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REAL BUSINESS CYCLES IN AS SMALL OPEN

ECONOMY: THE CANADIAN CASE

by

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This paper investigates an open-economy extension of real business cycle theory. The equilibrium stochastic process of an artificial economy is numerically computed in order to calculate measures of volatility, comovement and persistence of the main macro-aggregates. These population moments are compared with actual sample moments from Canadian data. The results show that the open-economy model requires smaller and less persistent technological disturbances than the closed-economy prototypes. The model matches closely the behavior of most variables, except private investment. An interpretation that accounts for the volatile behavior of investment in the artificial economy is provided.

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THE CANADIAN CASE.

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This paper investigates an open-economy extension of real business cycle theory. The equilibrium stochastic process of an artificial economy is numerically computed in order to calculate measures of volatility, comovement and persistence of the main macro-aggregates. These population moments are compared with actual sample moments from Canadian data. The results show that the open-economy model requires smaller and less persistent technological disturbances than the closed-economy prototypes. The model matches closely the behavior of most variables, except private investment. An interpretation that accounts for the volatile behavior of investment in the artificial economy is provided.

I.- **Introduction.**

Real models of economic fluctuations, such as Kydland and Prescott (1982), Prescott (1986) and Hansen (1985), have been very successful in reproducing some of the observed features of postwar U.S. business cycles. In these models, random productivity disturbances generate intertemporal income and substitution effects that induce optimizing agents to adjust investment and smooth consumption. These effects, in conjunction with the volatility and persistence of the disturbances, cause the models to exhibit a pattern of dynamic behavior similar to that observed in actual business cycles. The empirical relevance of this theory is evaluated by comparing the statistical moments of the endogenous variables in the artificial economies with the actual sample moments.

This paper extends the basic real business cycle model to a small open-economy environment. The resulting prototype is numerically solved and calibrated in order to evaluate the model's ability to reproduce the stylized facts of Canadian business cycles.

The study of the open-economy case is an interesting addition to the existing work on real business cycle theory, since the models developed to date analyze economies where the domestic capital stock is the only vehicle used to accumulate savings. In an open-economy framework, agents in the domestic economy can also accumulate savings in the form of foreign financial assets. Thus, the role played by international asset trading in the intertemporal allocation of consumption is stressed here.

The empirical evidence suggests that foreign asset accumulation is important for the understanding of the dynamics of savings. Trade balance and current account fluctuations are large in size, negatively correlated with

domestic output and display positive persistence. In the case of Canada, the volatility of net foreign interest payments reaches 15.25% whereas the variability of domestic private investment is only 9.82%[1]. Hence, the integration of the foreign sector appears to be an interesting additional way of evaluating the empirical performance of real business cycle theory.

The present work is also motivated by the desire to empirically implement a general model of dynamic open-economy macroeconomics. Although the pure analysis of international finance issues from the perspective of optimizing models has been the subject of numerous studies (as reviewed in Frenkel and Razin (1987) and Kimbrough (1987)), the empirical testing of these models is rarely pursued[2]. In fact, as Hercowitz (1986b) noted, some of the observed features of a small open economy do not seem to fit well with the predictions of a simple dynamic-optimizing framework[3]. By constructing a model that studies a stochastic production economy with an infinite life-horizon, the present paper provides a hopefully useful tool for the empirical analysis of the relationship between savings and investment in a small open economy[4].

This paper studies an abstract economy where the existence of an international, perfectly competitive capital market is allowed. The dynamic behavior of the balance of trade and the current account is the outcome of the optimal intertemporal choices with respect to foreign financial asset accumulation. The interaction between domestic capital and international assets, as alternative vehicles of savings, is highlighted.

In the spirit of the work by Obstfeld (1981), the model presented here features an endogenous rate of time preference. This is utilized to determine a well-behaved stationary equilibrium for the holdings of international assets. Accordingly, the stationary cardinal utility function (SCU),

formulated by Epstein (1983), is adopted as the time-recursive expression of preferences. SCU assumes that the instantaneous rate of impatience is an increasing function of past consumption levels. A set of restrictions is imposed on this utility function to allow the use of dynamic programming techniques, and to ensure the existence of a unique stationary distribution of the state variables in the stochastic steady-state of the economy. These restrictions also imply that the use of the stationary cardinal utility does not constitute a major deviation from the standard time-separable setup[5].

The equilibrium stochastic process of the model economy is numerically computed following a procedure suggested in Bertsekas (1976). This methodology was utilized by Sargent (1980) and was introduced to the real business cycle literature by Greenwood, Hercowitz and Huffman (1988). The procedure is based on value function iteration defined on a discretized version of the state space[6]. Using the optimal decision rules obtained in such way, the exact joint limiting distribution of the state variables is computed. The endogenous time-preference framework is required here to guarantee the existence, uniqueness and stationarity of such limiting distribution.

In accordance with the theoretical analysis of the model, the numerical investigation performed here confirms that the variability of domestic capital depends on the volatility and persistence of the exogenous technological disturbances[7]. It also illustrates that the model's ability to reproduce the countercyclical behavior of the foreign assets and the balance of trade depends on the expected permanence of the shocks.

The model analyzed in the present paper mimics fairly well the stochastic behavior of GNP, consumption, foreign interest payments, savings

and the trade balance-output ratio. In contrast, the volatility, persistence and output-correlation of private investment are poorly approximated. As will be explained later, the model's lack of accuracy in duplicating the behavior of investment is connected to the frictionless manner in which it can be undertaken. Thus, the results indicate that the simple view of capital accumulation, as a process free of adjustment costs or irreversibility restrictions, may be unsatisfactory for the study of small open-economy real business cycles.

The paper proceeds as follows: The next section describes the structure of the model, the functional forms employed and the manner in which the stochastic disturbances are treated. Section III discusses the solution procedure and the specification of the set of parameters used to calibrate the model. Section IV presents the results of various simulation exercises, comparing the performance of the open-economy model with the results obtained for closed-economy prototypes. Some concluding remarks are presented in section V.

II.- A Stochastic Growth Model for a Small Open Economy.

The representative agent economy studied here produces an internationally tradable composite commodity. The production technology is described in standard Cobb-Douglas form

$$\epsilon_t F(K_t, L_t) = \exp(e_t) \; K_t^\alpha \; L_t^{1-\alpha} \qquad\qquad 0 < \alpha < 1 \qquad\qquad (1)$$

where L_t are labor services, K_t is the capital stock, α is the capital's share in output and ϵ_t is a technology disturbance that follows a stochastic process to be described later. Following Greenwood (1983), it is possible to assume

that GDP is only an exportable commodity and that the disturbances represent terms of trade shocks.

Agents in this model economy have access to a perfectly competitive, international capital market. In this market, foreign financial assets $\dot{A}_t$ paying the non-random real rate of return r^* can be traded with the rest of the world. There are no restrictions on the international flow of goods and services, and the domestic economy is assumed to be small relative to the size of the world's capital market. Thus, r^* is taken as given by the agents when formulating their optimal intertemporal plans[8].

The resources available for consumption, denoted C_t, investment, I_t, and foreign asset accumulation, CA_t, are obtained from domestic production adding (substracting) foreign interest income (payments) in the amount r^*A_{t-1}. Hence, domestic absorption plus the current account cannot exceed GNP:

$$C_t + I_t + CA_t \leq \exp(e_t) K_t^\alpha L_t^{1-\alpha} + r^*A_{t-1} \tag{2}$$

The laws of motion for international assets and domestic capital are given by the following two equations:

$$K_{t+1} = (1-\delta)K_t + I_t \qquad 0 \leq \delta \leq 1 \tag{3}$$

$$A_t = CA_t + A_{t-1} \tag{4}$$

The agent allocates C_t and L_t intertemporally so as to maximize the expected value of his lifetime utility as given by

$$E_0 \left[\sum_{t=0}^{\infty} \{ u(C_t, L_t) \exp[-\sum_{\tau=0}^{t-1} v(C_\tau, L_\tau)] \} \right] \tag{5}$$

Instantaneous utility is given the isoelastic form

$$u[C_t - G(L_t)] = (1-\gamma)^{-1} [(C_t - L_t^\omega/\omega)^{1-\gamma} - 1] \qquad \omega > 1 \qquad \gamma > 1 \qquad (6)$$

with the impatience function being specified in the logarithmic form

$$v[C_t - G(L_t)] = \beta \ \text{Ln} \ [1 + C_t - L_t^\omega/\omega] \qquad \beta > 0 \qquad (7)$$

The functions $u(\cdot)$ and $v(\cdot)$ are constructed so as to make the marginal rate of substitution between C and L dependent on the latter only. Hence, the wealth effect on labor supply is eliminated. This pair of functions also satisfies the restrictions required by Epstein (1983):

$$u(\cdot)<0 \ , \ u'(\cdot)>0 \ , \ u'(0) = \infty \qquad (8.1)$$

$$v(\cdot)>0, \ v'(\cdot)>0, \ v''(\cdot)<0 \qquad (8.2)$$

$$u'(\cdot)\exp[v(\cdot)] \ \text{non-increasing} \qquad (8.3)$$

$$\text{Log}[-u(\cdot)] \ \text{convex} \qquad (8.4)$$

The logarithmic form in (7) ensures that (8.1)-(8.4) are all satisfied as long as $\beta \leq \gamma$.

As will be explained later, the particular values for γ (coefficient of relative risk aversion), ω (1 plus the inverse of the intertemporal elasticity in labor supply), α (capital's share in output), δ (depreciation rate) and β (the consumption elasticity of the rate of time preference), are selected using long-run actual averages and the estimates obtained in the relevant empirical literature[9].

The stochastic structure of the problem is simplified following the same strategy utilized by Greenwood, Hercowitz and Huffman (1988). Accordingly, the disturbances are assumed to follow a two-point Markov process, so that in any given period the productivity shock can take one of two values

$$e_t \in E = \{ e^1 , e^2 \} \tag{9}$$

The transition probabilities of the disturbance starting in state "s" and moving to state "r" in one period are denoted as π_{sr}, for s,r=1,2. The usual properties that $0 \leq \pi_{sr} \leq 1$ and $\pi_{s1} + \pi_{s2} = 1$ for s,r=1,2 must be satisfied. Then the assumptions that $\pi_{11} = \pi_{22} = \pi$ and $e^1 = e^2 = e$ are adopted. These imply that the asymptotic standard deviation, σ_e, and the first-order autocorrelation coefficient, ρ_e, that characterize the stochastic shock are given by "e" and $2\pi - 1$ respectively.

At each date there are four goods that the agents can trade: gross domestic output, foreign financial assets, labor services and domestic capital[10]. The distortion-free nature of the model ensures that both basic theorems of welfare economics hold. The Pareto optimum corresponds therefore to a competitive equilibrium which is the solution to the dynamic problem of maximizing (5) subject to (2)[11]. The state of the economy at any given date is described by the observed value of the disturbance, the initial capital stock and the initial holdings of international assets (i.e. the triple (e^s_t, K_t, A_{t-1})). Given this, agents decide how much to consume, how much labor services to supply and how much to save in the form of both domestic capital and foreign assets (i.e. they must choose C_t, L_t, K_{t+1}, and A_t).

This intertemporal optimization problem can be solved using dynamic programming techniques to find a solution to Bellman's functional equation. The structure introduced above implies that in this case the functional equation problem adopts the following form:

$$V(K_t, A_{t-1}, e^{s}t) = \max \{ (1-\gamma)^{-1} [(C_t - \hat{L}_t^{\omega}/\omega)^{1-\gamma} - 1] +$$

$$\exp[-\beta \ \text{Ln}(1+C_t - \hat{L}_t^{\omega}/\omega)] [\sum_{r=1}^{2} \pi_{sr} V(K_{t+1}, A_t, e^{r}_{t+1})] \}$$

(10)

s.t.
$$C_t = \exp(et) K_t^{\alpha} \hat{L}_t^{1-\alpha} - K_{t+1} + K_t(1-\delta) + (1+r^*)A_{t-1} - A_t$$

with:
$$\hat{L}_t = \text{argmax} \{ \exp(et) K_t^{\alpha} L_t^{1-\alpha} - L_t^{\omega}/\omega \}$$

The concavity of the value function in this case is guaranteed by the properties of $u(\cdot)$, $v(\cdot)$ and $F(\cdot)$[12].

III.- Numerical Solution of the Model.

Since this is a case where the value function cannot be solved for analytically, a numerical procedure based on a discretization of the state space is utilized. As mentioned before, this methodology follows the works of Bertsekas (1976), Sargent (1980) and Greenwood, Hercowitz and Huffman (1988). This technique makes use of the contraction property that mappings of the kind defined by functional equations like (10) exhibit. The functional equation problem is solved by the method of successive approximations starting from an initial guess $V^0(\cdot)$. A computer algorithm performs the iterations on (10), starting with $V^0(\cdot)$ on the right-hand side to obtain a new guess $V^1(\cdot)$ as the result of a maximization routine. The process continues until the stochastic decision rules for capital and assets converge[13].

Sargent (1980) and Greenwood, Hercowitz and Huffman (1988) discuss how this procedure obtains the exact joint p.d.f. of the state variables in the stochastic stationary equilibrium of the economy. What follows here is a brief review of how the procedure is adapted to the case of a small open

economy.

The first step is to define the discrete state space used to analyze the problem. Capital and assets can take any value contained in the corresponding finite, time invariant, grid: $K = \{ K_1, \ldots, K_N \}$ and $A = \{ A_1, \ldots, A_M \}$. Hence, the state space for the model is the discrete set defined by the product $K \times A \times E$ of dimensions $N \times M \times 2$. The definition of these grids is an important part of the solution, since they contain the specific set of numbers used to evaluate (10) in search for the maximum. The aim is to define K and A so as to capture the ergodic set for the joint stationary distribution of K, A and "e", refining the grids until the covariances among the state variables converge[14]. The grids are initially centered around the deterministic stationary equilibrium values of domestic capital and foreign assets. In this model, steady-state holdings of A and K are determined by a simultaneous equality between the rate of time preference, the net marginal productivity of capital and the world's real interest rate[15].

The algorithm solves for the value function and determines unique, state-contingent decision rules for both capital and foreign assets. These policy rules have the following form:

$$K_{t+1} = k(A_{t-1}, K_t, e^s_t) \in K \qquad (11)$$

$$A_t = a(A_{t-1}, K_t, e^s_t) \in A \qquad (12)$$

Both are used to construct the one step transition probabilities of moving from any initial triple (K^m, A^n, e^s) to any other triple (K^p, A^q, e^r) in one period. Each of these transitions is determined using the following property of the decision rules:

$$\Pr[\ K_{t+1}=K^p,\ A_t=A^q\ |\ K_t=K^m,\ A_{t-1}=A^n,\ e_t=e^s]\ =\ 1.0 \qquad (13)$$

only if   a) $K^p\ =\ k(\ A^n,\ K^m,\ e^s\ )$

and   b) $A^q\ =\ a(\ A^n,\ K^m,\ e^s\ )$

and zero otherwise. The corresponding transition probability ($P_{pqr,mns}$) is found by multiplying (13) by its corresponding π_{sr}. Following this procedure, the (2MN×2MN) transition matrix denoted as "P" is created. This matrix has most of its components set to zero, and the rest are the π_{sr}'s located at the coordinates defined by the decision rules on both grids.

Next, the unique joint stationary distribution function for the state variables is obtained using the fact that the sequence defined by $\rho^1=\rho^0 P$ is also a contraction. Making an initial guess for ρ^0 is possible to iterate repeatedly until the sequence converges to a limiting fixed point ρ^*. This 1×2MN vector contains the set of stationary probabilities for each triple of foreign assets, domestic capital and the stochastic disturbances. Such limiting distribution is then used to compute population moments of all variables in the system, since they are all functions of the decision rules (11) and (12). Consequently, expected values, variances, autocorrelations and correlations of C, I, GDP, GNP, L, K, r^*A, S (savings) and TB/Y (the trade balance-output ratio) can be computed.

Finally, the probabilistic structure of the model is calibrated so as to mimic a particular subset of the actual moments that characterize Canadian business cycles, and then the rest of the moments from the artificial economy are compared with those actually observed. Once the structural parameters are fixed, the calibration exercise is performed by adjusting the size of the shock "e" (i.e. the standard deviation σ_e) and its transition probability π

(i.e. the first-order autocorrelation coefficient ρ_e). The model has been calibrated to mimic the percentage standard deviation and first-order autocorrelation of annual, postwar Canadian GDP.

The structural parameters for the Canadian economy have been assigned the following values:

$$\alpha = 0.32 \quad \beta = 0.11 \quad \gamma = 1.001, 2.0 \quad \delta = 0.1 \quad \omega = 1.455 \quad r^* = 0.04$$

The parameter "α" was determined from the long-run average of the ratio of labor income to net national income at factor prices. The rate of depreciation $\delta=0.1$ corresponds to the one commonly used in the real business cycle literature. With this value, the average investment-output ratio is 23.0%, which is very close to the 21.5% observed in the data[16].

The value of "ω" is in the range of the estimates of the intertemporal elasticity of substitution in labor supply ($1/(\omega-1)$) obtained by the studies of MaCurdy (1981) and Heckman and MaCurdy (1980, 1982). MaCurdy (1981) estimated that, for adult males not younger than 25 years, the intertemporal elasticity of substitution in labor supply at the intensive margin is around 0.3. Heckman and MaCurdy (1980,1982) obtained corresponding estimates for females at both the intensive and extensive margins of about 2.2. The value of this parameter in the present model is 2.19, which is very close to the upper bound of the range (0.3,2.2)[17]. A sensitivity analysis of the model's statistical moments to changes in the value of this parameter showed that the behavior of all the statistics, except the volatility of labor supply, is generally independent of the particular number assigned to it.

The value of r^* corresponds to what Prescott (1986) and Kydland and Prescott (1982) consider to be the long-run annual real interest rate. The

parameter "β" is then determined combining the deterministic steady-state equation for assets, the specified values for the other parameters and the actual long-run ratio of GNP to GDP over the sample period. Hence, this parameter is set so as to equalize the rate of time preference with the world's interest rate in the non-random, stationary equilibrium.

Following Greenwood, Hercowitz and Huffman (1988) -from here on GHH-, two alternative sizes of the risk aversion parameter were used. The value of 1.001 is consistent with the findings of Hansen and Singleton (1983) and the value of 2.0 is taken from the results obtained by Friend and Blume (1975), although the models analyzed by these authors are not entirely consistent with the one studied here. A sensitivity analysis of the model's behavior to changes in this parameter was also performed.

Preliminary simulations illustrated how some of the theoretical implications of the model are correctly replicated by this numerical procedure. First, the volatility of domestic capital is directly related to the serial correlation of the disturbances. With a 1.18% serially uncorrelated shock the percentage standard deviation of K is only 0.83%, and it increases to 4.6% when a shock of the same size exhibits 0.99 first-order autocorrelation. Second, larger variations in the holdings of international assets relative to the domestic capital stock are always required. In the case of the 1.18% shock, the volatility of foreign assets is 15.7% when the shock is serially uncorrelated and 42.3% when the shock is highly persistent.

These results can be interpreted as follows: In the deterministic version of the model, a transitory productivity improvement induces optimizing agents to adjust their holdings of foreign assets as required to obtain the desired adjustment in the consumption path. Investment is not affected

because a once-and-for-all productivity change cannot affect its real rate of return. But when the productivity improvement has some persistence, capital accumulation fluctuates according to the difference in the marginal returns of K and A. Consequently, in the stochastic representation of the model, the lower the serial correlation of the shock, the less variable capital is relative to foreign financial assets[18].

IV.- Simulation Results and Analysis.

A series of numerical simulations were performed in order to calibrate the model and evaluate its sensitivity to changes in the various parameters specified before. The most important aspects of the results obtained are reviewed next.

The first exercise was designed to achieve a close comparison with the existing work on closed-economy real business cycles. The risk aversion parameter was set to the value $\gamma=2.0$, which corresponds to what GHH consider best to fit a closed economy with variable utilization and investment shocks. Two evenly spaced grids containing 22 points each were chosen to contain the admissible values of capital and foreign assets, the K grid spans the interval [3.25,3.56] and the A grid the interval [-1.42,0.08]. Hence, there are a total of 484 different options for the allocation of savings.

The actual statistics used as references to evaluate and calibrate the model are obtained from per-capita, postwar, annual data in terms of the 15+ population, logged and detrended with a linear quadratic time trend.

Given the parameter and grid specifications mentioned before, the model mimics the percentage standard deviation and serial correlation of GDP with a technological disturbance that exhibits 1.18% standard deviation and 0.35

first-order autocorrelation. The complete results obtained from the artificial economy and the statistics from the actual data are reported in table 1. The corresponding marginal probability density for capital and foreign assets is depicted in figure 1.

Although comparisons with the existing work are complicated by differences in solution methods, filtering procedures and the location of the disturbances, this numerical analysis shows that the required exogenous persistence and variability of the productivity shocks appears to be lower than comparable values found in closed-economy models. The shocks in the works of Prescott (1986) and Hansen (1985) follow a stochastic process close to a random walk, the second author estimates their quarterly correlation to be 0.95, or 0.81 annually. In the investment shock-endogenous utilization model of GHH, the required annual persistence of the disturbances is 0.51 (when the coefficient of relative risk aversion is given a value of 2). In contrast, the present model necessitates only a yearly autocorrelation of 0.35. Furthermore, following the criterion of GHH for comparing the size of the disturbances using the ratios of their required percentage volatility to that of output, the model of Hansen (1985) produces the values 1.3 and 1.7 and the GHH prototype gives 1.47. The open-economy model generates a ratio equal to only 0.42.

This observation contradicts a familiar result obtained with the IS-LM-BP model. In that model, output in an open economy with perfect capital mobility and flexible exchange rates should be less responsive to real domestic shocks than in a closed economy. Since the interest rate is fixed at its world level, a positive domestic shock shifts the IS to the right causing an output expansion and a balance of payments surplus. Then the exchange rate

appreciates and the IS curve moves back to its original position. Thus, this model predicts that a small open economy should require larger shocks than a closed economy in order to replicate the variability of output.

The inspection of table 1 illustrates that the model performs well in reproducing the volatility and persistence of most aggregates. The model is consistent with the facts in generating a consumption process which is less volatile than either domestic or national output, gross savings, investment and foreign interest payments. It is also compatible with actual observations in that S, I and r*A are the most volatile variables[19].

As column A of the model's results shows, the percentage standard deviation of the majority of the aggregates is quite well approximated, except those of savings, investment and foreign interest payments. The volatility of savings in the artificial economy is only 5.8%, which is below the 7.3% observed in the data. In contrast, the model exaggerates the variability of both investment and foreign interest payments (21.0% instead of 9.8% for the former and 19.6% instead of 15.3% for the latter).

The first-order serial correlations reported in column B also mimic the majority of the actual statistics, except in the cases of investment, capital and the trade balance-output ratio. The results reproduce the fact that C and r^*A possess the highest serial autocorrelation coefficients, whereas S, I and K have the lowest. The artificial economy also matches the facts in generating foreign interest payments as the most serially correlated variable, and is very close in duplicating the persistence of savings and consumption.

Table 1

Statistical Moments: Canadian Data and Artificial Economy[a].

| Variables	Canadian Data 1946-1985			Artificial Economy $\gamma=2.0$, $\sigma_\epsilon=1.18\%$, $\rho_\epsilon=0.356$		
	(A)[b]	(B)[c]	(C)[d]	(A)[b]	(B)[c]	(C)[d]
1) GDP	2.810	0.615	1.000	2.810	0.615	1.000
2) GNP	2.950	0.643	0.995	2.821	0.619	0.990
3) C	2.460	0.701	0.586	2.086	0.693	0.944
4) S	7.306	0.542	0.662	5.772	0.599	0.932
5) I	9.820	0.314	0.639	21.056	-0.319	0.235
6) K	1.380	0.649	-0.384	1.980	0.377	0.669
7) L	2.020	0.541	0.799	1.936	0.615	1.000
8) r^*A	15.250	0.727	-0.175	19.566	0.886	-0.198
9) TB/Y	0.019	0.623	-0.129	0.046	-0.312	0.032
	CORR(S,I) = 0.434			CORR(S,I) = 0.251		

[a]The data are measured in per-capita terms of the 15+ population, logged and detrended with a linear quadratic time trend. 1)-3), 5) and 8) are the totals from the national income accounts in 1981 dollars. 6) was obtained from the end-of-period net stocks of fixed non-residential capital in manufacturing and non-manufacturing industries, in 1981 prices. The labor data is an index of man hours worked by paid workers with 1981=100.0. Savings in 4) is generated as investment plus the trade balance surplus. The source of all is the CANSIM data retrieval.

[b]Percentage standard deviation (except 9)).

[c]First-order autocorrelation coefficient.

[d]Coefficient of correlation with GDP.

MARGINAL PROBABILITY DENSITY FOR CAPITAL AND ASSETS
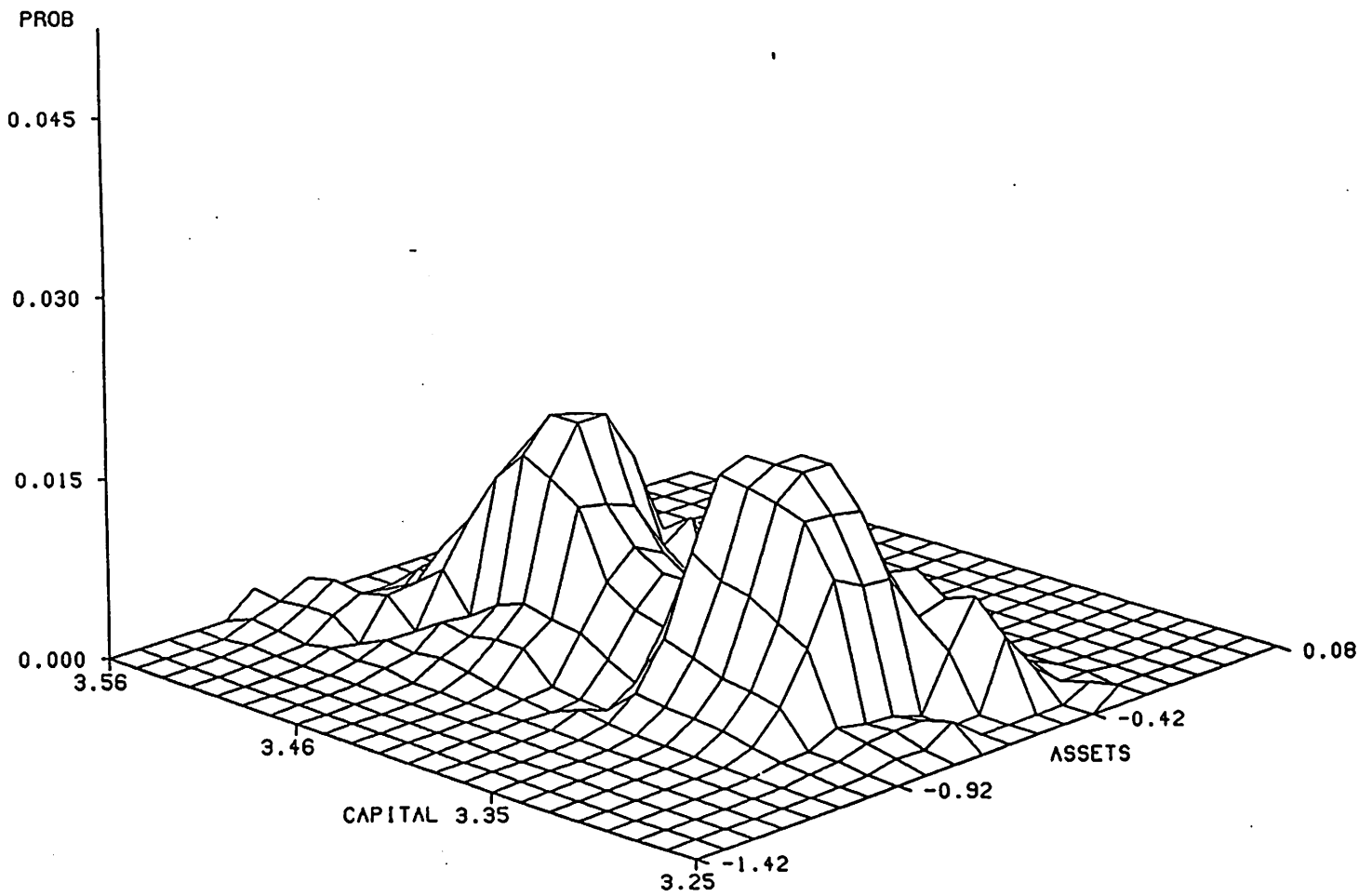
In contrast, this model does not prove to be as accurate in recreating the comovement features of Canadian business cycles. The results listed in column C of table 1 show that only the GDP correlations of GNP, r^*A and TB/Y are well approximated. This by itself is an important achievement, since to date the existing work in dynamic international finance has not been very successful in reproducing the countercyclical behavior of the balance of trade and foreign assets[20].

In comparison with the findings for closed-economy real business cycle models, consumption is almost perfectly correlated with GDP. Surprisingly, this fact is consistent with the predictions of deterministic international finance models that assume the existence of exogenous output endowments. In these models, output and consumption are perfectly correlated because homothetic preferences (as embodied in the isoelastic utility function) imply that agents desire to consume a constant fraction of their wealth each period. In a stochastic framework, where the output endowments follow a Markovian process, expected wealth is a fixed proportion of the current-period's endowment, and hence consumption and output would also tend to be highly correlated[21]. An interpretation of this result in the context of the present model is discussed later.

From table 1 it is also possible to observe that the model generates a correlation coefficient between savings and investment of 0.25, which is lower than the 0.43 actually observed. This, however, must not be regarded as an implication of the perfect capital mobility assumption exclusively. In fact, as will be explained later, the absence of adjustment costs in the process of accumulating domestic capital is also responsible for the low comovement between S and I.

The fact that labor (i.e. hours) has perfect positive correlation with domestic output is an implication of the Cobb-Douglas production technology and the utility function that were adopted. The common serial autocorrelation coefficient of both GDP and L also follows from this fact.

A discussion of the factors that cause the volatile behavior of investment, the high positive correlation between consumption and GDP and the low persistence of the trade balance-output ratio in the artificial economy is carried out next.

As mentioned before, the main difference between closed and open economies is the ability that agents have in the latter to accumulate savings in the form of foreign financial assets, in this case paying a fixed real rate of return. In such an environment, it is no longer necessary to gradually adjust the domestic capital stock in response to the wealth and substitution effects caused by technological disturbances. Specifically, this numerical analysis shows that the wealth effect pushing for the smoothing of consumption is concentrated in the process of foreign asset accumulation, and domestic capital is rapidly adjusted in order to ensure the equality of expected marginal returns in utility terms[22].

Since there are no restrictions on the international flow of commodities, nor there is any cost of adjustment in the process of expanding or contracting the capital stock, the behavior of investment is very volatile. Thus, the high percentage variability of investment reflects the fast rate at which K is intertemporally readjusted in response to random productivity changes.

Similarly, the negative autocorrelation of I and its weak positive correlation with GDP follow from the relatively small and short-lived technological shocks. This can be illustrated considering a deterministic

version of the model where the evolution of the disturbances is known with certainty and the equality of marginal returns holds in strict sense:

$$\exp(e_{t+1})\hat{F}'(K_{t+1},L_{t+1}) - \delta = r^* \tag{14}$$

Figures 2 and 3 display the equilibrium time profiles of private investment and gross domestic output as generated by the experiment described below.

Consider a 1.18% productivity shock with a serial correlation parameter of 0.35 that hits the economy at date 2. Since $e_{t+1} = 0.35e_t$, this shock affects the productivity of investment (i.e. the marginal product of capital in the following period) by only 0.42%. At date 3 this investment shock declines to only 0.15%. Thus, in period 2 investment is enlarged as K_3 is adjusted upward to ensure that the equality in (14) is maintained. In the following period, as the shock starts to vanish, optimizing agents reduce net investment so as to accommodate the capital stock to its declining productivity (see figure 2). In the stochastic version of the model, where positive or negative technological shocks occur randomly, investment would tend to exhibit a pattern of negative first-order serial autocorrelation.

The weak comovement between GDP and investment is also imposed by (14). This condition implies that domestic output increases when the shock hits the economy, and then falls monotonically towards its starting value as the effect of the disturbance disappears. Investment, to the contrary, falls below its stationary equilibrium when the economy reaches period 3 (after the expansion in I_2), and from then on increases gradually until it has returned back to it (see figures 2 and 3). If this kind of behavior prevails in the stochastic environment, it would be reasonable to expect that the correlation between investment and output would be weaker than in the closed-economy models.
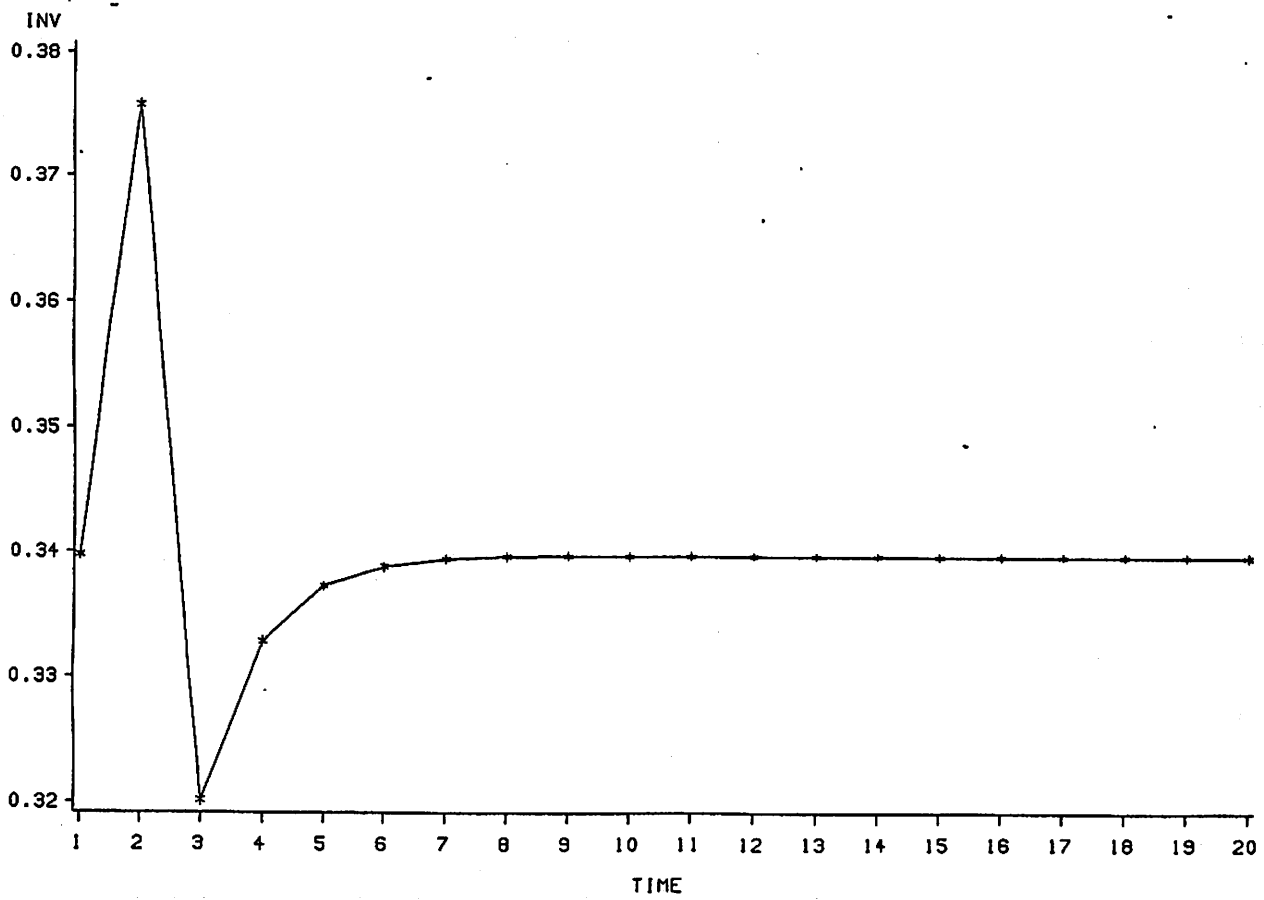
DETERMINISTIC INVESTMENT PATH
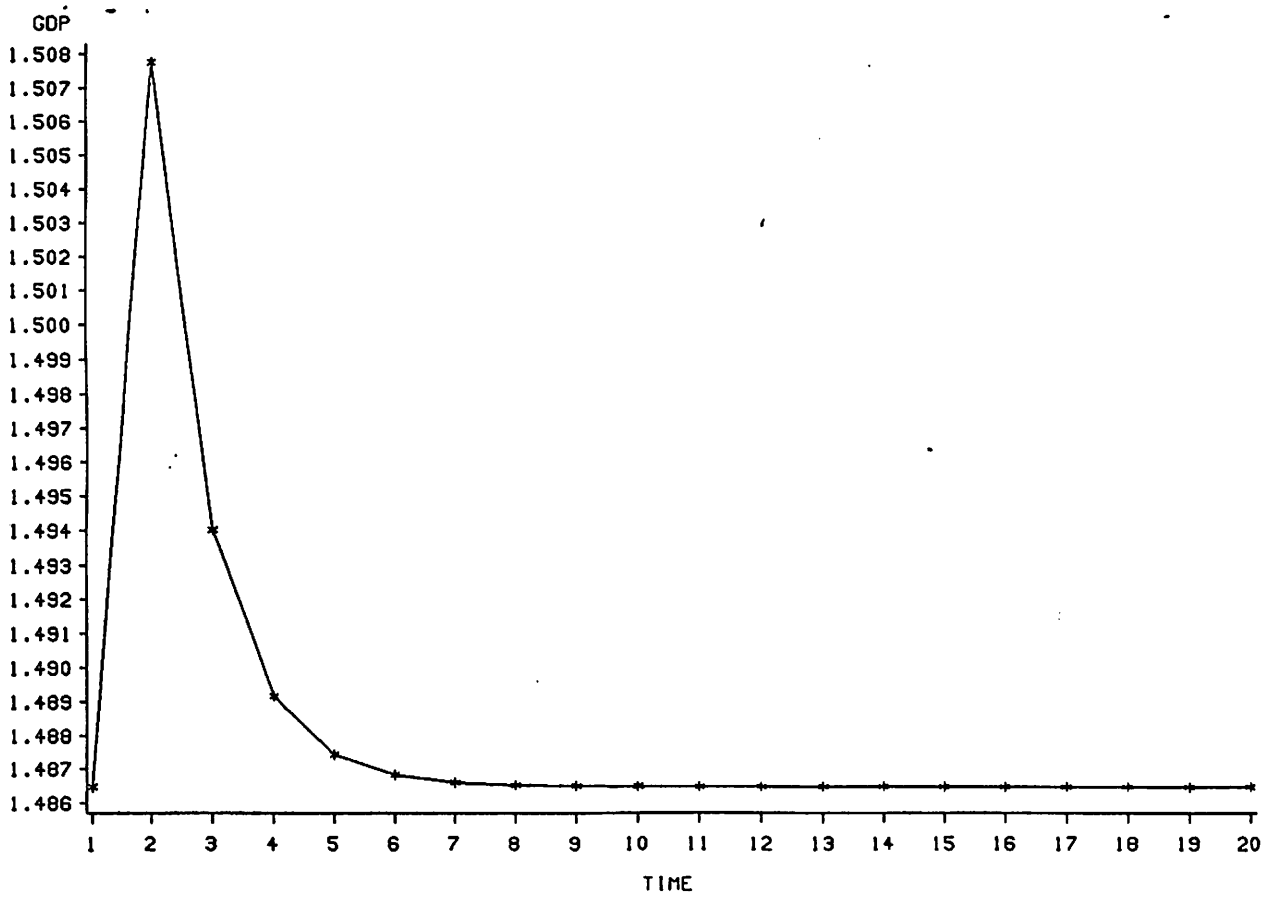

Figure 2.

DETERMINISTIC GDP PATH

Figure 3.

It is important to mention that domestic capital irreversibility constraints, of the type considered by Sargent (1980), are not relevant to generate the type of behavior observed in the investment process of the artificial economy. In all the numerical experiments performed, total gross investment in every possible state of nature is always positive.

The almost perfect correlation between C and GDP, and hence the high correlation of S and GDP, has the following theoretical justification. In this artificial economy, only the wealth effect caused by the disturbances affects the behavior of consumption. The intertemporal consumption substitution effect, that operates in a closed economy when a favorable persistent shock causes the interest rate to increase, is no longer at work[23]. The relative price of consumption at different dates is exogenously determined in the world's capital market.

The negative serial autocorrelation of the trade balance-output ratio is due to the consumption-smoothing effect present in the model. A favorable, temporary shock motivates optimizing agents to increase foreign asset accumulation in order to smooth consumption. The balance of trade improves in the same period that the shock occurs, but is negatively affected in all subsequent periods -as agents deplete foreign assets to finance the extra imports required to increase consumption permanently.

Since this model also incorporates an endogenous rate of time preference, it is important to establish whether the behavior of the aggregates deviates from that observed in the closed-economy models because of the introduction of foreign asset trading or because of the variable discount factor. It has been mentioned that simulating the GHH model using stationary cardinal utility does not dramatically affect the behavior of the results[24]. Also, in the

present model the expected value of the discount factor is 0.96 and its variability is less than 0.06%. However, to investigate the issue in more detail, variations of the model for the cases of a closed economy with SCU preferences and an open economy with constant discounting have been simulated. The analysis showed unambiguously that the introduction of the world's capital market is responsible for the volatile behavior of investment.

In the case of an open economy with constant discounting, the percentage variability of I is 16.0%, its serial autocorrelation is -0.42 and its comovement with GDP is only 0.22[25]. The correlation of GDP and consumption is still very high, 0.96, and the serial correlation of TB/Y is still negative at -0.47. In contrast, the closed-economy model with variable time preference delivered an investment process with 5.1% volatility, 0.43 autocorrelation and 0.94 output correlation. Therefore, this numerical evidence supports the argument that the distinctive features of the model's behavior are not related to the endogeneity of the discount factor, but to the availability of an alternative, risk-free vehicle of savings.

Table 2 reproduces the actual statistics and the results obtained with the artificial economy using a risk aversion parameter close to 1.0. A 1.18% productivity shock with 0.34 serial autocorrelation was required to calibrate the model in this case. The grids of admissible choices for domestic capital and foreign assets span the intervals [3.30,3.50] and [-1.14,-0.23] respectively; again, each grid contains 22 evenly spaced points. The results reported in table 2 are very similar to those obtained before, with the exception of the volatility, comovement and persistence of foreign asset holdings, which are all reduced[26].

Table 2

Statistical Moments: Canadian Data and Artificial Economy[a].

Variables	Canadian Data 1946-1985			Artificial Economy $\gamma=1.001$, $\sigma_\epsilon=1.18\%$, $\rho_\epsilon=0.34$		
	(A)[b]	(B)[c]	(C)[d]	(A)[b]	(B)[c]	(C)[d]
1) GDP	2.810	0.615	1.000	2.810	0.615	1.000
2) GNP	2.950	0.643	0.995	2.795	0.611	0.997
3) C	2.460	0.701	0.586	2.105	0.669	0.961
4) S	7.306	0.542	0.662	5.483	0.590	0.947
5) I	9.820	0.314	0.639	21.050	-0.343	0.266
6) K	1.380	0.649	-0.384	1.913	0.322	0.669
7) L	2.020	0.541	0.799	1.931	0.615	1.000
8) r^*A	15.250	0.727	-0.175	11.064	0.632	-0.354
9) TB/Y	0.019	0.623	-0.129	0.046	-0.338	-0.013
	CORR(S,I) = 0.434			CORR(S,I) = 0.268		

[a] See note "a" in table 1 for sources and calculations.

[b] Percentage standard deviation (except 9)).

[c] first-order autocorrelation coefficient.

[d] GDP correlation coefficient.

Unlike the case of the GHH model, the procyclical behavior of consumption does not depend on the value of the risk aversion parameter. This result follows from the fact that, in a small open economy, domestic productivity disturbances cannot affect the intertemporal relative price of consumption that is determined in the global capital market. Hence, private consumption appears to be proportional to wealth regardless of the degree of risk aversion.

The percentage variability of foreign interest payments is reduced to 11.1%, which is now lower than the 15.3% actually observed. The serial correlation coefficient of r^*A drops to 0.63 and its GDP correlation falls to -0.35. These results indicate that the appropriate parameter value for "γ" should be between 1.001 and 2.0. The direction of the movements in these three statistical moments is explained by the increase in the intertemporal elasticity of substitution in consumption (i.e. the reduction in the risk aversion coefficient). This enables individuals to attain optimal consumption behavior without having to resort as much to the insurance that foreign assets provide against domestic productivity disturbances[27].

V.- Concluding Remarks.

This paper investigated real business cycles in a framework where the existence of an international, perfectly competitive capital market is allowed. The artificial economy studied is characterized by a highly flexible savings mechanism. Both domestic capital and foreign financial assets are used to allocate resources intertemporally, and it is assumed that there are no restrictions on international borrowing and lending nor there is any adjustment cost in the process of investment.

The results indicate that the process of foreign asset accumulation may play an important role in the understanding of equilibrium real business cycles. In comparison with closed-economy prototypes, a relatively less volatile and less persistent technological shock is required to mimic the observed variability and persistence of output. The equilibrium stochastic processes of the majority of the macro-aggregates are fairly well reproduced, as the close match between the observed sample moments and the population moments obtained with the artificial economy suggests.

The model performed poorly in reproducing the stylized facts of investment, the comovement of consumption and the persistence of the trade balance-output ratio. These inaccurate results are related to the excessive flexibility of the savings mechanism embodied in the model, particularly the costless manner in which capital accumulation and depletion can be undertaken. Hence, the numerical exploration performed here indicates that frictions in the investment process may play an important role in open-economy real business cycle theory.

Extensions to incorporate adjustment costs in the process of accumulating capital and stochastic fluctuations in the world's real interest rate are proposed for further research.

FOOTNOTES

1. See table 1 in section IV.

2. With the exception of works like Ahmed (1986) or Hercowitz (1986b).

3. Hercowitz (1986b) concluded that an exchange economy, dynamic optimizing model cannot explain the weak countercyclical behavior of the Israeli foreign debt. This is attributed to a strong procyclical behavior of consumption, which is in turn related to either the assumption of exogenous endowments or the existence of capital controls.

4. This kind of setup differs from the one often used in dynamic international finance, where only deterministic exchange economies are studied.

5. Epstein (1983) showed theoretically that the stochastic growth model with SCU preferences generates similar comovement and persistence features as those obtained with the constant-discount framework. A numerical experiment performed on a variation of Greenwood, Hercowitz and Huffman (1988) illustrated that the variability of the discount factor is negligible. The model's results are almost identical to those obtained with the standard time-separable utility function. The only changes were a 0.2 reduction in the consumption-GNP correlation and a 2.4 point increase in the percentage variability of investment.

6. This procedure differs from other research strategies used in real business cycles based on quadratic approximations of the objective function (see Hansen (1985) and Prescott (1986)).

7. A detailed analysis of the theoretical properties of the proposed model is presented in Mendoza (1988). Various comparative statics exercises were undertaken to investigate impact and dynamic effects of stochastic and deterministic technological disturbances.

8. This kind of environment can be motivated by considering a large set of similar economies that constitute the world economy, each with its own technology disturbances. Then the domestic economy has access to a perfectly diversified portfolio since there is a security for each possible state of nature. The assumption that r* does not fluctuate is supported by arguing that national disturbances cancel out in the global average.

9. The coefficient β, which is not to be confused with the constant-discount factor, measures the elasticity of the rate of time preference with respect to Ln[1+C-G(L)].

10. The role of international risk sharing, arising from different economies exchanging contingent claims, is not explicitly modelled. This is partially replaced with foreign asset trading. These assets can be reinterpreted as a perfectly diversified portfolio consisting of shares

of the capital stock in various countries. Furthermore, the numerical analysis of Cole and Obstfeld (1988) suggests that, for some specifications of tastes and technology, the competitive allocations are independent of the completeness of financial markets.

11. The solution must also satisfy the transversality condition that, in the limit, foreign debt cannot grow faster than the market discount factor. The stability of the steady-state equilibrium of foreign asset holdings ensures that debt does not grow in the long-run. Furthermore, the real return on assets can be interpreted as the result of world production, which is assumed to possess the same concavity properties of the domestic technology. Thus, it is not possible to support an ever growing consumption path with an ever growing trade balance deficit.

12. Following the analysis of Epstein (1983), a formal proof of the concavity of the value function was presented in Appendix II of Mendoza (1988). In fact, not only is $V(\cdot)$ concave but it is also true that $Log[-V(\cdot)]$ is convex.

13. The use of the SCU function implies that the algorithm suggested by Bertsekas (1976) to speed up the convergence of the decision rules can no longer be used. This, combined with the large memory requirements, increases considerably the time and cost of running the program in a regular mainframe, and seriously restricts the size of the state space than can be analyzed. These technical difficulties were eliminated using an ETA-10P supercomputer with a vector-Fortran compiler.

14. The gradual redefinition of the grids is a complicated process. Since agents are allowed to substitute assets for capital and vice versa, the discretization of the state space restricts not only the divisibility of each savings instrument, but also the degree to which one can be substituted for the other.

15. This steady-state system of equations is studied in more detail in Mendoza (1988).

16. An alternative depreciation rate of 5.2% is determined using the capital evolution equation and the data on net capital stocks and gross investment, computing the long-run average as in Hercowitz (1986a). With such depreciation rate, the model generates an investment series with 43.0% standard deviation and an average I/Y ratio of only 18.0%.

17. Values of $\omega=4.3$ and 4.9 (intertemporal elasticity of labor supply=0.3 and 0.26) were estimated by Hercowitz (1986a) for the Canadian economy. The author points out, however, that these estimates are not very reliable because one of the reduced form coefficients involved is imprecisely estimated. In the present model, a value of 4.3 generates a percentage standard deviation of hours worked of just 0.76%, far below the 2.02% that is actually observed.

18. A more detailed analysis of the economics supporting these results is presented in Mendoza (1988).

19. Since in the present model r^* is non-random, both A and r^*A exhibit the same volatility, persistence and comovement properties.

20. See note 3.

21. Levhari and Srinivasan (1969) showed that a similar result holds for a stochastic, exchange economy where the return on assets is an i.i.d. process that satisfies $ER_t<1/ß$ (with ß being the constant discount factor).

22. This conclusion is supported by the results of the sensitivity analysis for changes in the coefficient of relative risk aversion, the depreciation rate and the intertemporal elasticity in labor supply.

23. The increasing rate of time preference, turning individuals relatively more impatient when a positive productivity shock takes place, is also partially responsible for the higher correlation between C and GDP.

24. See note 5.

25. In this case, the steady state value of A is not determined and hence there is no well-behaved limiting distribution of foreign assets. The limiting distribution of domestic capital is still stationary and nicely-behaved.

26. The sensitivity analysis for the risk aversion parameter showed that "γ" and the volatility, serial correlation and GDP correlation of assets are directly related. Increasing the value of "γ" to 3.0 increased the volatility of assets to 23.7%, their serial correlation to 0.93 and their GDP correlation to -0.15. The behavior of the rest of the aggregates is not significantly affected by the changes in the value of this parameter.

27. Optimal consumption behavior is characterized by the equality between the expected marginal rate of substitution and (1+r*), the former must consider the impatience effect induced by changes in the rate of time preference.

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