exposed to one or two components of productivity shocks (permanent and transitory). A baseline model with both rigidities and both productivity components is chosen

for two reasons. First, it provides the best joint fit of macroeconomic and yield curve properties among the different tested specifications. Second, there is ample empirical evidence of both types of rigidities for the United States. Implications of models with only one type of rigidity may not survive in a model with both rigidities, and then do not apply to the U.S. economy.

Table 4 reports results for the alternative estimations. Parameter values are chosen following the procedure for the baseline estimation explained above. Column (10) corresponds to the baseline estimation, except for the parameter γ .²⁶ The objective values in Panel A show that having two components in productivity shocks instead of one has a significant effect matching the macroeconomic moments under study. The model with the best (lowest) objective value is the one with no rigidities and two productivity components in column (13). The improved performance is driven by the implied positive autocorrelation of consumption growth. However, this model is unable to capture a positive average slope in the nominal yield curve, implying both negative real term and inflation risk premia and low short-term yield volatility. On the other hand, models in columns (10) to (12), with two productivity components and only one or both rigidities, imply a positive nominal bond spread. The one with both rigidities generates the highest one.

A comparison of columns (2) and (6) shows the different effects of permanent and transitory productivity shocks under both rigidities. Panels B and C show that both models imply positive real term and inflation risk premia. These premia are significantly smaller in the model with only a transitory component. The models in columns (3) and (7) with only wage rigidities and a permanent or transitory productivity component, respectively, have a significant ability to capture a positive 5-year nominal bond spread. However, the model in column (7) implies a lower spread and yield volatility. Columns (4) and (5) show the inability of the models with permanent shocks and price or no rigidities, respectively, to capture significant bond risk premia. The model with only price rigidities and a transitory component in column (8) captures positive bond risk premia,

²⁶The parameter values for the alternative estimations are not reported to save space. They are available upon request. For comparison purposes, a value of $\gamma = 400$ was used for all estimations in the table. Using the baseline estimation value of $\gamma = 720$, it was not possible to match the average levels of inflation and the interest rate for some specifications.

but its magnitude is small in comparison to the model in column (12). The model with no rigidities and a transitory productivity component in column (9) is able to capture significantly positive bond risk premia. In this model, however, monetary policy has no effects on the real yield curve and there is a low correlation between real and nominal yields.

It is convenient to compare these results with those of other models in the literature. The model in Rudebusch and Swanson (2012) has only price rigidities and only a transitory component in productivity shocks, as the one in column (8). However, their estimation captures much larger bond risk premia. Possible reasons for this difference are targeting different moments in the estimation, or heterogeneity in capital, which is absent in our framework. The model in Andreasen (2012) is similar to the one in column (12) and is shown to fit U.S. data well. A direct comparison, however, is difficult since the model has additional elements such as habit persistence and other shocks, the estimation methods are different and unconditional moments are not reported. Dew-Becker (2014) estimates a model with permanent shocks and both price and wage rigidities similar to the one in column (2), but with additional elements such as habit persistence, time-varying risk aversion, and capital accumulation. As described above, this model has a significant ability to capture positive bond risk premia. Finally, Kung (2014) calibrates a model with only price rigidities and transitory shocks, as in column (8), but with capital accumulation and endogenous growth. It generates positive inflation risk premia but negative real term premia.

4.3 Bond Risk Premia and Yield Volatility

This section describes the contribution of nominal rigidities and shocks to the dynamics of real and nominal bond yields and risk premia in the baseline model. The findings are explained using tables 5 and 6. Table 5 reports variance decompositions of relevant variables to understand the contribution of each shock to their dynamics. Table 6 presents statistics of models that share the same parameter values with the baseline estimation, except for rigidity or shock parameters. It highlights the contribution of each rigidity and shock to the quantitative results. Columns (2)-(5)

in this table are related to parameterizations where price or wage rigidities or both are shut down, but all shocks affect the economy. Columns (6)-(9) are related to parameterizations where the economy is exposed only to one source of risk, but both rigidities are active.²⁷

There are two main findings. First, positive real term and inflation risk premia are mainly a compensation for permanent productivity shocks as a result of wage rigidities. Table 5 shows that permanent productivity shocks generate most of the variability in consumption growth and the real and nominal pricing kernels. Consistent with this, column (6) in Table 6 shows that the 5-year real term and inflation risk premia for these shocks are 77 and 84 bps. respectively, while columns (7)-(9) show that these premia are minor or negligible for the other shocks. A similar pattern is seen in bond spreads and expected excess returns. Column (2)-(4) show that only wage rigidities generate positive real and nominal risk premia in the baseline model. This finding also applies to the alternative estimations in Table 4. The model with only wage rigidities and only a permanent component in productivity has the best ability to generate positive bond risk premia.

The second finding is that monetary policy shocks and inflation target shocks have a minor contribution to risk premia, but generate significant volatility in real and nominal bond yields, respectively. From Table 5, monetary policy shocks explain more than 75% of the variability in real and nominal interest rates and yield spreads. Inflation target shocks explain almost 15% of the variability of the short-term nominal interest rate but do not affect the real counterpart. This is consistent with the results in Panel B of Table 6. From column (8), real and nominal interest rate volatility of interest rates generated by monetary policy shocks is close to their total volatility in the baseline model. From column (9), inflation target shocks generate similar volatility in nominal short- and long-term bond yields.

²⁷For comparison purposes, β and g_π are adjusted across parametrizations to match the average inflation and short-term nominal rates. This adjustment has a minor effect on second moments.

4.4 The Mechanism Behind Real Term and Inflation Risk Premia

In the model, positive real term and inflation risk premia are generated by wage rigidities as compensations for permanent productivity shocks. This section explores the economic mechanism behind this result. The effects of the rigidities on consumption through labor are explained first to understand the joint dynamics of the pricing kernel, bond returns, and inflation. The link between real term and inflation risk premia imposed by endogenous inflation is highlighted. To gain intuition, closed-form solutions are obtained assuming log-normality and homoscedasticity for all variables. Monetary policy and inflation target shocks are shut down to focus on productivity shocks. For simplicity, the interest-rate smoothing component in the policy rule is ignored ($\rho = 0$). To confirm the intuition from the approximate analytical solutions, impulse responses for the baseline model are computed using a second-order perturbation solution.²⁸

4.4.1 Labor dynamics and consumption growth

The effect of nominal rigidities on labor play a crucial role in driving the model results. A first-order approximation of the solution implies that labor $n_t \equiv \log N_t^s$ and consumption growth are given by

$$n_t = \bar{n}_{t-1} + n_a \Delta a_t + n_z z_t, \quad \text{and} \quad \Delta c_t = \Delta a_t + \Delta z_t + \Delta n_t,$$

respectively, where the term \bar{n}_{t-1} , and the constants n_a and n_z are determined in equilibrium.

Consider first an economy without nominal rigidities. It follows from equations (11) and (17) that

$$n_t = \bar{n} + \frac{1 - \varphi}{\omega + \varphi} z_t.$$

Labor does not react to the permanent component of productivity ($n_a = 0$), since wages adjust freely after a permanent shock to keep labor constant. On the other hand, labor reacts positively

²⁸The impulse responses are generated applying the pruning method of Kim et al. (2008) to compute simulations of the second-order approximation of the solution.

to transitory shocks ($n_z > 0$) if $\varphi < 1$, and negatively ($n_z < 0$) if $\varphi > 1$. Since φ is larger than one in the baseline model, labor decreases after a positive transitory shock. When productivity increases, more output is produced for the same amount of labor. Since the shock is transitory and the elasticity of intertemporal substitution of consumption is low, the representative household works less to smooth consumption. Consumption increases ($1 + n_z > 0$), since the lower labor does not completely offset the increase in productivity.

In the presence of rigidities, labor supply reacts positively to a permanent productivity shock ($n_a > 0$), and negatively to a transitory productivity shock ($n_z < 0$). This result is driven by the household's consumption smoothing incentives. After a negative permanent shock, productivity is low. Since the permanent productivity component is positively autocorrelated ($\phi_a > 0$), future expected productivity becomes even lower. To smooth consumption, the household works less the current period, resulting in a procyclical labor supply. On the contrary, after a negative transitory shock, consumption is low at the current period, but expected to increase in the future since the effect is mean-reverting. Consequently, the household works more to smooth consumption, resulting in a counter-cyclical labor supply.

The impulse responses from the second-order solution of the model confirm the analysis above. Figure 2 shows that after a negative permanent shock, labor supply in the model with no rigidities stays constant, while it decreases in the models with price or wage rigidities. Figure 3 shows that, after a negative transitory shock, labor supply increases under all specifications. Consumption decreases after a negative permanent or transitory shock.

4.4.2 The real pricing kernel and risk-free rate

Appendix E shows that the real pricing kernel in equation (6) can be approximated as

$$m_{t,t+1} = \bar{m} - \varphi \Delta c_{t+1} - (\gamma - \varphi) \sum_{s=0}^{\infty} \eta_{vc}^s (\mathbb{E}_{t+1} - \mathbb{E}_t) [\Delta c_{t+1+s} + \eta_n n_{t+1+s} + \eta_z z_{t+1+s}], \quad (26)$$

where \bar{m} , $0 < \eta_{vc} < 1$, η_n , and η_z are constants. The second term in the pricing kernel captures the link between consumption growth and its marginal rate of substitution under constant relative risk aversion ($\varphi = \gamma$). The third term is the dependence of this rate on continuation utility under recursive preferences ($\varphi \neq \gamma$). It captures revisions in expectations of consumption growth, labor, and transitory shocks. It depends on labor supply (and transitory shocks) as a result of the household's preference on labor. In the absence of rigidities, the labor terms are proportional to consumption and the pricing kernel becomes identical to the one with no labor preferences. In the presence of rigidities, the proportionality breaks down.²⁹ However, the effect of labor is quantitatively minor in the baseline model. The pricing kernel impulse responses in Figures 2 and 3 show that they are not substantially different from each other with and without rigidities.

From equation (7), the real risk-free rate is approximated as $r_t = \bar{r} + \varphi \mathbb{E}_t[\Delta c_{t+1}]$, similar to the one in a CRRA economy. The recursive preferences terms only generate (constant) precautionary savings terms since, by definition, expected revisions in expectations are zero.

4.4.3 Real Term Premia

The risk compensation for holding long-term real bonds is considerably affected by nominal rigidities through the dynamics of labor and consumption growth above. For intuition, consider first the one-period real term premium in a 2-period bond. From equation (23), this premium depends on the covariance between the real pricing kernel and the real risk-free rate one-period in the future, $rTP_t^{(2)} = \text{cov}_t(m_{t,t+1}, r_{t+1}) = -\text{cov}_t(m_{t,t+1}, \mathbb{E}_{t+1}[m_{t+1,t+2}])$. Thus, if the pricing kernel is negatively autocorrelated, the real term premium is positive, and vice versa. Since the pricing kernel is dominated by consumption growth in the baseline model, the sign of the real risk premium depends on the autocorrelation of consumption growth as in Piazzesi and Schneider (2007)).

More generally, consider a real console bond that pays one unit of consumption every period.

²⁹Specifically, labor preferences affect the household's continuation utility and then the return on wealth. Wealth is a claim on all future consumption and the opportunity cost of leisure. Since labor income is proportional to consumption in the absence of rigidities, the return on wealth is proportional to the return on the consumption claim. This is not the case in the presence of rigidities.

This bond can be seen as a portfolio of real zero-coupon bonds with maturities from one to infinity. The console bond risk premium is then a weighted average of the real risk premia of these bonds. Appendix E shows that a log-linear approximation of the return on the console bond can be written as

$$r_{\infty,t+1}^c = \bar{\eta}_{\infty}^c + \eta_{\infty} p_{\infty,t+1}^c - p_{\infty,t}^c, \quad \text{where} \quad p_{\infty,t}^c = \bar{p}_{\infty}^c + p_{\infty,a}^c \Delta a_t + p_{\infty,z}^c z_t$$

is the log-bond price, for approximation constants $\bar{\eta}_{\infty}^c$, $0 < \eta_{\infty} < 1$, and equilibrium constants \bar{p}_{∞}^c ,

$$p_{\infty,a}^c = -\frac{\varphi(\phi_a - (1 - \phi_a)n_a)}{1 - \eta_{\infty}\phi_a}, \quad \text{and} \quad p_{\infty,z}^c = \frac{\varphi(1 - \phi_z)(1 + n_z)}{1 - \eta_{\infty}\phi_z}. \quad (27)$$

The one-period real term premium of the console bond is $rTP_t^{\infty} = -\text{cov}_t(m_{t,t+1}, r_{\infty,t+1}^c)$. The impulse responses in Figures 2 and 3 show that (real) marginal utility increases under negative permanent and transitory shocks. Therefore, the real term premium is positive if the return on the console bond decreases after negative productivity shocks, i.e., $p_{\infty,a}^c > 0$ and $p_{\infty,z}^c > 0$.

Consider first the permanent component of productivity. The coefficient $p_{\infty,a}^c$ in equation (27) is positive if the response of labor to permanent shocks is strong enough, i.e., $n_a > \frac{\phi_a}{1 - \phi_a}$. This condition characterizes the autocorrelation of consumption growth induced by the permanent component. In the absence of rigidities, labor is not affected by permanent shocks ($n_a = 0$), consumption growth inherits the properties of productivity growth and then is positively autocorrelated. Low consumption today implies low expected future consumption and low bond yields. Real bonds are hedging instruments with negative term premia since bond returns are high in periods of high marginal utility. In the presence of price and/or wage rigidities, labor supply becomes procyclical ($n_a > 0$) and mean-reverting, and the autocorrelation of consumption growth declines and becomes negative if $n_a > \frac{\phi_a}{1 - \phi_a}$. After a negative permanent productivity shock, prices and/or wages do not decline as much as in an economy with flexible prices and wages, and reduce labor. The decreased labor reduces consumption by more than in an economy with no rigidities.

Over time, prices and/or wages gradually decline, increasing labor. If the effect is strong enough, expected future consumption growth is positive, bond yields increase, and bond returns are low in periods of high marginal utility. Therefore, real term premia for permanent productivity shocks can become positive if the rigidities induce strong effects on labor. The impulse responses in Figure 2 show that models with no rigidities or only price rigidity generate a negative real term premium, since the excess return of the five year-bond $XR^{c,(20)}$ has a positive reaction to a negative permanent shock. Only the models with wage rigidity or both rigidities generate a positive real term premium.

Similarly, the sign of the real term premium for transitory productivity shocks depends on the autocorrelation of consumption growth induced by these shocks. This autocorrelation depends on the term $-(1 - \phi_z)(1 + n_z)$ in equation (27). It is negative as long as $n_z > -1$. This condition is always satisfied with or without nominal rigidities, and then the term premium for transitory shocks is positive. Since nominal rigidities affect the magnitude of the response of labor to these shocks, the magnitude of the real term premia depends on the rigidities. The impulse responses of $XR^{c,(20)}$ to a negative transitory shock in Figure 3 indicate that models under all specifications generate a positive real term premium for these shocks. The magnitude is small relative to the premium for permanent shocks since marginal utility has a small response to transitory shocks.

4.4.4 Inflation Risk Premia

The compensation for inflation risk in nominal bonds depends on the dynamics of the real pricing kernel and inflation. Appendix E shows that the inflation process can be approximated as

$$\pi_t = \bar{\pi}_{t-1} + \pi_a \Delta a_t + \pi_z z_t,$$

where $\bar{\pi}_{t-1}$,

$$\pi_a = \frac{\varphi(\phi_a - (1 - \phi_a)n_a) - \iota_x n_a}{\iota_\pi - \phi_a}, \quad \text{and} \quad \pi_z = \frac{-\varphi(1 - \phi_z)(1 + n_z) - \iota_x \left(n_z - \frac{1 - \varphi}{\omega + \varphi} \right)}{\iota_\pi - \phi_z}, \quad (28)$$

are determined in equilibrium. For intuition, consider first the one-period bond inflation risk premium $\pi T P_t^{(1)} = \text{cov}_t(m_{t,t+1}, \pi_{t+1})$. From equation (26), it is

$$\pi T P_t^{(1)} = -\varphi \text{cov}_t(\Delta c_{t+1}, \pi_{t+1}) - (\gamma - \varphi) \sum_{s=0}^{\infty} \eta_{vc}^s \text{cov}_t(\mathbb{E}_{t+1}[\Delta c_{t+1+s} + \eta_n n_{t+1+s} + \eta_z z_{t+1+s}], \pi_{t+1}),$$

Under constant relative risk aversion ($\varphi = \gamma$), it is positive as long as consumption growth and inflation are negatively autocorrelated. Under recursive preferences ($\varphi \neq \gamma$), the premium also depends on the correlations between inflation and expected future consumption growth and labor. All these terms are affected by nominal rigidities.

More generally, consider a nominal console bond that pays one unit of nominal consumption every period. Appendix E shows that the inflation risk premium can be approximated as

$$\pi T P_t^\infty = \pi T P_t^{(1)} - \eta_\infty \text{cov}_t(m_{t,t+1}, p_{\infty,t}^s - p_{\infty,t}^c), \quad \text{where} \quad p_{\infty,t}^s - p_{\infty,t}^c = -\frac{\phi_a \pi_a}{1 - \eta_\infty \phi_a} \Delta a_t - \frac{\phi_z \pi_z}{1 - \eta_\infty \phi_z} z_t.$$

The inflation risk premium depends on the one-period bond inflation risk premium and the correlation between the real pricing kernel and differences in the valuation of the nominal and real console bonds, $p_{\infty,t}^s - p_{\infty,t}^c$, which in turn depends on the inflation process. The inflation risk premia for permanent and transitory productive shocks are positive if $\pi_a < 0$ and $\pi_z < 0$, respectively, since the pricing kernel reacts negatively to both types of shocks. In the baseline model, ι_x is small and the values of π_a and π_z are mainly determined by the first term of the numerators in equations (28). The denominators are positive since $\iota_\pi > 1$. The equation for π_a indicates that inflation reacts negatively to the permanent shock ($\pi_a < 0$) if $n_a > \frac{\phi_a}{1 - \phi_a}$, resulting in a positive inflation risk premium for permanent shocks. The equation for π_z shows that inflation reacts negatively to

transitory shocks since $n_z > -1$, resulting in a positive inflation risk premium for this shock.

The response of inflation to permanent productivity shocks can be explained as follows. After a negative permanent shock, productivity is low and expected to be even lower in the future. In the absence of nominal rigidities, wages and prices decrease to keep labor and product markups constant. In the presence of rigidities, labor positively co-moves with the shocks ($n_a > 0$) and becomes mean-reverting. Product prices can increase or decrease after a negative shock depending on the labor response. Under wage rigidities, prices can adjust upwards to keep product markups constant. This occurs if $n_a > \frac{\phi_a}{1-\phi_a}$. Figure 2 shows that, in the baseline model, the labor response n_a is significant as a result of wage rigidities, inflation co-moves negatively with permanent shocks, and then nominal bonds embed a positive inflation risk premium.

Nominal rigidities amplify the response of inflation to transitory shocks, but not its sign. The impulse responses of inflation in Figure 3 show that product prices always increase after a negative shock. This is the result of the mean-reverting effects on productivity and, then, consumption of these shocks, with and without nominal rigidities. The positive response of inflation to a negative shock, decreases the value of nominal bonds in periods of high marginal utility, generating a positive inflation risk premium.

4.4.5 The policy rule and the link between real term and inflation risk premia

As explained above, real term premia for permanent and transitory productivity shocks are positive if $n_a > \frac{\phi_a}{1-\phi_a}$ and $n_z > -1$, respectively. Notice that a similar condition applies for positive inflation risk premia, as long as the response to the output gap ι_x in the policy rule is small. This is the result of the endogenous inflation process implied by the interest rate policy rule. For illustration, consider the real term premium of a two-period bond, $rTP_t^{(2)} = -\text{cov}_t(m_{t,t+1}, \mathbb{E}_{t+1}[m_{t+1,t+2}])$, and the one-period inflation risk premium $\pi TP_t^{(1)}$. From the policy rule equation (18), the inflation

risk premium can be written as

$$\pi TP_t^{(1)} = -\frac{\text{COV}_t(m_{t,t+1}, \mathbb{E}_{t+1}[m_{t+1,t+2}])}{\iota_\pi} - \frac{\iota_x \text{COV}_t(m_{t,t+1}, x_{t+1})}{\iota_\pi}.$$

Therefore, both real term and inflation risk premia depend on the autocorrelation of the pricing kernel. If $\iota_x = 0$, both premia are positive if the real pricing kernel is negatively autocorrelated. That is, inflation in equilibrium links real term and inflation risk premia, as emphasized by Gallmeyer et al. (2007, 2008). Appendix E presents a general specification for this link.

Relative to term structure models with exogenous inflation, the interest policy rule acts as an additional constraint to determine the joint properties of real and nominal yield curves. Tables 4 and 6 show that, in most tested model specifications, the real term and inflation risk premia share the same sign. This is explained by the joint dynamics of consumption growth, hence the pricing kernel, and inflation. Specifically, real term premia depend mostly on the autocorrelation of consumption growth, which also plays an important role determining the response of inflation to shocks, and then inflation risk premia. In models with exogenous inflation, real term and inflation risk premia can have different signs. This can be achieved in the model if the response in the policy rule to the output gap is strong enough. However, the values for these parameters used in the literature seem to be small, and imply the same sign for both types of premia.

4.5 Bond Yields and Risk Premia and Monetary Policy

The analysis above shows that both systematic and non-systematic responses in monetary policy affect the dynamics of bond yields and risk premia. The response to economic conditions in the policy rule affect both real term and inflation risk premia. Yield volatility for real and nominal bonds is affected by monetary policy and inflation target shocks. This section explores these findings in more detail.

4.5.1 Yield Volatility

Policy shocks have a minor effect on bond risk premia, but a significant one on the volatility of bond yields. The effect of these shocks on the real rate is a result of price and wage rigidities. The impulse responses in Figure 4 show that the real risk-free rate does not react to policy shocks in the absence of rigidities. Changes in the policy rule are completely reflected in inflation changes and have no effect on real activity. On the other hand, the figure shows that nominal rigidities induce a positive response in the real risk-free rate to positive policy shocks. Since prices and/or wages do not adjust optimally to changes in the nominal rate, output and consumption are affected. A positive policy shock, increases the real risk-free rate, which has a transitory negative effect on output. Since real bond prices depend on the expected evolution of short-term real rates, policy shocks also increase the volatility of the entire real yield curve.

Inflation target shocks have a considerable effect on inflation, but a minor effect on the output gap and, thus, bond risk premia. These shocks, however, have a significant contribution to the volatility of nominal long-term yields. The high persistence in inflation target shocks generates volatility in long-term nominal yields similar to the volatility of the one-period nominal rate. On the other hand, these shocks have a reduced effect on real yield volatility, even in the presence of nominal rigidities. Therefore, changes in the properties of inflation target shocks may explain differences between real and nominal yield volatility over time, while not affecting the main dynamics of real economic variables.

4.5.2 Bond Risk Premia and the Monetary Policy Rule

The monetary policy interest-rate rule affects bond risk premia. To quantify these effects, comparative statics for policy rule parameters are computed. These parameters are modified individually, keeping the remaining parameters at their baseline model levels. Selected moments are computed and compared with the baseline estimation counterparts in Table 7. Column (3) reports statistics when the response to inflation in the policy rule ι_π is increased to 1.7 from 1.5 in the baseline

estimation. Similarly, column (4) reports statistics when the response to the output gap ι_x is increased to 0.25 from 0.125. A comparison of both columns with the baseline estimation in column (2) shows opposite effects of the two policy changes. While a stronger response to inflation decreases inflation volatility and increases real and nominal bond spreads and expected excess returns, a stronger response to the output gap increases inflation volatility slightly, and reduces real and nominal bond spreads and expected excess returns. For instance, an increase in ι_π of 0.2 is reflected in an increase in expected excess returns on real and nominal 5-year bonds of 23 and 17 bps., respectively. An increase in ι_x of 0.125 reduces these expected returns by 9 and 10 bps., respectively. Changes in expected excess returns are explained by the effects of the policy rule on labor dynamics. An increase in the response to inflation, increases the response of labor to productivity shocks and real term premia. An increase in the response to the output gap has the opposite effect. Column (5) presents statistics for a policy that increases the interest-rate smoothing coefficient ρ from the baseline value of 0.62 to 0.72. Similar to a reduction in the response to inflation, this policy increases inflation volatility and decreases real and nominal bond spreads, expected excess returns, and Sharpe ratios. In addition, the reduction in the smoothing coefficient increases dramatically the volatility of the real rate, but has a minor effect on the volatility of nominal yields. Finally, column (6) presents statistics when the autocorrelation of the inflation target is reduced from 0.9999 to 0.9. This change only affects the volatility of long-term nominal yields. The ratio of long- to short-term yield volatility decreases from 0.4 to 0.13 with this change.

5 Model Limitations and Extensions

The baseline model captures important macroeconomic and nominal yield curve properties, but with important limitations. This section extends the model to address three of these limitations: (i) a negative autocorrelation of consumption growth to explain positive bond risk premia, (ii) the absence of capital accumulation in the economy, and (iii) the lack of significant time variation in

bond risk premia.³⁰

5.1 Habit Persistence and the Autocorrelation of Consumption Growth

A model with external habit persistence in preferences is able to match the positive autocorrelation of consumption growth in the data, while preserving the previous results for bond risk premia. Consider the modified household's utility specification of equation (1) given by

$$V_t = (1 - \beta)U(C_{h,t}, N_t^s)^{1-\varphi} + \beta\mathbb{E}_t \left[V_{t+1}^{\frac{1-\gamma}{1-\varphi}} \right]^{\frac{1-\varphi}{1-\gamma}},$$

where the habit-adjusted consumption $C_{h,t} \equiv C_t - b_h \tilde{C}_{t-1}$ replaces consumption. The habit is represented by lagged aggregate consumption \tilde{C}_{t-1} , equal to C_{t-1} but not determined directly by the household. This is a simplified Campbell and Cochrane (1999) habit specification. Notice that $b_h = 0$ corresponds to the baseline case. The utility over the habit-adjusted consumption and labor $U(C_{h,t}, N_t^s)$ follows the functional form in equation (2). The real pricing kernel is isomorphic to the one in equation (6), but replacing consumption with the habit-adjusted consumption. Table 8 presents results for an estimation of this model using the procedure in Section 4. The parameter values from the GMM estimation are very similar to those of the baseline model (available upon request). The habit parameter $b_h = 0.42$ is within the range of values reported in the literature. As in the baseline model, unreported results show that the best model specification with habits incorporates both price and wage rigidities, and permanent and transitory components in productivity. The lower objective value in the estimation implies better macroeconomic performance than in the baseline model, driven mainly by the ability to match the positive autocorrelation of consumption growth in the data. A comparison of the bond yield and risk premia statistics shows that the two models have very similar implications for similar coefficients of relative risk

³⁰A third- or higher- order perturbation solution method is required to capture variability in expected excess returns and risk premia. The third-order solution of the optimality conditions in the baseline model implies a negligible volatility in these variables.

aversion around 29.^{31,32} The pricing kernel dynamics important for bond pricing are generated by the habit-adjusted consumption growth and not by consumption growth, providing additional flexibility to match the autocorrelation of consumption growth in the data.

5.2 Capital Accumulation and Bond Risk Premia

The baseline model economy has an only-labor production function. It is of interest to learn whether the bond risk premia mechanism and the results above hold under capital accumulation. Table 8 reports results for an estimation of a model with capital K_t and the production function $Y_t = (A_t N_t^d)^{1-\alpha} K_t^\alpha$. Capital follows the process

$$K_{t+1} = (1 - \delta)K_t + \Phi \left(\frac{J_t}{K_t} \right) K_t, \quad \text{where} \quad \Phi \left(\frac{J_t}{K_t} \right) = b_1 + \frac{b_2}{1 - 1/\zeta} \left(\frac{J_t}{K_t} \right)^{1-1/\zeta}$$

captures capital adjustment costs through $\zeta \geq 0$, J_t is investment, δ is the depreciation rate, and b_1 and b_2 are parameters chosen to preserve balanced growth.³³ The model has a reasonable macroeconomic performance using an adjustment cost $\zeta = 4$, similar to values reported in the literature. It matches the volatility of output and investment growth, the positive autocorrelation of consumption growth, and the negative correlation of consumption growth and inflation. However, the output gap is highly volatile, and the correlations between real and nominal yields are lower. As in the baseline estimation, the real and nominal average yield curves are upward sloping, but with substantially smaller and larger, respectively, real term and inflation risk premia. That is,

³¹The same can be said about the models with only one and no rigidities, and only a permanent or a transitory component in shocks. A difference, however, is that, in the model with habits, it is easier to capture a sizable risk premium in these specifications.

³²A parameter value of $\gamma = 400$ was used for comparison purposes. This value allows us to match the average levels of the inflation and short-term nominal rate in the data. The average coefficient of relative risk aversion for the model with habits is

$$\frac{\varphi}{1 - b_h e^{-g_a} + \frac{\varphi}{\omega} \frac{\mu_w}{\mu_p}} + \frac{\gamma - \varphi}{1 - b_h e^{-g_a} - \frac{1-\varphi}{1+\omega} \frac{\mu_w}{\mu_p}}.$$

³³The complete model specification, equilibrium conditions and estimated parameters are available under request.

capital accumulation weakens the strong link between real term and inflation risk premia imposed by the policy rule, by generating a large volatility in the output gap. An economic analysis of the additional mechanisms at work under capital accumulation is beyond the scope of the paper.

5.3 Stochastic Volatility and Time-Varying Bond Risk Premia

The low volatility in bond risk premia in the baseline model is at odds with the well-documented empirical evidence on deviations from the expectations hypothesis and bond return predictability. Adding time-varying volatility to productivity shocks captures substantial variation in bond risk premia. Consider the modified specifications for productivity shocks in equation (15) given by

$$\Delta a_{t+1} = (1 - \phi_a)g_a + \phi_a \Delta a_t + \sigma_a e^{\nu_a \Delta a_t} \varepsilon_{a,t+1}, \quad \text{and} \quad z_{t+1} = \phi_z z_t + \sigma_z e^{\nu_z z_t} \varepsilon_{z,t+1},$$

where $\nu_a \neq 0$ and $\nu_z \neq 0$ capture time-variation in the conditional volatility of the shocks. Volatility depends on the level of the productivity component, avoiding the need for new state variables. Table 9 reports results for two specifications with only time-varying volatility in only one productivity component at a time: $\nu_a = -100$, or $\nu_z = -100$, respectively. The negative signs capture the fact that volatility tends to increase during periods of high marginal utility. The magnitude implies that the level of volatility is 40% higher if a positive shock of size σ_a or σ_z , respectively, hits the economy. The table shows that bond risk premia become time-varying in specifications with stochastic volatility.³⁴ In particular, volatility in permanent shocks in the model with habit persistence generates the largest variability in bond risk premia. Real term premia are more or less volatile than inflation risk premia depending on whether the stochastic volatility is in the permanent or transitory productivity components, respectively.

³⁴A third-order perturbation of the model solution is required to capture the effects of time-varying volatility. The model moments are computed based on simulations that use the pruning method described in Andreasen, Fernández-Villaverde and Rubio-Ramírez (2014).

6 Conclusion

This paper provides a quantitative analysis of the link between nominal rigidities, monetary policy, and long-term real and nominal bond yields and risk premia. A key contribution is the analysis of the restrictions on the joint properties of real and nominal risk premia imposed by endogenous inflation. The model estimation implies an average upward sloping real curve, volatile long-term rates, and positive real term and inflation risk premia. These properties are consistent with those observed in the United States for inflation-linked and nominal bonds in recent years. There are three main findings. First, nominal rigidities increase term and inflation risk premia in real and nominal bonds, respectively, as a compensation for permanent productivity shocks. This is explained by a procyclical mean-reverting labor demand induced by the rigidities: It simultaneously generates high marginal utility of consumption, high inflation, and low returns on real and nominal bonds. Second, the model transitory shocks do not have significant effects on risk premia, but important effects on yield volatility. Monetary policy and inflation-target shocks increase the volatility of real and nominal bond yields, respectively. Third, a stronger response to inflation or a weaker response to output in the interest-rate policy rule increase real term and inflation risk premia. The strong link between the two premia is the result of this rule and its equilibrium effects on the pricing kernel and inflation.

The analysis can be extended in several dimensions. First, an empirical study of the model testable implications across countries. For instance, the model predicts lower real yield curve slopes in economies with more flexible wages. This is consistent with the average inverted real yield curve in the United Kingdom, and the findings in Smith (2000) and Dickens et al. (2007) of less rigid wages in United Kingdom than in United States. Second, a model with capital accumulation reduces the link between real and nominal yields and risk premia. This model and its underlying economic mechanisms deserve further analysis. Finally, the framework can be used to learn about the effects of optimal monetary policy on real rates and their economic content.

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A Household's Utility Maximization under Wage Rigidities

The household's problem is

$$\max_{\{C_t, N_t^s, W_t^*\}} V_t = U_t + \beta \mathbb{V}_t^{\frac{1-\varphi}{1-\gamma}}$$

where

$$U_t = \frac{C_{h,t}^{1-\varphi}}{1-\varphi} - \kappa_t \frac{(N_t^s)^{1+\omega}}{1+\omega}, \quad \text{and} \quad \mathbb{V}_t = \mathbb{E}_t \left[V_{t+1}^{\frac{1-\gamma}{1-\varphi}} \right],$$

subject to the budget constraint

$$\mathbb{E}_t \left[\sum_{\tau=0}^{\infty} M_{t,t+\tau}^{\$} P_{t+\tau} C_{t+\tau} \right] \leq \mathbb{E}_t \left[\sum_{\tau=0}^{\infty} M_{t,t+\tau}^{\$} P_{t+\tau} (LI_{t+\tau} + D_{t+\tau}) \right],$$

where LI_t and D_t are aggregate labor income and firm profits, respectively. The Lagrangian associated with this problem is

$$\mathcal{L} = \frac{C_{h,t}^{1-\varphi}}{1-\varphi} - \kappa_t \frac{(N_t^s)^{1+\omega}}{1+\omega} + \beta \mathbb{V}_t^{\frac{1-\varphi}{1-\gamma}} + \lambda \mathbb{E}_t \left[\sum_{\tau=0}^{\infty} M_{t,t+\tau}^{\$} P_{t+\tau} (LI_{t+\tau} + D_{t+\tau} - C_{t+\tau}) \right].$$

It can be shown that utility maximization implies $\lambda = \frac{C_{h,t}^{-\varphi}}{P_t}$, and

$$\begin{aligned} M_{t,t+1}^{\$} &= \frac{\partial V_t / \partial C_{t+1}}{\partial V_t / \partial C_t} \frac{P_t}{P_{t+1}} = \beta \frac{\frac{\partial \mathbb{V}_t}{\partial C_{t+1}} \frac{\partial V_t}{\partial \mathbb{V}_t}}{C_{h,t}^{-\varphi}} \frac{P_t}{P_{t+1}} \\ &= \beta \left(\frac{C_{h,t+1}}{C_{h,t}} \right)^{-\varphi} \left(\frac{V_{t+1}^{1/(1-\varphi)}}{\mathbb{V}_t^{1/(1-\gamma)}} \right)^{\varphi-\gamma} \frac{P_t}{P_{t+1}}. \end{aligned}$$

The τ -period nominal pricing kernel is

$$M_{t,t+\tau}^{\$} = \prod_{s=1}^{\tau} M_{t,t+s}^{\$}.$$

The household cannot change wages for α_w fraction of labor types. For the remaining $1 - \alpha_w$ fraction of labor types k , the household chooses wages $W_t^*(k)$ to maximize V_t . We assume that the wage choice for one labor type has negligible effects on the aggregate wage index and the aggregate labor demand. To see the impact of $W_t^*(k)$ on the household's utility, we rewrite the labor supply at $t + \tau$ as

$$N_{t+\tau}^s = \int_0^1 N_{t+\tau}^s(k) dk = N_{t+\tau}^d \int_0^1 \left(\frac{W_{t+\tau}(k)}{W_{t+\tau}} \right)^{-\theta_w} dk,$$

and the aggregate labor income at $t + \tau$ as

$$LI_{t+\tau} = \int_0^1 \frac{W_{t+\tau}(k)}{P_{t+\tau}} N_{t+\tau}^s(k) dk = \frac{N_{t+\tau}^d W_{t+\tau}}{P_{t+\tau}} \int_0^1 \left(\frac{W_{t+\tau}(k)}{W_{t+\tau}} \right)^{1-\theta_w} dk.$$

For the wage of type k labor at $t + \tau$, there are $\tau + 2$ possible values:

$$W_{t+\tau}(k) = \begin{cases} W_{t+\tau-s}^*(k), & \text{with prob} = (1 - \alpha_w)\alpha_w^s \text{ for } s = 0, 1, \dots, \tau \\ W_{t-1}\Lambda_{w,t-1,t+\tau}, & \text{with prob} = \alpha_w^{\tau+1}. \end{cases}$$

We obtain derivatives

$$\begin{aligned} \frac{\partial N_{t+\tau}^s}{W_t^*(k)} &= N_{t+\tau}^d (1 - \alpha_w)\alpha_w^\tau \left(\frac{-\theta_w}{W_t^*(k)} \right) \left(\frac{W_t^*(k)\Lambda_{w,t,t+\tau}}{W_{t+\tau}} \right)^{-\theta_w}, \\ \frac{\partial LI_{t+\tau}}{\partial W_t^*(k)} &= \frac{N_{t+\tau}^d}{P_{t+\tau}} (1 - \alpha_w)\alpha_w^\tau (1 - \theta_w) \left(\frac{W_t^*(k)\Lambda_{w,t,t+\tau}}{W_{t+\tau}} \right)^{-\theta_w}. \end{aligned}$$

The first-order condition of the Lagrangian with respect to $W_t^*(k)$ is given by

$$\frac{\partial \mathcal{L}}{\partial W_t^*(k)} = \frac{\partial V_t}{\partial W_t^*(k)} + \lambda \mathbb{E}_t \left[\sum_{\tau=0}^{\infty} M_{t,t+\tau}^s P_{t+\tau} \frac{\partial LI_{t+\tau}}{\partial W_t^*(k)} \right] = 0,$$

where

$$\frac{\partial V_t}{\partial W_t^*(k)} = -\mathbb{E}_t \left[\sum_{\tau=0}^{\infty} M_{t,t+\tau}^s \frac{P_{t+\tau}}{P_t} \left(\frac{C_{h,t+\tau}}{C_{h,t}} \right)^\varphi \kappa_{t+\tau} (N_{t+\tau}^s)^\omega \frac{\partial N_{t+\tau}^s}{\partial W_t^*(k)} \right].$$

Rearranging terms, we get

$$\mathbb{E}_t \left[\sum_{\tau=0}^{\infty} M_{t,t+\tau}^s \Lambda_{w,t,t+\tau} \alpha_w^\tau W_{t+\tau}^{\theta_w} N_{t+\tau}^d \frac{W_t^*(k)}{P_t} C_{h,t}^{-\varphi} \right] = \mathbb{E}_t \left[\sum_{\tau=0}^{\infty} M_{t,t+\tau}^s \Lambda_{w,t,t+\tau} \alpha_w^\tau \left(\frac{P_{t+\tau}}{P_t} \right) W_{t+\tau}^{\theta_w} N_{t+\tau}^d \mu_w \kappa_{t+\tau} (N_{t+\tau}^s)^\omega \left(\frac{C_{h,t+\tau}}{C_{h,t}} \right)^\varphi \right].$$

Since all labor types face the same demand curve, we have $W_t^*(k) = W_t^*$ for all k . We can write the left-hand side of the equation as

$$LHS = C_{h,t}^{-\varphi} W_t^{\theta_w} N_t^d H_{w,t} \frac{W_t^*}{P_t},$$

where

$$H_{w,t} = 1 + \alpha_w \mathbb{E}_t \left[M_{t,t+1}^s \Lambda_{w,t,t+1} \left(\frac{N_{t+1}^d}{N_t^d} \right) \left(\frac{W_t}{W_{t+1}} \right)^{-\theta_w} H_{w,t+1} \right].$$

Similarly, the right-hand side of the first-order condition can be written as

$$RHS = \mu_w W_t^{\theta_w} N_t^d (N_t^s)^\omega G_{w,t} = \mu_w W_t^{\theta_w} N_t^d \kappa_t (N_t^s)^\omega G_{w,t}$$

where

$$G_{w,t} = 1 + \alpha_w \mathbb{E}_t \left[M_{t,t+1}^s \Lambda_{w,t,t+1} \left(\frac{P_{t+1}}{P_t} \right) \left(\frac{C_{h,t+1}}{C_{h,t}} \right)^\varphi \left(\frac{N_{t+1}^d}{N_t^d} \right) \left(\frac{\kappa_{t+1}}{\kappa_t} \right) \left(\frac{N_{t+1}^s}{N_t^s} \right)^\omega \left(\frac{W_t}{W_{t+1}} \right)^{-\theta_w} G_{w,t+1} \right].$$

The optimal real wage and the optimal wage markup $\mu_{w,t}$ are then given by

$$\frac{W_t^*}{P_t} = \mu_{w,t} C_{h,t}^\varphi \kappa_t (N_t^s)^\omega \quad \text{and} \quad \mu_{w,t} = \mu_w \frac{G_{w,t}}{H_{w,t}}.$$

B Profit Maximization under Price Rigidities

Consider the Dixit-Stiglitz aggregate (3) as a production function, and a competitive “producer” of a differentiated good facing the problem

$$\max_{\{C_t(j)\}} P_t C_t - \int_0^1 P_t(j) C_t(j) dj$$

subject to (3). Solving the problem, we find the demand function

$$P_t(j) = P_t \left(\frac{C_t(j)}{C_t} \right)^{-1/\theta_p} \quad (29)$$

The zero-profit condition implies

$$P_t C_t = \int_0^1 P_t(j) C_t(j) dj = \int_0^1 P_t C_t \left(\frac{P_t(j)}{P_t} \right)^{-\theta_p} dj.$$

Solving for P_t , it follows that

$$P_t = \left[\int_0^1 P_t(j)^{1-\theta_p} dj \right]^{\frac{1}{1-\theta_p}}, \quad (30)$$

which can be written as the demand function for each differentiated good

$$C_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-\theta_p} C_t. \quad (31)$$

Therefore, when prices are flexible, prices of all differentiated goods are the same.

The profit maximization problem is

$$\max_{\{P_t(j)\}} \mathbb{E}_t \left[\sum_{\tau=0}^{\infty} M_{t,t+\tau}^{\$} \alpha^\tau \left[\Lambda_{p,t,t+\tau} P_t(j) Y_{t+\tau|t}(j) - W_{t+\tau|t}(j) N_{t+\tau|t}^d(j) \right] \right]$$

subject to

$$Y_{t+\tau|t}(j) = Y_{t+\tau} \left(\frac{P_t(j) \Lambda_{p,t,t+\tau}}{P_{t+\tau}} \right)^{-\theta_p}, \quad \text{and} \quad Y_{t+\tau|t}(j) = A_t N_{t+\tau|t}^d(j).$$

The first-order condition of this problem with respect to $P_t(j)$ is

$$\mathbb{E}_t \left[\sum_{\tau=0}^{\infty} M_{t,t+\tau}^{\$} \alpha^\tau Y_{t+\tau|t}(j) \Lambda_{p,t,t+\tau} P_t^*(j) \right] = \mathbb{E}_t \left[\sum_{\tau=0}^{\infty} M_{t,t+\tau}^{\$} \alpha^\tau Y_{t+\tau|t}(j) \mu_p \frac{W_{t+\tau|t}(j)}{A_{t+\tau}} \right].$$

The left-hand side (LHS) of the equation can be written recursively as

$$LHS = P_t^* \left(\frac{P_t^*}{P_t} \right)^{-\theta_p} Y_t H_{p,t},$$

where

$$H_{p,t} = 1 + \alpha_p \mathbb{E}_t \left[M_{t,t+1}^{\$} \Lambda_{p,t,t+1}^{1-\theta_p} \left(\frac{Y_{t+1}}{Y_t} \right) \left(\frac{P_t}{P_{t+1}} \right)^{-\theta_p} H_{p,t+1} \right].$$

Similarly, the right-hand side (RHS) of the equation can be written as

$$RHS = \frac{\mu_p}{A_t} Y_t \left(\frac{P_t^*}{P_{I,t}} \right)^{-\theta_p} \frac{W_t}{P_t} P_t G_{p,t},$$

where

$$G_{p,t} = 1 + \alpha_p \mathbb{E}_t \left[M_{t,t+1}^s \Lambda_{p,t,t+\tau}^{-\theta_p} \left(\frac{Y_{t+1}}{Y_t} \right) \left(\frac{P_t}{P_{t+1}} \right)^{-\theta_p} \left(\frac{W_{t+1}}{W_t} \right) \left(\frac{A_t}{A_{t+1}} \right) G_{p,t+1} \right].$$

The optimal price is hence given by

$$\left(\frac{P_t^*}{P_t} \right) H_{p,t} = \frac{\mu_p}{A_t} \frac{W_t}{P_t} G_{p,t}.$$

Here, $P_t^*(j) = P_t^*$ because all firms changing prices face the same demand curve and hence the same optimization problem. Based on the definition of markup, the optimal time-varying product markup is given by

$$\mu_{p,t} = \mu_p \frac{G_t}{H_t} \quad \text{and} \quad P_t^* = \mu_{p,t} \frac{W_t}{A_t}.$$

Price inflation is given by

$$1 = (1 - \alpha_p) \left(\frac{P_t^*}{P_t} \right)^{1-\theta_p} + \alpha_p \Lambda_{p,t-1,t}^{1-\theta_p} \left(\frac{P_{t-1}}{P_t} \right)^{(1-\theta_p)}.$$

C Labor Market Clearing Conditions

The total supply of type k labor is given by

$$N_t^s(k) = \int_0^1 N_t^s(j, k) dj = \int_0^1 N_t^d(j, k) dj = \left(\frac{W_t(k)}{W_t} \right)^{-\theta_w} \int_0^1 N_t^d(j) dj.$$

From the production function $Y_t(j) = A_t N_t^d(j)$, we obtain

$$N_t^s(k) = \left(\frac{W_t(k)}{W_t} \right)^{-\theta_w} \int_0^1 \frac{Y_t(j)}{A_t} dj = \left(\frac{W_t(k)}{W_t} \right)^{-\theta_w} \frac{Y_t}{A_t} \int_0^1 \left(\frac{P_t(j)}{P_t} \right)^{-\theta_p} dj.$$

where the second equality follows from the product demand function $Y_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-\theta_p} Y_t$. Defining the price and wage dispersion aggregators by

$$F_{p,t} \equiv \int_0^1 \left(\frac{P_t(j)}{P_t} \right)^{-\theta_p} dj, \quad \text{and} \quad F_{w,t} \equiv \int_0^1 \left(\frac{W_t(k)}{W_t} \right)^{-\theta_w} dk,$$

respectively, it follows that aggregate labor supply is $N_t^s = \frac{Y_t F_{p,t} F_{w,t}}{A_t}$. From the resource constraint $N_t^d = \int_0^1 N_t^d(j) dj$, it can be shown that $N_t^d = N_t^s / F_{w,t} = \frac{Y_t F_{p,t}}{A_t}$. Note that the wage dispersion $F_{w,t}$ is bounded below by one.

$$F_{w,t} = \int_0^1 \left[\left(\frac{W_t(k)}{W_t} \right)^{1-\theta_w} \right]^{\frac{-\theta_w}{1-\theta_w}} dk \geq \left[\int_0^1 \left(\frac{W_t(k)}{W_t} \right)^{1-\theta_w} dk \right]^{\frac{-\theta_w}{1-\theta_w}} = 1^{-\theta_w} = 1,$$

where the second equality is due to Jensen's inequality for $\frac{-\theta_w}{1-\theta_w} > 1$. Similarly, we can show that $F_{p,t}$ is bounded below by one.

D Equilibrium Conditions

This appendix provides a summary of the equilibrium equations for the model. These conditions need to be expressed in terms of de-trended variables. In order to obtain balanced growth, $\kappa_t \equiv \kappa_0(A_t^p)^{1-\varphi}$. This condition ensures that Y_t , W_t , W_t^* , C_t , and $C_{h,t}$ share the same average trend. Therefore, the equations can be written in stationary form in terms of $\hat{Y}_t = \frac{Y_t}{A_t^p}$, $\hat{W}_t = \frac{W_t}{A_t^p}$, $\hat{W}_t^* = \frac{W_t^*}{A_t^p}$, $\hat{C}_t = \frac{C_t}{A_t^p}$, and $\hat{C}_{h,t} = \frac{C_{h,t}}{A_t^p}$.

Wage setting

$$\frac{W_t^*}{P_t} = \mu_w \kappa_t (N_t^s)^\omega C_{h,t}^\varphi \frac{G_{w,t}}{H_{w,t}}.$$

$$H_{w,t} = 1 + \alpha_w \mathbb{E}_t \left[M_{t,t+1}^s \Lambda_{w,t,t+1}^{-\theta_w} \left(\frac{N_{t+1}^d}{N_t^d} \right) \left(\frac{W_t}{W_{t+1}} \right)^{-\theta_w} H_{w,t+1} \right],$$

$$\text{and } G_{w,t} = 1 + \alpha_w \mathbb{E}_t \left[M_{t,t+1}^s \Lambda_{w,t,t+1}^{-\theta_w} \left(\frac{P_{t+1}}{P_t} \right) \left(\frac{C_{h,t+1}}{C_{h,t}} \right)^\varphi \left(\frac{N_{t+1}^d}{N_t^d} \right) \left(\frac{\kappa_{t+1}}{\kappa_t} \right) \left(\frac{N_{t+1}^s}{N_t^s} \right)^\omega \left(\frac{W_t}{W_{t+1}} \right)^{-\theta_w} G_{w,t+1} \right].$$

Price dispersion

$$F_{p,t} = \int_0^1 \left(\frac{P_t(j)}{P_t} \right)^{-\theta_p} dj = (1 - \alpha_p) \left(\frac{P_t^*}{P_t} \right)^{-\theta_p} + \alpha_p \Lambda_{p,t-1,t}^{-\theta_p} \left(\frac{P_{t-1}}{P_t} \right)^{-\theta_p} F_{p,t-1}.$$

Wage dispersion

$$F_{w,t} = \int_0^1 \left(\frac{W_t(k)}{W_t} \right)^{-\theta_w} dk = (1 - \alpha_w) \left(\frac{W_t^*}{W_t} \right)^{-\theta_w} + \alpha_w \Lambda_{w,t-1,t}^{-\theta_w} \left(\frac{W_{t-1}}{W_t} \right)^{-\theta_w} F_{w,t-1}.$$

Wage aggregator

$$\left(\frac{W_t}{P_t} \right)^{1-\theta_w} = \int_0^1 \left(\frac{W_t(k)}{P_t} \right)^{1-\theta_w} dk = (1 - \alpha_w) \left(\frac{W_t^*}{P_t} \right)^{1-\theta_w} + \alpha_w \Lambda_{w,t-1,t}^{1-\theta_w} \left(\frac{P_{t-1}}{P_t} \right)^{1-\theta_w} \left(\frac{W_{t-1}}{P_{t-1}} \right)^{1-\theta_w},$$

Price setting

$$\left(\frac{P_t^*}{P_t} \right) H_{p,t} = \frac{\mu_p W_t}{A_t P_t} G_{p,t},$$

$$H_{p,t} = 1 + \alpha_p \mathbb{E}_t \left[M_{t,t+1}^s \Lambda_{p,t,t+1}^{1-\theta_p} \left(\frac{Y_{t+1}}{Y_t} \right) \left(\frac{P_t}{P_{t+1}} \right)^{-\theta_p} H_{p,t+1} \right],$$

$$\text{and } G_{p,t} = 1 + \alpha_p \mathbb{E}_t \left[M_{t,t+1}^s \Lambda_{p,t,t+1}^{-\theta_p} \left(\frac{Y_{t+1}}{Y_t} \right) \left(\frac{P_t}{P_{t+1}} \right)^{-\theta_p} \left(\frac{W_{t+1}}{W_t} \right) \left(\frac{A_t}{A_{t+1}} \right) G_{p,t+1} \right].$$

Price aggregator

$$1 = (1 - \alpha_p) \left(\frac{P_t^*}{P_t} \right)^{1-\theta_p} + \alpha_p \Lambda_{p,t-1,t}^{1-\theta_p} \left(\frac{P_{t-1}}{P_t} \right)^{1-\theta_p}.$$

Aggregate labor supply and demand

$$N_t^s = F_{w,t} N_t^d, \quad N_t^d = \frac{Y_t}{A_t} F_{p,t}.$$

Pricing kernel

$$\begin{aligned} M_{t,t+1} &= \left[\beta \left(\frac{C_{h,t+1}}{C_{h,t}} \right)^{-\varphi} \right]^{\frac{1-\gamma}{1-\varphi}} \left(\frac{1}{R_{Q,t+1}} \right)^{1-\frac{1-\gamma}{1-\varphi}}, \\ R_{Q,t+1} &= (1 - \nu_t) R_{C_h,t+1} + \nu_t R_{LI^*,t+1}, \\ R_{C_h,t+1} &= \frac{C_{h,t+1} + S_{C_h,t+1}}{S_{C_h,t}}, \quad R_{LI^*,t+1} = \frac{LI_{t+1}^* + S_{LI^*,t+1}}{S_{LI^*,t}}, \\ \nu_t &= \frac{\bar{\nu} S_{LI^*,t}}{\bar{\nu} S_{LI^*,t} - S_{C_h,t}}. \end{aligned}$$

Real and nominal bond yields

$$\exp(-nr_t^{(n)}) = \mathbb{E}_t \left[M_{t,t+1} \exp(-(n-1)r_{t+1}^{(n-1)}) \right], \quad \exp(-ni_t^{(n)}) = \mathbb{E}_t \left[M_{t,t+1}^{\$} \exp(-(n-1)i_{t+1}^{(n-1)}) \right].$$

Indexation

$$\log \Lambda_{p,t,t+1} = \pi_t^*, \quad \log \Lambda_{w,t,t+1} = g_a + \pi_t^*.$$

Policy rule

$$i_t = \rho i_{t-1} + (1 - \rho) [\bar{i} + \nu_\pi (\pi_t - \pi_{t-1}^*) + \nu_x (x_t - x_{ss})] + u_t.$$

Goods market clearing

$$Y_t = C_t.$$

Habit

$$C_{h,t} = C_t - b_h C_{t-1},$$

Flexible price and wage economy

$$\begin{aligned} C_{h,t}^f &= C_t^f - b_h C_{t-1}^f, \\ Y_t^f &= C_t^f, \\ (Y_t^f)^\omega (C_{h,t}^f)^\varphi &= \frac{A_t^{1+\omega}}{\mu_p \mu_w \kappa_t}. \end{aligned}$$

Output steady state

$$\begin{aligned}
1 &= \mu_p \mu_w \kappa_t (Y_{ss})^\omega (C_{ss} - b_h C_{ss})^\varphi \frac{G_{w,ss}}{H_{w,ss}}, \\
1 &= \mu_p \mu_w \kappa_t \left(Y_{ss}^f\right)^\omega \left(C_{ss}^f - b_h C_{ss}^f\right)^\varphi, \\
Y_{ss} &= C_{ss}, \\
x_{ss} &= y_{ss} - y_{ss}^f.
\end{aligned}$$

E Bond Risk Premia

E.1 Real Term and Inflation Risk Premia

Consider the no arbitrage equation for the n -period real bond:

$$B_t^{c,(n)} = e^{-nr_t^{(n)}} = \mathbb{E}_t \left[M_{t,t+1} B_{t+1}^{c,(n-1)} \right] = \mathbb{E}_t \left[e^{m_{t,t+1} - (n-1)r_{t+1}^{(n-1)}} \right],$$

where $m_{t,t+1} \equiv \log M_{t,t+1}$. Assuming normality and homoscedasticity for the log-pricing kernel and bond yields, it follows that

$$e^{-nr_t^{(n)}} = \mathbb{E}_t [e^{m_{t,t+1}}] \mathbb{E}_t \left[e^{-(n-1)r_{t+1}^{(n-1)}} \right] e^{-\text{cov}_t(m_{t,t+1}, (n-1)r_{t+1}^{(n-1)})}.$$

From equation (22), the real term premium in equation (23) follows. The equation above also implies

$$nr_t^{(n)} = r_t - \frac{1}{2} \text{var}_t \left((n-1)r_{t+1}^{(n-1)} \right) + rTP_t^{(n)} + \mathbb{E}_t \left[(n-1)r_{t+1}^{(n-1)} \right].$$

Solving for the last term iteratively and applying unconditional expectations, the spread in equation (23) follows.

Consider the inflation risk premium in equation (24) for $n = 1$,

$$\pi TP_t^{(1)} = \text{cov}_t(m_{t,t+1}, \pi_{t+1}) = i_t - r_t + \log \mathbb{E}_t[\exp(-\pi_{t,t+1})]. \quad (32)$$

In general, the inflation risk premium in equation (24) can be written in terms of bond yields in equations (21) as

$$\begin{aligned}
\pi TP_t^{(n)} &= n(i_t^{(n)} - r_t^{(n)}) + \log \mathbb{E}_t \left[e^{-(n-1)i_{t+1}^{(n-1)}} \right] - \log \mathbb{E}_t \left[e^{-(n-1)r_{t+1}^{(n-1)}} \right] \\
&+ \log \mathbb{E}_t [e^{-\pi_{t,t+1}}] + \text{cov}_t \left((n-1)i_{t+1}^{(n-1)}, \pi_{t+1} \right).
\end{aligned}$$

From equation (32), the recursive bond pricing equation

$$e^{-ni_t^{(n)}} = e^{-i_t} \mathbb{E}_t \left[e^{-(n-1)i_{t+1}^{(n-1)}} \right] e^{-\text{cov}_t(m_{t,t+1}^\$, (n-1)i_{t+1}^{(n-1)})},$$

where $m_{t,t+1}^\$ \equiv \log M_{t,t+1}^\$$, and a similar equation for the comparable real bond yield, it follows that

$$\begin{aligned}
\pi TP_t^{(n)} &= \pi TP_t^{(1)} + \text{cov}_t \left(m_{t,t+1}^\$, (n-1)i_{t+1}^{(n-1)} \right) - \text{cov}_t \left(m_{t,t+1}, (n-1)i_{t+1}^{(n-1)} \right) + \text{cov}_t \left(\pi_{t+1}, (n-1)i_{t+1}^{(n-1)} \right) \\
&= \pi TP_t^{(1)} + \text{cov}_t \left(m_{t,t+1}^\$, (n-1) \left(i_{t+1}^{(n-1)} - r_{t+1}^{(n-1)} \right) \right),
\end{aligned}$$

where the second equality follows from $m_{t,t+1} = m_{t,t+1}^\$ + \pi_{t+1}$. Realizing that under log-normality and homoskedas-

ticity assumptions the nominal-real bond spread is

$$(n-1) \left(i_{t+1}^{(n-1)} - r_{t+1}^{(n-1)} \right) = \sum_{s=1}^{n-1} \mathbb{E}_t[\pi_{t+s}] - \frac{1}{2} \text{var}_t \left(\sum_{s=1}^{n-1} \pi_{t+s} \right) - \text{cov}_t \left(\sum_{s=1}^{n-1} m_{t,t+s}, \sum_{s=1}^{n-1} \pi_{t+s} \right).$$

Since the variance and covariance terms are constant, it follows that

$$\pi T P_t^{(n)} = \text{cov}_t \left(m_{t,t+1}, \sum_{s=1}^{n-1} \pi_{t+s} \right).$$

Computing the unconditional expectation of the nominal-real bond spread above and replacing the covariance terms for the one-period inflation risk premia, the spread in equation (25) follows.

E.2 Understanding the Mechanism

The labor-only linear production technology in equation (13) implies that aggregate consumption is

$$C_t = A_t^p Z_t \frac{N_t^d}{F_{p,t} F_{w,t}},$$

where the difference-stationary shocks $a_t \equiv \log A_t^p$ and the trend-stationary shocks $z_t \equiv \log Z_t$ follow the processes in equations (15), and $F_{p,t}$ and $F_{w,t}$ are distortions generated by price and wage rigidities, respectively. These distortions and the complete set of equilibrium conditions are presented in appendix D. It can be shown that a first-order approximation of the distortions implies $F_{p,t} \approx 1$ and $F_{w,t} \approx 1$. It implies that $N_t^s = N_t^d = N_t$.

Notice that when prices and wages are perfectly flexible, consumption growth becomes

$$\Delta c_t = \Delta a_t + \left(\frac{1+\omega}{\omega+\varphi} \right) \Delta z_t, \quad \text{and} \quad Q_t = C_t \left[1 - \left(\frac{1-\varphi}{1+\omega} \right) \left(\frac{1}{\mu_p \mu_w} \right) \right].$$

That is, the dividend of the wealth portfolio is proportional to consumption and, then, the return on wealth is a “levered” claim on the return on the consumption claim.

Consider the recursive preferences on consumption and labor in equation (1) and its associated real pricing kernel in equation (6). Under the change of variable $(1-\varphi)\tilde{v}_t \equiv \log[(1-\varphi)V_t/C_t^{1-\varphi}]$, these preferences can be written as

$$(1-\varphi)\tilde{v}_t = \log \left[(1-\beta) \left(1 - \frac{1-\varphi}{1+\omega} e^{(\omega+\varphi)n_t - (1-\varphi)z_t} \right) + \beta e^{\left(\frac{1-\varphi}{1+\omega}\right) \log \mathbb{E}_t[\exp((1-\gamma)(\tilde{v}_{t+1} + \Delta c_{t+1}))]} \right].$$

A log-linear approximation of this term implies

$$\begin{aligned} \tilde{v}_t + \Delta c_t &= \text{constant} + \eta_n n_t + \eta_z z_t + \eta_{vc} \mathbb{E}_t[\tilde{v}_{t+1} + \Delta c_{t+1}] \\ &= \text{constant} + \sum_{s=0}^{\infty} \eta_{vc}^s \mathbb{E}_t[\eta_{vc} \Delta c_{t+s} + \eta_n n_{t+s} + \eta_z z_{t+s}], \end{aligned} \quad (33)$$

where η_n , η_z , and η_{vc} are appropriate approximation constants, and the second equality follows from solving the first equation recursively. The term $\frac{V_{t+1}^{1/(1-\varphi)}}{\mathbb{E}_t[V_{t+1}^{(1-\gamma)/(1-\varphi)]^{1/(1-\gamma)}}$ in the pricing kernel can be written in log-form as

$$\tilde{v}_{t+1} + \Delta c_{t+1} - \frac{1}{1-\gamma} \log \mathbb{E}_t[\exp((1-\gamma)(\tilde{v}_{t+1} + \Delta c_{t+1}))].$$

Replacing equation (33) in the pricing kernel equation, equation (26) follows.

This pricing kernel also can be written in terms of the return on wealth $R_{Q,t}$ as

$$M_{t,t+1} = \left[\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\varphi} \right]^{\frac{1-\gamma}{1-\varphi}} \left[\frac{1}{R_{Q,t+1}} \right]^{1-\frac{1-\gamma}{1-\varphi}}, \quad \text{where } Q_t = C_t \left[1 - \left(\frac{1-\varphi}{1+\omega} \right) \kappa_0 \left(\frac{N_t^{\varphi+\omega}}{Z_t^{1-\varphi}} \right) \right]$$

is the dividend associated to the wealth portfolio. The log-pricing kernel can be written as

$$m_{t,t+1} = \left(\frac{1-\gamma}{1-\varphi} \right) \log \beta - \varphi \left(\frac{1-\gamma}{1-\varphi} \right) \Delta c_{t+1} + \left(\frac{\varphi-\gamma}{1-\varphi} \right) r_{q,t+1}.$$

The log-return on wealth, $r_{q,t+1}$ can be approximated as

$$r_{q,t+1} = \bar{\eta}_q + \eta_q p_{q,t+1} + \Delta q_{t+1} - p_{q,t}, \quad \text{where } \Delta q_t = \Delta c_t - \left(\frac{1-\varphi}{1+\omega} \right) (\omega + \varphi) \bar{\kappa} \Delta n_t + \frac{(1-\varphi)^2}{1+\omega} \bar{\kappa} \Delta z_t$$

is the wealth-dividend ratio for appropriate approximation constants $\bar{\eta}_q$, η_q , and $\bar{\kappa}$.

Assume that labor follows the process $n_t = \bar{n} + n_a \Delta a_t + n_z z_t$, where \bar{n} , n_a , and n_z are determined in equilibrium. From this process, the consumption growth processes $\Delta c_t = \Delta a_t + \Delta n_t$, and the no-arbitrage pricing equation $1 = \mathbb{E}_t[\exp(m_{t,t+1} + r_{q,t+1})]$, it can be shown that the wealth-dividend ratio can be approximated as

$$p_{q,t} = \bar{p}_q + p_{q,a} \Delta a_t + p_{q,z} z_t,$$

where

$$\begin{aligned} p_{q,a} &= \left(\frac{1-\varphi}{1-\eta_q \phi_a} \right) \left[\phi_a - (1-\phi_a) n_a \left(1 - \bar{\kappa} \left(\frac{\omega + \varphi}{1+\omega} \right) \right) \right], \\ \text{and } p_{q,z} &= -\frac{(1-\phi_z)(1-\varphi)}{1-\eta_q \phi_z} \left[1 + n_z + \bar{\kappa} \left(\frac{1-\varphi - (\omega + \varphi) n_z}{1+\omega} \right) \right]. \end{aligned}$$

E.2.1 The real console bond

Consider the real console bond that pays one unit of consumption every period. The price of this bond can be written recursively as

$$B_t^{c,\infty} = \mathbb{E}_t [M_{t,t+1} (1 + B_{t+1}^{c,\infty})].$$

Its one-period log-return can be written as

$$r_{\infty,t+1}^c = \log \left(\frac{1 + \exp(p_{\infty,t+1}^c)}{\exp(p_{\infty,t}^c)} \right) \approx \bar{\eta}_{\infty}^c + \eta_{\infty}^c p_{\infty,t+1}^c - p_{\infty,t}^c,$$

where $p_{\infty,t}^c \equiv \log B_t^{c,\infty}$, and $\bar{\eta}_{\infty}^c$, and $\eta_{\infty}^c < 1$, are appropriate approximation constants. From the pricing equation $1 = \mathbb{E}_t [\exp(m_{t,t+1} + r_{\infty,t+1}^c)]$, it can be shown that the log-bond price follows the linear function

$$p_{\infty,t}^c = \bar{p}_{\infty}^c + p_{\infty,a}^c \Delta a_t + p_{\infty,z}^c z_t$$

where

$$p_{\infty,a}^c = \frac{\varphi[(1-\phi_a)n_a - \phi_a]}{1-\eta_{\infty}^c \phi_a}, \quad \text{and } p_{\infty,z}^c = \frac{(1-\phi_z)(1+n_z)\varphi}{1-\eta_{\infty}^c \phi_z}.$$

E.2.2 Inflation dynamics

Consider the interest-rate policy rule says that the current interest rate depends on the lagged interest rate as follows

$$i_t = \bar{i} + \iota_\pi(\pi_t - \pi^*) + \iota_x(x_t - x_{ss}) + u_t,$$

where the response to the lagged interest rate i_{t-1} is $\rho = 0$. Under nominal rigidities, the output gap is given by

$$x_t = y_t - y_t^f = n_t - n_t^f + \frac{\log(\mu_w \mu_p)}{\omega + \varphi},$$

where n_t^f denotes labor under no price and wage rigidities. The output gap can be written as

$$x_t = \bar{x} + n_a \Delta a_t + \left(n_z - \frac{1 - \varphi}{\omega + \varphi} \right) z_t,$$

where \bar{x} is a constant not important for the analysis, and the term $\frac{1 - \varphi}{\omega + \varphi}$ is the sensitivity of labor to transitory shocks under flexible prices and wages. From the pricing equation $\mathbb{E}_t[\exp(m_{t,t+1} - \pi_{t+1} + i_t)] = 1$, and guessing that

$$\pi_t = \bar{\pi} + \pi_a \Delta a_t + \pi_z z_t,$$

it can be shown that

$$\pi_a = \frac{-\varphi[(1 - \phi_a)n_a - \phi_a] - \iota_x n_a}{\iota_\pi - \phi_a}, \quad \text{and} \quad \pi_z = \frac{-\varphi(1 - \phi_z)(1 + n_z) - \iota_x \left(n_z - \frac{1 - \varphi}{\omega + \varphi} \right)}{\iota_\pi - \phi_z}.$$

E.2.3 The interest rate rule and the link between real term and inflation risk premia

Let $m_{t+1} \equiv m_{t,t+1}$. Consider the process for the one-period pricing kernel

$$-m_{t+1} = \bar{m} + m_s^\top \mathbf{s}_t + \lambda^\top \Sigma^{1/2} \varepsilon_{t+1},$$

where \mathbf{s}_t is a set of state variables that follows the process

$$\mathbf{s}_{t+1} = (\mathbb{I} - \Phi)\bar{\mathbf{s}} + \Phi \mathbf{s}_t + \Sigma^{1/2} \varepsilon_{t+1},$$

and the interest-rate policy rule

$$i_t = \bar{i} + \iota_\pi \pi_t + \iota_x x_t.$$

The no-arbitrage price of a one-period bond is

$$e^{-i_t} = \mathbb{E}_t[e^{m_{t+1} - \pi_{t+1}}].$$

Guess $\pi_t = \bar{\pi} + \pi_s^\top \mathbf{s}_t$, and $x_t = \bar{x} + x_s^\top \mathbf{s}_t$.

Using the method of undetermined coefficients, it can be shown that

$$\pi_s = (\iota_\pi \mathbb{I} - \Phi^\top)^{-1} (m_s - \iota_x x_s).$$

The one-period real term premium of a real console bond with price $B_t^{c,\infty} \equiv \exp(p_t^{c,\infty})$ is

$$rTP_t^\infty = -\text{cov}_t(m_{t+1}, \log(1 + B_{t+1}^{c,\infty})) \approx -\text{cov}_t(m_{t+1}, \eta_\infty^c p_{t+1}^{c,\infty}),$$

where $0 < \eta_\infty^c < 1$. From the bond pricing equation, it can be shown that

$$p_{t+1}^{c,\infty} = \bar{p}^c + p_s^c \mathbf{s}_t = \bar{p}^c - m_s^\top (\mathbb{I} - \eta_\infty^c \Phi)^{-1} \mathbf{s}_t,$$

and the premium becomes

$$rTP_t^\infty = -\eta_\infty^c \lambda^\top \Sigma (\mathbb{I} - \eta_\infty^c \Phi^\top)^{-1} m_s.$$

Consider the one-period inflation risk premium in a nominal console bond with price $B_t^{\$, \infty} \equiv \exp(p_t^{\$, \infty})$, given by

$$\begin{aligned} \pi TP_t^\infty &= \text{cov}_t(m_{t+1}, \pi_{t+1}) - \text{cov}_t\left(m_{t+1}, \log\left(1 + B_{t+1}^{\$, \infty}\right)\right) + \text{cov}_t\left(m_{t+1}, \log\left(1 + B_{t+1}^{c, \infty}\right)\right). \\ &\approx \text{cov}_t(m_{t+1}, \pi_{t+1}) - \text{cov}_t\left(m_{t+1}, \eta_\infty^{\$} p_{t+1}^{\$, \infty}\right) + \text{cov}_t\left(m_{t+1}, \eta_\infty^c p_{t+1}^{c, \infty}\right). \end{aligned}$$

The first term is the one-period inflation risk premium in the nominal risk-free bond. It is

$$\text{cov}_t(m_{t+1}, \pi_{t+1}) = -\lambda^\top \Sigma \pi_s = -\lambda^\top \Sigma (\iota_\pi \mathbb{I} - \Phi^\top)^{-1} (m_s - \iota_x x_s).$$

Since $m_{t+1}^{\$} = m_{t+1} - \pi_{t+1}$, the nominal pricing kernel is

$$-m_{t+1}^{\$} = \bar{m}^{\$} + (m_s + \Phi^\top \pi_s)^\top \mathbf{s}_t + (\lambda + \pi_s)^\top \Sigma^{1/2} \varepsilon_{t+1},$$

and then the nominal console bond is characterized by the process

$$p_{t+1}^{\$, \infty} = \bar{p}^{\$} + p_s^{\$} \mathbf{s}_t = \bar{p}^c - (m_s + \Phi^\top \pi_s)^\top (\mathbb{I} - \eta_\infty^{\$} \Phi)^{-1} \mathbf{s}_t,$$

The second term in the inflation risk premium is

$$-\text{cov}_t\left(m_{t+1}, \eta_\infty^{\$} p_{t+1}^{\$, \infty}\right) = \eta_\infty^{\$} \lambda^\top \Sigma p_s^{\$} = -\eta_\infty^{\$} \lambda^\top \Sigma (\mathbb{I} - \eta_\infty^{\$} \Phi^\top)^{-1} (m_s + \Phi^\top \pi_s).$$

The third term is the negative of the one-period real term premium of the real console bond

$$\text{cov}_t(m_{t+1}, \eta_\infty^c p_{t+1}^{c, \infty}) = \eta_\infty^c \lambda^\top \Sigma (\mathbb{I} - \eta_\infty^c \Phi^\top)^{-1} m_s.$$

Assuming that $\eta_\infty^c = \eta_\infty^{\$} = \eta_\infty$, it follows that the inflation risk premium is

$$\begin{aligned} \pi TP_t^\infty &= -\lambda^\top \Sigma \pi_s - \eta_\infty \lambda^\top \Sigma (\mathbb{I} - \eta_\infty \Phi^\top)^{-1} \Phi^\top \pi_s \\ &= -\lambda^\top \Sigma [\mathbb{I} + \eta_\infty (\mathbb{I} - \eta_\infty \Phi^\top)^{-1} \Phi^\top] (\iota_\pi \mathbb{I} - \Phi^\top)^{-1} (m_s - \iota_x x_s). \end{aligned}$$

Consider the particular case $\mathbf{s}_t \equiv m_t$. In this case,

$$m_{t+1} = (1 - \phi_m) \bar{m} + \phi_m m_t + \sigma_m \varepsilon_{m, t+1},$$

and the equations above for real term and inflation risk premia can be written as

$$rTP_t^\infty = -\frac{\eta_\infty^c \phi_m}{1 - \eta_\infty^c \phi_m} \text{var}_t(m_{t+1}),$$

and

$$\pi TP_t^\infty = -\left(\frac{\iota_x x_m + \phi_m}{(\iota_\pi - \phi_m)(1 - \eta_\infty \phi_m)}\right) \text{var}_t(m_{t+1}) = \left(\frac{1}{\eta_\infty (\iota_\pi - \phi_m)}\right) \left(1 + \frac{\iota_x x_m}{\phi_m}\right) rTP_t^\infty,$$

respectively. If $\iota_x = 0$, the sign of the inflation risk premium is entirely determined by the autocorrelation of the pricing kernel. If the autocorrelation is negative, the inflation risk premium is positive. If $\iota_x \neq 0$, the sign of the inflation risk premium also depends on the correlation between the pricing kernel and the output gap.

Table 1: Descriptive Statistics of U.K. and U.S. Government Inflation-Linked and Nominal Bond Yields and Excess Returns, Consumption Growth, and Inflation

Yields are annualized rates. Statistics are quarterly, non-annualized. Consumption growth is denoted by Δc , inflation by π_t , and the 3-month nominal rate by i_t . Excess returns on inflation-linked bonds are computed as $\log P_{linker,t+1} - \log P_{linker,t} + \pi_{t+1} - i_t$. Excess returns on nominal bonds are computed as $\log P_{nom,t+1} - \log P_{nom,t} - i_t$. The row labeled “2-2.5 years” contains statistics related to the 2- and 2.5-year bonds for the United States and the United Kingdom, respectively. Values not reported are not available. Information for 2-year TIPS is only available since 2004.

*Note: The three month rate in the “Inflation-linked” and “TIPS” columns correspond to the three month real rate estimated using the methodology described in Pflueger and Viceira (2011).

	United Kingdom				United States			
	1985:Q1 - 2008:Q3		1999:Q1 - 2008:Q3		1985:Q1 - 2008:Q3		1999:Q1 - 2008:Q3	
	Inflation-Linked	Nominal	Inflation-Linked	Nominal	Nominal	TIPS	Nominal	
Panel A: Bond Yields								
<i>Average</i>								
3-month rate*	3.92	7.14	2.65	4.71	4.62	0.30	3.19	
2-2.5 years	2.85	7.07	2.15	4.77	5.37		3.70	
5 years	2.88	7.14	2.04	4.81	5.92	2.27	4.24	
10 years	2.94	7.13	1.91	4.76	6.48	2.64	4.92	
<i>Standard Deviations</i>								
3-month rate*	1.64	3.06	0.39	0.74	2.04	1.51	1.76	
2-2.5 years	0.94	2.54	0.80	0.73	2.07		1.54	
5 years	0.86	2.43	0.49	0.60	1.93	1.14	1.10	
10 years	0.98	2.38	0.40	0.38	1.75	0.88	0.76	
Panel B: Bond Excess Returns								
<i>Average</i>								
2-2.5 years	-0.22	0.15	0.00	0.06	0.34		0.26	
5 years	-0.17	0.72	-0.04	0.08	0.74	0.79	0.53	
10 years	-0.03	0.36	-0.01	0.02	1.22	1.02	0.77	
<i>Sharpe Ratios</i>								
2-2.5 years	-0.18	0.11	0.00	0.05	0.29		0.24	
5 years	-0.10	0.12	-0.03	0.04	0.25	0.33	0.20	
10 years	-0.01	0.12	-0.01	0.00	0.24	0.30	0.18	
Panel C: Correlations with Macroeconomic Variables and Stock Returns								
<i>Yields and Consumption Growth</i>								
2-2.5 years	0.28	0.54	0.06	0.47	0.30		0.51	
5 years	0.39	0.54	0.16	0.44	0.28	0.42	0.54	
10 years	0.48	0.56	0.28	0.39	0.24	0.42	0.51	
<i>Yields and Inflation</i>								
2-2.5 years	0.19	0.46	-0.18	-0.35	0.24		-0.08	
5 years	0.27	0.46	-0.14	-0.33	0.23	-0.34	-0.18	
10 years	0.33	0.47	-0.13	-0.28	0.24	-0.37	-0.27	
<i>Excess Bond and Stock Returns</i>								
2-2.5 years	0.15	0.21	0.03	-0.19	-0.21		-0.63	
5 years	0.20	0.25	0.09	-0.17	-0.17	-0.36	-0.60	
10 years	0.20	0.30	0.15	-0.15	-0.06	-0.37	-0.51	
Panel D: Macroeconomic Variables								
	Δc	π	Δc	π	Δc	π	Δc	π
Average	1.47	0.77	1.10	0.57	0.43	0.78	0.36	0.78
Std. Deviation	0.90	0.62	0.72	0.49	0.37	0.33	0.37	0.32
Autocorrelation	0.45	0.40	0.11	-0.22	0.40	0.43	0.61	0.18
corr($\Delta c, \pi$)	0.26		-0.21		-0.24		-0.32	

Table 2: **Model Parameter Values**

Parameter values for the baseline estimation of the economic model. Standard errors are reported for the variables that are estimated.

Parameter	Description	Value	Std. Error
Panel A: Preferences			
β	Subjective discount factor	0.96833	
φ	Inverse of elasticity of intertemporal substitution	20	1.556
γ	Risk aversion parameter	720	
ω	Inverse of Frisch labor elasticity	0.50	
Panel B: Product and Labor Rigidities and Elasticities			
α_p	Price rigidity parameter	0.66	
θ_p	Elasticity of substitution of differentiated goods	6	
α_w	Wage rigidity parameter	0.78	
θ_w	Elasticity of substitution of labor types	6	
Panel C: Interest Rate Rule			
ρ	Interest-rate smoothing coefficient in policy rule	0.6159	0.074
ι_π	Response to inflation in the policy rule	1.5	
ι_x	Response to output gap in the policy rule	0.125	
Panel D: Policy and Productivity Shocks			
ϕ_u	Autocorrelation of policy shock	0.396	0.130
$\sigma_u \times 10^2$	Conditional vol. of policy shock	0.4152	0.010
ϕ_a	Autocorrelation of permanent productivity shock	0.145	0.004
$\sigma_a \times 10^2$	Conditional vol. of permanent productivity shock	0.342	0.011
ϕ_z	Autocorrelation of transitory productivity shock	0	0.119
$\sigma_z \times 10^2$	Conditional vol. of transitory productivity shock	1.728	0.068
Panel E: Growth Rates and Inflation Target			
$g_a \times 10^2$	Unconditional mean of productivity growth	0.4695	
g_{π^*}	Unconditional mean of inflation target	0.2617	
ϕ_{π^*}	Autocorrelation of inflation target	0.9999	
$\sigma_{\pi^*} \times 10^2$	Conditional volatility of inflation target	0.001	

Table 3: **Data and Baseline Model Implied Statistics**

The data statistics are for the 1982:Q1 to 2008:Q3 period. The parameter values of the baseline model are reported in Table 2. The operators $\mathbb{E}[\cdot]$, $\sigma(\cdot)$, and $AC(\cdot)$ denote the unconditional mean, volatility, and first-order autocorrelation, respectively. $rTP^{(20)}$ and $\pi TP^{(20)}$ are the 5-year bond real term and inflation risk premia, respectively. The mean of the baseline model corresponds to the closed-form average of the second-order approximation of the solution. The columns labeled “5%” and “95%” provide the confidence interval for the statistic computed from the average of 1,000 simulations of samples with size of 107 quarters using the second-order approximation of the solution. Volatilities, yields, and (excess) returns are in percentage terms. The inflation rate, yields, excess returns, and risk premia are annualized. The data statistics related to the real rate r are obtained from the estimated real rate. Values not reported are not available. $R_{r,\Delta c}^2$ is the R^2 of a regression of the real rate on one-quarter ahead consumption growth. $R_{\Delta i^{(20)},\Delta c}^2$ and $R_{\Delta i^{(20)},\pi}^2$ are the R^2 's of the regressions of the 5-year nominal bond yield on contemporaneous and three quarter lags of consumption growth and inflation, respectively.

Statistic	Data	Baseline Model		
		Mean	5%	95%
Panel A: Macroeconomic Variables				
$\sigma(\Delta c)$	0.38	0.41	0.36	0.45
$\sigma(\pi)$	1.36	1.68	1.24	1.60
$\sigma(\Delta w)$	0.66	0.42	0.37	0.47
$\sigma(x)$		0.16	0.13	0.17
$AC(\Delta c)$	0.42	-0.01	-0.17	0.15
$AC(\pi)$	0.42	0.39	0.22	0.57
$AC(\Delta w)$	0.17	0.18	-0.07	0.27
$\text{corr}(\Delta c, \pi)$	-0.15	-0.10	-0.28	0.04
Panel B: Real and Nominal Yield Curves				
$\mathbb{E}[i]$	5.20	5.20	4.46	5.92
$\mathbb{E}[i^{(20)} - i]$	1.38	1.12	1.51	1.64
$\mathbb{E}[r]$	1.98	1.38	0.43	2.29
$\mathbb{E}[r^{(20)} - r]$		0.82	0.52	0.58
$\sigma(i)$	2.59	2.29	1.72	2.43
$\sigma(r)$	2.09	2.84	2.29	3.28
$\sigma(r)/\sigma(i)$	0.81	1.24	1.32	1.37
$\text{corr}(i, r)$	0.99	0.92	1.00	1.00
$\sigma(i^{(20)})/\sigma(i)$	1.02	0.40	0.12	0.14
$\sigma(r^{(20)})/\sigma(r)$		0.13	0.12	0.13
Panel C: Expected Excess Returns and Risk Premia				
$\mathbb{E}[XR^{\$, (20)}]$	4.28	1.97	1.97	1.97
$\mathbb{E}[XR^{c, (20)}]$		1.17	1.17	1.17
$SR^{\$, (20)}$	0.32	0.32	0.16	0.50
$SR^{c, (20)}$		0.18	0.02	0.36
$\mathbb{E}[rTP^{(20)}]$		1.03	1.03	1.03
$\mathbb{E}[\pi TP^{(20)}]$		0.86	0.86	0.86
Panel D: Additional Implications				
$\sigma(\Delta d)$	8.10	11.14	9.64	12.68
$\mathbb{E}[XR_d]$	7.51	9.06	7.92	10.22
$R_{r,\Delta c}^2$	6.92	1.01	0.01	6.16
$R_{\Delta i^{(20)},\Delta c}^2$	8.19	10.97	3.41	22.15
$R_{\Delta i^{(20)},\pi}^2$	1.17	7.64	2.91	15.39

Table 4: Data and Model Implied Statistics for Alternative Estimations

The data statistics are for the 1982:Q1 to 2008:Q3 period. The operators $\mathbb{E}[\cdot]$, $\sigma(\cdot)$, and $AC(\cdot)$ denote the unconditional mean, volatility, and first-order autocorrelation, respectively. $rTP^{(20)}$ and $\pi TP^{(20)}$ are the 5-year bond real term and inflation risk premia, respectively. “BR” indicates an economy with both price and wage rigidities. “WR” indicates no price rigidities ($\alpha_p = 0$). “PR” indicates no wage rigidities ($\alpha_w = 0$). “NR” indicates no price and wage rigidities ($\alpha_p = \alpha_w = 0$). “ A^P ” indicates that productivity shocks have only a transitory component. “ Z ” indicates that productivity shocks have only a transitory component. “ A^P and Z ” indicates that productivity shocks have both permanent and transitory components. The model statistic corresponds to the closed-form average of the second-order approximation of the solution. Volatilities, yields, and (excess) returns are in percentage terms. The inflation rate, yields, excess returns, and risk premia are annualized. The data statistics related to the real rate r are obtained from the estimated real rate. Values not reported are not available. All estimations use $\gamma = 400$. The objective value is the sum of squared percentage differences between the model- and data-implied moments targeted in the estimation.

Statistic	(1) Data	Model											
		A^P				Z				A^P and Z			
		(2) BR	(3) WR	(4) PR	(5) NR	(6) BR	(7) WR	(8) PR	(9) NR	(10) BR	(11) WR	(12) PR	(13) NR
Panel A: Macroeconomic Variables													
Objective value		3.83	1.79	4.21	1.90	1.92	3.31	2.33	2.43	1.33	1.32	1.40	0.97
$\sigma(\Delta c)$	0.38	0.53	0.40	0.49	0.45	0.37	0.37	0.38	0.37	0.41	0.45	0.41	0.43
$\sigma(\pi)$	1.36	1.32	2.02	0.84	1.62	1.89	1.90	1.66	1.45	1.60	1.70	1.86	1.35
$\sigma(\Delta w)$	0.66	0.41	0.39	0.25	0.23	0.36	0.30	0.19	0.18	0.37	0.27	0.20	0.17
$\sigma(x)$		0.16	0.13	0.10	0.00	0.68	0.39	0.25	0.00	0.16	0.14	0.14	0.00
$AC(\Delta c)$	0.42	0.00	-0.02	0.16	0.20	-0.08	-0.14	0.06	0.00	0.00	-0.01	0.00	0.15
$\text{corr}(\Delta c, \pi)$	-0.15	-0.04	-0.13	0.01	0.00	-0.14	-0.16	-0.21	-0.15	-0.12	-0.11	-0.15	-0.13
Panel B: Real and Nominal Yield Curve													
$\mathbb{E}[i]$	5.20	5.20	5.20	5.20	5.20	5.20	5.20	5.20	5.20	5.20	5.20	5.20	5.20
$\mathbb{E}[i^{(20)} - i]$	1.38	0.48	0.90	0.00	-0.01	0.03	0.50	0.02	0.30	0.73	0.44	0.20	-0.78
$\mathbb{E}[r]$	1.98	1.74	1.25	1.93	1.94	1.94	1.39	1.93	1.84	1.62	1.56	1.83	3.27
$\mathbb{E}[r^{(20)} - r]$		0.07	0.23	0.00	0.00	0.01	0.16	0.00	0.03	0.12	0.10	0.04	-0.37
$\sigma(i)$	2.59	2.78	2.47	0.84	0.99	2.07	1.66	1.35	1.08	2.34	1.89	1.77	1.01
$\sigma(r)$	2.09	3.36	3.21	0.00	0.00	2.52	2.06	0.86	0.24	2.83	2.16	1.10	0.62
$\sigma(r)/\sigma(i)$	0.81	1.21	1.30	0.00	0.00	1.22	1.24	0.63	0.22	1.21	1.14	0.62	0.61
$\text{corr}(i, r)$	0.99	0.95	0.93	0.00	0.00	0.88	0.82	0.38	0.62	0.92	0.87	0.80	0.49
$\sigma(i^{(20)})/\sigma(i)$	1.02	0.34	0.38	1.00	0.88	0.47	0.58	0.82	0.97	0.40	0.51	0.51	0.86
$\sigma(r^{(20)})/\sigma(r)$		0.13	0.11	-	-	0.16	0.14	0.46	0.92	0.13	0.12	0.23	0.21
Panel C: Expected Excess Returns													
$\mathbb{E}[XR^{s,(20)}]$	4.28	0.87	1.27	0.00	-0.01	0.06	0.59	0.05	0.61	1.10	0.70	0.29	-0.87
$\mathbb{E}[XR^{c,(20)}]$		0.46	1.15	0.00	0.00	0.09	0.76	0.03	0.21	0.70	0.53	0.21	-1.64
$SR^{s,(20)}$	0.32	0.12	0.23	0.00	0.00	0.01	0.19	0.01	0.37	0.20	0.16	0.07	-0.49
$SR^{c,(20)}$		0.06	0.18	-	-	0.01	0.17	0.01	0.38	0.11	0.12	0.07	-1.08
$\mathbb{E}[rTP^{(20)}]$		0.41	1.07	0.00	0.00	0.09	0.75	0.03	0.21	0.65	0.49	0.20	-1.62
$\mathbb{E}[\pi TP^{(20)}]$		0.38	0.63	0.00	0.00	-0.02	0.37	0.04	0.47	0.55	0.41	0.18	-0.56
Panel D: Additional Implications													
$\sigma(\Delta d)$	8.10	1.31	0.40	0.51	0.45	10.98	0.37	3.00	0.37	11.27	0.45	7.04	0.43
$\mathbb{E}[XR_d]$	7.51	7.00	3.22	5.69	4.94	0.09	0.48	0.04	1.08	4.06	4.08	2.95	3.33

Table 5: **Variance Decompositions for the Baseline Model**

The unconditional variance decompositions for the baseline model correspond to the closed-form variance decompositions of the second-order approximation of the solution, for the four model shocks: ε_a , ε_z , ε_u , and ε_{π^*} . Variance decompositions are in percentage terms. The parameter values of the baseline model are reported in Table 2.

	ε_a	ε_z	ε_u	ε_{π^*}
Panel A: Macroeconomic Variables				
Δc	96.28	0.27	3.45	0.00
π	1.28	43.01	28.24	27.47
Δw	38.25	40.86	20.88	0.00
Δd	0.72	98.63	0.65	0.00
x	15.91	48.38	31.85	3.86
Panel B: Real and Nominal Yield Curve				
i	0.55	9.04	75.64	14.77
$i^{(20)} - i$	0.58	11.94	87.48	0.00
r	0.17	11.55	88.28	0.00
$r^{(20)} - r$	0.16	12.82	87.02	0.00
Panel C: Excess Returns and Pricing Kernel				
$XR^{\$, (20)}$	0.94	4.76	94.03	0.28
$XR^{(20)}$	0.23	6.11	93.66	0.00
XR_d	17.74	37.52	44.74	0.00
$M^{\$}$	99.88	0.11	0.01	0.00
M	99.87	0.12	0.01	0.00

Table 6: **Data and Baseline Model Implied Statistics - The Effect of Rigidities and Shocks**

The data statistics are for the 1982:Q1 to 2008:Q3 period. The parameter values of the baseline model are reported in Table 2. The operators $\mathbb{E}[\cdot]$, $\sigma(\cdot)$, and $AC(\cdot)$ denote the unconditional mean, volatility, and first-order autocorrelation, respectively. $rTP^{(20)}$ and $\pi TP^{(20)}$ are the 5-year bond real term and inflation risk premia, respectively. “Baseline” indicates an economy with both price and wage rigidities and all four exogenous shocks. “WR” indicates no price rigidities ($\alpha_p = 0$). “PR” indicates no wage rigidities ($\alpha_w = 0$). “NR” indicates no price and wage rigidities ($\alpha_p = \alpha_w = 0$). “Only A^p ” indicates only permanent productivity shocks ($\sigma_z = \sigma_u = \sigma_{\pi^*} = 0$). “Only Z ” indicates only transitory productivity shocks ($\sigma_a = \sigma_u = \sigma_{\pi^*} = 0$). “Only u ” indicates only policy shocks ($\sigma_a = \sigma_z = \sigma_{\pi^*} = 0$). “Only π^* ” indicates only shocks to the inflation target ($\sigma_a = \sigma_z = \sigma_u = 0$). The baseline model statistic corresponds to the closed-form average of the second-order approximation of the solution. Volatilities, yields, and (excess) returns are in percentage terms. The inflation rate, yields, excess returns, and risk premia are annualized. The data statistics related to the real rate r are obtained from the estimated real rate. Values not reported are not available. The values of β and g_π are adjusted across columns to match the average inflation and short-term nominal rate.

Statistic	Model								
	(1)	Rigidities				Shocks			
		(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Data	Baseline	WR	PR	NR	Only A^p	Only Z	Only u	Only π^*	
Panel A: Macroeconomic Variables									
$\sigma(\Delta c)$	0.38	0.41	0.45	0.39	0.39	0.40	0.02	0.08	0.00
$\sigma(\pi)$	1.36	1.68	9.92	2.70	11.70	0.19	0.93	0.69	0.87
$\sigma(\Delta w)$	0.66	0.42	2.47	1.99	2.47	0.26	0.19	0.11	0.00
$\sigma(x)$		0.16	0.11	0.13	0.00	0.06	0.11	0.10	0.02
$AC(\Delta c)$	0.42	-0.01	-0.09	0.01	0.01	0.00	-0.27	-0.11	0.77
$\text{corr}(\Delta c, \pi)$	-0.15	-0.10	-0.41	0.28	-0.12	-0.86	-0.94	0.25	-0.01
Panel B: Real and Nominal Yield Curve									
$\mathbb{E}[i]$	5.20	5.20	5.20	5.20	5.20	5.20	5.20	5.20	5.20
$\mathbb{E}[i^{(20)} - i]$	1.38	1.12	2.14	-2.10	-6.36	0.97	0.00	0.04	0.00
$\mathbb{E}[r]$	1.98	1.38	-0.59	3.91	11.68	1.47	1.93	1.94	1.93
$\mathbb{E}[r^{(20)} - r]$		0.82	3.01	-2.47	-11.25	0.61	0.00	0.04	0.00
$\sigma(i)$	2.59	2.29	4.67	1.72	5.50	0.17	0.59	2.20	0.87
$\sigma(r)$	2.09	2.84	10.87	2.48	10.87	0.12	0.82	2.72	0.01
$\sigma(r)/\sigma(i)$	0.81	1.24	2.33	1.44	1.98	0.69	1.38	1.24	0.01
$\text{corr}(i, r)$	0.99	0.92	0.97	0.85	0.97	0.84	1.00	1.00	0.00
$\sigma(i^{(20)})/\sigma(i)$	1.02	0.40	0.20	0.50	0.16	0.18	0.07	0.14	1.00
$\sigma(r^{(20)})/\sigma(r)$		0.13	0.05	0.11	0.05	0.21	0.08	0.14	0.11
Panel C: Expected Excess Returns									
$\mathbb{E}[XR^{\$, (20)}]$	4.28	1.97	3.50	-3.34	-9.60	1.69	0.00	0.07	0.00
$\mathbb{E}[XR^{c, (20)}]$		1.17	3.85	-3.08	-13.30	0.87	0.00	0.09	0.00
$SR^{\$, (20)}$	0.32	0.32	0.66	-0.85	-1.18	2.83	0.00	0.02	0.00
$SR^{c, (20)}$		0.18	0.37	-0.61	-1.06	2.83	0.00	0.01	0.00
$\mathbb{E}[rTP^{(20)}]$		1.03	3.39	-2.72	-11.80	0.77	0.00	0.08	0.00
$\mathbb{E}[\pi TP^{(20)}]$		0.85	1.60	-1.56	-4.38	0.84	0.00	-0.02	0.00

Table 7: **Data and Baseline Model Implied Statistics - The Effects of Monetary Policy**

The data statistics are for the 1982:Q1 to 2008:Q3 period. The parameter values of the baseline model are reported in Table 2. The model columns report statistics for the baseline model estimation and for parametrizations where individual parameters in the policy rule

$$i_t = \rho i_{t-1} + (1 - \rho) [\bar{i} + \iota_\pi (\pi_t - \pi_{t-1}^*) + \iota_x (x_t - x_{ss})] + u_t$$

are modified to the values reported in each column. The operators $\mathbb{E}[\cdot]$, $\sigma(\cdot)$, and $AC(\cdot)$ denote the unconditional mean, volatility, and first-order autocorrelation, respectively. $rTP^{(20)}$ and $\pi TP^{(20)}$ are the 5-year bond real term and inflation risk premia, respectively. The model statistic corresponds to the closed-form average of the second-order approximation of the solution. Volatilities, yields, and (excess) returns are in percentage terms. The inflation rate, yields, excess returns, and risk premia are annualized. The data statistics related to the real rate r are obtained from the estimated real rate. Values not reported are not available.

Statistic	Model					
	(1) Data	(2) Baseline	(3) $\iota_\pi = 1.7$	(4) $\iota_x = 0.25$	(5) $\rho = 0.72$	(6) $\phi_{\pi^*} = 0.90$
Panel A: Macroeconomic Variables						
$\sigma(\Delta c)$	0.38	0.41	0.41	0.41	0.41	0.41
$\sigma(\pi)$	1.36	1.68	1.55	1.69	1.99	1.43
$\sigma(\Delta w)$	0.66	0.42	0.40	0.42	0.46	0.42
$\sigma(x)$		0.16	0.15	0.16	0.18	0.15
$AC(\Delta c)$	0.42	-0.01	-0.01	0.00	-0.01	-0.01
$\text{corr}(\Delta c, \pi)$	-0.15	-0.10	-0.12	-0.10	-0.06	-0.12
Panel B: Real and Nominal Yield Curve						
$\mathbb{E}[i]$	5.20	5.20	5.20	5.20	5.20	5.20
$\mathbb{E}[i^{(20)} - i]$	1.38	1.12	1.31	1.05	0.97	1.12
$\mathbb{E}[r]$	1.98	1.38	1.36	1.38	1.42	1.38
$\mathbb{E}[r^{(20)} - r]$		0.82	0.99	0.73	0.77	0.82
$\sigma(i)$	2.59	2.29	2.25	2.29	2.34	2.11
$\sigma(r)$	2.09	2.84	2.80	2.82	3.30	2.84
$\sigma(r)/\sigma(i)$	0.81	1.24	1.24	1.23	1.41	1.34
$\text{corr}(i, r)$	0.99	0.92	0.95	0.91	0.93	1.00
$\sigma(i^{(20)})/\sigma(i)$	1.02	0.40	0.33	0.42	0.40	0.13
$\sigma(r^{(20)})/\sigma(r)$		0.13	0.13	0.13	0.15	0.13
Panel C: Expected Excess Returns						
$\mathbb{E}[XR^{s,(20)}]$	4.28	1.97	2.14	1.87	1.74	1.97
$\mathbb{E}[XR^{c,(20)}]$		1.17	1.40	1.06	1.17	1.17
$SR^{s,(20)}$	0.32	0.32	0.38	0.30	0.25	0.32
$SR^{c,(20)}$		0.18	0.22	0.17	0.14	0.18
$\mathbb{E}[rTP^{(20)}]$		1.03	1.24	0.94	1.03	1.03
$\mathbb{E}[\pi TP^{(20)}]$		0.86	0.90	0.87	0.66	0.85

Table 8: Data and Model Implied Statistics - Habit Persistence and Capital

The data statistics are for the 1982:Q1 to 2008:Q3 period. The parameter values of the baseline model are reported in Table 2. The operators $\mathbb{E}[\cdot]$, $\sigma(\cdot)$, and $AC(\cdot)$ denote the unconditional mean, volatility, and first-order autocorrelation, respectively. $rTP^{(20)}$, and $\pi TP^{(20)}$ are the 5-year bond real term and inflation risk premia, respectively. “Baseline” indicates an economy with both price and wage rigidities and all four exogenous shocks. “Habit” indicates an economy with external habit persistence in household preferences. “Capital” indicates an economy with capital accumulation. The model statistic corresponds to the closed-form average of the second-order approximation of the solution. Volatilities, yields, and (excess) returns are in percentage terms. The inflation rate, yields, excess returns, and risk premia are annualized. The data statistics related to the real rate r are obtained from the estimated real rate. Values not reported are not available. All estimations use $\gamma = 400$. The objective value is the sum of squared percentage differences between the model- and data-implied moments targeted in the estimation.

Statistic	Data	Model		
		Baseline	Habit	Capital
Panel A: Parameter values				
b_h		0.00	0.42	0.90
ζ		-	-	4.00
Panel A: Macroeconomic Variables				
Objective value		1.33	0.29	0.67
$\sigma(\Delta c)$	0.38	0.41	0.35	0.34
$\sigma(\pi)$	1.36	1.60	1.73	1.90
$\sigma(\Delta w)$	0.66	0.38	0.51	0.29
$\sigma(\Delta y)$	0.65	0.41	0.35	0.82
$\sigma(\Delta j)$	2.45	-	-	2.46
$\sigma(x)$		0.16	0.14	1.55
$AC(\Delta c)$	0.42	0.00	0.41	0.40
$AC(\Delta c_h)$	0.00	0.00	0.00	-0.23
$\text{corr}(\Delta c, \pi)$	-0.15	-0.12	-0.08	-0.12
Panel B: Real and Nominal Yield Curve				
$\mathbb{E}[i]$	5.20	5.20	5.20	5.20
$\mathbb{E}[i^{(20)} - i]$	1.38	0.73	0.65	0.58
$\mathbb{E}[r]$	1.98	1.62	1.61	1.65
$\mathbb{E}[r^{(20)} - r]$		0.12	0.11	0.05
$\sigma(i)$	2.59	2.34	2.36	1.49
$\sigma(r)$	2.09	2.83	2.91	1.23
$\sigma(r)/\sigma(i)$	0.81	1.21	1.24	0.83
$\text{corr}(i, r)$	0.99	0.92	0.93	0.35
$\sigma(i^{(20)})/\sigma(i)$	1.02	0.40	0.39	0.88
$\sigma(r^{(20)})/\sigma(r)$		0.13	0.13	0.08
Panel C: Expected Excess Returns				
$\mathbb{E}[XR^{S,(20)}]$	4.28	1.10	1.14	0.94
$\mathbb{E}[XR^{(20)}]$		0.70	0.64	0.21
$SR^{S,(20)}$	0.32	0.20	0.18	0.31
$SR^{(20)}$		0.11	0.10	0.11
$\mathbb{E}[rTP^{(20)}]$		0.65	0.56	0.20
$\mathbb{E}[\pi TP^{(20)}]$		0.55	0.54	0.98
Panel D: Additional Implications				
$\sigma(\Delta d)$	8.10	11.27	11.96	7.08
$\mathbb{E}[XR_d]$	7.51	4.06	7.41	2.32

Table 9: **Data and Model Implied Statistics - The Effects of Stochastic Volatility in Shocks**

The operators $\mathbb{E}[\cdot]$, and $\sigma(\cdot)$ denote the unconditional mean and volatility, respectively. $rTP^{(20)}$, and $\pi TP^{(20)}$ are the 5-year bond real term and inflation risk premia, respectively. “Baseline” indicates an economy with both price and wage rigidities and all four exogenous shocks. “Habit” indicates an economy with external habit persistence in household preferences. “Capital” indicates an economy with capital accumulation. Columns labeled as “No SV” corresponds to the case $\nu_a = \nu_z = 0$. Columns labeled $\nu_a = -100$ and $\nu_z = 100$ correspond to the specifications with stochastic volatility in the permanent and transitory components in productivity, respectively. The model statistic corresponds to the simulated average statistics for a sample of 1,000 periods of the third-order approximation of the solution. Volatilities, yields, (excess) returns, and risk premia are in percentage terms. The excess returns, and risk premia are annualized. All estimations use $\gamma = 400$.

	Baseline			Habit			Capital		
	No SV	$\nu_a = -100$	$\nu_z = -100$	No SV	$\nu_a = -100$	$\nu_z = -100$	No SV	$\nu_a = -100$	$\nu_z = -100$
Panel A: Means									
$\mathbb{E}[XR^{\$, (20)}]$	1.10	1.11	1.10	1.15	1.15	1.15	0.94	0.94	0.94
$\mathbb{E}[XR^{c, (20)}]$	0.69	0.69	0.69	0.64	0.64	0.64	0.20	0.20	0.21
$\mathbb{E}[rTP^{c, (20)}]$	0.65	0.65	0.65	0.56	0.56	0.56	0.20	0.20	0.21
$\mathbb{E}[\pi TP^{c, (20)}]$	0.55	0.55	0.55	0.54	0.54	0.54	0.98	0.98	0.98
Panel B: Standard Deviations									
$\sigma(XR^{\$, (20)})$	0.01	0.33	0.06	0.03	0.40	0.17	0.12	0.13	0.12
$\sigma(XR^{c, (20)})$	0.01	0.18	0.09	0.02	0.18	0.21	0.01	0.10	0.08
$\sigma(rTP^{(20)})$	0.01	0.17	0.08	0.02	0.16	0.18	0.01	0.10	0.08
$\sigma(\pi TP^{(20)})$	0.01	0.19	0.04	0.02	0.22	0.08	0.13	0.15	0.13

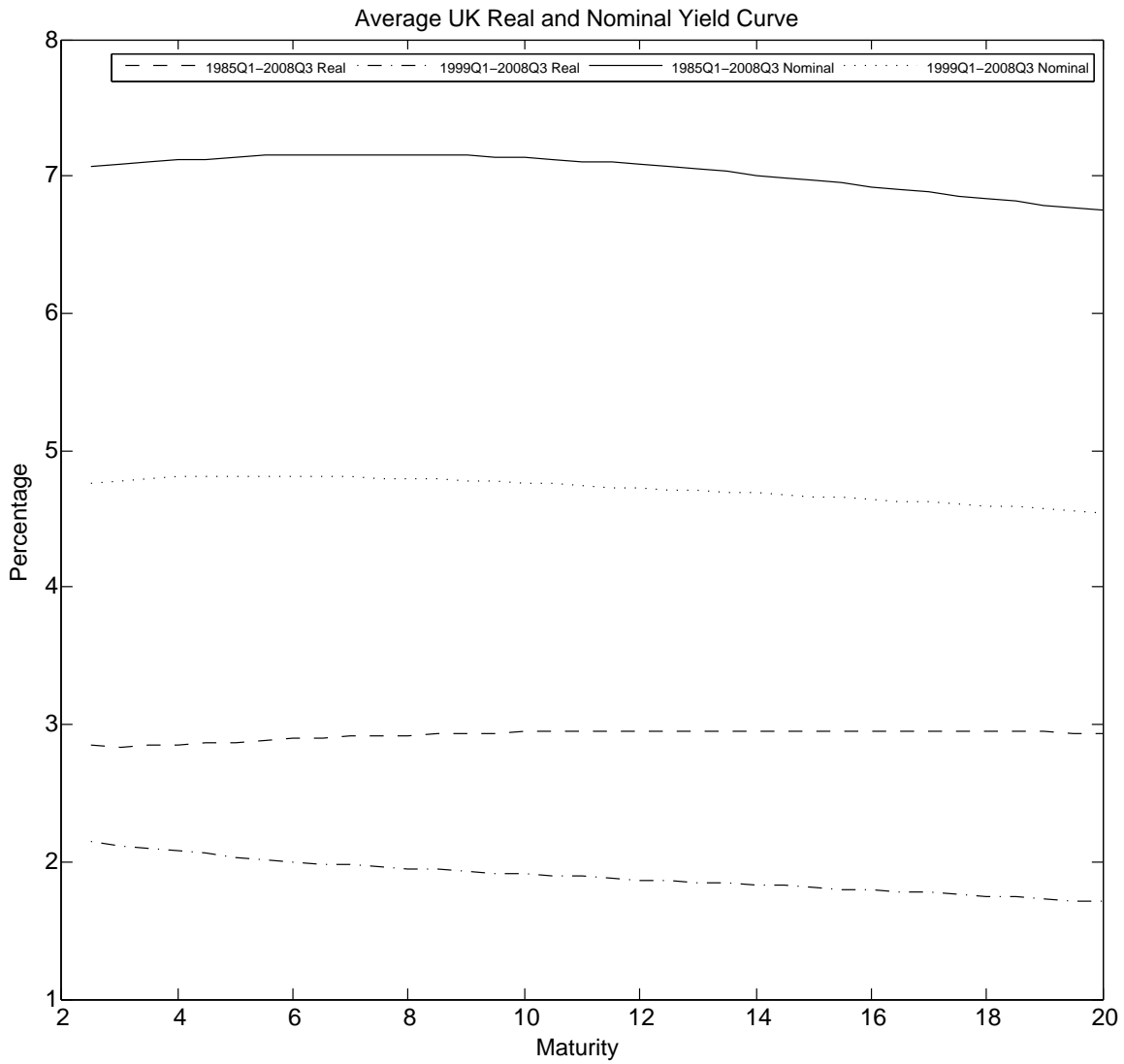


Figure 1: Average yield curve for United Kingdom inflation-indexed and nominal bonds.

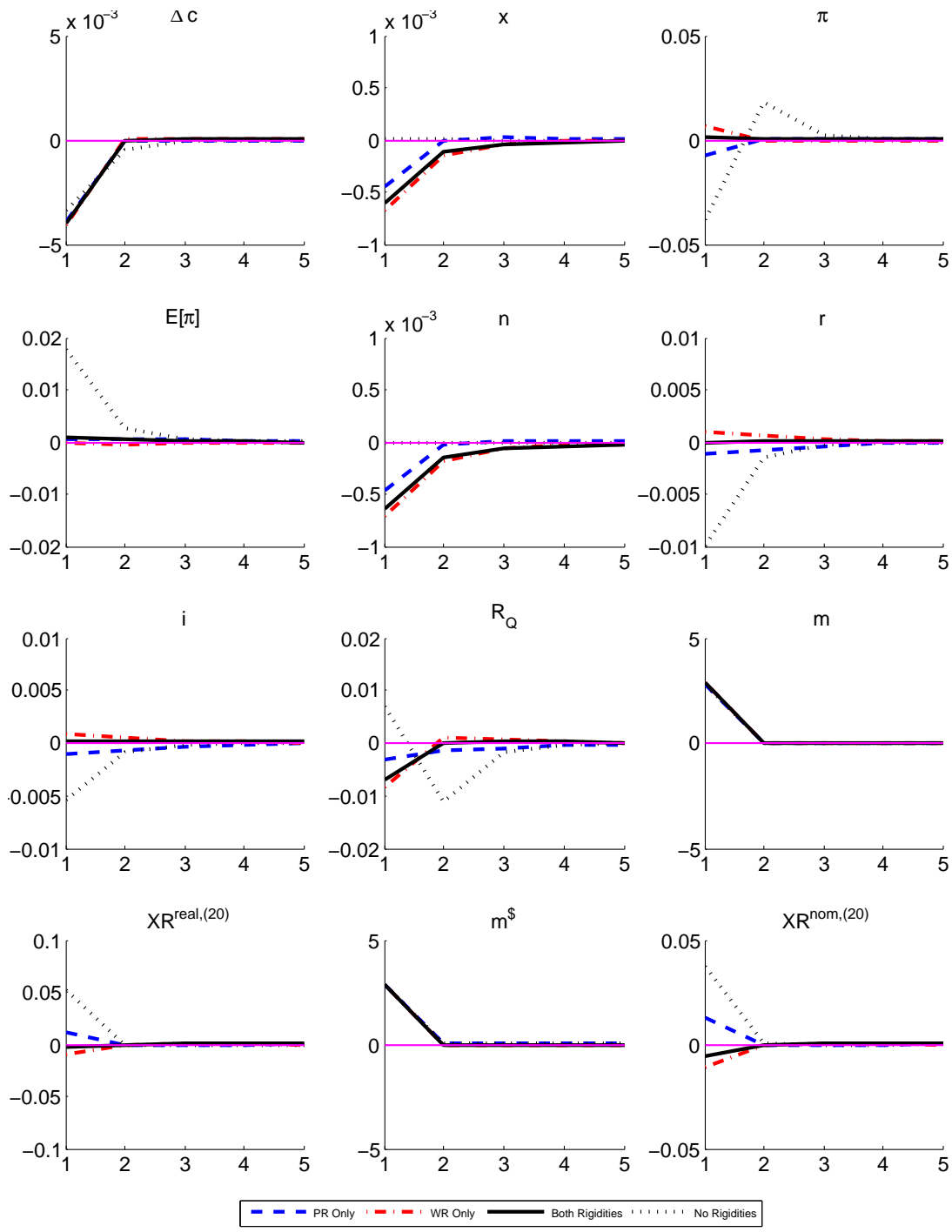


Figure 2: Impulse responses to a one-standard deviation negative permanent productivity shock.

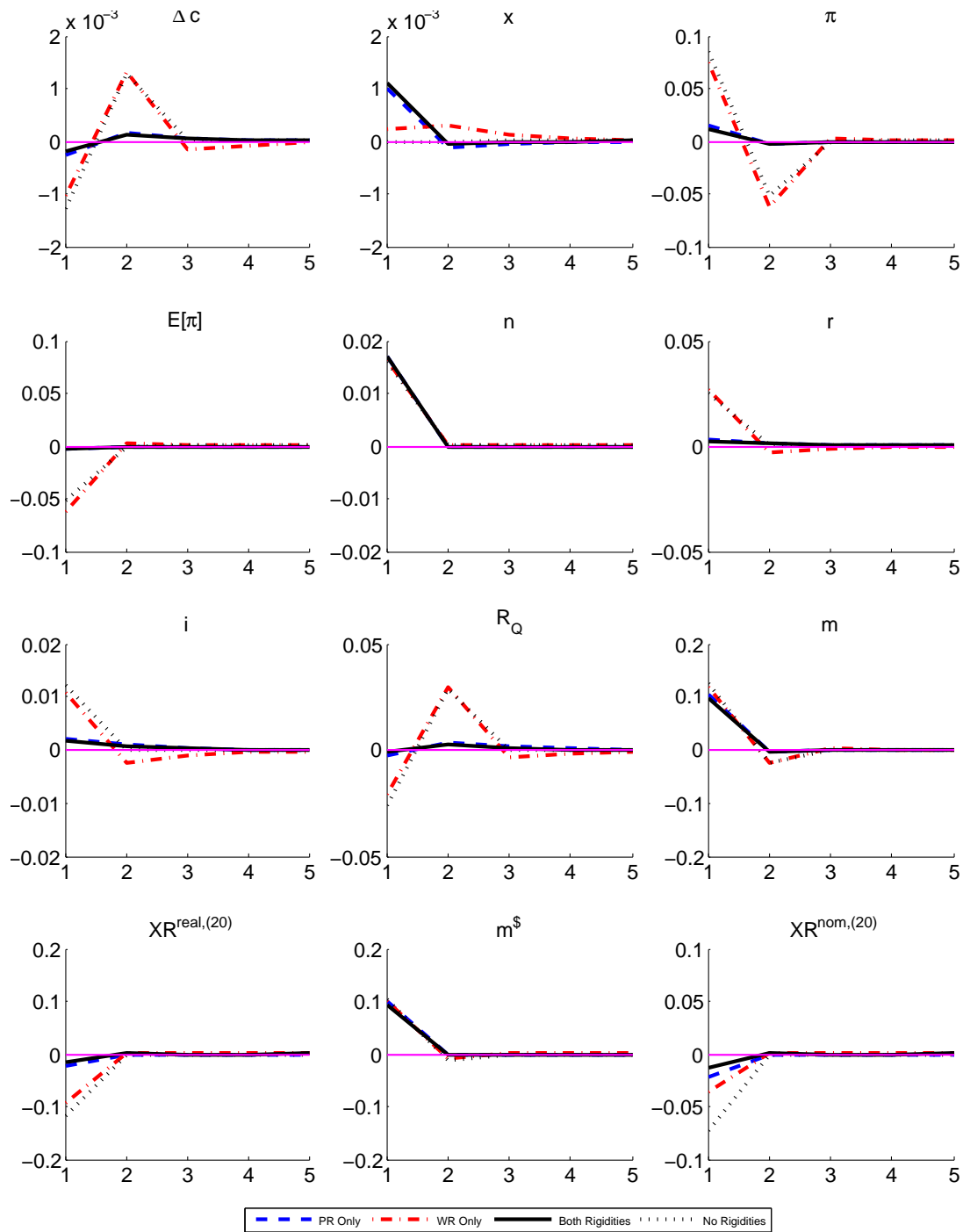


Figure 3: Impulse responses to a one-standard deviation negative transitory productivity shock.

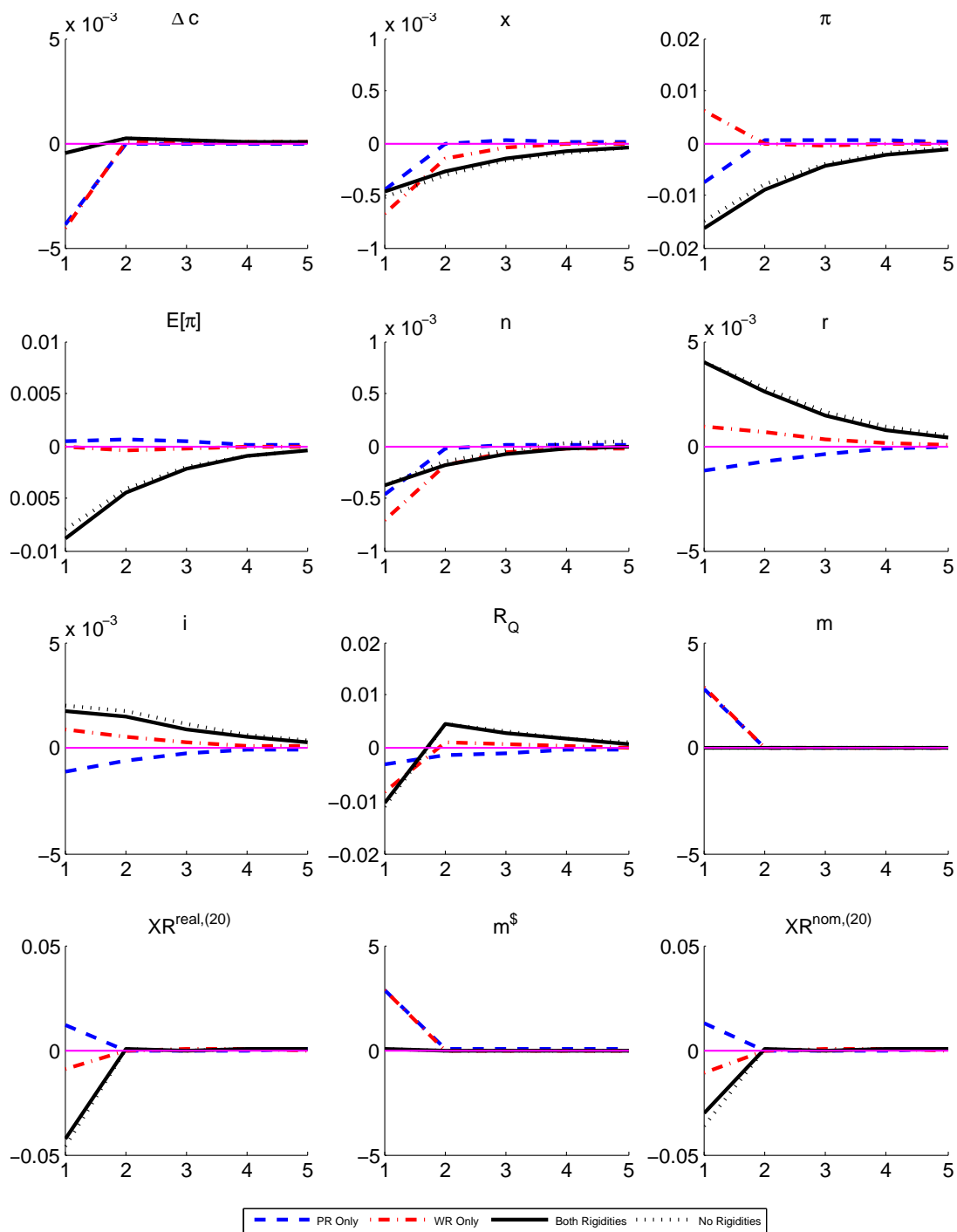


Figure 4: Impulse responses to a one-standard deviation positive monetary policy shock.